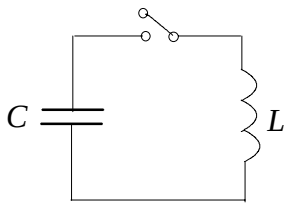
Idealni električni nihajni krog (*R* = 0)

$$\boxed{U_C(t=0)=U_0}$$



$$U_L + U_C = 0$$

$$-L\frac{\mathrm{d}I}{\mathrm{d}t} - \frac{e}{C} = 0 \quad / \frac{\mathrm{d}}{\mathrm{d}t}$$

$$-L\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} - \frac{I}{C} = 0 \ , \qquad \text{kjer smo upoštevali } I = \frac{\mathrm{d}e}{\mathrm{d}t}$$

$$\boxed{\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = -\frac{1}{LC}\, I} \qquad (1a)$$

Nastavek za rešitev:

$$\boxed{I = I_0 \sin(\omega_0 t)} \qquad (2)$$

$$\frac{dI}{dt} = I_0\, \omega_0\, \cos(\omega_0 t)$$

$$\frac{d^2 I}{dt^2} = -I_0\, \omega_0^2 \sin(\omega_0 t) = -\omega_0^2\, I \qquad (1b)$$

Iz primerjave med enačbama (1a) in (1b) sledi:

$$\omega_0^2 = \frac{1}{LC} \quad \Rightarrow \quad \boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \ \text{lastna krožna frekvenca} \qquad (3)$$

$$U_L = -L\frac{dI}{dt} = -L\, I_0\, \omega_0 \cos(\omega_0 t) \qquad (4)$$

$$U_C = U_L = L\, I_0\, \omega_0 \cos(\omega_0 t) \qquad (5)$$

torej:

$$\boxed{U_0 = L\, I_0 \omega_0} \qquad (6)$$

- **Skupna energija idealnega električnega nihajnega kroga**

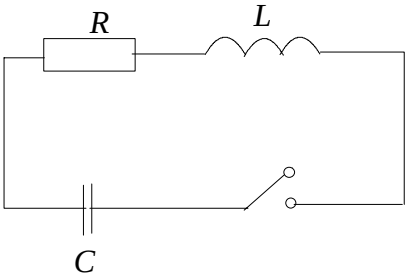
$$W=\frac{1}{2}LI^2+\frac{1}{2}CU_C^2=\frac{1}{2}LI_0^2\sin^2(\omega_0 t)+\frac{1}{2}CU_0^2\cos^2(\omega_0 t)=$$
$$=\frac{1}{2}LI_0^2\sin^2(\omega_0 t)+\frac{1}{2}CL^2I_0^2\omega_0^2\cos^2(\omega_0 t)=\frac{1}{2}LI_0^2\sin^2(\omega_0 t)+\frac{1}{2}LI_0^2\cos^2(\omega_0 t)=$$
$$=\frac{1}{2}LI_0^2\left\{\sin^2(\omega_0 t)+\cos^2(\omega_0 t)\right\}=\frac{1}{2}LI_0^2=\frac{1}{2}CU_0^2,$$

kjer smo upoštevali:

$$I_0=\frac{U_0}{L\omega_0} \quad \text{in} \quad \omega_0^2=\frac{1}{LC} \quad .$$

Zaključek: skupna energija idealnega nihajnega kroga je konstantna.

Dušeni električni nihajni krog



$$U_L + U_R + U_C = 0 \,, \tag{1}$$

$$-L\frac{\mathrm{d}I}{\mathrm{d}t} - RI - \frac{e}{C} = 0 \quad / \frac{\mathrm{d}}{\mathrm{d}t} \quad ,$$

$$\boxed{-L\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} - R\frac{\mathrm{d}I}{\mathrm{d}t} - \frac{I}{C} = 0} \,, \tag{2}$$

kjer smo upoštevali $I = \frac{\mathrm{d}e}{\mathrm{d}t}$.

Nastavek za rešitev:

$$\boxed{I = I_0\, e^{-\beta t} \cos\left(\omega_0^{'} t\right)} \,, \quad \text{kjer je } \omega_0^{'} \text{ krožna frekvenca} \tag{3a}$$

ali v kompleksnem:

$$I = I_0\, e^{-\beta t}\, e^{-i\omega_0^{'} t} = I_0\, e^{\left(i\omega_0^{'} - \beta\right)t} \tag{3b}$$

torej:

$$\frac{\mathrm{d}I}{\mathrm{d}t} = I_0 \left(i\,\omega_0^{'} - \beta\right) e^{\left(i\omega_0^{'} - \beta\right)t} \,, \tag{4}$$

$$\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = I_0 \left(i\omega_0^{'} - \beta\right)^2 e^{\left(i\omega_0^{'} - \beta\right)t} = \left(-\omega_0^{'2} - 2\beta\,\omega_0^{'}\, i + \beta^2\right) I_0\, e^{\left(i\omega_0^{'} - \beta\right)t} \,. \tag{5}$$

Izraze (3b), (4) in (5) vstavimo v enačbo (2), torej:

$$\begin{aligned} & I_0\,\beta^2\, e^{-\beta t}\, e^{i\omega_0^{'} t} - I_0\, e^{-\beta t}\, \omega_0^{'2}\, e^{i\omega_0^{'} t} - 2I_0\,\beta i\, \omega_0^{'}\, e^{-\beta t}\, e^{i\omega_0^{'} t} + \\ & + \frac{R}{L}\left(-I_0\beta\, e^{-\beta t}\, e^{i\omega_0^{'} t} + I_0\, e^{-\beta t} i\,\omega_0^{'}\, e^{i\omega_0^{'} t}\right) + \frac{1}{LC} I_0\, e^{-\beta t}\, e^{i\omega_0^{'} t} = 0 \end{aligned} \tag{6}$$

Iz enačbe (6) sledi:

$$I_0\, e^{\left(i\,\omega_0^{'} - \beta\right)t} \left[\beta^2 - \omega_0^{'2} - \beta\frac{R}{L} + i\,\omega_0^{'} \left\{-2\beta + \frac{R}{L}\right\} + \frac{1}{LC}\right] = 0 \,,$$

zato mora v splošnem veljati:

$$\left[\beta^2 - \omega_0^{'2} - \beta\frac{R}{L} + \omega_0^2 + i\,\omega_0^{'}\left\{-2\beta + \frac{R}{L}\right\}\right] = 0\ , \tag{7}$$

kjer smo upoštevali $\omega_0^2 = \frac{1}{LC}$.

Imaginarni in realni del enačbe (7) morata biti nič vsak posebej, torej:

$$-2\beta + \frac{R}{L} = 0 \tag{8}$$

$$-\beta^2 - \omega_0^{'2} - \beta\frac{R}{L} + \omega_0^2 = 0 \tag{9}$$

Iz enačbe (8) sledi:

$$\boxed{\beta = \frac{1}{2}\frac{R}{L}}\ . \tag{10}$$

Ob upoštevanju enačbe (10) zapišemo enačbo (8) v obliki:

$$-\beta^2 - \omega_0^{'2} - 2\beta^2 + \omega_0^2 = 0\ ,$$

od koder sledi:

$$\boxed{\omega_0^{'2} = \sqrt{\omega_0^2 - \beta^2}}\ . \tag{11}$$

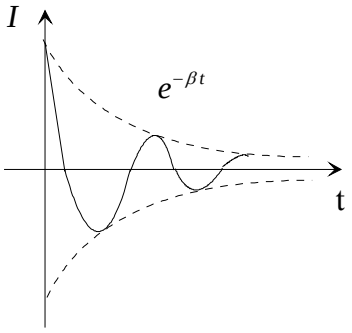
Torej:

1. če $\beta < \omega_0$: $I = I_0\, e^{-\beta t}\cos\left(\omega_0^{'}\, t + \delta\right)$

2. če $\beta > \omega_0$: $\omega_0^{'} = i\sqrt{\left(\beta^2 - \omega_0^2\right)} = i\,\alpha\,, \alpha = \sqrt{\left(\beta^2 - \omega_0^2\right)}$
   $I = I_0\, e^{-\beta t}\, e_0^{i\omega_0^{'} t} = I_0\, e^{-\beta t}\, e^{-\alpha t} = I_0\, e^{-(\alpha+\beta)t}$

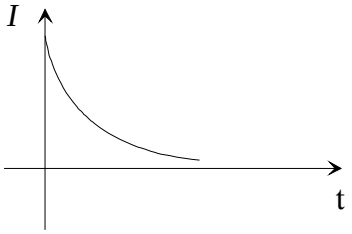
Primer 1 ($\beta < \omega_0$):

ZMERNO DUŠENJE



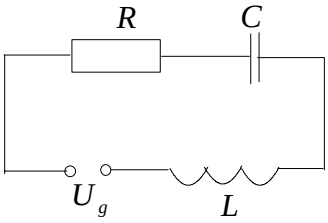
Primer 2 ($\beta > \omega_0$):

MOČNO DUŠENJE



Vsiljeno nihanje električnega nihajnega kroga

Zaporedna vezava



Napetost generatorja: $U_g = U_0 \cos(\omega t)$

$$U_g + U_C + U_L + U_R = 0 \tag{1}$$

$$U_g - \frac{e}{C} - L\frac{\mathrm{d}I}{\mathrm{d}t} - IR = 0 \quad / \frac{\mathrm{d}}{\mathrm{d}t}$$

$$\boxed{\frac{\mathrm{d}U_g}{\mathrm{d}t} - \frac{I}{C} - L\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} - R\frac{\mathrm{d}I}{\mathrm{d}t} = 0}\ , \tag{2}$$

kjer smo upoštevali $I = \frac{\mathrm{d}e}{\mathrm{d}t}$. V nadaljevanju iščemo rešitev enačbe (2). Predpostavimo, da člen zaradi lasnega nihanja, ki je sorazmeren $e^{-\beta t}\cos(\omega_0' t + \delta)$ po zadosti dolgem času izgine. Zato takrat nihajni krog niha le z vsiljeno frekvenco. Torej iščemo rešitev enačbe (2) z nastavkom (v kompleksnem):

$$I = I_0 e^{i\omega t}, \tag{3}$$

kjer je

$$U_g = U_0 e^{i\omega t}. \tag{4}$$

Vstavimo izraza (3) in (4) v enačbo (2). Tako dobimo:

$$i\omega_0 U_0 - \frac{I_0}{C} e^{i\omega t} + L\omega^2 I_0 e^{i\omega t} - R i\omega I_0 e^{i\omega t} = 0 \tag{5}$$

Enačbo (5) preuredimo v obliko**:**

$$\left(i\omega U_0 - \frac{I_0}{C} + L\omega^2 I_0 - R i\omega I_0\right) e^{i\omega t} = 0. \tag{6}$$

Vidimo, da v splošnem velja:

$$i\omega_0 U_0 - \frac{I_0}{C} + L\omega^2 I_0 - R i\omega I_0 = 0 \ , \tag{7}$$

torej:

$$U_0 = \left(-\frac{\omega L}{i} + R + \frac{1}{i\omega C} \right) I_0 = \left(R + i\omega L - i\frac{1}{\omega C} \right) I_0 \ . \tag{8}$$

Vpeljemo impedance:

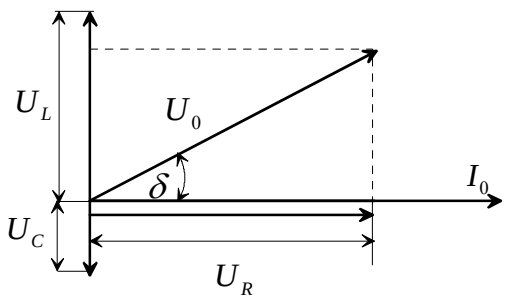
$$Z_R = R \ , \tag{9}$$

$$Z_L = i\omega L \ , \tag{10}$$

$$Z_C = -i\frac{1}{\omega C} \ , \tag{11}$$

Ker teče skozi upornik, tuljavo in kondenzator enak tok predpostavimo, da je I_0 realno število, U_0 pa kompleksno število.

