

PRIMER VAJE 1

Realizirajte le s pomočjo Piercovih (Shefferjevih) operatorjev naslednjo funkcijo!

$$f(x_1, x_2, x_3, x_4) = x_1 \bar{x}_2 + x_2 \bar{x}_3 x_4 + \bar{x}_1 \bar{x}_3 x_4 + x_1 x_2 x_3 \bar{x}_4 + \bar{x}_1 \bar{x}_2 x_3 x_4$$

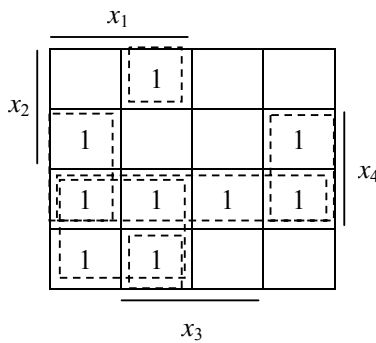
REŠITEV:

Produkte moramo zapisati v popolni obliki. (npr.: $x_1 \bar{x}_2 = x_1 \bar{x}_2 x_3 x_4 + x_1 \bar{x}_2 x_3 \bar{x}_4 + x_1 \bar{x}_2 \bar{x}_3 x_4 + x_1 \bar{x}_2 \bar{x}_3 \bar{x}_4$)

$$f_{PDNO}(x_1, x_2, x_3, x_4) = x_1 \bar{x}_2 x_3 x_4 + x_1 \bar{x}_2 x_3 \bar{x}_4 + x_1 \bar{x}_2 \bar{x}_3 x_4 + x_1 \bar{x}_2 \bar{x}_3 \bar{x}_4 + x_1 x_2 \bar{x}_3 x_4 + \bar{x}_1 x_2 \bar{x}_3 x_4 + \bar{x}_1 x_2 \bar{x}_3 \bar{x}_4 + x_1 x_2 x_3 \bar{x}_4 + \bar{x}_1 x_2 x_3 x_4$$

$$f_{PDNO}(x_1, x_2, x_3, x_4) = m_{11} + m_{10} + m_9 + m_8 + m_{13} + m_5 + m_1 + m_{14} + m_3$$

Za zapis s Shefferjevimi operatorji moramo poiskati minimalno disjunktivno normalno obliko (MDNO) s pomočjo Veichovega diagrama:



$$f_{MDNO}(x_1, x_2, x_3, x_4) = \bar{x}_2 x_4 + \bar{x}_3 x_4 + x_1 \bar{x}_2 + x_1 x_3 \bar{x}_4$$

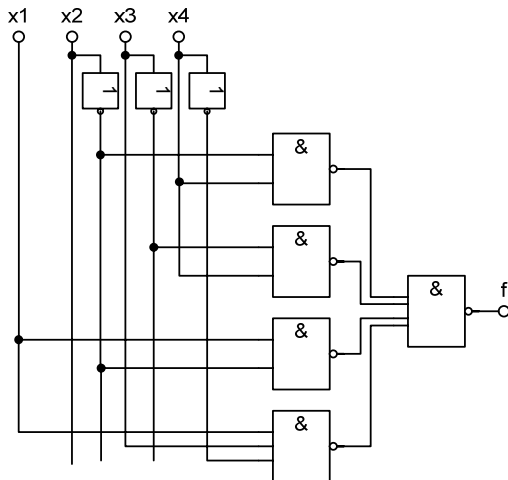
Uporabimo De Morganov teorem:

$$f(x_1, x_2, x_3, x_4) = \overline{\overline{\bar{x}_2 x_4 + \bar{x}_3 x_4 + x_1 \bar{x}_2 + x_1 x_3 \bar{x}_4}}$$

