Haarove temeljne funkcije

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Temeljne funkcije:

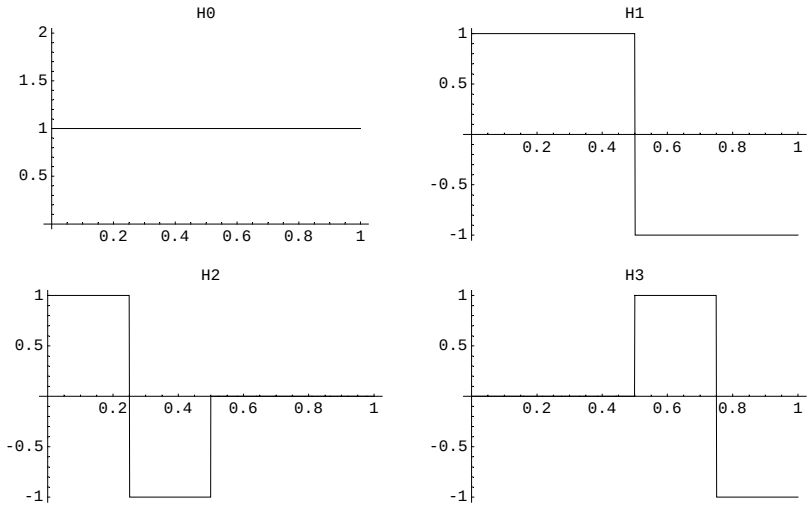
In[131]:=
```
H0[t_] := 1;
H1[t_] := Which[t ≥ 0 && t ≤ 1 / 2, 1,
                t > 1 / 2 && t ≤ 1, -1,
                t < 0 || t > 1, 0];
H2[t_] := Which[t ≥ 0 && t ≤ 1 / 4, 1,
                t > 1 / 4 && t ≤ 1 / 2, -1,
                t > 1 / 2 && t ≤ 1, 0,
                t < 0 || t > 1, 0];
H3[t_] := Which[t ≥ 1 / 2 && t ≤ 3 / 4, 1,
                t > 3 / 4 && t ≤ 1, -1,
                t ≥ 0 && t < 1 / 2, 0,
                t < 0 || t > 1, 0];
```

Koeficienti:

In[91]:=
```
K0 := 1;
K1 := 1;
K2 := 1/2;
K3 := 1/2;
```

Izris funkcij:

In[95]:=
```
<< Graphics`Graphics`
DisplayTogetherArray[{{
   Plot[H0[t], {t, 0, 1}, PlotRange → All, PlotLabel → "H0"],
   Plot[H1[t], {t, 0, 1}, PlotRange → All, PlotLabel → "H1"]},
  {Plot[H2[t], {t, 0, 1}, PlotRange → All, PlotLabel → "H2"],
   Plot[H3[t], {t, 0, 1}, PlotRange → All, PlotLabel → "H3"]}}]
```



Out[96]= - GraphicsArray -

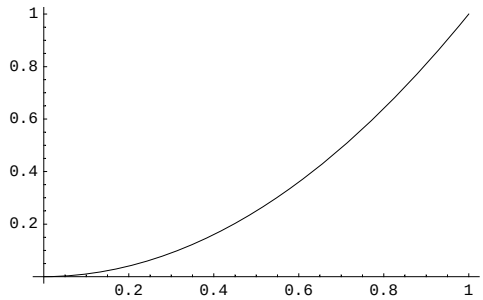
Aproksimacija signala na osnovnem intervalu (naloga 1)

Naloga 1: Signal x(t) = t^2 na intervalu [0,1] izrazite s približkom prvih štirih Haarovih temeljnih funkcij.
Določite še napako aproksimacije in skicirajte približek.

Rešitev:

In[97]:=
```
x[t_] := t^2;
```

In[98]:=
```
Plot[x[t], {t, 0, 1}, PlotRange → All];
```



Izračun koeficientov:

In[99]:=
```
C0 = 1/1 * ∫_0^1 t^2 * 1 dt
```

Out[99]= $\frac{1}{3}$

In[100]:= $C1 = \frac{1}{1} * \left(\int_0^{1/2} t^2 * 1 \, dt + \int_{1/2}^{1} t^2 * (-1) \, dt \right)$

Out[100]= $-\frac{1}{4}$

In[101]:= $C2 = \frac{1}{\frac{1}{2}} * \left(\int_0^{1/4} t^2 * 1 \, dt + \int_{1/4}^{1/2} t^2 * (-1) \, dt \right)$

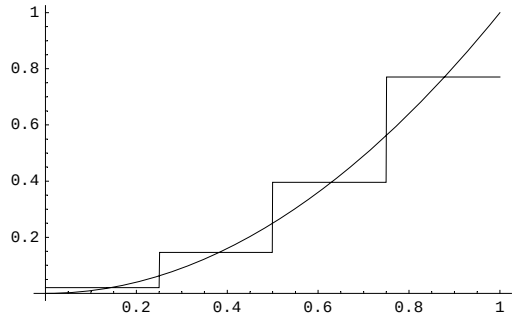
Out[101]= $-\frac{1}{16}$

In[102]:= $C3 = \frac{1}{\frac{1}{2}} * \left(\int_{1/2}^{3/4} t^2 * 1 \, dt + \int_{3/4}^{1} t^2 * (-1) \, dt \right)$

