Kompleksna Fourierjeva vrsta

$$f[t\_] := \sum_{n=-\infty}^{\infty} F_n * \mathrm{Exp}[i * n * \omega * t];$$

$$\omega := \frac{2 * \pi}{T};$$

$$F_n := \frac{1}{T} * \int_{t_0}^{t_0+T} f[t] * \mathrm{Exp}[-i * n * \omega * t]\, dt;$$

Realna Fourierjeva vrsta

$$f[t\_] := \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n * \mathrm{Cos}[n * \omega * t] + b_n * \mathrm{Sin}[n * \omega * t];$$

$$\omega := \frac{2 * \pi}{T};$$

$$a_n := \frac{2}{T} * \int_{t_0}^{t_0+T} f[t] * \mathrm{Cos}[n * \omega * t]\, dt;$$

$$b_n := \frac{2}{T} * \int_{t_0}^{t_0+T} f[t] * \mathrm{Sin}[n * \omega * t]\, dt;$$

Amplitudni in fazni spekter

$$F_n = P_n + jQ_n = |F_n| * e^{j\Theta_n}$$

Amplitudni spekter

$$|F_n| = \sqrt{F_n * \overline{F_n}} = \sqrt{P_n^2 + Q_n^2} = \frac{1}{2}\sqrt{a_n^2 + b_n^2}$$

Fazni spekter

$$\phi_n = \begin{cases} \arctan \frac{Q_n}{P_n} & P_n > 0 \\ \arctan \frac{Q_n}{P_n} \pm \pi & P_n < 0 \end{cases}$$

$$= \begin{cases} -\arctan \frac{b_n}{a_n} & a_n > 0 \\ -\arctan \frac{b_n}{a_n} \pm \pi & a_n < 0 \end{cases}$$

Naloga 2:

Naloga:
Razvij periodični signal f(t) v realno in kompleksno F.v.
Določi in nariši tudi amplitudni in fazni spekter.

Signal:

$$x[t\_] := \begin{cases} \frac{A}{2*\pi} * t & 0 < t \le 2\pi \\ 0 & \text{True} \end{cases} \qquad A > 0$$

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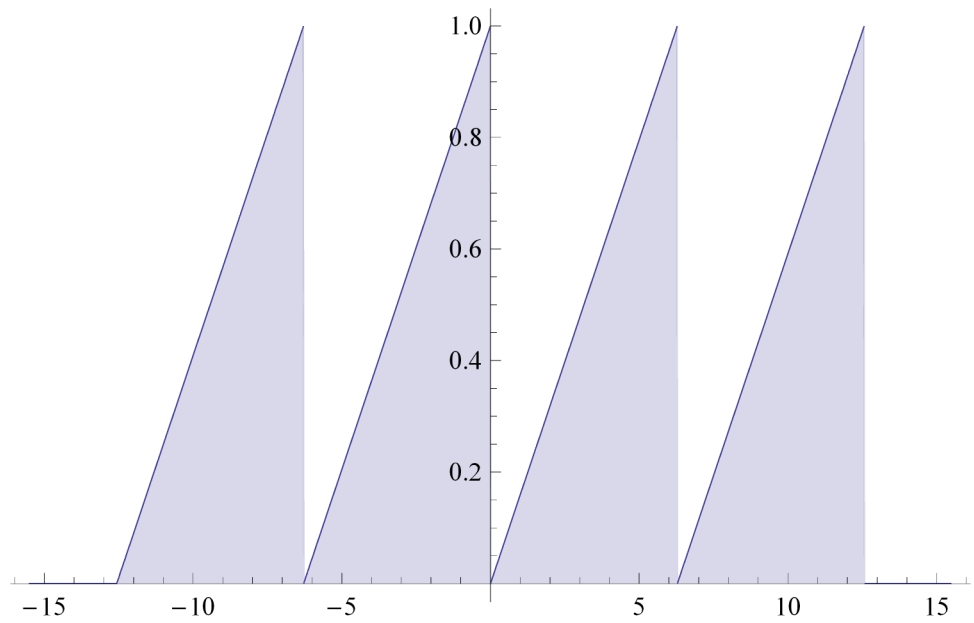
A = 1; Plot[x[t + 4 * π] + x[t + 2 * π] + x[t] + x[t - 2 * π], {t, -15.5, 15.5},
 PlotRange → All, Filling → Axis]
```



Rešitev:

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T = 2 * π; ω = 1;
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Izberemo $t_0 = 0$.

$$a_n = \frac{2}{2*\pi} * \int_0^{2*\pi} \frac{A}{2*\pi} * t * \text{Cos}[n * \omega * t]\, dt$$

Pomagamo si z:

$$\int t * \text{Cos}[a * t]\, dt$$

$$\frac{\text{Cos}[a\,t]}{a^2} + \frac{t\,\text{Sin}[a\,t]}{a}$$

$a_0 = A;$
$a_n = 0;$

$$b_n = \frac{2}{2*\pi} * \int_0^{2*\pi} \frac{A}{2*\pi} * t * \text{Sin}[n * \omega * t]\, dt$$

Pomagamo si z:

$$\int t * \text{Sin}[a * t]\, dt$$

$$-\frac{t\,\text{Cos}[a\,t]}{a} + \frac{\text{Sin}[a\,t]}{a^2}$$

$$b_0 = 0;$$
$$b_n = \frac{A}{-\pi * n};$$

Realna F. v.

$$f[t] = \frac{A}{2} - \frac{A}{\pi}\sum_{n=1}^{\infty}\frac{1}{n} * \mathrm{Sin}[n * t];$$

Preverimo, kako je v točkah nezveznosti :

$$f[2 * k * \pi] = \frac{A}{2} - \frac{A}{\pi}\sum_{n=1}^{\infty}\frac{1}{n} * \mathrm{Sin}[n * 2 * k * \pi] = \frac{A}{2}$$

Kompleksna F. v.

Pomagamo si z :

$$F_n = \frac{a_n - i * b_n}{2}, \quad \text{in} \quad F_{-n} = \overline{F_n}, \text{ ker je vhodni signal realen.}$$

$$F_n = \begin{cases} \frac{i*A}{2*\pi*n} & n \neq 0 \\ \frac{A}{2} & n = 0 \end{cases}$$

Amplitudni spekter:

$$|F_n| = \begin{cases} \frac{A}{2*\pi*|n|} & n \neq 0 \\ \frac{A}{2} & n = 0 \end{cases}$$

Fazni spekter:

$$\phi_n = \begin{cases} \pm\frac{\pi}{2} & n \neq 0 \\ 0 & n = 0 \end{cases}$$

Zaradi realnega signala mora biti fazni spekter liha funkcija. Zato :

$$= \begin{cases} +\frac{\pi}{2} & n > 0 \\ -\frac{\pi}{2} & n < 0 \\ 0 & n = 0 \end{cases}$$