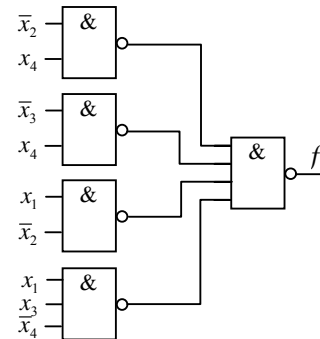
$$f(x_1, x_2, x_3, x_4) = \overline{(\overline{\bar{x}_2 x_4})(\overline{\bar{x}_3 x_4})(\overline{x_1 \bar{x}_2})(\overline{x_1 x_3 \bar{x}_4})}$$

$$f(x_1, x_2, x_3, x_4) = (\bar{x}_2 | x_4) | (\bar{x}_3 | x_4) | (x_1 | \bar{x}_2) | (x_1 | x_3 | \bar{x}_4)$$

Realizacija:



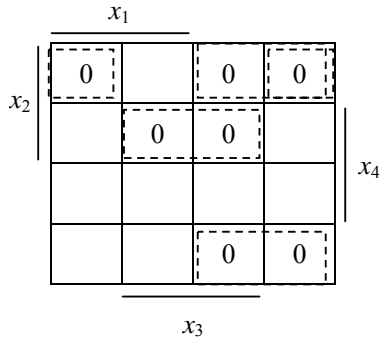
ali



Realizacijski par tega vezja je (8, 16).

Realizacijski par je vektor, ki pove koliko elementov potrebujemo za realizacijo in število vhodov (število operatorjev, število vhodov)

Za zapis s Piercovimi operatorji moramo poiskati minimalno konjunktivno normalno obliko (MKNO) s pomočjo Veichovega diagrama:



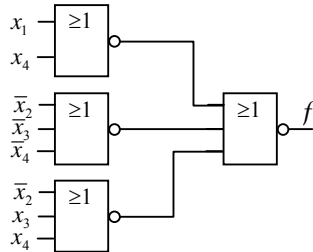
$$f_{MKNO}(x_1, x_2, x_3, x_4) = (x_1 + x_4)(\bar{x}_2 + \bar{x}_3 + \bar{x}_4)(\bar{x}_2 + x_3 + x_4)$$

$$f(x_1, x_2, x_3, x_4) = \overline{(x_1 + x_4)(\bar{x}_2 + \bar{x}_3 + \bar{x}_4)(\bar{x}_2 + x_3 + x_4)}$$

$$= \overline{(x_1 + x_4) + (\bar{x}_2 + \bar{x}_3 + \bar{x}_4) + (\bar{x}_2 + x_3 + x_4)}$$

$$= (x_1 \downarrow x_4) \downarrow (\bar{x}_2 \downarrow \bar{x}_3 \downarrow \bar{x}_4) \downarrow (\bar{x}_2 \downarrow x_3 \downarrow x_4)$$

Realizacija:



Realizacijski par (7, 14)

Če bi želeli realizirati minimalno konjunktivno obliko s Shefferjevimi operatorji, bi potrebovali en nivo več in obratno, če bi hoteli realizirati minimalno disjunktivno obliko s Piercovimi operatorji, bi potrebovali en nivo več.

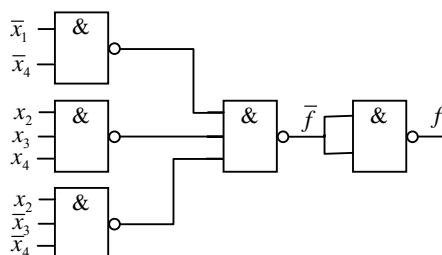
Primer:

$$f_{MKNO}(x_1, x_2, x_3, x_4) = (x_1 + x_4)(\bar{x}_2 + \bar{x}_3 + \bar{x}_4)(\bar{x}_2 + x_3 + x_4)$$

$$= (\bar{x}_1 \bar{x}_4)(x_2 x_3 x_4)(x_2 \bar{x}_3 \bar{x}_4)$$

$$= (\bar{x}_1 | \bar{x}_4) | (x_2 | x_3 | x_4) | (x_2 | \bar{x}_3 | \bar{x}_4)$$

