Izražava signalov s temeljnimi funkcijami

Haarove temeljne funkcije

Definicija prvih štirih funkcij

In[1]:=

$$H_0[t\_] := \begin{cases} 1 & 0 \le t \le 1 \\ 0 & \text{True} \end{cases}$$

$$H_1[t\_] := \begin{cases} 1 & 0 < t \le \frac{1}{2} \\ -1 & \frac{1}{2} < t \le 1 \\ 0 & \text{True} \end{cases}$$

$$H_2[t\_] := \begin{cases} 1 & 0 < t \le \frac{1}{4} \\ -1 & \frac{1}{4} < t \le \frac{1}{2} \\ 0 & \text{True} \end{cases}$$

$$H_3[t\_] := \begin{cases} 1 & \frac{1}{2} < t \le \frac{3}{4} \\ -1 & \frac{3}{4} < t \le 1 \\ 0 & \text{True} \end{cases}$$

Koeficienti:

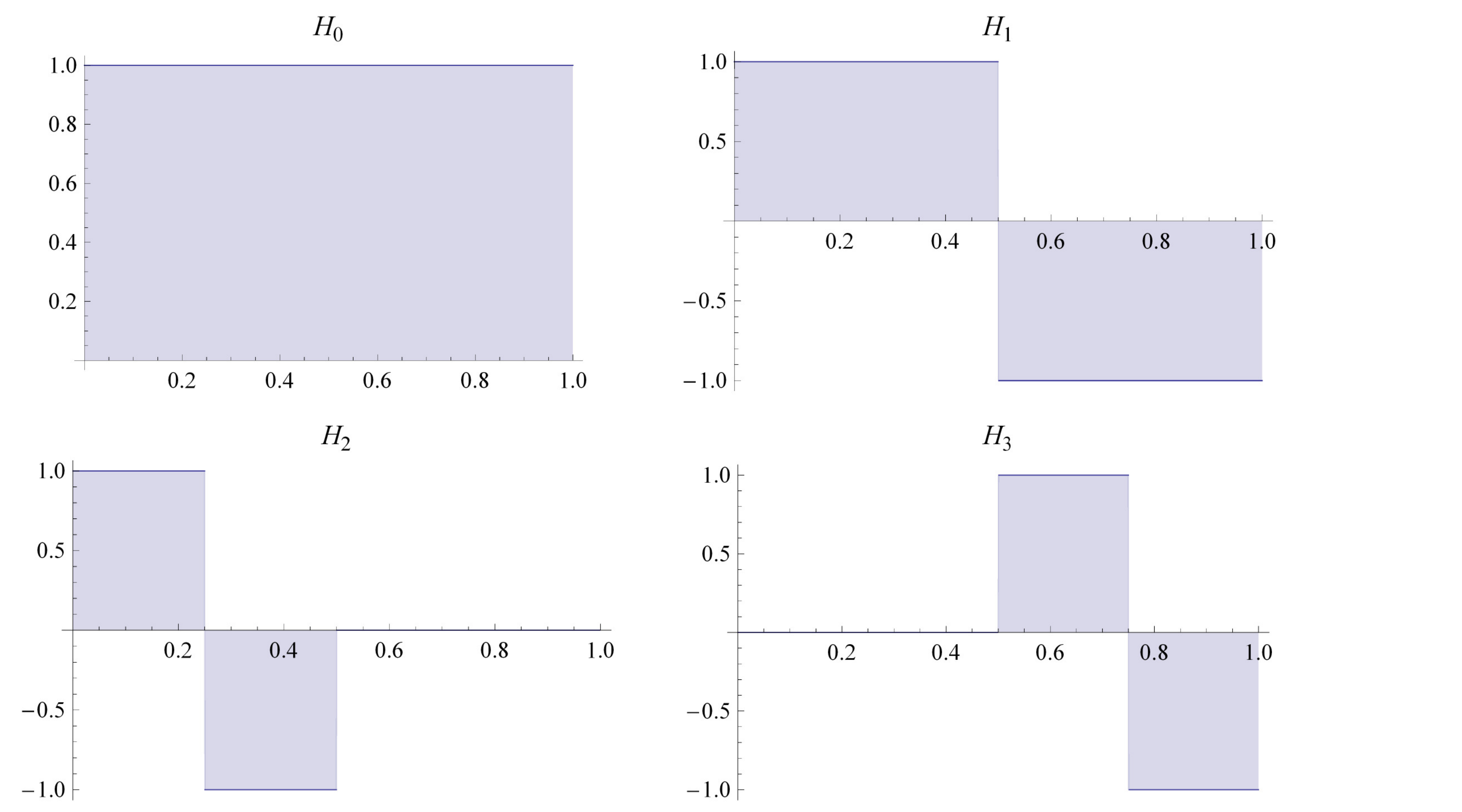
In[5]:=

$$K_0 := 1;$$
$$K_1 := 1;$$
$$K_2 := \frac{1}{2};$$
$$K_3 := \frac{1}{2};$$

Izris funkcij:

In[9]:=
```
gw0 = Plot[H0[t], {t, 0, 1}, PlotRange → All, PlotLabel → "H0", Filling → Axis];
gw1 = Plot[H1[t], {t, 0, 1}, PlotRange → All, PlotLabel → "H1", Filling → Axis];
gw2 = Plot[H2[t], {t, 0, 1}, PlotRange → All, PlotLabel → "H2", Filling → Axis];
gw3 = Plot[H3[t], {t, 0, 1}, PlotRange → All, PlotLabel → "H3", Filling → Axis];
GraphicsGrid[{{gw0, gw1}, {gw2, gw3}}]
```

Out[13]=



Aproksimacija signala (naloga 1)

Naloga:

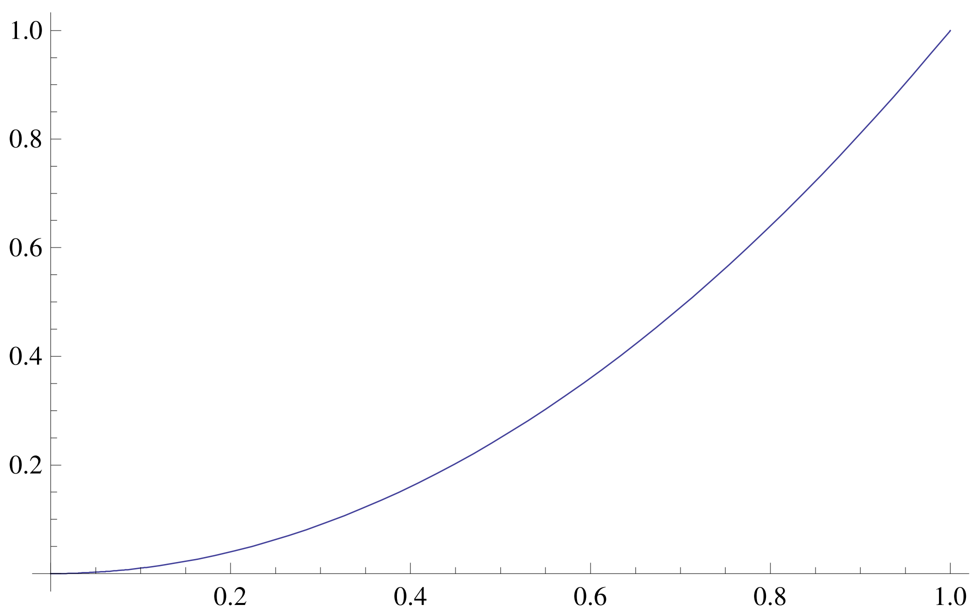
Signal $\mathbf{x(t) = t^2}$ na intervalu **[0,1]** izrazite s približkom *prvih štirih Haarovih* temeljnih funkcij.
Določite še napako aproksimacije in skicirajte približek.

Rešitev:

In[16]:=
```
x[t_] := t^2;
```

In[17]:=
```
Plot[x[t], {t, 0, 1}, PlotRange → All]
```

Out[17]=



Izračun koeficientov:

In[18]:=

$$C_0 = \frac{1}{1} * \int_0^1 t^2 * 1 \, dt$$

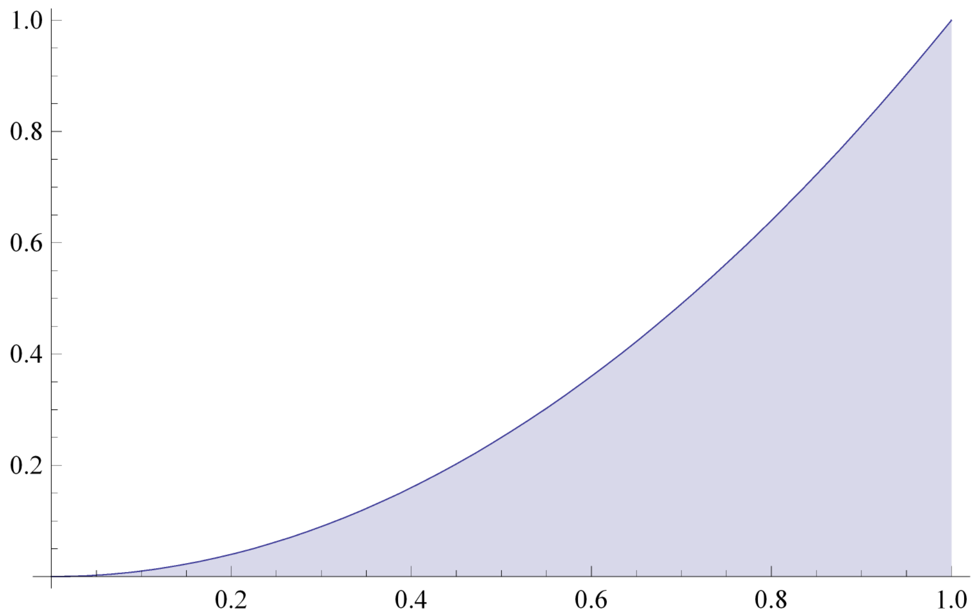
Out[18]=

$$\frac{1}{3}$$

In[19]:=

```
Plot[{x[t] * H0[t]}, {t, 0, 1}, Filling → Axis]
```

Out[19]=



In[20]:=

$$C_1 = \frac{1}{1} * \left(\int_0^{1/2} t^2 * 1 \, dt + \int_{1/2}^1 t^2 * (-1) \, dt \right)$$