Torej:



Iz slike ter enačbe (8) sledi:

$$tg\,\delta = \frac{\left(\omega L - \frac{1}{\omega C} \right)}{R} \tag{12}$$

in

$$|U_0|^2 = U_0 U_0^* = I_0^2 \left[R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2 \right] . \tag{13}$$

Iz enačbe (13) sledi:

$$I_0 = \frac{|U_0|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \tag{14}$$

Vidimo, da je I_0 maksimalen, če je:

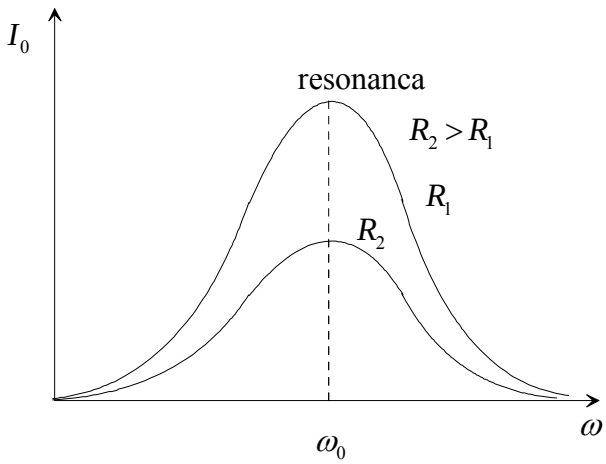
$$\omega L - \frac{1}{\omega C} = 0 \ , \tag{15}$$

torej

$$\omega L = \frac{1}{\omega C} \ ,$$

oziroma

$$\boxed{\omega^2 = \frac{1}{LC} = \omega_0^2 \ ,} \tag{16}$$



Če je $\omega = \omega_0$ velja:

$$tg\,\delta = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R} = 0 \ , \tag{17}$$

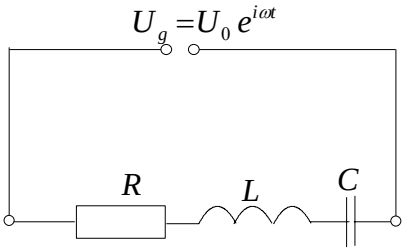
torej $\delta = 0$. Ker je v resonanci $\delta = 0$ in I_0 maksimalen je takrat porabljena povprečna moč:

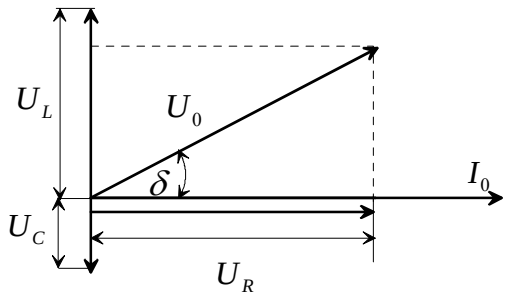
$$\overline{P} = \frac{1}{2} I_0 |U_0| \cos\delta = \frac{1}{2} \frac{|U_0|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} |U_0| \cos\delta = \frac{1}{2} \frac{|U_0|^2}{R} \cos\delta = \frac{1}{2} \frac{|U_0|^2}{R} \qquad (18)$$

največja možna. V izpeljavi smo upoštevali enačbi (15) in (17).

Zaključek:

A) Zaporedna vezava





$$e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$$

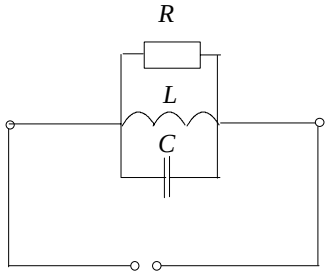
Vzamemo: I_0 je realen, U_0 je kompleksen

Kompleksne impedance:

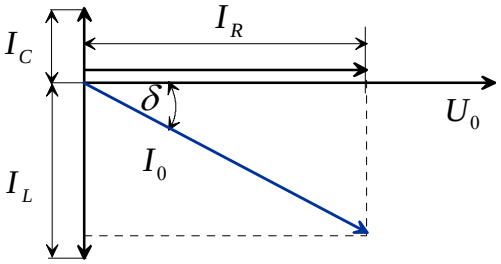
$$\boxed{\begin{aligned} Z_R &= R \\ Z_L &= i\,\omega L \\ Z_C &= -i \frac{1}{\omega C} \end{aligned}}$$

$$\boxed{\begin{aligned} Z &= Z_R + Z_L + Z_C \\ U_0 &= Z\, I_0 \end{aligned}}$$

B) Vzporedna vezava



$$U_g = U_0\, e^{i\omega t}$$



$$\boxed{U_0 = Z\, I_0}$$

$$\boxed{\frac{1}{Z} = \frac{1}{R} + \frac{1}{i\omega L} - \frac{\omega C}{i}}$$

Vzamemo: U_0 je realen, I_0 je kompleksen

$$I_0 = \frac{1}{Z} U_0 = \left(\frac{1}{R} + \frac{1}{i\omega L} + i\,\omega C \right) U_0 = U_0 \left[\frac{1}{R} + i \left(\omega C - \frac{1}{\omega L} \right) \right]$$

$$|I_0| = \sqrt{I_0 \cdot I_0^*} = U_0 \left[\left(\frac{1}{R} \right)^2 + \left(\omega C - \frac{1}{\omega L} \right)^2 \right]^{1/2}$$

$$\boxed{tg\,\delta = \frac{\mathrm{Im}(I_0)}{\mathrm{Re}(I_0)} = \left(\omega C - \frac{1}{\omega L} \right) R}$$

Moč: izmenični tok

Napetost prehiteva tok v fazi za δ.

$$I = I_0 \cos \omega t$$
$$U = U_0 \cos(\omega t + \delta)$$

$$P = IU = I_0 U_0 \cos \omega t \cos(\omega t + \delta) = I_0 U_0 \cos \omega t [\cos \omega t \cos \delta - \sin \omega t \sin \delta] =$$
$$= I_0 U_0 [\cos^2 \omega t \cos \delta - \sin \omega t \cos \omega t \sin \delta] \Rightarrow$$
$$\overline{P} = I_0 U_0 \left[\overline{\cos^2 \omega t} \cos \delta - \overline{\frac{1}{2} \sin(2 \omega t)} \sin \delta \right],$$

kjer smo upoštevali:

$$\sin \omega t \cos \omega t = \frac{1}{2} \sin(2\omega t).$$

Velja:

$$\overline{\cos^2 \omega t} = 1/2$$

$$\overline{\frac{1}{2} \sin(2 \omega t)} = 0,$$

torej:

$$\boxed{\overline{P} = \frac{1}{2} I_0 U_0 \cos \delta}$$

oziroma:

$$\overline{P} = I_{ef} U_{ef} \cos \delta, \quad \text{kjer} \quad I_{ef} = \frac{I_0}{\sqrt{2}} \quad in \quad U_{ef} = \frac{U_0}{\sqrt{2}}$$

Elektromagnetno valovanje

- **Faradayev zakon** (ind. zakon)**:**

$$\oint \vec{E}\cdot \mathrm{d}\vec{S} = -\frac{\partial}{\partial t}\int \vec{B}\cdot \mathrm{d}\vec{S} \;\Rightarrow\; \boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t},} \qquad \vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

- **Amperov zakon:**

$$\int \vec{H}\cdot \mathrm{d}\vec{S} = \int \vec{j}_e \mathrm{d}\vec{S} + \int \frac{\partial \vec{D}}{\partial t}\mathrm{d}\vec{S} \;\Rightarrow\; \boxed{\vec{\nabla}\times\vec{H} = \vec{j}_e + \frac{\partial \vec{D}}{\partial t}}$$

  - če $j_e = 0$ (ni dielektričnih tokov)

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \frac{\partial \vec{D}}{\partial t}$$

  - **vakuum:** $\varepsilon = 1, \quad \mu = 1$

$$B = \mu_0 H$$

$$D = \varepsilon_0 E$$

torej:

$$\boxed{\begin{aligned} \vec{\nabla}\times\vec{E} &= -\mu_0 \frac{\partial \vec{H}}{\partial t} \\ \vec{\nabla}\times\vec{H} &= \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \end{aligned}}$$

  - poseben primer: raven elektromagnetni val, $\vec{E}$ in $\vec{H}$ odvisna samo od koordinate x, $H_x = E_x = 0$:

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \left(0, -\frac{\partial E_z}{\partial x}, \overset{a}{\overset{\downarrow}{\frac{\partial E_y}{\partial x}}}\right) = -\mu_0 \left(\frac{\partial H_x}{\partial t}, \frac{\partial H_y}{\partial t}, \overset{a}{\overset{\downarrow}{\frac{\partial H_z}{\partial t}}}\right)$$

$$\vec{\nabla} \times \vec{H} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \left(0, \overset{b}{\downarrow} -\frac{\partial H_z}{\partial x}, \frac{\partial H_y}{\partial x} \right) = \varepsilon_0 \left(\frac{\partial E_x}{\partial t}, \overset{b}{\downarrow} \frac{\partial E_y}{\partial t}, \frac{\partial E_z}{\partial t} \right) \quad \leftarrow \quad a$$

Enačbi a in b:

1. $$\frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \quad / \frac{\partial}{\partial t} \quad \Rightarrow \quad \frac{\partial^2 E_y}{\partial x \partial t} = -\mu_0 \frac{\partial^2 H_z}{\partial t^2} \qquad (1)$$

$$-\frac{\partial H_z}{\partial x} = \varepsilon_0 \frac{\partial E_y}{\partial t} \quad / \frac{\partial}{\partial x} \quad \Rightarrow \quad -\frac{\partial^2 H_z}{\partial x^2} = \varepsilon_0 \frac{\partial^2 E_y}{\partial t \partial x} \qquad (2)$$

torej:

$$(1), \quad (2) \quad \Rightarrow \quad +\mu_0 \frac{\partial^2 H_z}{\partial t^2} = +\frac{1}{\varepsilon_0} \frac{\partial^2 H_z}{\partial x^2}$$, oziroma:

VALOVNA ENAČBA

$$\boxed{\frac{\partial^2 H_z}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 H_z}{\partial x^2}} \qquad (3)$$

2. $$\frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \quad / \frac{\partial}{\partial x} \quad \Rightarrow \quad \frac{\partial^2 E_y}{\partial x^2} = -\mu_0 \frac{\partial^2 H_z}{\partial t \partial x} \qquad (4)$$

$$-\frac{\partial H_z}{\partial x} = \varepsilon_0 \frac{\partial E_y}{\partial t} \quad / \frac{\partial}{\partial t} \quad \Rightarrow \quad -\frac{\partial^2 H_z}{\partial t \partial x} = \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \qquad (5)$$

torej:

$$(4), \quad (5) \quad \Rightarrow \quad -\frac{1}{\mu_0} \frac{\partial^2 E_y}{\partial x^2} = -\varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$, oziroma:

VALOVNA ENAČBA

$$\boxed{\frac{1}{\mu_0 \varepsilon_0}\frac{\partial^2 E_y}{\partial x^2} = \frac{\partial^2 E_y}{\partial t^2}} \qquad (6)$$

Reševanje valovnih enačb (3) in (6).

Nastavek za rešitev enačbe (6): $E_y = E_{y0}\cos(\omega t - kx)$. (7)

$$\frac{\partial^2 E_y}{\partial t^2} = -\omega^2 \cos(\omega t - kx),$$

$$\frac{\partial^2 E_y}{\partial x^2} = -k^2 \cos(\omega t - kx), \text{ torej:}$$

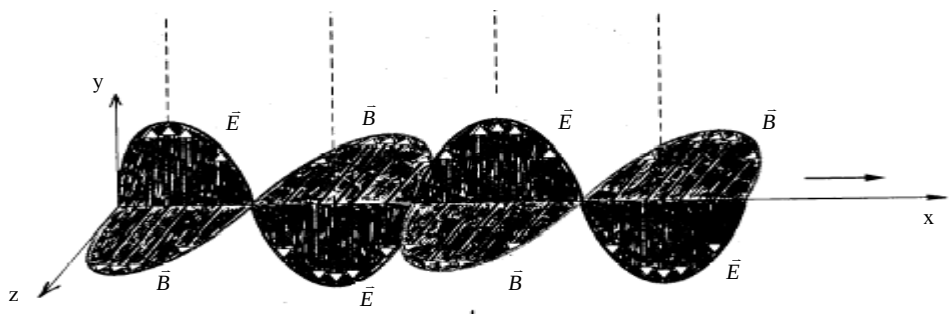
$$-\omega^2 \cos(\omega t - kx) = \frac{-1}{\varepsilon_0 \mu_0} k^2 \cos(\omega t - kx), \text{ oziroma}$$

$$\omega^2 = \frac{1}{\varepsilon_0 \mu_0} k^2 . \qquad (8)$$

Ker velja $k = \frac{\omega}{c_o}$ iz enačbe (8) sledi: $\boxed{c_0^2 = \frac{1}{\varepsilon_0 \mu_0}}$ hitrost širjenja EM valovanja v vakuumu.