Realizacija:



PRIMER VAJE 2

Realizirajte naslednjo preklopno funkcijo z 8 vhodnimi skalarnimi multiplekserji:

$$f(x_1, x_2, \dots, x_8) = \bar{x}_1 \bar{x}_2 \bar{x}_3 x_4 \bar{x}_6 \bar{x}_8 + \bar{x}_1 \bar{x}_2 \bar{x}_3 x_6 x_7 \bar{x}_8 + \bar{x}_1 \bar{x}_2 \bar{x}_3 x_4 \bar{x}_6 x_8 + \bar{x}_1 \bar{x}_2 \bar{x}_3 x_5 x_6 x_8 + \bar{x}_1 \bar{x}_2 x_3 x_5 x_6 x_8 + \bar{x}_1 \bar{x}_2 x_3 x_4 x_6 \bar{x}_8 + \bar{x}_1 \bar{x}_2 x_3 x_4 \bar{x}_6 x_7 x_8 + \bar{x}_1 \bar{x}_2 \bar{x}_3 \bar{x}_4 + \bar{x}_1 x_2 \bar{x}_3 \bar{x}_4 + \bar{x}_1 x_2 x_3 x_4 \bar{x}_6 + x_1 \bar{x}_2 x_3 x_7 + x_1 x_2 x_3 x_4 x_5 \bar{x}_6 + x_1 x_2 x_3 \bar{x}_4 x_5 x_8 + x_1 x_2 x_3 x_4 \bar{x}_5 \bar{x}_7 + x_1 x_2 x_3 \bar{x}_4 \bar{x}_5 x_7 \bar{x}_8$$

REŠITEV:

$$f(x_1, x_2, \dots, x_8) = \bar{S}_0 \bar{S}_1 \bar{S}_2 a_0 + S_0 \bar{S}_1 \bar{S}_2 a_1 + \bar{S}_0 S_1 \bar{S}_2 a_2 + S_0 S_1 \bar{S}_2 a_3 + \bar{S}_0 \bar{S}_1 S_2 a_4 + S_0 \bar{S}_1 S_2 a_5 + \bar{S}_0 S_1 S_2 a_6 + S_0 S_1 S_2 a_7$$

a_0, a_1, \dots, a_7 so podatkovni vhodi in S_2, S_1 in S_0 so naslovni vhodi multiplekserja.

$$f = x_1 (\bar{x}_2 x_3 x_7 + x_2 x_3 x_4 x_5 \bar{x}_6 + x_2 x_3 \bar{x}_4 x_5 x_8 + x_2 x_3 x_4 \bar{x}_5 \bar{x}_7 + x_2 x_3 \bar{x}_4 \bar{x}_5 x_7 \bar{x}_8) + \bar{x}_1 \left(\bar{x}_2 \bar{x}_3 x_4 \bar{x}_6 \bar{x}_8 + \bar{x}_2 \bar{x}_3 x_6 x_7 \bar{x}_8 + \bar{x}_2 \bar{x}_3 x_4 \bar{x}_6 x_8 + \bar{x}_2 \bar{x}_3 x_5 x_6 x_8 + \bar{x}_2 x_3 x_5 x_6 x_8 + \bar{x}_2 x_3 x_4 x_6 \bar{x}_8 + \bar{x}_2 x_3 x_4 \bar{x}_6 x_7 x_8 + x_2 \bar{x}_3 \bar{x}_4 + x_2 x_3 x_4 \bar{x}_6 \right)$$