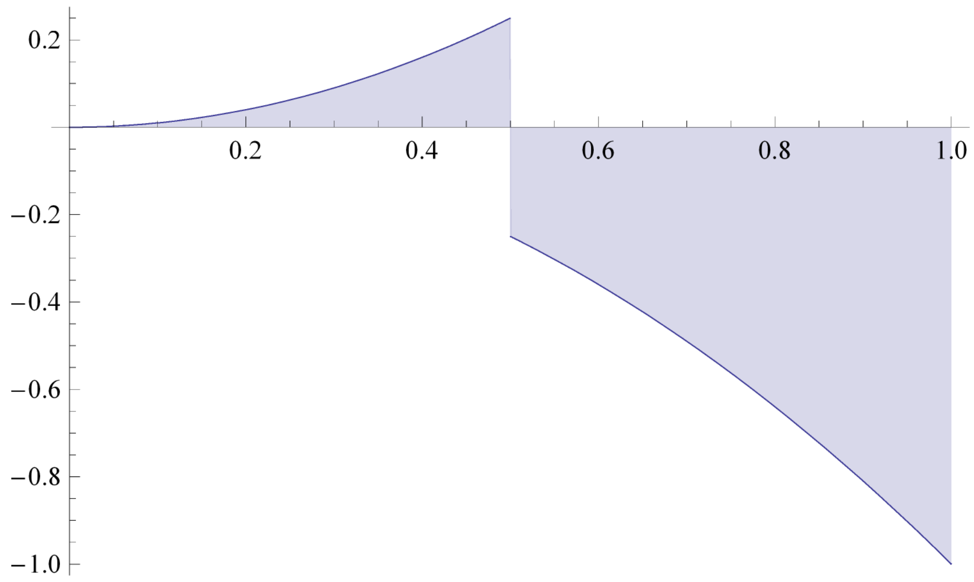
Out[20]=

$$-\frac{1}{4}$$

In[21]:=

```
Plot[{x[t] * H1[t]}, {t, 0, 1}, Filling → Axis]
```

Out[21]=



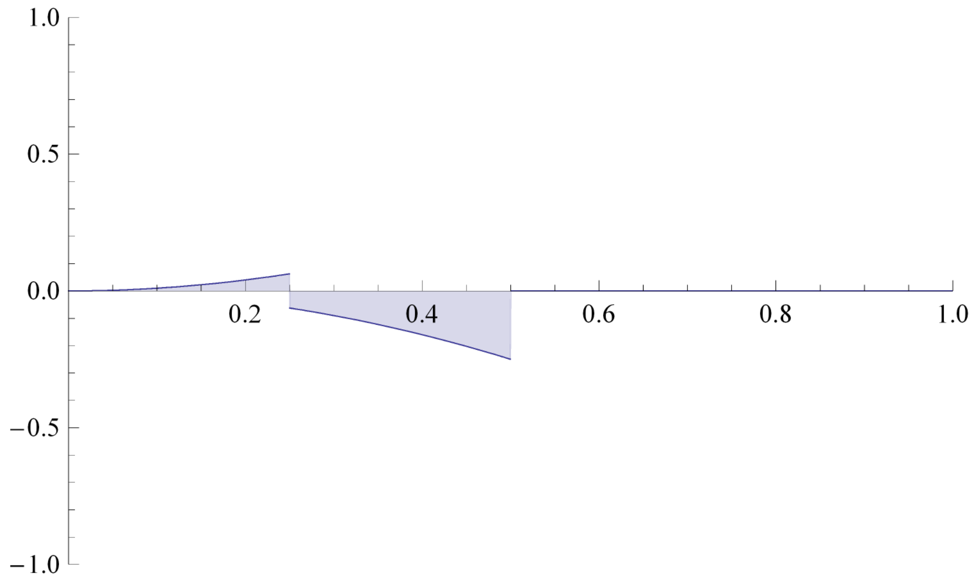
In[23]:=

$$C_2 = \frac{1}{\frac{1}{2}} * \left(\int_0^{1/4} t^2 * 1 \, dt + \int_{1/4}^{1/2} t^2 * (-1) \, dt \right)$$

Out[23]=

$$-\frac{1}{16}$$

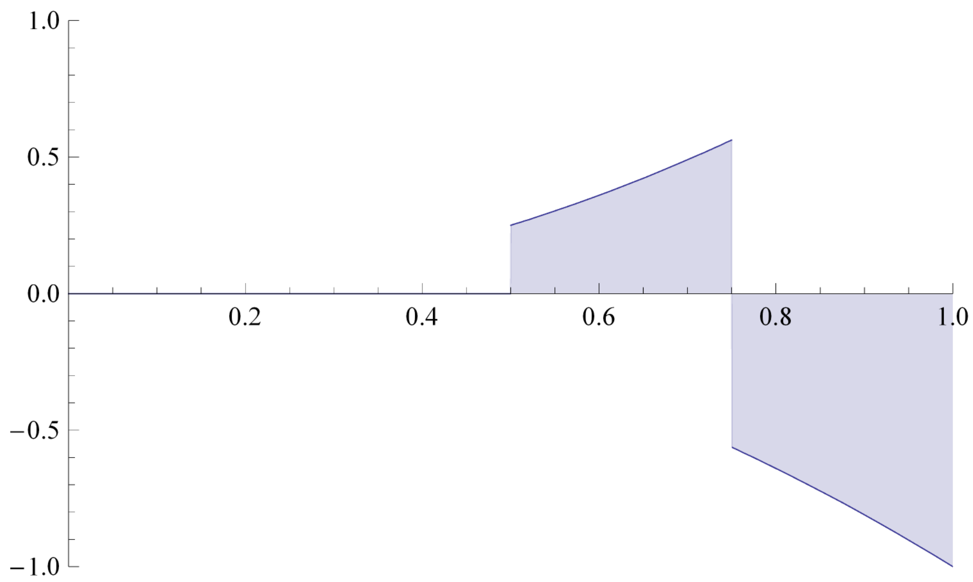
In[25]:=
```
Plot[{x[t] * H2[t]}, {t, 0, 1}, Filling → Axis, PlotRange → {{0, 1}, {-1, 1}}]
```

Out[25]=


In[27]:=
$$C_3 = \frac{1}{\frac{1}{2}} * \left(\int_{\frac{1}{2}}^{\frac{3}{4}} t^2 * 1\, dt + \int_{\frac{3}{4}}^{1} t^2 * (-1)\, dt\right)$$

