Kompleksna Fourierjeva vrsta

$$f[t\_] := \sum_{n=-\infty}^{\infty} F_n * \mathrm{Exp}[i * n * \omega * t];$$

$$\omega := \frac{2 * \pi}{T};$$

$$F_n := \frac{1}{T} * \int_{t_0}^{t_0+T} f[t] * \mathrm{Exp}[-i * n * \omega * t] \, dt;$$

Realna Fourierjeva vrsta

$$f[t\_] := \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n * \mathrm{Cos}[n * \omega * t] + b_n * \mathrm{Sin}[n * \omega * t];$$

$$\omega := \frac{2 * \pi}{T};$$

$$a_n := \frac{2}{T} * \int_{t_0}^{t_0+T} f[t] * \mathrm{Cos}[n * \omega * t] \, dt;$$

$$b_n := \frac{2}{T} * \int_{t_0}^{t_0+T} f[t] * \mathrm{Sin}[n * \omega * t] \, dt;$$

Amplitudni in fazni spekter

$$F_n = P_n + jQ_n = |F_n| * e^{j\Theta_n}$$

Amplitudni spekter

$$|F_n| = \sqrt{F_n * \overline{F_n}} = \sqrt{P_n^2 + Q_n^2} = \frac{1}{2} \sqrt{a_n^2 + b_n^2}$$

Fazni spekter

$$\phi_n = \begin{cases} \arctan \frac{Q_n}{P_n} & P_n > 0 \\ \arctan \frac{Q_n}{P_n} \pm \pi & P_n < 0 \end{cases}$$

$$= \begin{cases} -\arctan \frac{b_n}{a_n} & a_n > 0 \\ -\arctan \frac{b_n}{a_n} \pm \pi & a_n < 0 \end{cases}$$

Vpliv premika osnovnega signala na F.v.

Če so $F[n]$ koeficienti F.v. signala $f[t]$, kako je s premaknjenim signalom $f[t - t_1]$?

Označimo s $F_1[n]$ koeficiente F. v. signala $f[t - t_1]$.

Velja :

$F_1[n] = e^{-jn\omega t_1} * F[n]$

Sledi :

$|F_1[n]| = |F[n]|$

$\phi_1[n] = \phi[n] - n\omega t_1$

Naloga 2:

Naloga:

Izrazi periodično funkcijo f(t) s kompleksno F. v.
Določi tudi koeficiente realne F.v. ter zapiši realno vrsto.
Na koncu določi in nariši tudi amplitudni in fazni spekter.

Signal:

$f[t\_] := \mathrm{Sin}\left[\frac{\pi * t}{4} + \frac{\pi}{4}\right];$

Rešitev:

Zgornji signal lahko razumemo kot premaknjen osnovni signal:

$f_1[t] = \mathrm{Sin}\left[\frac{\pi * t}{4}\right]$

$f[t] = f_1[t - t_1]$,

pri tem je $\frac{\pi * t}{4} + \frac{\pi}{4} = \frac{\pi * (t - t_1)}{4}$,

sledi $t_1 = -1$

$\omega = \frac{\pi}{4} = \frac{2 * \pi}{T} => T = 8$

Realna F. v. f_1

$a_0 = 0$

$a_1 = 0$

$b_1 = 1$

$a_n = b_n = 0, \quad n > 1$

Kompleksna F. v. f_1

Ker velja:

$F_1[n] = \frac{a_n - j * b_n}{2}$, in $F_{-n} = \overline{F_n}$, ker je vhodni signal realen.

$F_1[1] = \frac{a_1 - j * b_1}{2} = \frac{-j}{2}$

$F_1[-1] = \overline{F_1[1]} = \frac{j}{2}$

Kompleksna F. v. f

Upoštevamo premik $f[t] = f_1[t - t_1]$, $t_1 = -1$

$$F[1] = e^{-jn\omega t_1} * F_1[1] = \frac{-j}{2} * e^{-j\frac{\pi}{4}(-1)} = \frac{-j}{2} * e^{j\frac{\pi}{4}}$$

$$F[-1] = e^{-jn\omega t_1} * F_1[-1] = \frac{j}{2} * e^{-j\frac{\pi}{4}(-1)} = \frac{j}{2} * e^{j\frac{\pi}{4}}$$

Realna F. v. f

$$F[1] = \frac{-j}{2} * e^{j\frac{\pi}{4}} = \frac{-j}{2} * \left(\mathrm{Cos}\left[\frac{\pi}{4}\right] + j * \mathrm{Sin}\left[\frac{\pi}{4}\right]\right) = \frac{\sqrt{2}}{4} - j * \frac{\sqrt{2}}{4}$$

$$F[-1] = \frac{\sqrt{2}}{4} + j * \frac{\sqrt{2}}{4}$$

$$a_0 = 0$$

$$a_1 = F[1] + \overline{F[1]} = \frac{\sqrt{2}}{4} - j * \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} + j * \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$$

$$b_1 = j\left(F[1] - \overline{F[1]}\right) = j\left(\frac{\sqrt{2}}{4} - j * \frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{4} - j * \frac{\sqrt{2}}{4}\right) = j\left(-j\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}$$

$$a_n = b_n = 0, \quad n > 0$$

Amplitudni spekter:

$$|F_n| = \begin{cases} \frac{1}{2} & n = 1 \\ \frac{1}{2} & n = -1 \\ 0 & n \neq \pm 1 \end{cases}$$

Fazni spekter:

$$\phi_n = \begin{cases} +\frac{\pi}{4} & n = -1 \\ -\frac{\pi}{4} & n = 1 \\ 0 & n \neq \pm 1 \end{cases}$$