$$f = x_1 x_2 (x_3 x_4 x_5 \bar{x}_6 + x_3 \bar{x}_4 x_5 x_8 + x_3 x_4 \bar{x}_5 \bar{x}_7 + x_3 \bar{x}_4 \bar{x}_5 x_7 \bar{x}_8) + x_1 \bar{x}_2 (x_3 x_7) + \bar{x}_1 x_2 (x_3 x_4 \bar{x}_6 + \bar{x}_3 \bar{x}_4) + \bar{x}_1 \bar{x}_2 (\bar{x}_3 x_4 \bar{x}_6 \bar{x}_8 + \bar{x}_3 x_6 x_7 \bar{x}_8 + \bar{x}_3 x_4 \bar{x}_6 x_8 + \bar{x}_3 x_5 x_6 x_8 + x_3 x_5 x_6 x_8 + x_3 x_4 x_6 \bar{x}_8 + x_3 x_4 \bar{x}_6 x_7 x_8)$$

$$f = x_1 x_2 x_3 (x_4 x_5 \bar{x}_6 + \bar{x}_4 x_5 x_8 + x_4 \bar{x}_5 \bar{x}_7 + \bar{x}_4 \bar{x}_5 x_7 \bar{x}_8) + x_1 x_2 \bar{x}_3 (0) + x_1 \bar{x}_2 x_3 (x_7) + x_1 \bar{x}_2 \bar{x}_3 (0) + \bar{x}_1 x_2 x_3 (x_4 \bar{x}_6) + \bar{x}_1 x_2 \bar{x}_3 (\bar{x}_4) + \bar{x}_1 \bar{x}_2 x_3 (x_5 x_6 x_8 + x_4 x_6 \bar{x}_8 + x_4 \bar{x}_6 x_7 x_8) + \bar{x}_1 \bar{x}_2 \bar{x}_3 (x_4 \bar{x}_6 \bar{x}_8 + x_6 x_7 \bar{x}_8 + x_4 \bar{x}_6 x_8 + x_5 x_6 x_8)$$

	S_2	S_1	S_0	
a_i	x_1	x_2	x_3	funkcijski ostanki
1	0	0	0	$f_{000}(x_4, x_5, \dots, x_8) = x_4 \bar{x}_6 \bar{x}_8 + x_6 x_7 \bar{x}_8 + x_4 \bar{x}_6 x_8 + x_5 x_6 x_8$
2	0	0	1	$f_{001}(x_4, x_5, \dots, x_8) = x_5 x_6 x_8 + x_4 x_6 \bar{x}_8 + x_4 \bar{x}_6 x_7 x_8$
3	0	1	0	$f_{010}(x_4, x_5, \dots, x_8) = \bar{x}_4$
4	0	1	1	$f_{011}(x_4, x_5, \dots, x_8) = x_4 \bar{x}_6$
5	1	0	0	$f_{100}(x_4, x_5, \dots, x_8) = 0$
6	1	0	1	$f_{101}(x_4, x_5, \dots, x_8) = x_7$
7	1	1	0	$f_{110}(x_4, x_5, \dots, x_8) = 0$
8	1	1	1	$f_{111}(x_4, x_5, \dots, x_8) = x_4 x_5 \bar{x}_6 + \bar{x}_4 x_5 x_6 + x_4 \bar{x}_5 \bar{x}_7 + \bar{x}_4 \bar{x}_5 x_7 \bar{x}_8$

$$\begin{aligned} f_0 &= x_4 \bar{x}_6 \bar{x}_8 + x_6 x_7 \bar{x}_8 + x_4 \bar{x}_6 x_8 + x_5 x_6 x_8 \\ &= x_6 (x_5 x_8 + x_7 \bar{x}_8) + \bar{x}_6 (x_4 \bar{x}_8 + x_4 x_8) \\ &= x_6 x_8 (x_5) + x_6 \bar{x}_8 (x_7) + \bar{x}_6 x_8 (x_4) + \bar{x}_6 \bar{x}_8 (x_4) \end{aligned}$$