Influenčna konstanta: $\varepsilon_0 = 8.85 \cdot 10^{-12}$ As/Vm
Indukcijska konstanta: $\mu_0 = 4\pi \cdot 10^{-7}$ Vs/Am

$\} \Rightarrow c_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \cong 3 \cdot 10^8 \frac{\text{m}}{\text{s}}$



primer: raven elektromagnetni val

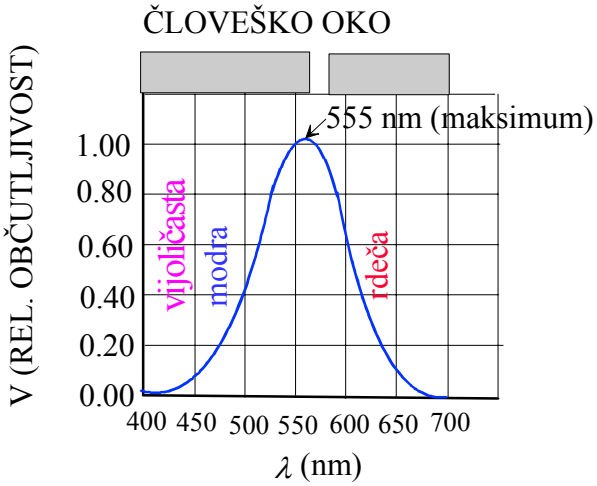
VRSTA VALOV	VALOVNE DOLŽINE (λ)
Radijski valovi	od 1 m do več 1000 km
mikrovalovi	od 0.1 mm do ~ 1 m
infra rdeče sevanje	od ~ 780 nm do 0.1 mm
vidna svetloba	od 380 do 780 nm
UV svetloba	od 5 ~ 380 nm
Rtg žarki	od 1 pm ~ 10 nm
γ žarki	od 0 do 1 pm

- **SPEKTER ELEKTROMAGNETNEGA VALOVANJA**

Fiziološka enota za $jS = P$

1W=680 lm pri $\lambda = 555\,\text{nm}$

$$\frac{680\,\text{lm}\cdot V(\lambda)}{1\,\text{W}}$$



- **Elektromagnetno valovanje v snovi – lomni količnik**

$$\boxed{c = \frac{1}{\sqrt{\varepsilon\,\mu\,\varepsilon_0\,\mu_0}}}$$

Snovi, po katerih se širi elektromagnetno valovanje, so diamagnetne ali paramagnetne, ki imajo $\mu \approx 1$:

$$\boxed{c = \frac{1}{\sqrt{\varepsilon\ \ \varepsilon_0\ \ \mu_0}} = \frac{c_0}{n}} \quad . \qquad (9)$$

Definiramo lomni kvocient:

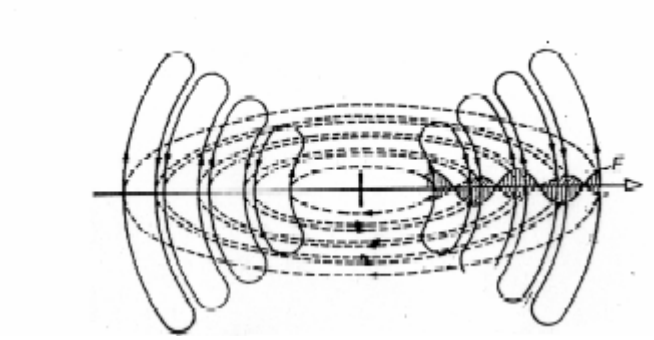
$$\boxed{n = \sqrt{\varepsilon}} \tag{10}$$

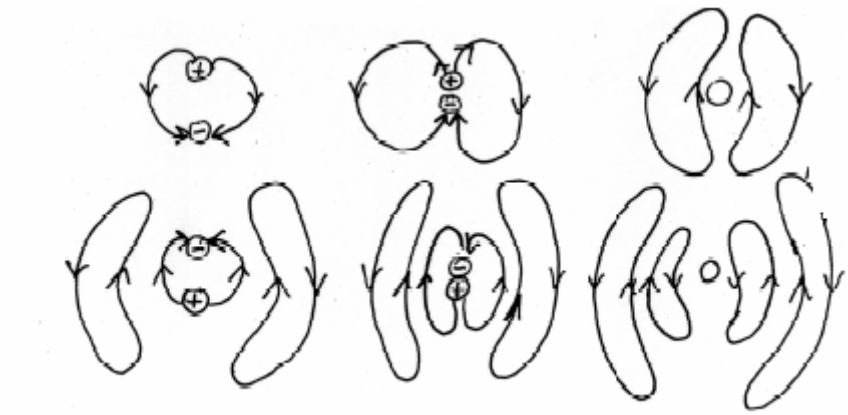
$n = \sqrt{\varepsilon}$ je odvisen od valovne dolžine λ

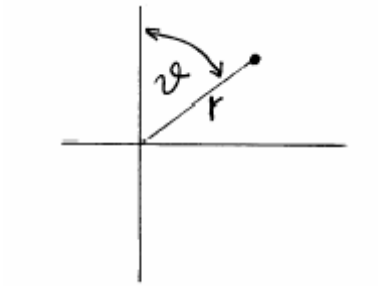
Iz tega sledi, **da je tudi hitrost EM valovanja** (svetlobe) odvisna od valovne dolžine.

$$\boxed{c = \frac{c_0}{n(\lambda)}} \tag{11}$$

SEVANJE DIPOLA (iz knjige J. Strnad: Fizika II)







$$j = c\,\overline{w} = \frac{r_0\,\omega}{32\pi^2\,\varepsilon_0\,c_0^3}\cdot\frac{\sin\vartheta}{r^2}$$

$$P = \int j\,\mathrm{d}S = \frac{p_0^2\,\omega^4}{12\pi\,\varepsilon_0\,c_0^3}$$

Gostota energijskega toka v elektromagnetnem valovanju

$$\boxed{\overline{j} = \overline{w}\, c_0} \tag{12}$$

Gostota energije

$$\boxed{\begin{aligned} w_E &= \frac{1}{2}\varepsilon_0 E^2 \\ w_B &= \frac{1}{2}\mu_0 H^2 = \frac{1}{2}\frac{B^2}{\mu_0} \end{aligned}}$$

$$\boxed{w = w_E + w_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}} \tag{13}$$

$$\boxed{\vec{E} = \vec{B} \times \vec{c}_0}$$

$$\left.\begin{aligned} \text{če:}\quad E &= E_0 \cos(\omega t - kx) \\ B &= B_0 \cos(\omega t - kx) \end{aligned}\right\} \Rightarrow \boxed{E_0 = B_0\, c_0}$$

$$\overline{w} = \frac{1}{2}\varepsilon_0 E_0^2\, \overline{\cos^2(\omega t - kx)} + \frac{1}{2\mu_0} B_0^2\, \overline{\cos^2(\omega t - kx)} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2 =$$

$$= \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4\mu_0}\frac{E_0^2}{c_0^2} \quad \text{torej:}$$

$$\overline{w} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2\mu_0} B_0^2, \tag{14}$$

kjer smo upoštevali:

$$\overline{\cos^2(\omega t - kx)} = \frac{1}{2}$$

$$\overline{\cos^2(\omega t - kx)} = \frac{1}{2}$$

$$B_0 = \frac{E_0}{c_0}$$

$$c_0^2 = \frac{1}{\varepsilon_0 \mu_0}$$

- torej:

$$\boxed{\bar{j} = \bar{w} c_0 = \frac{1}{2} \varepsilon_0 E_0^2 c_0 = \frac{1}{2} \frac{B_0^2}{\mu_0} c_0} \qquad (15)$$

- če antena oddaja izotropno:

$$\boxed{\bar{j} = \frac{P}{4\pi r^2} \propto \frac{1}{r^2}} \qquad (16)$$

- Poyntingov vektor:

$$\boxed{\vec{P} = \vec{E} \times \vec{H}} \qquad (17)$$

podaja:

- velikost gostote energijskega toka
- smer razširjanja valovanja

Dokaz prve trditve:

če $E = E_0 \cos(\omega t - kx)$, $H = H_0 \cos(\omega t - kx)$ in $B_0 = \mu_0 H_0$ velja:

$$\overline{|\vec{P}|} = E_0 H_0 \overline{\cos^2 (\omega t - kx)} = \frac{1}{2} E_0 H_0 = \frac{1}{2} E_0 \frac{B_0}{\mu_0} = \frac{1}{2} E_0 \frac{E_0}{\mu_0 c_0} \cdot \frac{c_0}{c_0} = \frac{1}{2} E_0 \frac{E_0}{\mu_0 c_0^2} c_0 =$$

$$= \frac{1}{2} E_0 \frac{\varepsilon_0 \mu_0}{\mu_0} E_0 c_0 = \frac{1}{2} \varepsilon_0 E_0^2 c_0 .$$

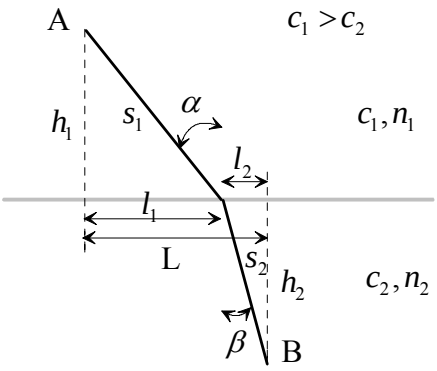
kjer smo upoštevali:

$$B_0 = \frac{E_0}{c_0} \quad \text{in} \quad c_0^2 = \frac{1}{\varepsilon_0 \mu_0} .$$

Torej:

$$\boxed{\bar{j} = \frac{1}{2} \varepsilon_0 E_0^2 c_0}$$

Lom: Fermatov princip
(minimalen čas za pot svetlobe iz točke A v točko B)



$$t = \frac{s_1}{c_1} + \frac{s_2}{c_2} \Rightarrow$$

$$t = \frac{h_1}{c_1 \cos\alpha} + \frac{h_2}{c_2 \cos\beta},$$ kjer smo upoštevali:

$$\cos\alpha = \frac{h_1}{s_1},\ \cos\beta = \frac{h_2}{s_2}$$

$$tg\,\alpha = \frac{l_1}{h_1},\ tg\,\beta = \frac{l_2}{h_2}$$

$$\mathrm{d}t = \frac{\partial t}{\partial \alpha}\mathrm{d}\alpha + \frac{\partial t}{\partial \beta}\mathrm{d}\beta = 0 \quad \text{(pogoj za ekstrem)}$$

$$\mathrm{d}t = \frac{h_1 \cdot \sin\alpha}{c_1 \cdot \cos^2\alpha}\mathrm{d}\alpha + \frac{h_2 \cdot \sin\beta}{c_2 \cdot \cos^2\beta}\mathrm{d}\beta = 0, \text{torej:}$$

$$\frac{d\beta}{d\alpha} = \frac{c_2 h_1}{c_1 h_2} \cdot \frac{\sin\alpha}{\sin\beta}\left(\frac{\cos\beta}{\cos\alpha}\right)^2 \qquad (1)$$

$$L = h_1 \cdot tg\alpha + h_2 \cdot tg\beta$$

$$dL = h_1 \frac{1}{\cos^2\alpha} d\alpha + h_2 \frac{1}{\cos^2\beta} d\beta = 0, \text{ker je L konstanta}$$

Torej:

$$\frac{d\beta}{d\alpha} = -\frac{h_1}{h_2}\left(\frac{\cos\beta}{\cos\alpha}\right)^2 \qquad (2)$$

Upoštevamo enačbi (1) in (2);

$$\frac{d\beta}{d\alpha} = \frac{d\beta}{d\alpha}:$$

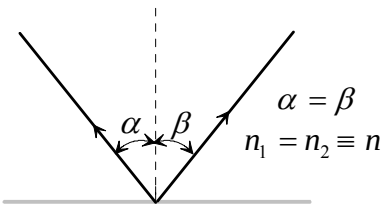
$$-\frac{c_2}{c_1} \cdot \frac{h_1}{h_2} \cdot \frac{\sin\alpha}{\sin\beta}\left(\frac{\cos\beta}{\cos\alpha}\right)^2 = -\frac{h_1}{h_2}\left(\frac{\cos\beta}{\cos\alpha}\right)^2,$$

torej: $\frac{\sin\alpha}{\sin\beta} = \frac{c_1}{c_2}$;

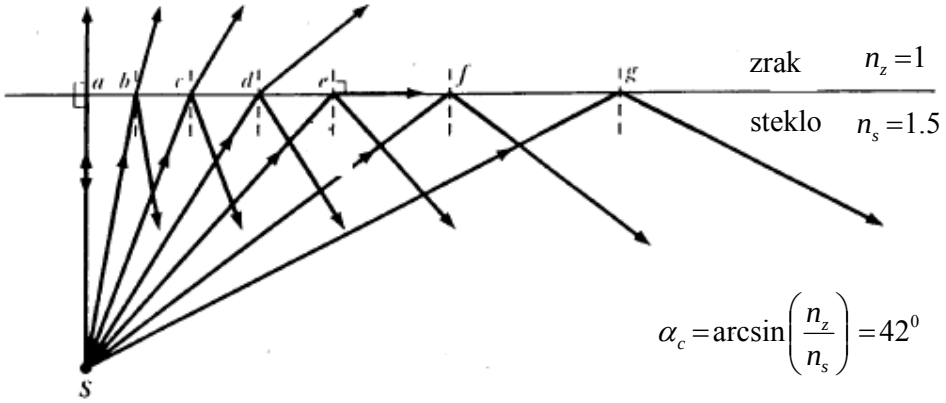
oziroma:

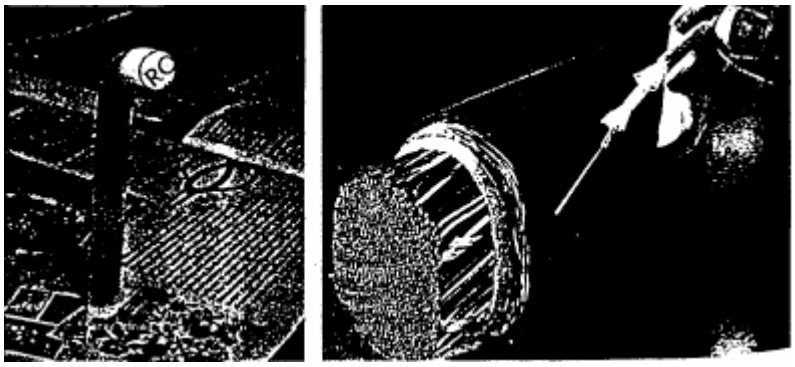
$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}$, kjer smo upoštevali $c_1 = \frac{c_0}{n_1}$, $c_2 = \frac{c_0}{n_2}$