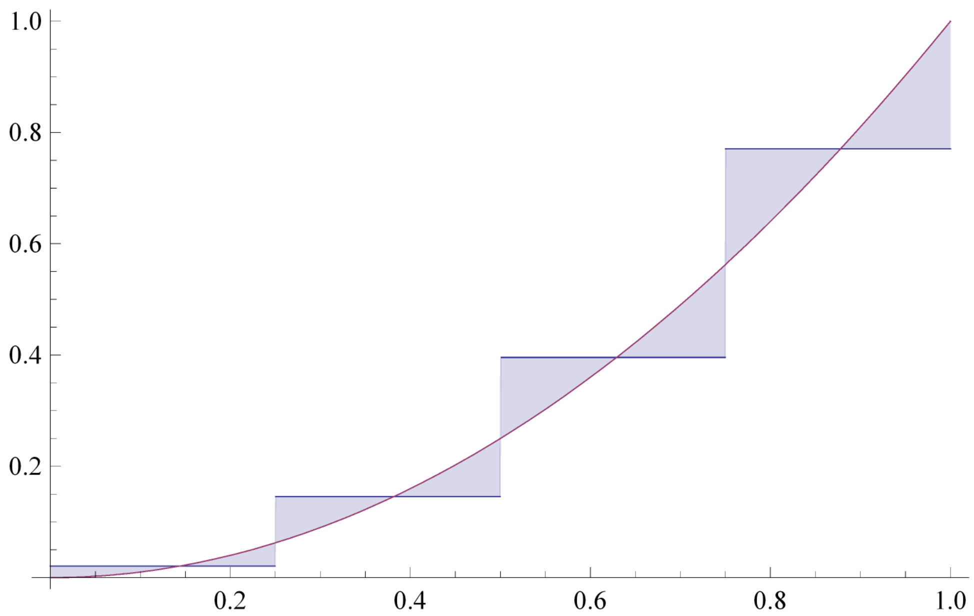
Out[27]=
$$-\frac{3}{16}$$

In[26]:=
```
Plot[{x[t] * H3[t]}, {t, 0, 1}, Filling → Axis, PlotRange → {{0, 1}, {-1, 1}}]
```

Out[26]=


Izris aproksimiranega signala:

In[28]:=
```
Plot[{C0 H0[t] + C1 H1[t] + C2 H2[t] + C3 H3[t], x[t]}, {t, 0, 1}, Filling → {1 → {2}}]
```

Out[28]=


Izračun napake:

In[29]:=
```
t1 = 0;
t2 = 1;
```
$$\epsilon = \frac{1}{t_2 - t_1} * \left(\int_0^1 t^2 * t^2 \, dt - \left(K_0 * C_0^2 + K_1 * C_1^2 + K_2 * C_2^2 + K_3 * C_3^2\right)\right)$$

Out[31]= $\frac{79}{11520}$

In[32]:=
```
N[%]
```
Out[32]= 0.00685764

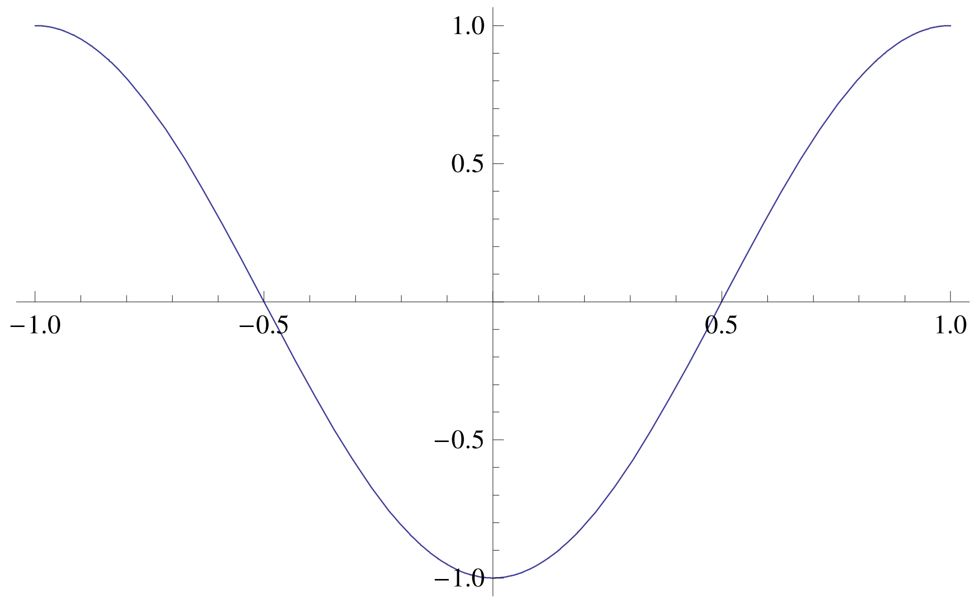
Aproksimacija signala na drugem intervalu (naloga 2)

Naloga:
Signal **x(t) = -Cos[πt]** na **intervalu [-1,1]** izrazite s približkom prvih štirih Haarovih temeljnih funkcij.
Določite še **razliko v napaki** aproksimacije, če aproksimiramo s prvimi štirimi Haarovimi funkcijami ali samo s prvimi tremi Haarovimi funkcijami.

