Lastnosti Fourierjeve transformacije

Linearnost

$\mathcal{F}(\alpha\, f(t) + \beta\, g(t)) = \alpha\, F(\omega) + \beta\, G(\omega)$

Premik

$\mathcal{F}(f(t - t_0)) = F(\omega) e^{-j\omega t_0}$

$\mathcal{F}^{-1}(F(\omega - \omega_0)) = f(t) e^{j\omega_0 t}$

Modulacija

$f(t) = g(t) * \cos(\omega_0 t) = g(t) * [\frac{1}{2}(e^{j\omega_0 t} + e^{-j\omega_0 t})]$

$F(\omega) = \frac{1}{2}[G(\omega + \omega_0) + G(\omega - \omega_0)]$

$f(t) = g(t) * \sin(\omega_0 t) = g(t) * [-\frac{j}{2}(e^{j\omega_0 t} - e^{-j\omega_0 t})]$

$F(\omega) = \frac{j}{2}[G(\omega + \omega_0) - G(\omega - \omega_0)]$

Lastnosti odvoda

$\mathcal{F}(\frac{d^n f(t)}{dt^n}) = (j\omega)^n F(\omega)$

Odvodi v točkah nezveznosti

$\frac{df(t)}{dt}|_{t=t_0} = \delta(t - t_0)\,[f^+(t_0) - f^-(t_0)]$

Ker velja $\mathcal{F}(a\,\delta(t - t_0)) = a\, e^{-j\omega t_0}$, sledi:

$\mathcal{F}(\frac{df(t)}{dt}|_{t=t_0}) = e^{-j\omega t_0}[f^+(t_0) - f^-(t_0)]$

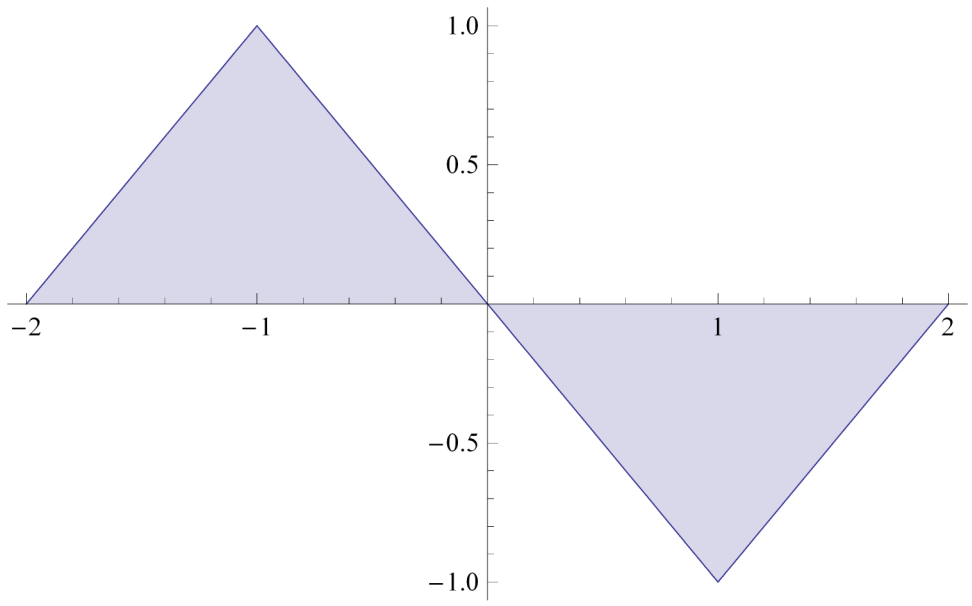
Naloga 1:

Naloga:

Izračunaj kompleksni spekter ter spekter amplitudne in močnostne gostote za signal podan na sliki:

Signal:

```
Plot[f[t], {t, -2.0, 2.0}, PlotRange → All, Filling → Axis]
```



$$f[t\_] := \begin{cases} t+2 & -2 \le t \le -1 \\ -t & -1 \le t \le 1 \\ t-2 & 1 \le t \le 2 \\ 0 & \text{True} \end{cases}$$

Fourierjeva transformacija *(z odvajanjem)*

$$\frac{d^2 f(t)}{dt^2} = \delta(t+2)[1-0] + \delta(t+1)[-1-1] + \delta(t-1)[1+1] + \delta(t-2)[0-1]$$

$$\frac{d^2 f(t)}{dt^2} = \delta(t+2) - 2\,\delta(t+1) + 2\,\delta(t-1) - \delta(t-2)$$

Izračunamo F.t. leve in desne strani:

$$(j\omega)^2 F(\omega) = \left(e^{j2\omega} - 2\,e^{j\omega} + 2\,e^{-j\omega} - e^{-j2\omega}\right)$$

$$F(\omega) = -\frac{1}{\omega^2}\left(e^{j2\omega} - 2\,e^{j\omega} + 2\,e^{-j\omega} - e^{-j2\omega}\right)$$