Odboj



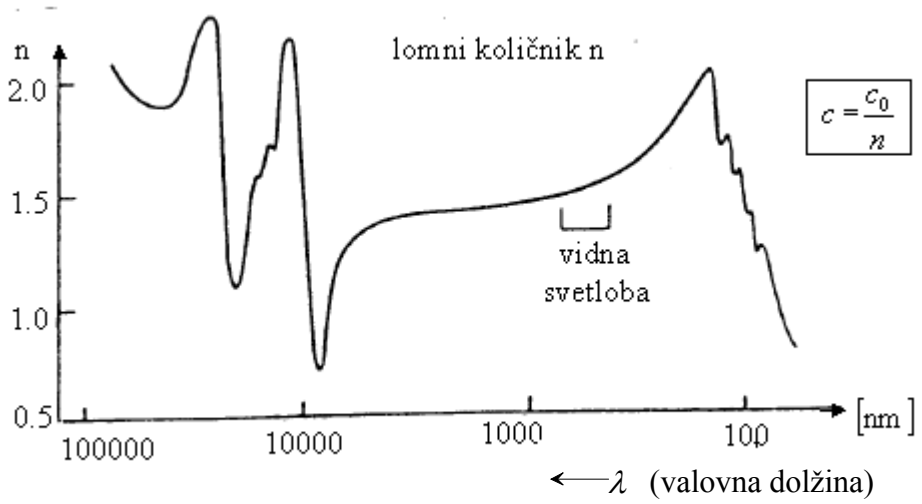
Totalni odboj





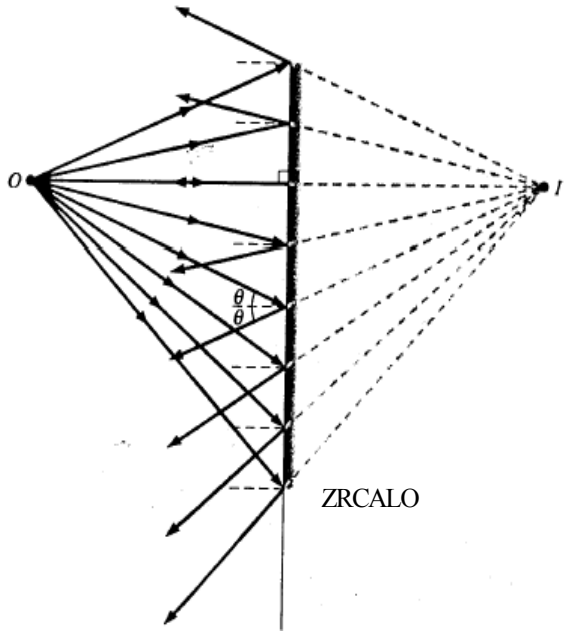
optična vlakna

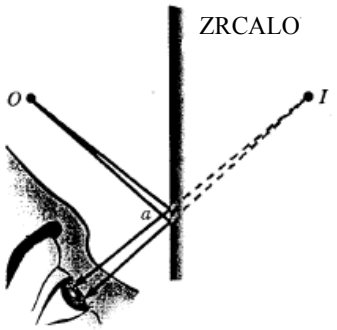
Disperzija



Disperzijska krivulja za neobravano steklo (J. Strnad, Fizika II)

ZRCALO





Slike: - Serway: Principles of Physics
- Halliday et al.: Fundamentals of Physics

Koherentnost izvorov EM valovanj in interferenca

- povprečna gostota energijskega toka v vakuumu:

$$\overline{j} = \overline{w} c_0 = \left(\overline{w}_E + \overline{w}_B\right) c_0 = \varepsilon_0 \, \overline{E^2} \, c_0 \tag{1}$$

- **eno** ravno EM valovanje: $E = E_0 \cos\left(\omega t - kx\right)$

$$\boxed{\overline{j} = \overline{w} c_0 = \varepsilon_0 \, \overline{E_0^2} \, c_0 = \varepsilon_0 \, E_0^2 \, \overline{\cos^2\left(\omega t - kx\right)} c_0 = \frac{1}{2} \varepsilon_0 \, E_0^2 \, c_0 \ ,}$$

kjer smo upoštevali $\overline{\cos^2\left(\omega t - kx\right)} = \frac{1}{2}$.

- **dve** ravni EM valovanji (fazna razlika δ):

V izbrani točki prostora x_0 uporabimo načelo superpozicije:

$$E_1 = E_{01} \cos\left(\omega t - k\, x_0 - \frac{1}{2}\delta\right)$$

$$E_2 = E_{02} \cos\left(\omega t - k\, x_0 + \frac{1}{2}\delta\right)$$

$$E = E_1 + E_2$$

torej:

$$\left(E_1 + E_2\right)^2 = \left[E_{01} \cos\left(\omega t - k\, x_0 - \frac{1}{2}\delta\right) + E_{02} \cos\left(\omega t - k\, x_0 + \frac{1}{2}\delta\right)\right]^2 =$$

$$= E_{01}^2 \cos^2\left(\omega t - k\, x_0 - \frac{1}{2}\delta\right) + 2\, E_{01}\, E_{02} \cos\left(\omega t - k\, x_0 - \frac{1}{2}\delta\right) \cos\left(\omega t - k\, x_0 + \frac{1}{2}\delta\right) + \tag{2}$$

$$+ E_{02}^2 \cos^2\left(\omega t - k\, x_0 + \frac{1}{2}\delta\right)$$

Poglejmo mešani člen v enačbi (2):

$$2\, E_{01} E_{02} \cos\left(\omega t - kx_0 - \frac{1}{2}\delta\right) \cos\left(\omega t - kx_0 + \frac{1}{2}\delta\right) =$$

$$= 2\, E_{01} E_{02} \left[\cos\left(\omega t - kx_0\right) \cos\left(\frac{1}{2}\delta\right) + \sin\left(\omega t - kx_0\right) \sin\left(\frac{1}{2}\delta\right)\right] \cdot$$

$$\cdot \left[\cos\left(\omega t - kx_0\right) \cos\left(\frac{1}{2}\delta\right) - \sin\left(\omega t - kx_0\right) \sin\left(\frac{1}{2}\delta\right)\right] =$$

$$= 2\, E_{01} E_{02} \left[\cos^2\left(\omega t - kx_0\right) \cos^2\left(\frac{1}{2}\delta\right) - \sin^2\left(\omega t - kx_0\right) \sin^2\left(\frac{1}{2}\delta\right)\right]. \tag{3}$$

Iz enačbe (2) in (3) sledi:

$$\overline{(E_1+E_2)^2} = E_{01}^2\,\overline{\cos^2\left(\omega t - k\,x_0 - \frac{1}{2}\delta\right)} + E_{02}^2\,\overline{\cos^2\left(\omega t - k\,x_0 + \frac{1}{2}\delta\right)} +$$
$$+2\,E_{01}E_{02}\left[\overline{\cos^2(\omega t - kx_0)}\;\overline{\cos^2\left(\frac{1}{2}\delta\right)} - \overline{\sin^2(\omega t - kx_0)}\,\overline{\sin^2\left(\frac{1}{2}\delta\right)}\right]. \tag{4}$$

Če upoštevamo

$$\overline{\cos^2 z} = \frac{1}{2} \quad \text{in} \quad \overline{\sin^2 z} = \frac{1}{2}$$

Iz enačbe (4) sledi:

$$\overline{(E_1+E_2)^2} = \frac{1}{2}E_{01}^2 + \frac{1}{2}E_{02}^2 + E_{01}E_{02}\left[\overline{\cos^2\left(\frac{1}{2}\delta\right)} - \overline{\sin^2\left(\frac{1}{2}\delta\right)}\right],$$

oziroma

$$\overline{(E_1+E_2)^2} = \frac{1}{2}E_{01}^2 + \frac{1}{2}E_{02}^2 + E_{01}E_{02}\,\overline{\cos\delta}\,, \tag{5}$$

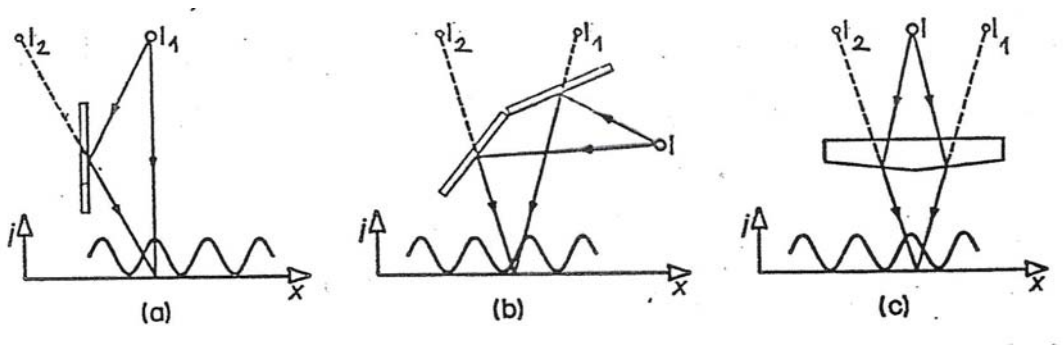
Torej:

$$\boxed{\overline{j} = \overline{w}c_0 = \varepsilon_0\,\overline{(E_1+E_2)^2}\,c_0 = \frac{1}{2}\varepsilon_0\,E_{01}^2\,c_0 + \frac{1}{2}\varepsilon_0\,E_{02}^2\,c_0 + \varepsilon_0\,E_{01}\,E_{02}c_0\,\overline{\cos\delta}.} \tag{6}$$

Zaključek:

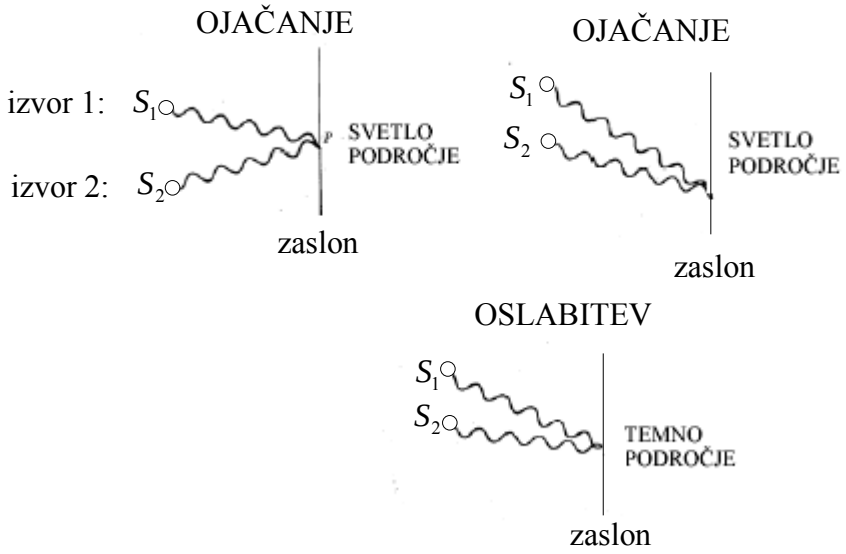
- nekoherentna izvora, če $\delta = \delta(t)$ naključen $\Rightarrow$ $\overline{\cos\delta(t)} = 0$ $\Rightarrow$ ni interference
- koherentna izvora, če δ stalna $\Rightarrow$ $\overline{\cos\delta(t)} \neq 0$ $\Rightarrow$ je interferenca (glejte slike na naslednji strani)

Primer:



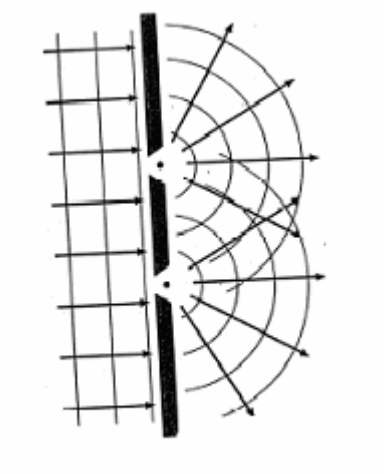
Nekaj naprav s katerimi dobimo navidezna izvira koherentnih delnih valovanj: Lloydovo zrcalo (a), Fresnelovi nagnjeni zrcali (b) in Fresnelova biprizma (c). (Iz knjige J. Strnad, Fizika II)

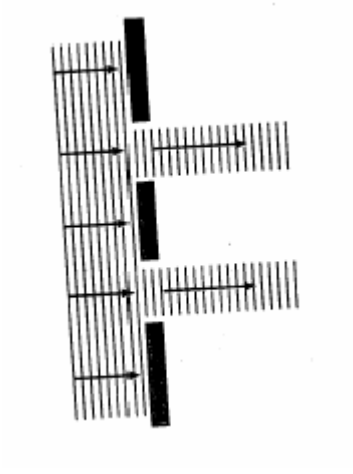
- **Interferenca EM valovanj** (2 koherentna izvora)



Uklon svetlobe, interferenca

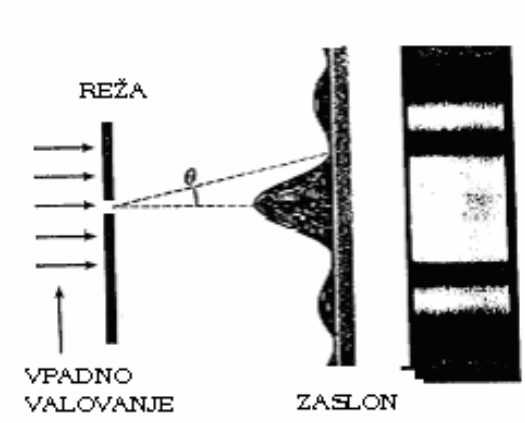
Young-ov poskus → uklon





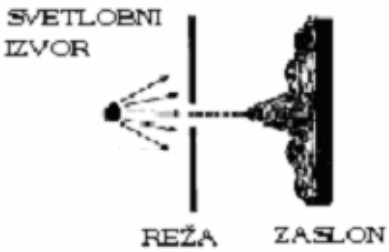
Huygensovo načelo: vsako točko valovne fronte obravnavamo kot izvor krogelnih (krožnih) valov)

Fraunhofer-jev uklon



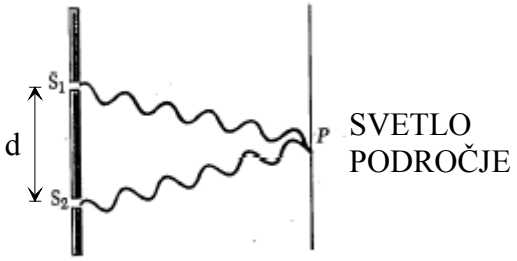
Zaslon je zelo oddaljen.