Rešitev:

In[33]:=
```
x[t_] := -Cos[π * t];
```

In[35]:=
```
Plot[x[t], {t, -1, 1}, PlotRange → All]
```

Out[35]=


Haarove funkcije imamo definirane na intervalu [0,1] zato jih moramo premakniti na interval [-1,1] in izvajati aproksimacijo s premaknjenimi Haarovimi t.f.

Premaknjene Haarove. t.f.

Poiščemo preslikavo **u: [-1,1] → [0,1]; u[t] = a*t+b**

In[54]:=
```
u[t_] := 1/2 * t + 1/2;
```

In[127]:=
```
Ĥ0[t_] := H0[u[t]];
Ĥ1[t_] := H1[u[t]];
Ĥ2[t_] := H2[u[t]];
Ĥ3[t_] := H3[u[t]];
```

$$a = \frac{1}{2};$$
$$\hat{K}_0 := \frac{K_0}{a}$$
$$\hat{K}_1 := \frac{K_1}{a}$$
$$\hat{K}_2 := \frac{K_2}{a}$$
$$\hat{K}_3 := \frac{K_3}{a}$$

In[136]:= $\{\hat{K}_0, \hat{K}_1, \hat{K}_2, \hat{K}_3\}$

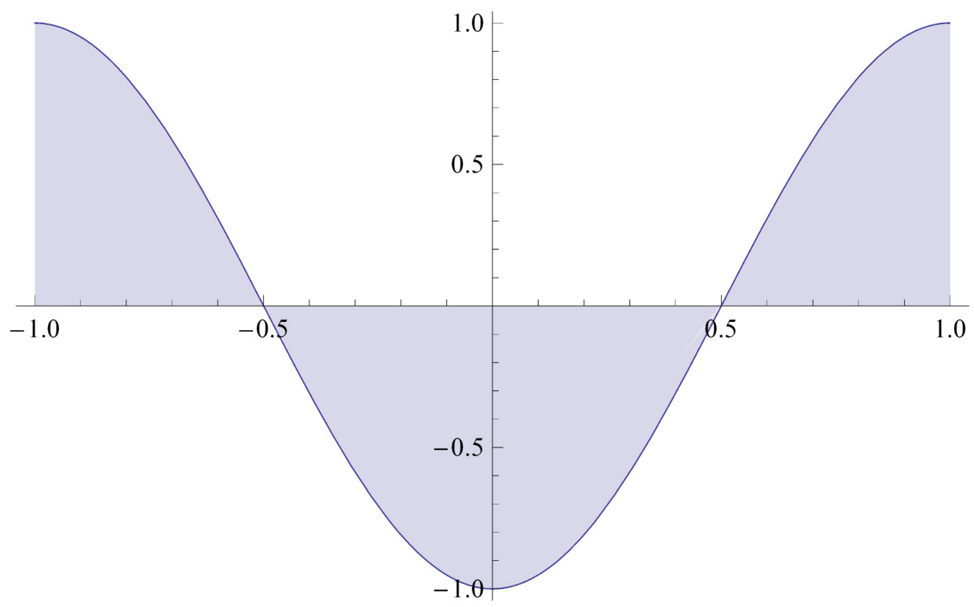
Out[136]= {2, 2, 1, 1}

In[137]:= $C_0 = \frac{1}{\hat{K}_0} * \int_{-1}^{1} x[t] * \hat{H}_0[t] \, dt$

Out[137]= 0

In[139]:= Plot[{x[t] * Ĥ0[t]}, {t, -1, 1}, Filling → Axis]

Out[139]=

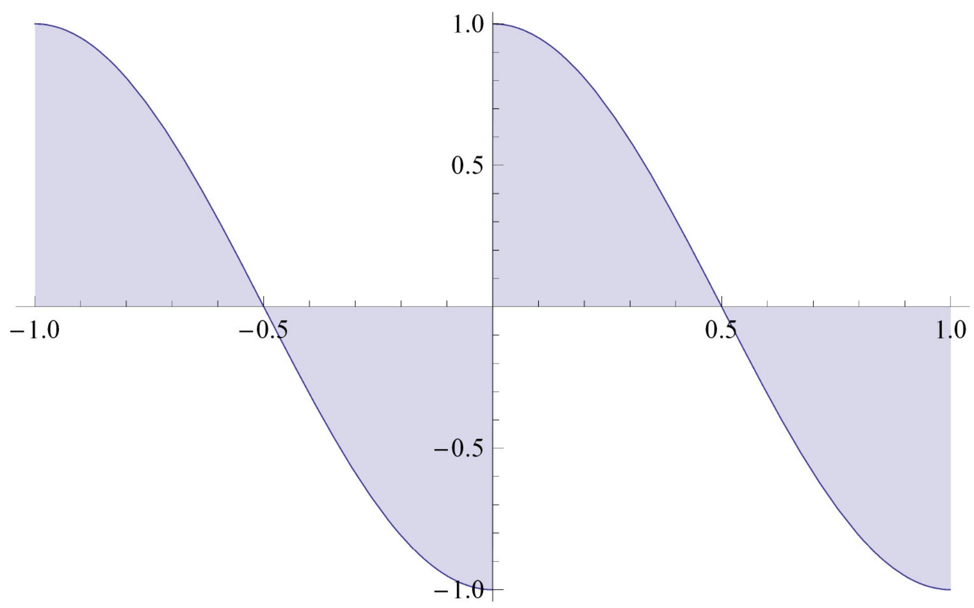


In[140]:= $C_1 = \frac{1}{\hat{K}_1} * \int_{-1}^{1} x[t] * \hat{H}_1[t] \, dt$

Out[140]= 0

In[141]:=
```
Plot[{x[t] * Ĥ₁[t]}, {t, -1, 1}, Filling → Axis]
```

Out[141]=



Pomoč pri izračunu koeficientov C_2 in C_3 :

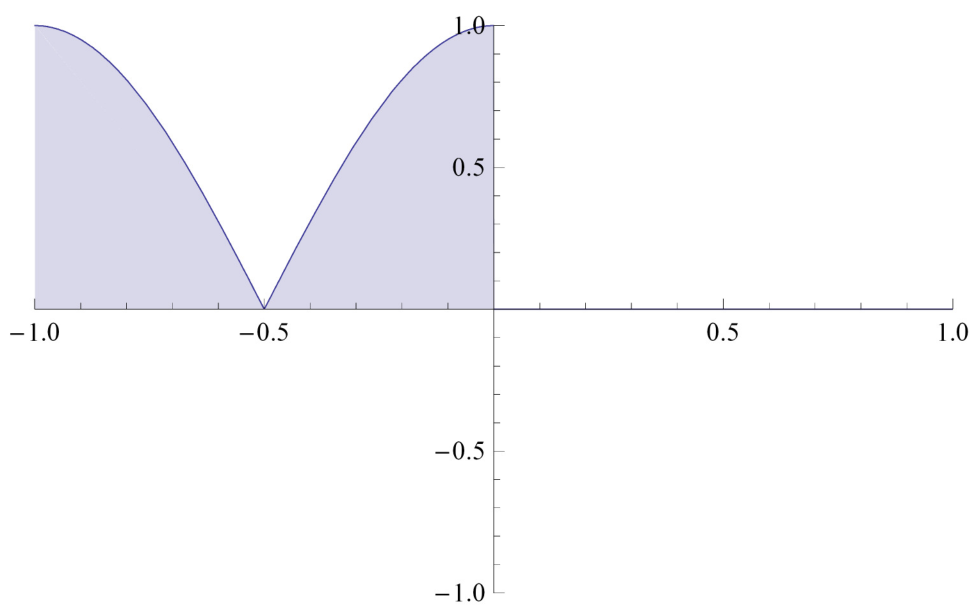
In[74]:= $\int \mathrm{Cos}[\pi * t]\, dt = \frac{\mathrm{Sin}[\pi t]}{\pi}$

In[142]:= $C_2 = \frac{1}{\hat{K}_2} * \int_{-1}^{1} x[t] * \hat{H}_2[t]\, dt$

Out[142]= $\frac{2}{\pi}$

In[146]:=
```
Plot[{x[t] * Ĥ₂[t]}, {t, -1, 1}, Filling → Axis, PlotRange → {{-1, 1}, {-1, 1}}]
```