Out[102]= $-\frac{3}{16}$

Izris aproksimiranega signala:

```
In[103]:= Plot[{C0 * H0[t] + C1 * H1[t] + C2 * H2[t] + C3 * H3[t], x[t]}, {t, 0, 1
```



Izrračun napake:

```
In[104]:= T = 1;
          ϵ = 1/T * (∫_0^1 t^2 * t^2 dt - (K0 * C0^2 + K1 * C1^2 + K2 * C2^2 + K3 * C3^2))
```

Out[105]= $\frac{79}{11520}$

```
In[106]:= N[%]
```

Out[106]= 0.00685764

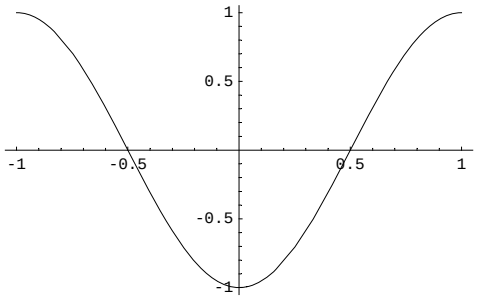
Aproksimacija signala na drugem intervalu (naloga 2)

Naloga 2: Signal x(t) = $-\cos[\pi t]$ na **intervalu [-1,1]** izrazite s približkom prvih štirih Haarovih temeljnih funkcij.
Določite še razliko v napaki aproksimacije, če aproksimiramo s prvimi štirimi Haarovimi funkcijami ali samo s prvimi tremi Haarovimi funkcijami.

Rešitev:

```
In[108]:= x[t_] := -Cos[π * t];
```

In[109]:=
```
Plot[x[t], {t, -1, 1}, PlotRange → All];
```



Haarove funkcije imamo definirane na intervalu [0,1] zato jih moramo premakr
in izvajati aproksimacijo s premaknjenimi Haarovimi t.f.

Premaknjene Haarove. t.f.

Poiščemo preslikavo **u: [-1,1] → [0,1]; u[t] = a*t+b**

In[110]:=
```
u[t_] := 1/2 * t + 1/2;
```

In[111]:=
```
Ĥ0[t_] := H0[u[t]];
Ĥ1[t_] := H1[u[t]];
Ĥ2[t_] := H2[u[t]];
Ĥ3[t_] := H3[u[t]];
```

```
a = 1/2;
K̂0 := K0/a
K̂1 := K1/a
K̂2 := K2/a
K̂3 := K3/a
```

In[120]:=
```
{K̂0, K̂1, K̂2, K̂3}
```
Out[120]= {2, 2, 1, 1}

Izračun koeficientov

In[121]:=
```
C0 = 1/K̂0 * ∫_-1^1 x[t] * Ĥ0[t] dt
```
Out[121]= 0

In[122]:= $C1 = \frac{1}{\hat{K1}} * \int_{-1}^{1} x[t] * \hat{H1}[t]\, dt$

Out[122]= 0

In[123]:= $C2 = \frac{1}{\hat{K2}} * \int_{-1}^{1} x[t] * \hat{H2}[t]\, dt$

Out[123]= $\frac{2}{\pi}$

In[124]:= $C3 = \frac{1}{\hat{K3}} * \int_{-1}^{1} x[t] * \hat{H3}[t]\, dt$

Out[124]= $-\frac{2}{\pi}$

Pomoč pri izračunu koeficientov C2 in C3 :

In[74]:=

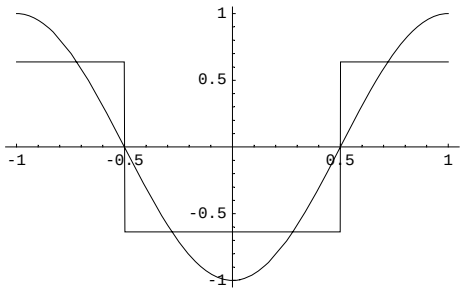
$$\int \text{Cos}[\pi * t]\, dt = \frac{\text{Sin}[\pi\, t]}{\pi}$$

In[125]:=

In[126]:=

Izračun aprokismiranega signala:

In[127]:=
```
Plot[{C0 * Ĥ0[t] + C1 * Ĥ1[t] + C2 * Ĥ2[t] + C3 * Ĥ3[t], x[t]},
  {t, -1, 1}];
```



In[128]:=

Izračun razlike napake, če aproksimiramo s prvimi 4-imi H.t.f. ali pa samo s 3-mi

In[77]:=

$$\Delta\epsilon = \epsilon 4 - \epsilon 3 = \frac{1}{t_2 - t_1} * K3 * C3 * C3;$$

V našem primeru:

In[129]:= $\Delta\epsilon = \frac{1}{2} * \left(-\frac{2}{\pi}\right) * \left(-\frac{2}{\pi}\right)$

Out[129]= $\frac{2}{\pi^2}$

In[130]:= **N[%]**

Out[130]= 0.202642