Spekter amplitudne gostote

Pomagamo si s pravilom:

$$-\frac{2}{j}\sin(z) = e^{jz} - e^{-jz} \quad ALI \quad 2\cos(z) = e^{jz} + e^{-jz}$$

$$F(\omega) = -\frac{1}{\omega^2}\left(-\frac{2}{j}\sin(2\omega) + \frac{4}{j}\sin(\omega)\right)$$

$$F(\omega) = -\frac{2}{\omega^2}(\sin(2\omega) - 2\sin(\omega))$$

$$|F(\omega)| = \frac{2}{\omega^2}\left|\sin(2\omega) - 2\sin(\omega)\right|$$

Spekter močnostne gostote

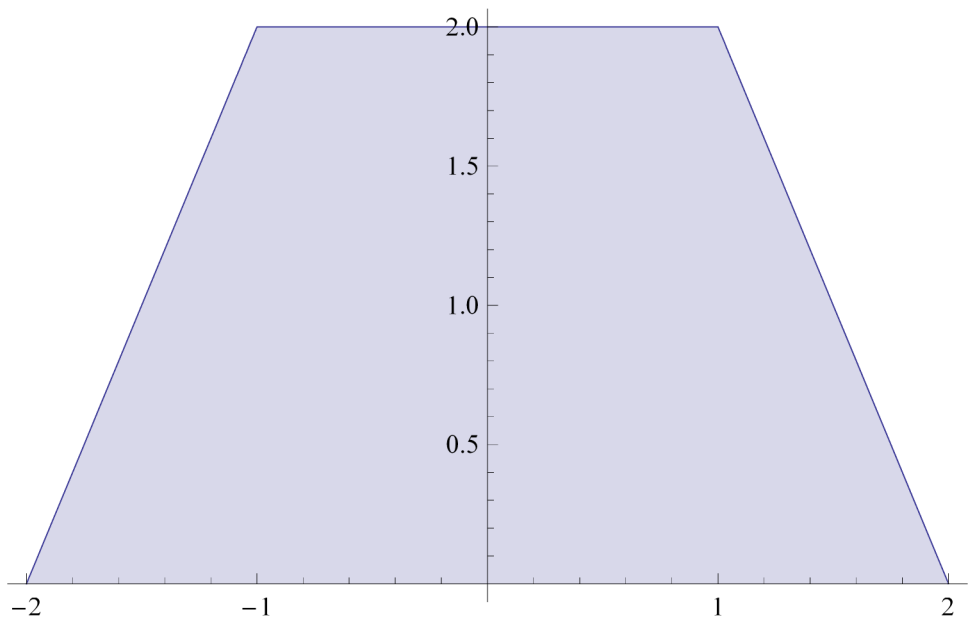
$$|F(\omega)|^2 = \frac{4}{\omega^4}(\sin(2\omega) - 2\sin(\omega))^2$$

Naloga 2:

Naloga:
Izračunaj kompleksni spekter ter spekter amplitudne in močnostne gostote za signal podan na sliki:

Signal:

```
Plot[f[t], {t, -2.0, 2.0}, PlotRange → All, Filling → Axis]
```



$$f[t\_] := \begin{cases} 2*t+4 & -2 \le t \le -1 \\ 2 & -1 \le t \le 1 \\ -2*t+4 & 1 \le t \le 2 \\ 0 & \text{True} \end{cases}$$

Fourierjeva transformacija *(z odvajanjem)*

$$\frac{d^2 f(t)}{dt^2} = \delta(t+2)[2-0] + \delta(t+1)[0-2] + \delta(t-1)[-2-0] + \delta(t-2)[0-(-2)]$$

$$\frac{d^2 f(t)}{dt^2} = 2\left(\delta(t+2) - \delta(t+1) - \delta(t-1) + \delta(t-2)\right)$$

Izračunamo F.t. leve in desne strani:

$$(j\omega)^2 F(\omega) = 2\left(e^{j2\omega} - e^{j\omega} - e^{-j\omega} + e^{-j2\omega}\right)$$

$$F(\omega) = -\frac{2}{\omega^2}\left(e^{j2\omega} + e^{-j2\omega} - e^{j\omega} - e^{-j\omega}\right)$$

Spekter amplitudne gostote

Pomagamo si s pravilom:

$-\frac{2}{j}\sin(z) = e^{jz} - e^{-jz}$ *ALI* $2\cos(z) = e^{jz} + e^{-jz}$

$$F(\omega) = -\frac{2}{\omega^2}(2\cos(2\omega) - 2\cos(\omega))$$

$$F(\omega) = -\frac{4}{\omega^2}(\cos(2\omega) - \cos(\omega))$$

$$|F(\omega)| = \frac{4}{\omega^2}\left|\cos(2\omega) - \cos(\omega)\right|$$

Spekter močnostne gostote

$$|F(\omega)|^2 = \frac{16}{\omega^4}(\cos(2\omega) - \cos(\omega))^2$$