Kompleksna Fourierjeva vrsta

$$f[t\_] := \sum_{n=-\infty}^{\infty} F_n * Exp[i * n * \omega * t];$$

$$\omega := \frac{2 * \pi}{T};$$

$$F_n := \frac{1}{T} * \int_{t_0}^{t_0+T} f[t] * Exp[-i * n * \omega * t] \, dt;$$

Realna Fourierjeva vrsta

$$f[t\_] := \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n * Cos[n * \omega * t] + b_n * Sin[n * \omega * t];$$

$$\omega := \frac{2 * \pi}{T};$$

$$a_n := \frac{2}{T} * \int_{t_0}^{t_0+T} f[t] * Cos[n * \omega * t] \, dt;$$

$$b_n := \frac{2}{T} * \int_{t_0}^{t_0+T} f[t] * Sin[n * \omega * t] \, dt;$$

Amplitudni in fazni spekter

$$F_n = P_n + jQ_n = \left| F_n \right| * e^{j\Theta_n}$$

Amplitudni spekter

$$\left| F_n \right| = \sqrt{F_n * \overline{F_n}} = \sqrt{P_n^2 + Q_n^2} = \frac{1}{2}\sqrt{a_n^2 + b_n^2}$$

Fazni spekter

$$\phi_n = \begin{cases} arctan \frac{Q_n}{P_n} & P_n > 0 \\ arctan \frac{Q_n}{P_n} \pm \pi & P_n < 0 \end{cases}$$

$$= \begin{cases} -arctan \frac{b_n}{a_n} & a_n > 0 \\ -arctan \frac{b_n}{a_n} \pm \pi & a_n < 0 \end{cases}$$

Naloga 1:

Naloga:
Izrazi periodično funkcijo f(t) s kompleksno F. v.
Določi tudi koeficiente realne F.v. ter zapiši realno vrsto.
Na koncu določi in nariši tudi amplitudni in fazni spekter.

▫ **Signal:**

$$f[t\_] := \operatorname{Sin}\left[\frac{\pi * t}{4} + \frac{\pi}{4}\right];$$

▫ **Rešitev:**

Adicijski izrek:

$$f[t] = \operatorname{Sin}\left[\frac{\pi * t}{4}\right] * \operatorname{Cos}\left[\frac{\pi}{4}\right] + \operatorname{Cos}\left[\frac{\pi * t}{4}\right] * \operatorname{Sin}\left[\frac{\pi}{4}\right]$$

$$\omega = \frac{\pi}{4} = \frac{2 * \pi}{T} = > T = 8$$

Realna F. v.

$$a_0 = 0$$

$$a_1 = \operatorname{Sin}\left[\frac{\pi}{4}\right] = \frac{\sqrt{2}}{2}$$

$$b_1 = \operatorname{Cos}\left[\frac{\pi}{4}\right] = \frac{\sqrt{2}}{2}$$

$$a_n = b_n = 0, \quad n > 1$$

Kompleksna F. v.

Ker velja:

$$F_n = \frac{a_n - i * b_n}{2}, \quad \text{in} \quad F_{-n} = \overline{F_n}, \text{ ker je vhodni signal realen}.$$

$$F_1 = \frac{a_1 - j * b_1}{2} =$$
$$\frac{\operatorname{Sin}\left[\frac{\pi}{4}\right] - j * \operatorname{Cos}\left[\frac{\pi}{4}\right]}{2} =$$
$$-\frac{j}{2}\left(\operatorname{Cos}\left[\frac{\pi}{4}\right] + j * \operatorname{Sin}\left[\frac{\pi}{4}\right]\right) = -\frac{j}{2}\operatorname{Exp}\left[j * \frac{\pi}{4}\right]$$

$$F_{-1} = \overline{F_1} = \frac{j}{2}\operatorname{Exp}\left[j * \frac{\pi}{4}\right]$$

Amplitudni spekter:

$$|F_n| = \begin{cases} \frac{1}{2} & n = 1 \\ \frac{1}{2} & n = -1 \\ 0 & n \neq \pm 1 \end{cases}$$

Fazni spekter:

$$\phi_n = \begin{cases} +\frac{\pi}{4} & n = -1 \\ -\frac{\pi}{4} & n = 1 \\ 0 & n \neq \pm 1 \end{cases}$$