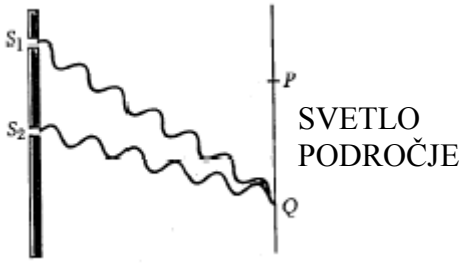
Fresnel- ov uklon

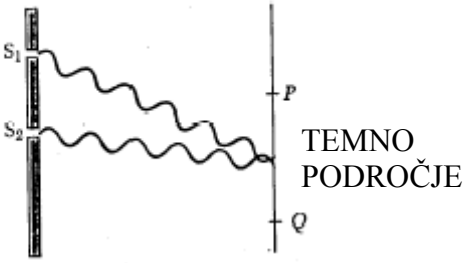


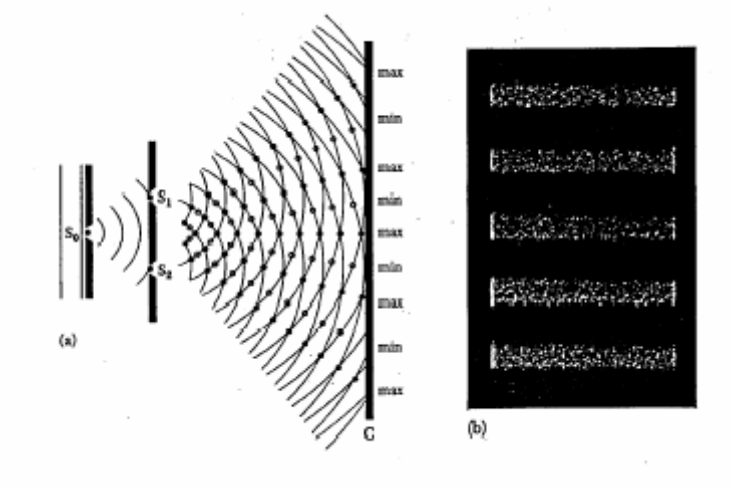
Zaslon je na končni razdalji (delna valovanja, ki interferirajo na izbranem delu zaslona ne smemo imeti za vzporedno).

Youngov poskus – interferenca:



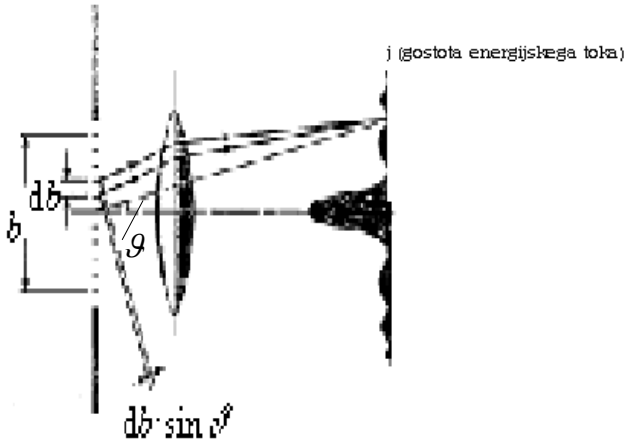


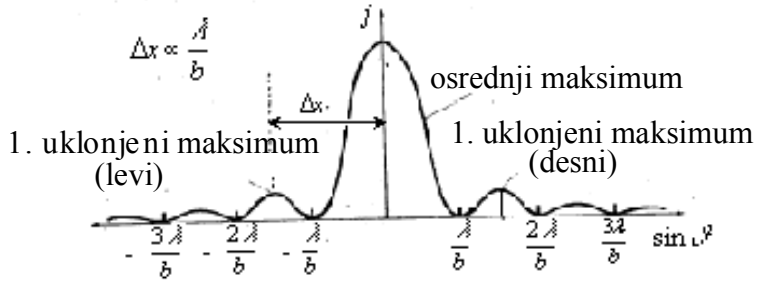




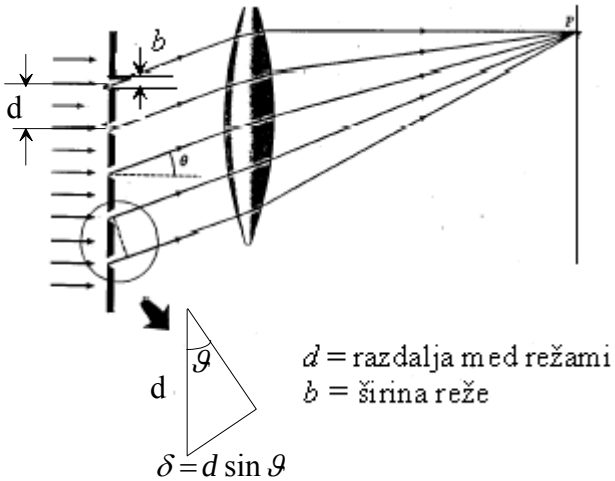
Uklon in interferenca na reži s širino b:

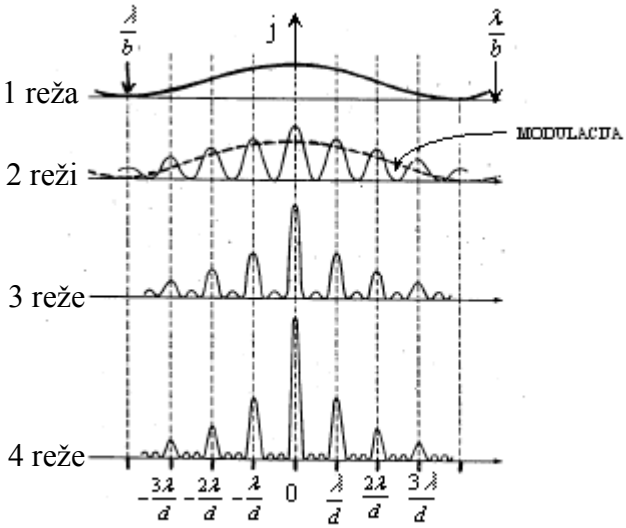
$$j = j_0 \frac{\sin^2(\pi b \sin\vartheta/\lambda)}{(\pi b \sin\vartheta/\lambda)^2}$$



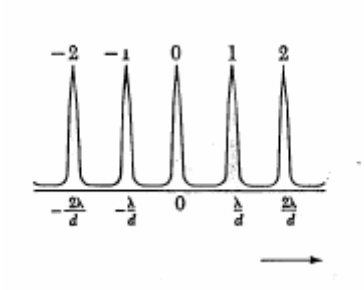


Uklon in interferenca na sistemu rež v enakih razmikih:





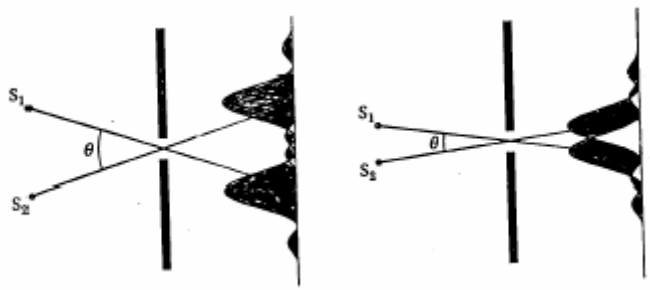
Slika uklonske mrežice

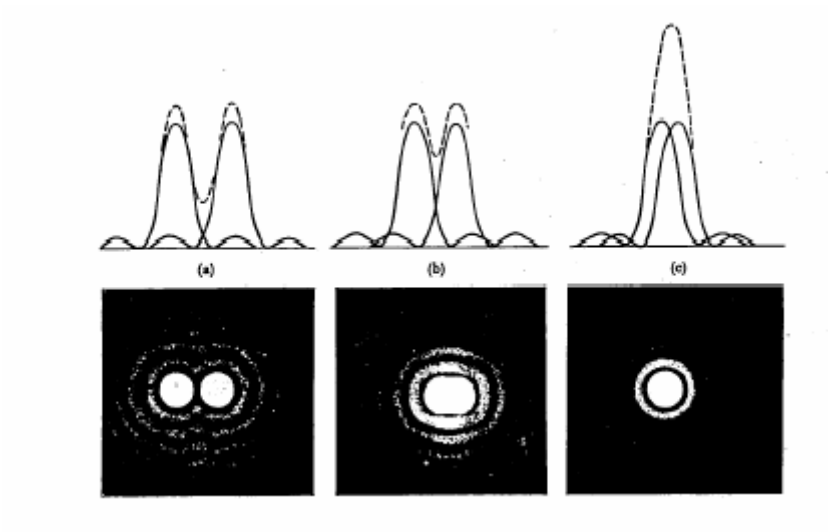


pogoj za ojačanje:

$d \sin \vartheta = \mathrm{N}\lambda, \quad \mathrm{N} = 0, 1, 2, 3, ...$

Ločljivost (zaradi uklona)



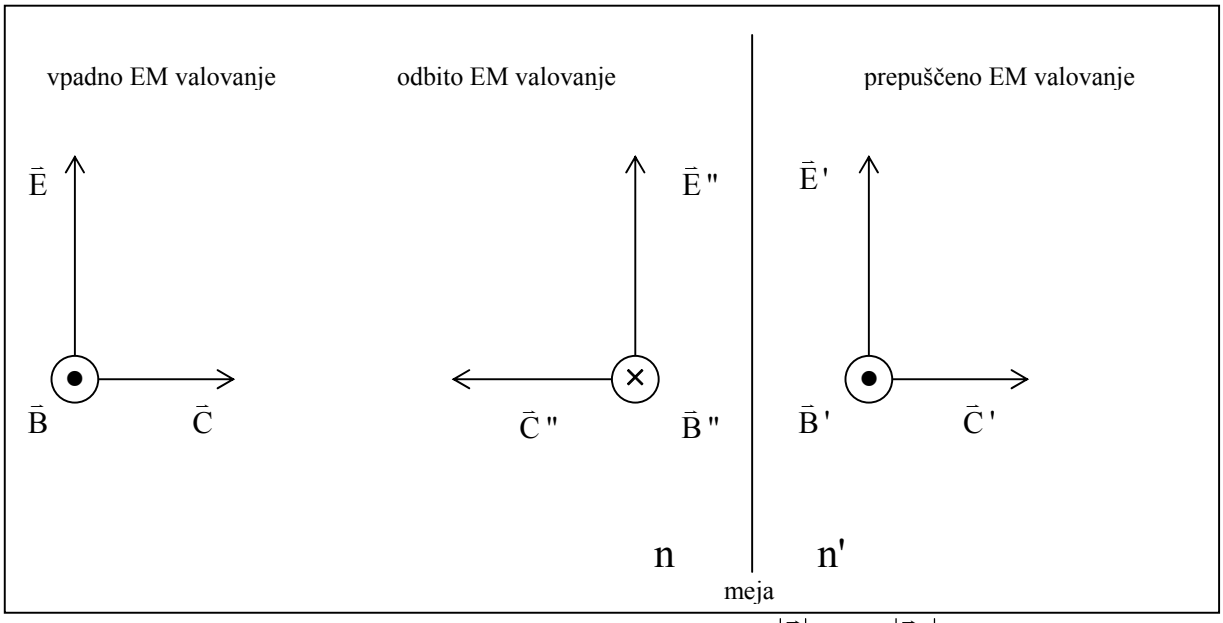


slike: J Strnad: Fizika II

Sprememba faze EM valovanja pri odboju

Študirali bomo odboj EM valovanja na optično redkejši snovi (n' < n) in odboj EM valovanja na optično gostejši snovi (n' > n), kjer sta $n = \sqrt{\varepsilon}$ in $n' = \sqrt{\varepsilon'}$ lomna količnika, ε in ε' pa ustrezni dielektrični konstanti. Upoštevamo $\mu \cong 1$.