	S_1	S_0	
a_i	x_6	x_8	funkcijski ostanki
1	0	0	$f_{00}(x_4, x_5, x_7) = x_4$
2	0	1	$f_{01}(x_4, x_5, x_7) = x_4$
3	1	0	$f_{10}(x_4, x_5, x_7) = x_7$
4	1	1	$f_{11}(x_4, x_5, x_7) = x_5$

$$\begin{aligned}
f_1 &= x_5 x_6 x_8 + x_4 x_6 \bar{x}_8 + x_4 \bar{x}_6 x_7 x_8 \\
&= x_6 (x_5 x_8 + x_4 \bar{x}_8) + \bar{x}_6 (x_4 x_7 x_8) \\
&= x_6 x_8 (x_5) + x_6 \bar{x}_8 (x_4) + \bar{x}_6 x_8 (x_4 x_7) \\
&= x_6 x_8 x_4 (x_5) + x_6 x_8 \bar{x}_4 (x_5) + x_6 \bar{x}_8 x_4 (1) + \bar{x}_6 x_8 x_4 (x_7)
\end{aligned}$$

a_i	S_2	S_1	S_0	funkcijski ostanki
	x_6	x_8	x_4	
1	0	0	0	$f_{000}(x_5, x_7) = 0$
2	0	0	1	$f_{001}(x_5, x_7) = 0$
3	0	1	0	$f_{010}(x_5, x_7) = 0$
4	0	1	1	$f_{011}(x_5, x_7) = x_7$
5	1	0	0	$f_{100}(x_5, x_7) = 0$
6	1	0	1	$f_{101}(x_5, x_7) = 1$
7	1	1	0	$f_{110}(x_5, x_7) = x_5$
8	1	1	1	$f_{111}(x_5, x_7) = x_5$

$$\begin{aligned}
f_3 &= x_4 \bar{x}_6 \\
&= \bar{x}_4 \bar{x}_6 (0) + \bar{x}_4 x_6 (0) + x_4 \bar{x}_6 (1) + x_4 x_6 (0)
\end{aligned}$$

a_i	S_1	S_0	funkcijski ostanki
	x_4	x_6	
1	0	0	$f_{00}(x_5, x_7, x_8) = 0$
2	0	1	$f_{01}(x_5, x_7, x_8) = 0$
3	1	0	$f_{10}(x_5, x_7, x_8) = 1$
4	1	1	$f_{11}(x_5, x_7, x_8) = 0$

$$\begin{aligned}
f_7 &= x_4 x_5 \bar{x}_6 + \bar{x}_4 x_5 x_6 + x_4 \bar{x}_5 \bar{x}_7 + \bar{x}_4 \bar{x}_5 x_7 \bar{x}_8 \\
&= x_4 (x_5 \bar{x}_6 + \bar{x}_5 x_7) + \bar{x}_4 (x_5 x_6 + \bar{x}_5 x_7 \bar{x}_8) \\
&= x_4 x_5 (\bar{x}_6) + x_4 \bar{x}_5 (x_7) + \bar{x}_4 x_5 (x_6) + \bar{x}_4 \bar{x}_5 (x_7 \bar{x}_8) \\
&= x_4 x_5 x_7 (\bar{x}_6) + x_4 x_5 \bar{x}_7 (\bar{x}_6) + x_4 \bar{x}_5 \bar{x}_7 (1) + \bar{x}_4 x_5 x_7 (x_6) + \bar{x}_4 x_5 \bar{x}_7 (x_6) + \bar{x}_4 \bar{x}_5 x_7 (\bar{x}_8)
\end{aligned}$$

a_i	S_2	S_1	S_0	funkcijski ostanki
	x_4	x_5	x_7	
1	0	0	0	$f_{000}(x_6, x_8) = 0$
2	0	0	1	$f_{001}(x_6, x_8) = \bar{x}_8$
3	0	1	0	$f_{010}(x_6, x_8) = x_6$
4	0	1	1	$f_{011}(x_6, x_8) = x_6$
5	1	0	0	$f_{100}(x_6, x_8) = 1$
6	1	0	1	$f_{101}(x_6, x_8) = 0$
7	1	1	0	$f_{110}(x_6, x_8) = \bar{x}_6$
8	1	1	1	$f_{111}(x_6, x_8) = \bar{x}_6$

Blokovni diagram:

