Kompleksna Fourierjeva vrsta

$$f[t\_] := \sum_{n=-\infty}^{\infty} F_n * \text{Exp}[i * n * \omega * t];$$

$$\omega := \frac{2 * \pi}{T};$$

$$F_n := \frac{1}{T} * \int_{t_0}^{t_0+T} f[t] * \text{Exp}[-i * n * \omega * t]\, dt;$$

Realna Fourierjeva vrsta

$$f[t\_] := \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n * \text{Cos}[n * \omega * t] + b_n * \text{Sin}[n * \omega * t];$$

$$\omega := \frac{2 * \pi}{T};$$

$$a_n := \frac{2}{T} * \int_{t_0}^{t_0+T} f[t] * \text{Cos}[n * \omega * t]\, dt;$$

$$b_n := \frac{2}{T} * \int_{t_0}^{t_0+T} f[t] * \text{Sin}[n * \omega * t]\, dt;$$

Amplitudni in fazni spekter

$$F_n = P_n + jQ_n = |F_n| * e^{j\Theta_n}$$

Amplitudni spekter

$$|F_n| = \sqrt{F_n * \overline{F_n}} = \sqrt{P_n^2 + Q_n^2} = \frac{1}{2}\sqrt{a_n^2 + b_n^2}$$

Fazni spekter

$$\phi_n = \begin{cases} \arctan \frac{Q_n}{P_n} & P_n > 0 \\ \arctan \frac{Q_n}{P_n} \pm \pi & P_n < 0 \end{cases}$$

$$= \begin{cases} -\arctan \frac{b_n}{a_n} & a_n > 0 \\ -\arctan \frac{b_n}{a_n} \pm \pi & a_n < 0 \end{cases}$$

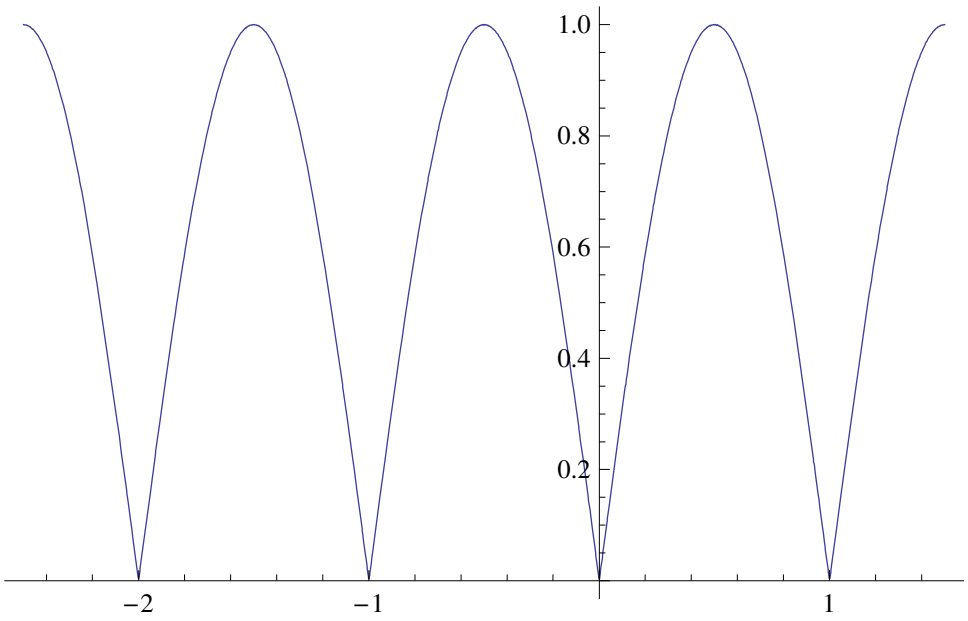
Naloga 1:

Naloga:
Izrazi periodično funkcijo f(t) s kompleksno F. v.
Določi tudi koeficiente realne F.v. ter zapiši realno vrsto.
Na koncu določi in nariši tudi amplitudni in fazni spekter.

Signal:

```
x[t_] := { A * Sin[π * t]   0 < t < 1
         {       0          True
```

```
A = 1; Plot[x[t - 1] + x[t] + x[t + 1] + x[t + 2] + x[t + 3], {t, -2.5, 1.5},
 PlotRange → All]
```



Rešitev:

```
T = 1;  ω = 2 * π;
```

Funkcija je soda: f(t)=f(-t). $F_n=P_n+i^*Q_n$ => $Q_n = 0$. => $F_n=P_n$.

$$F_n = \frac{1}{T} * \int_{t_0}^{t_0+T} x[t] * \text{Cos}[n * 2 * \pi * t] \, dt$$

Izberemo $t_0 = 0$.

$$F_n = \frac{1}{1} * \int_{0}^{1} A * \text{Sin}[\pi * t] * \text{Cos}[n * 2 * \pi * t] \, dt$$

$$= \frac{2\,A\,\text{Cos}[n\,\pi]^2}{\pi - 4\,n^2\,\pi} = \frac{2\,A}{\pi\,\left(1 - 4\,n^2\right)}$$

Pri izračunu integrala si pomagamo z:

$$\int \text{Sin}[a * t] * \text{Cos}[b * t] \, dt$$

$$-\frac{\text{Cos}[(a-b)\,t]}{2\,(a-b)} - \frac{\text{Cos}[(a+b)\,t]}{2\,(a+b)}$$

Kompleksna F. v.

$$f[t] = \frac{2\,A}{\pi} \sum_{n=-\infty}^{\infty} \frac{1}{\left(1 - 4\,n^2\right)} * \text{Exp}[i * n * 2 * \pi * t];$$

Realna F. v.

$$a_n = F_n + \overline{F_n} = \frac{4\,A}{\left(1 - 4\,n^2\right)\,\pi};$$

$$b_n = \dot{\imath} * \left(F_n - \overline{F_n}\right) = 0;$$

$$f[t] = \frac{2 * A}{\pi} + \frac{4 * A}{\pi} \sum_{n=1}^{\infty} \frac{1}{1 - 4\,n^2} * \text{Cos}[2 * n * \pi * t];$$

Amplitudni spekter:

$$|F_n| = \begin{cases} \frac{2\,A}{\left(4\,n^2 - 1\right)\,\pi} & n \neq 0 \\ \frac{2 * A}{\pi} & n = 0 \end{cases}$$

Fazni spekter:

$$\phi_n = \begin{cases} \pm\pi & n \neq 0 \\ 0 & n = 0 \end{cases}$$

Zaradi realnega signala mora biti fazni spekter liha funkcija. Zato :

$$= \begin{cases} +\pi & n > 0 \\ -\pi & n < 0 \\ 0 & n = 0 \end{cases}$$