Out[146]=



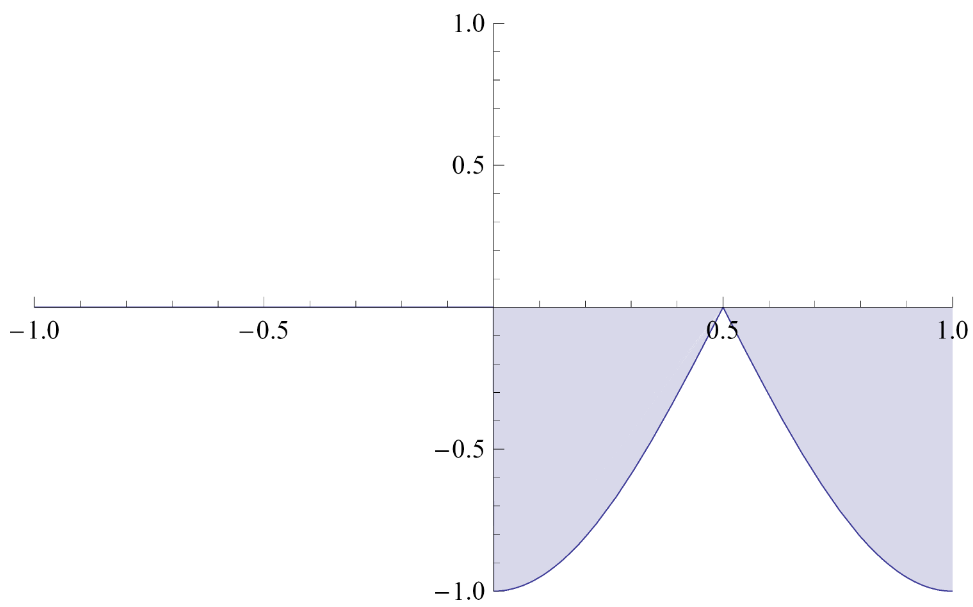
In[144]:= $C_3 = \frac{1}{\hat{K}_3} * \int_{-1}^{1} x[t] * \hat{H}_3[t]\, dt$

Out[144]= $-\frac{2}{\pi}$

In[147]:=

```
Plot[{x[t] * Ĥ3[t]}, {t, -1, 1}, Filling → Axis, PlotRange → {{-1, 1}, {-1, 1}}]
```

Out[147]=

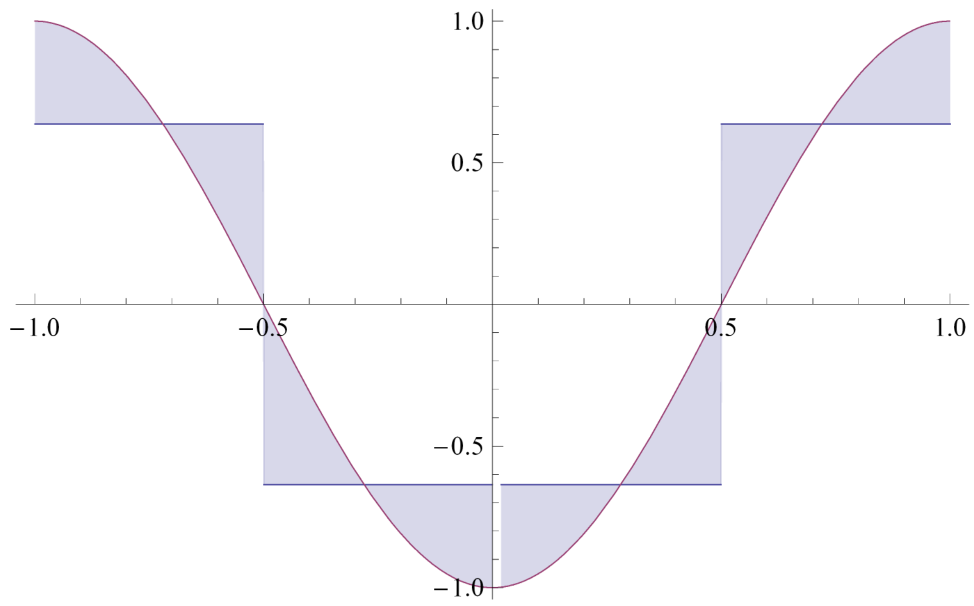


- *Izris aproksimiranega signala:*

In[148]:=

```
Plot[{C0 Ĥ0[t] + C1 Ĥ1[t] + C2 Ĥ2[t] + C3 Ĥ3[t], x[t]}, {t, -1, 1}, Filling → {1 → {2}}]
```

Out[148]=



- *Izračun razlike napake, če aproksimiramo s prvimi 4-imi H.t.f. ali pa samo s 3-mi H.t.f.*

$$\Delta\epsilon = \epsilon 4 - \epsilon 3 = \frac{1}{t_2 - t_1} * K_3 * C_3 * C_3;$$

V našem primeru:

$$\Delta\epsilon = \frac{1}{2} * \left(-\frac{2}{\pi}\right) * \left(-\frac{2}{\pi}\right)$$

Out[129]= $\frac{2}{\pi^2}$

In[130]:= `N[%]`

Out[130]= 0.202642