E , B	→	vpadno EM valovanje
E", B"	→	odbito EM valovanje
E' , B'	→	prepuščeno EM valovanje



Slika: Pravokotni vpad EM valovanja na mejo dveh sredstev, $|\vec{C}| = C = |\vec{C}''| = C''$.

Velja tudi

$$C' = \frac{C_0}{n'}, \tag{1}$$

$$C = C'' = \frac{C_0}{n}, \tag{2}$$

kjer je C_0 hitrost EM valovanja v vakuumu. Pri določanju robnih pogojev na meji dveh sredstev uporabimo dve izmed štirih Maxwell-ovih enačb v obliki:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{S} = 0, \tag{3}$$

kjer $\vec{B} \perp d\vec{S}$ in

$$\int \vec{H} \cdot d\vec{s} = \frac{d}{dt}\int_S \vec{D} \cdot d\vec{S} = 0, \tag{4}$$

kjer $\vec{D} \perp d\vec{S}$.

Iz enačbe (3) sledi (robni pogoj):

$$E + E'' = E' . \tag{5}$$

Iz enačbe (4) pa sledi (robni pogoj):

$$H - H'' = H' \tag{6}$$

Iz

$$\begin{aligned} B &= \mu_0 H, \\ B' &= \mu_0 H', \\ B'' &= \mu_0 H'', \end{aligned} \tag{7}$$

kjer smo upoštevali $\mu = \mu' = \mu'' \cong 1$, in iz enačbe (6) sledi

$$B - B'' = B' \tag{8}$$

Za EM valovanje velja $\vec{E} = \vec{B} \times \vec{C}$, torej

$$\begin{aligned} E &= B\,C, \\ E' &= B'\,C', \\ E'' &= B''C''. \end{aligned} \tag{9}$$

Iz enačb (8) in (9) tako sledi

$$\frac{E}{C} - \frac{E''}{C} = \frac{E'}{C'}, \tag{10}$$

od tod pa

$$E - E'' = E'\frac{C}{C'}. \tag{11}$$

Ob upoštevanju enačb (1) in (2) iz enačbe (11) sledi

$$E - E'' = E'\left(\frac{n'}{n}\right), \tag{12}$$

Enačbi (5) in (12) predstavljata sistem dveh enačb za dve neznanki E' in E'' (pri znani vrednosti E, n in n'). Iz enačbe (5) izrazimo E'':

$$E'' = E' - E \tag{13}$$

in vstavimo v enačbo (12). Tako dobimo:

$$E - E' + E = E'\left(\frac{n'}{n}\right),$$

od tod pa sledi

$$\boxed{E' = \frac{2n}{(n+n')} E \ .} \qquad (14)$$

Iz enačb (13) in (14) pa dobimo:

$$\boxed{E'' = \left(\frac{n-n'}{n+n'}\right) E \ .} \qquad (15)$$

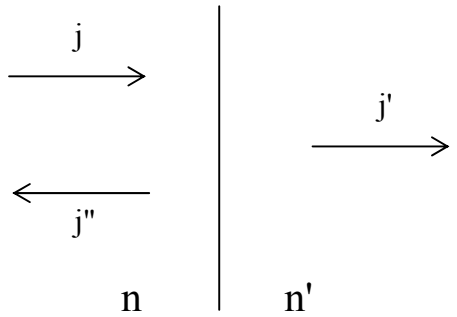
Na osnovi enačbe (15) torej sklepamo, da*:

1. se EM valovanje na optično redkejši snovi (n' < n) odbije z enako fazo (če E > 0 ⇒ E'' > 0)

in

2. se EM valovanje na optično gostejši snovi (n' > n) odbije z nasprotno fazo (če E > 0 ⇒ E'' < 0).

Izračunajmo še odbojnost (a) in prepustnost (b).



odbojnost (albedo) $\quad a = \frac{j''}{j}$, (16)

prepustnost $\quad b = \frac{j'}{j}$. (17)

---

* glejte še knjgo J. Strnad: *Fizika (drugi del)*, str. 471, 472.

Ob upoštevanju $\overline{j} = \overline{w}c = \frac{1}{2}\varepsilon\varepsilon_0 E_0^2 c$ iz enačb (14), (15), (16) in (17) sledi:

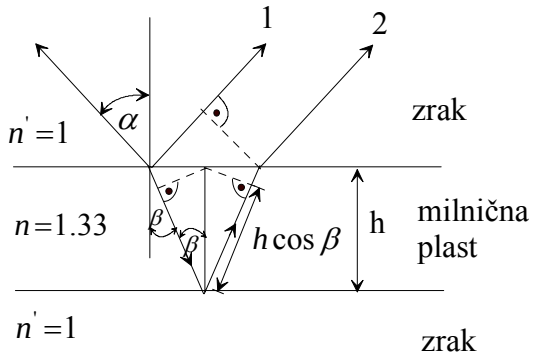
$$b = \frac{j'}{j} = \frac{\frac{1}{2}\varepsilon'\varepsilon_0 E'^2_0 c'}{\frac{1}{2}\varepsilon\varepsilon_0 E_0^2 c} = \frac{\varepsilon'}{\varepsilon}\cdot\frac{c'}{c}\left(\frac{E'_0}{E_0}\right)^2 = \frac{n'^2}{n^2}\cdot\frac{n}{n'}\left(\frac{E'_0}{E_0}\right)^2 = \frac{n'}{n}\left(\frac{E'_0}{E_0}\right)^2 = \frac{4nn'}{(n+n')^2} \quad , \tag{18}$$

$$a = \frac{j''}{j} = \frac{(n-n')^2}{(n+n')^2}, \tag{19}$$

kjer smo upoštevali tudi enačbe (1) in (2) ter $n = \sqrt{\varepsilon}$ in $n' = \sqrt{\varepsilon'}$.

Interferenca (odboj) na tankih plasteh

A) Interferenca na milnični opni



V milnični plasti je valovna dolžina svetlobe enaka $\lambda = \lambda_0 / n$. Razlika v poti (δ) med žarkom 1 in 2 je približno (glejte sliko):

$$\delta = 2h\cos\beta + \frac{\lambda}{2},$$

kjer smo z $\lambda/2$ upoštevali, da se žarek 1 odbije na optično gostejši snovi (n' > n). Pogoj za ojačanje zapišemo v obliki:

$$2h\cos\beta + \frac{\lambda}{2} = N\lambda,$$

kjer je $N = 1, 2, \ldots$ Upoštevamo $\lambda = \lambda_0 / n$ in dobimo:

$$2h\cos\beta + \frac{\lambda_0}{2n} = N\frac{\lambda_0}{2n},$$

od koder sledi:

$$\boxed{2nh\cos\beta = \frac{\lambda_0}{2n}(2N-1).} \qquad (1)$$

Ob upoštevanju lomnega zakona:

$$\frac{\sin\alpha}{\sin\beta} = n,$$

dobimo:

$$\frac{\sin^2\alpha}{\sin^2\beta} = \frac{\sin^2\alpha}{1-\cos^2\beta} = n^2 \ ,$$

od tod pa sledi:

$$\frac{1}{n^2}\sin^2\alpha \ = 1-\cos^2\beta \ ,$$

oziroma:

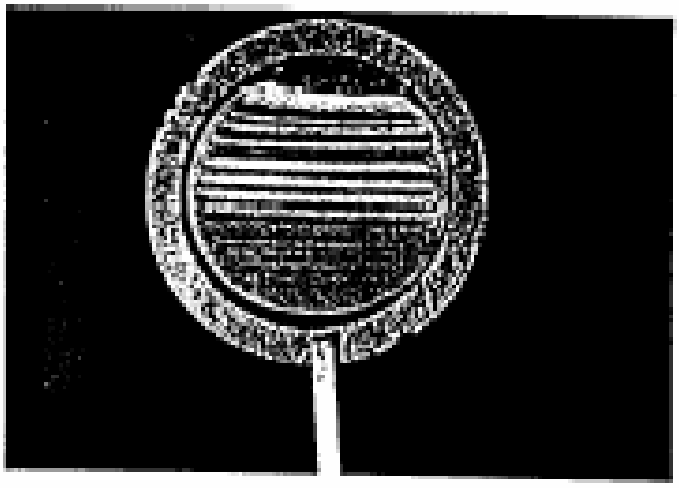
$$\cos\beta = \left(1-\frac{\sin^2\alpha}{n^2}\right)^{1/2} . \qquad (2)$$

Za N = 1 in ob upoštevanju enačbe (2) iz enačbe (1) sledi:

$$2nh\left(1-\frac{\sin^2\alpha}{n^2}\right)^{1/2} = \frac{\lambda_0}{2} \ , \qquad (3)$$

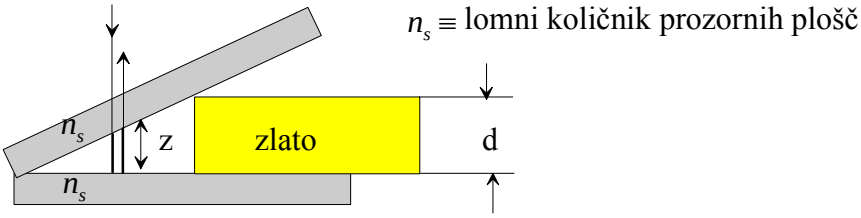
S pomočjo enačbe (3) izračunamo debelino milnične plasti:

$$h = \frac{\lambda_0}{4}\left(n^2-\sin^2\alpha\right)^{-1/2} \qquad (4)$$



Interference in a vertical soap film of variable thickness. The top of the film appears darkest where the film is thinnest. (© 1983 Larry Mulvehill, Photo Researchers)

B) Merjenje debeline zlatega lističa



Pogoj za ojačanje:

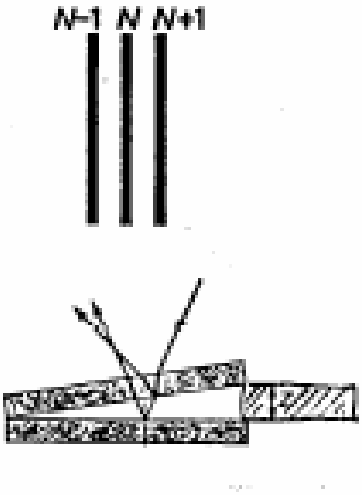
$$\delta = 2\,z + \frac{\lambda_0}{2} = N\,\lambda_0 \;,\;\; N = 1, 2, 3, ... \qquad (5)$$

Iz (5) sledi:

$$z + \frac{\lambda_0}{4} = N\,\frac{\lambda_0}{2} \;, \qquad (6)$$

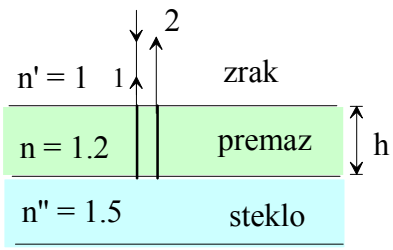
torej:

$$\boxed{z = \frac{1}{4}(2N-1)\lambda_0}$$



Iz knjige J. Strnad: Fizika II

C) Interferenčni filter



$\boxed{n'' > n > n'}$

Pogoj za oslabitev odbitih žarkov 1 in 2:

$$2h = N\frac{\lambda}{2}, \qquad N = 1,\ 3,\ 5 \tag{7}$$

kjer je

$$\lambda = \frac{\lambda_0}{n} \tag{8}$$

Torej:

$$\boxed{2h = N\frac{\lambda_0}{2n},} \qquad N = 1,\ 3,\ 5 \tag{9}$$

Oziroma:

$$2h = (2N+1)\frac{\lambda_0}{2n}, \quad N = 0,\ 1,\ 2. \tag{10}$$

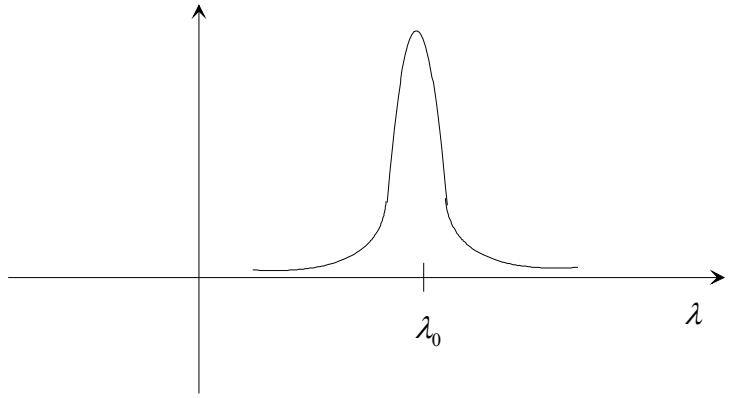
Debeline vrhnje plasti z lomnim količnikom n, ki ustrezajo oslabitvam odbite svetlobe:

$$\boxed{h = (2N+1)\frac{\lambda_0}{4n},} \qquad N = 0,\ 1,\ 2. \tag{11}$$

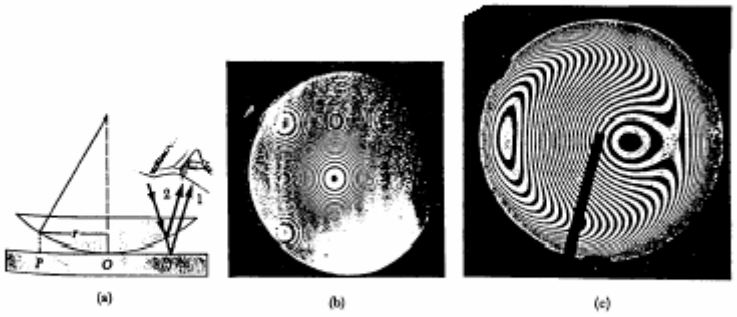
Najmanjša debelina je za N = 0:

$$\boxed{h_{\min} = \frac{\lambda_0}{4n},} \tag{12}$$

prepustnost interferenčnega filtra



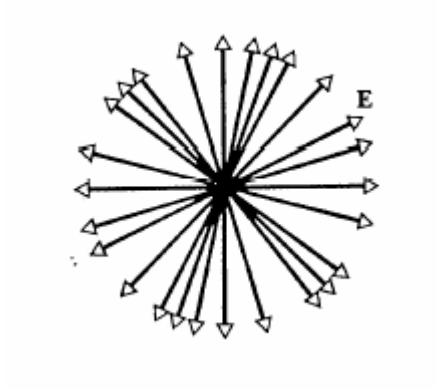
- Newtonovi kolobarji:



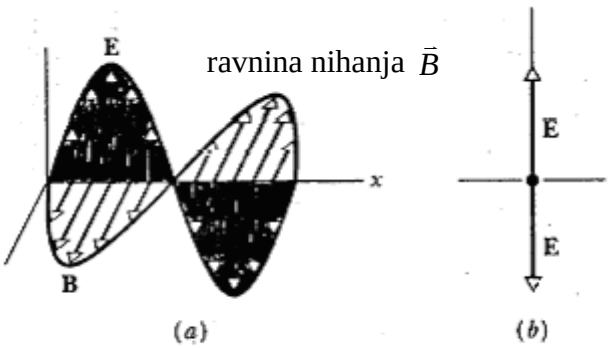
(a) The combination of rays reflected from the glass plate and the curved surface of the lens give rise to an interference pattern known as Newton's rings. (b) Photograph of Newton's rings. (Courtesy of Bausch and Lomb Optical Co.) (c) This asymmetrical interference pattern indicates imperfections in the lens. (From Physical Science Study Committee, *College Physics*, Lexington, Mass., Heath, 1968)

Polarizacija sončne svetlobe

- **Nepolarizirana svetloba (EM valovanje)**



- **Linearno polarizirano elektromagnetno valovanje (svetloba)**

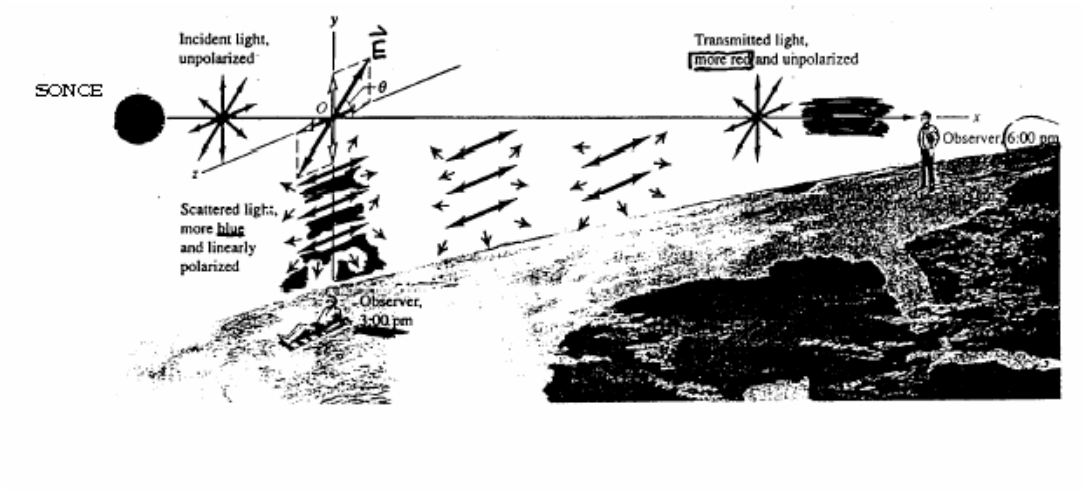


Polarizacija v mehaniki: Nepolarizirano (nv) in linearno polarizirano (lp) valovanje na prožni vrvici. P polarizator, A analizator. Za analitatorjem ni valovanja. Daljice kažejo smer gibanja delov vrvice. (J. Strnad, Fizika II)

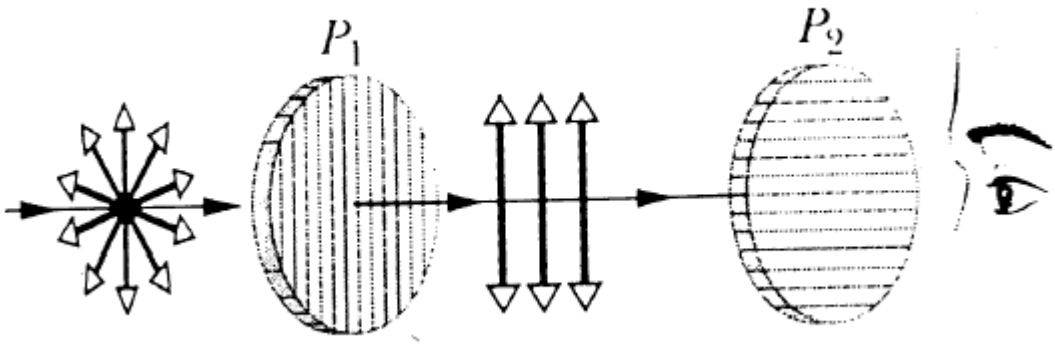
- **Sipanje sončne svetlobe**

Sipana svetloba (sevanje molekul kot dipolov)

$$\left.\begin{array}{l} j \propto \omega^4 \propto \dfrac{1}{\lambda^4} \\ j \propto \sin^2 \vartheta \end{array}\right\} \text{ gostota energijskega toka}$$

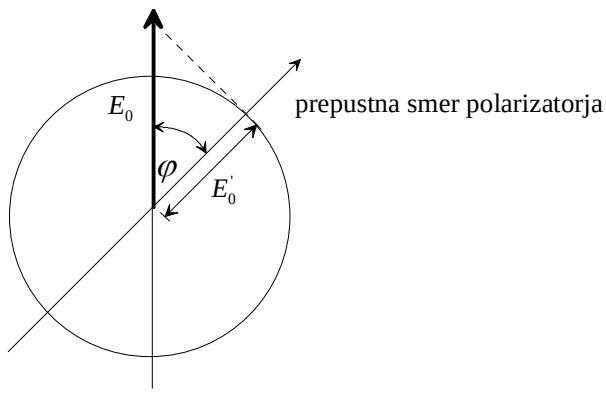


Prepustnost polarizatorja



popolnoma nepolarizirano EM valovanje

linearno polarizirano EM valovanje



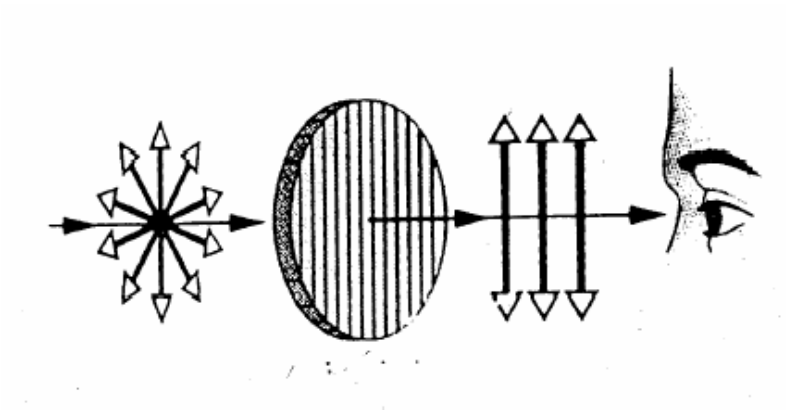
$$E_0^{'} = E_0 \cos\varphi$$

$$j_{prep} = \frac{1}{2}\varepsilon_0 E_0^{'2} c_0 = \frac{1}{2}\varepsilon_0 E_0^2 \left(\cos^2 \varphi\right) c_0$$

$$j_{vpad} = \frac{1}{2}\varepsilon_0 E_0^2 c_0$$

prepustnost polarizatorja $= \dfrac{j_{prep}}{j_{vpad}} = \cos^2 \varphi$

Primer: vpadno valovanje je popolnoma nepolarizirano

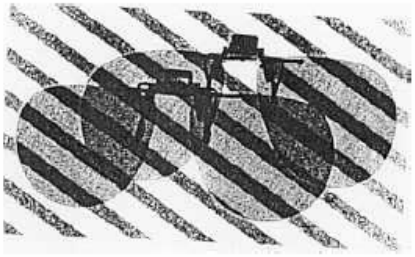


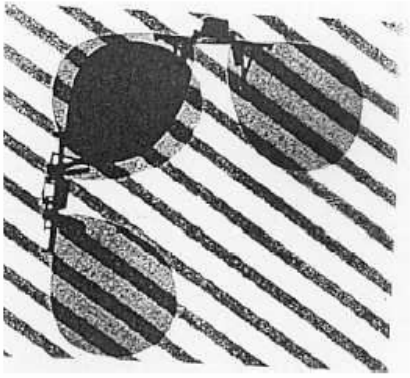
$$\bar{j}_{prep} = \frac{1}{2}\varepsilon_0 E_0^2 \overline{\cos^2 \varphi} c_0 = \frac{1}{2}\varepsilon_0 E_0^2 \frac{1}{2} c_0 ,$$

ker $\overline{\cos^2 \varphi} = \dfrac{1}{2}$

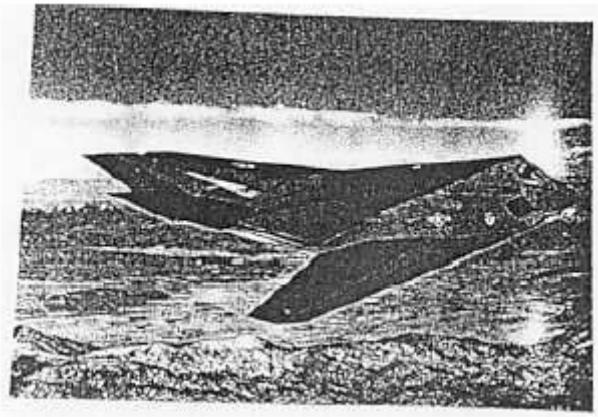
Torej: prepustnost je $\dfrac{1}{2}$.

- **Polarizacijska očala:**

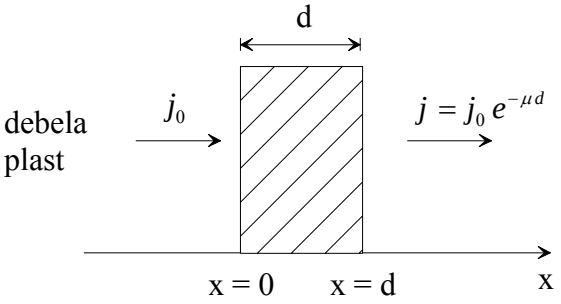




- **Odbojnost**

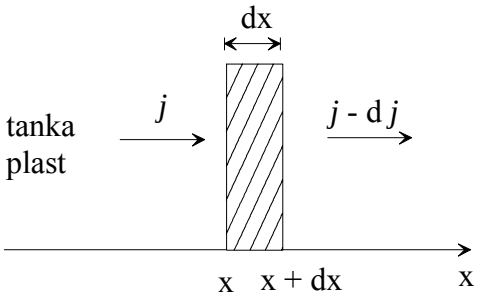


Absorpcija EM valovanja (svetlobe)



$\mu \equiv$ absorpcijski koeficient

Izpeljava:



$$\mathrm{d}j = -\mu\, j\, \mathrm{d}x, \tag{1}$$

$$\frac{\mathrm{d}j}{j} = -\mu\, \mathrm{d}x\,, \tag{2}$$

$$\int_{j_0}^{j(x)} \frac{\mathrm{d}j}{j} = -\mu \int_0^x \mathrm{d}x\,, \tag{3}$$

$$\ln j \Big|_{j_0}^{j(x)} = -\mu\, x\,, \tag{4}$$

$$\ln j(x) - \ln j_0 = -\mu\, x\,, \tag{5}$$

$$\ln\left(\frac{j(x)}{j_0}\right) = -\mu\, x\,, \tag{6}$$

$$\boxed{j(x) = j_0\, e^{-\mu x}} \tag{7}$$

Razpolovna debelina:

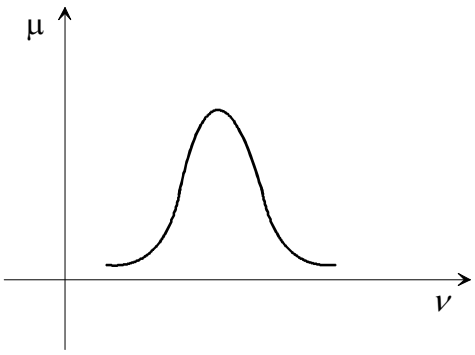
$$\frac{j_0}{2} = j_0\, e^{-\mu x_{1/2}} \ ,$$

$$\frac{1}{2} = e^{-\mu x_{1/2}} \ ,$$

$$\ln\left(\frac{1}{2}\right) = -\mu\, x_{1/2} \ ,$$

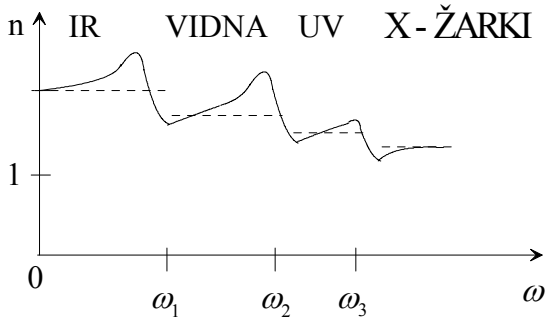
$$\boxed{x_{1/2} = \frac{\ln 2}{\mu}}$$

Najmočnejša absorpcija EM valovanja je pri frekvenci EM valovanja (ν), ki se ujema z eno izmed lastnih frekvenc:



Odvisnost lomnega količnika od valovne dolžine

$n = n(\lambda)$

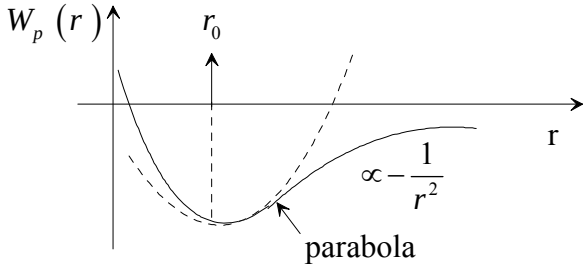


- **Primer: Razredčen vodikov (H) plin**

pozitivni del naboja: proton ($m_p \equiv$ masa protona)
negativni del naboja: elektron ($m_e \equiv$masa elektrona)

$$\boxed{m_e << m_p}$$

r $\equiv$ razdalja med protonom in nevtronom
$W_p \equiv$ potencialna energija med protonom in elektronom



$$\boxed{W_p \cong \frac{1}{2} K (r - r_0)^2 - C}$$

sila: $F = -\frac{\partial W_p}{\partial r} = -K(r - r_0)$

če $\boxed{r = r_0 + s}$

$$W_p = \frac{1}{2} K s^2 - C$$

$$\boxed{F = -Ks} \qquad (1)$$

Sistem opišemo v težiščnem sistemu, kjer je:

$$r_T = \frac{m_p\, r_p + m_e\, r_e}{\left(m_p + m_e\right)} = 0\,, \tag{2}$$

torej

$$m_p r_p + m_e r_e = 0\,. \tag{3}$$

Enačbo (3) odvajamo po času:

$$m_p \frac{\mathrm{d}r_p}{\mathrm{d}t} + m_e \frac{\mathrm{d}r_e}{\mathrm{d}t} = 0\,,$$

oziroma

$$m_p v_p + m_e v_e = 0\,. \tag{4}$$

kjer je $v_p = \frac{\mathrm{d}r_p}{\mathrm{d}t}$ hitrost protona in $v_e = \frac{\mathrm{d}r_e}{\mathrm{d}t}$ hitrost elektrona. Iz enačbe (4) sledi:

$$v_p = -\frac{m_e}{m_p} v_e\,. \tag{5}$$

Ker je $\frac{m_e}{m_p} << 1$ od tod sledi, da je v težiščnem sistemu protona in elektrona

$$\left|v_p\right| << \left|v_e\right| \tag{6}$$

Zato v prvem približku vzamemo, da proton miruje in zapišemo Newtonov zakon samo za elektron:

$$\boxed{m_e\, a = -Ks - e_0 E}\,, \tag{7}$$

kjer smo upoštevali $F = -Ks$ (enačba (1)).

$a = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$ $\equiv$ pospešek elektrona

$v = \frac{\mathrm{d}s}{\mathrm{d}t}$ $\equiv$ hitrost elektrona

E $\equiv$ jakost zunanjega električnega polja

Predpostavimo, da je jakost zunanjega električnega polja na mestu atomov posledica EM valovanja s krožno frekvenco ω, kjer je

$$E = E_0 \sin(\omega t) \,, \tag{8}$$

električno polje na mestu vodikovega atoma. Enačbo (8) vstavimo v enačbo (7) in dobimo:

$$\boxed{m_e \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} + K\, s + e_0\, E_0 \sin(\omega t) = 0} \tag{9}$$

Rešitev enačbe (9) iščemo z nastavkom:

$$s = s_0 \sin(\omega t + \delta) \;. \tag{10}$$

Enačbo (10) in

$$\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = -s_0 \omega^2 \sin(\omega t + \delta) \tag{11}$$

vstavimo v enačbo (9) in dobimo:

$$-m_e\, s_0\, \omega^2 \sin(\omega t + \delta) + K s_0 \sin(\omega t + \delta) = -e_0\, E_0 \sin(\omega t) \;. \tag{12}$$

V enačbo (12) vstavimo:

$$\sin(\omega t + \delta) = \sin(\omega t)\cos\delta \,+\, \cos(\omega t)\sin\delta$$

in dobimo:

$$\sin(\omega t)\left[\left(\omega_0^2 - \omega^2\right) s_0 \cos\delta + \frac{e_0\, E_0}{m_e}\right] + \left(\omega_0^2 - \omega^2\right) s_0 \sin\delta \cos(\omega t) = 0 \,, \tag{13}$$

kjer $\boxed{\omega_0^2 = \frac{K}{m_e}}$ lastna krožna frekvenca nihanja elektrona.

Enačba (13) mora veljati posebej za člen s $\sin(\omega t)$ in člen s $\cos(\omega t)$. Torej

$$\left(\omega_0^2 - \omega^2\right) s_0 \cos\delta + \frac{e_0\, E_0}{m_e} = 0 \,, \tag{14}$$

$$\sin\delta = 0, \quad \delta = 0, \pm\pi, \pm 2\pi \ldots . \tag{15}$$

Iz enačbe (14) sledi:

$$s_0 = \frac{e_0\, E_0}{m_e\left(\omega_0^2 - \omega^2\right)} \,, \tag{16}$$

kjer smo upoštevali $\delta = -\pi$. Izračunajmo še polarizacijo P_0:

$$P_0 = n\, e_0\, s_0 = \frac{n\, e_0^2\, E_0}{m_e\left(\omega_0^2 - \omega^2\right)} \cong \left(\varepsilon - 1\right)\varepsilon_0\, E_0 \ , \tag{17}$$

kjer smo upoštevali $\chi = \varepsilon - 1$, n pa je volumska gostota atomov. Iz enačbe (17) sledi:

$$\boxed{\varepsilon = 1 + \frac{n\, e_0^2}{m_e\, \varepsilon_0\left(\omega_0^2 - \omega^2\right)}} \ . \tag{18}$$

Iz zveze med lomnim količnikom in dielektrično konstanto ε dobimo:

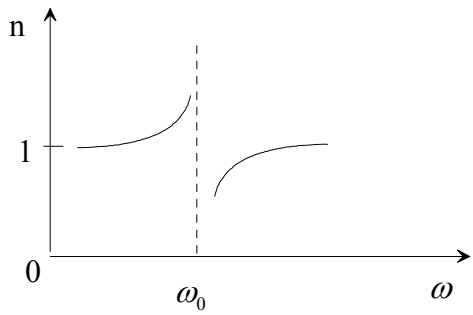
$$n = \sqrt{\varepsilon} = \left(1 + \frac{n\, e_0^2}{m_e\, \varepsilon_0\left(\omega_0^2 - \omega^2\right)}\right)^{1/2} \ . \tag{19}$$

S pomočjo razvoja $\left(1 + x\right)^{1/2} = 1 + \frac{1}{2}x + ...$ iz enačbe (19) sledi:

$$\boxed{n \cong 1 + \frac{1}{2}\frac{n\, e_0^2}{m_e\, \varepsilon_0\left(\omega_0^2 - \omega^2\right)}} \ . \tag{20}$$

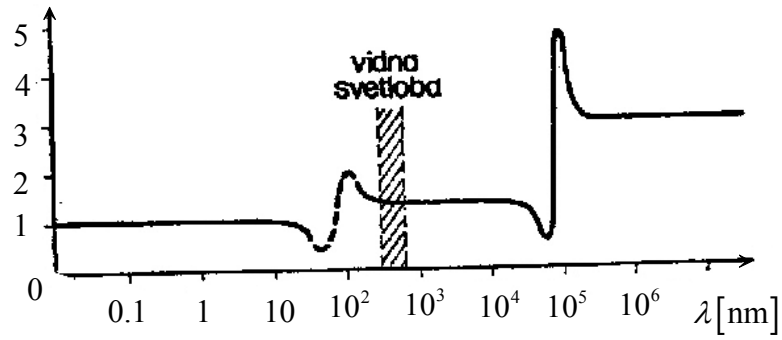
torej:

Opomba: ker v enačbi (7) nismo upoštevali člena zaradi dušenja, izraz za n (enačba (20)) pri $\omega = \omega_0$ divergira.



- **Primeri** (iz knjige J. Strnad, Fizika II)

1. Odvisnost lomnega količnika n od valovne dolžine (λ) za kameno sol (NaCl):



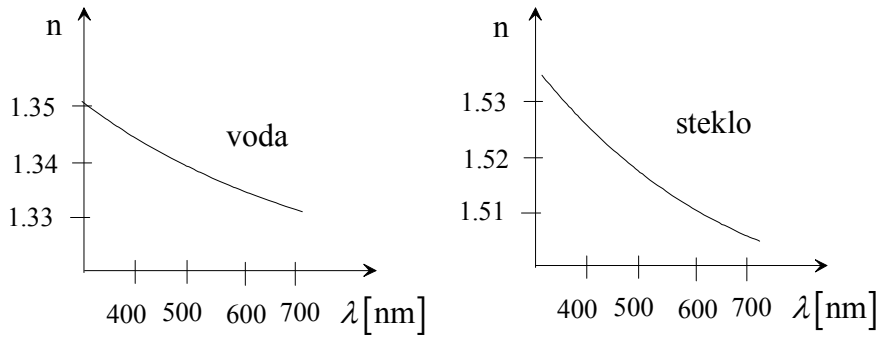
$$c = \nu \lambda$$

$$\omega = 2\pi \upsilon$$

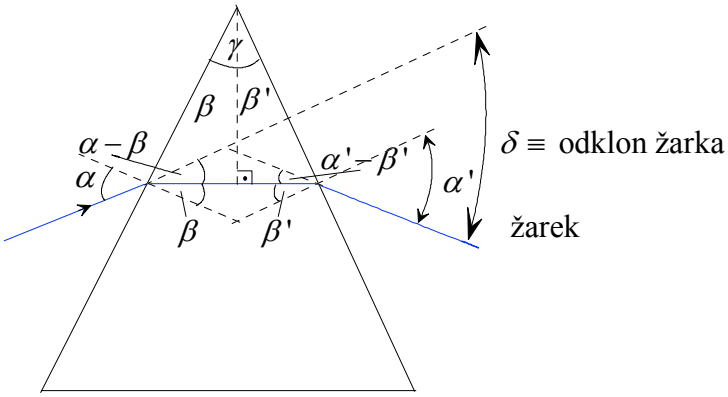
$$c = \frac{c_0}{n}$$

$$\lambda = \frac{2\pi c_0}{\omega}$$

2. Odvisnost lomnega količnika od valovne dolžine (λ) za vodo in steklo:



- **Lom svetlobe na prizmi**

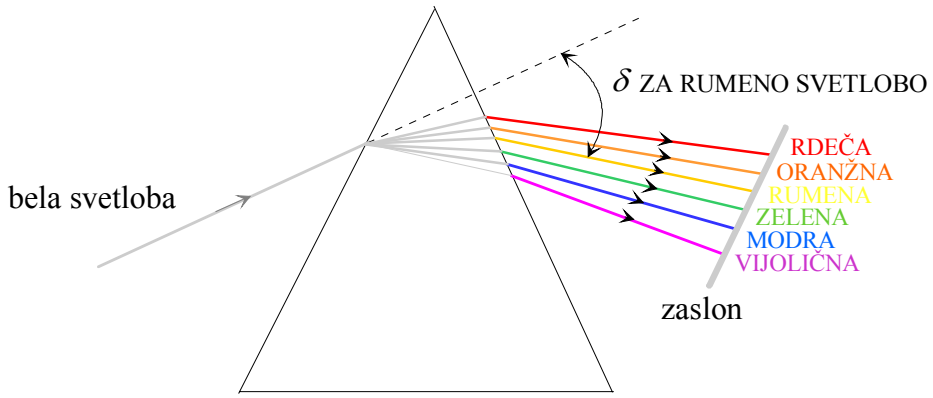


$$\frac{\sin\alpha}{\sin\beta} = n \qquad \frac{\sin\alpha'}{\sin\beta'} = n \qquad \gamma = \beta + \beta'$$

$$180^0 = 180^0 - \delta + \alpha - \beta + \alpha' - \beta' \Rightarrow$$

$$\boxed{\delta = \alpha - \beta + \alpha' - \beta' = \alpha + \alpha' - \gamma}\ .$$

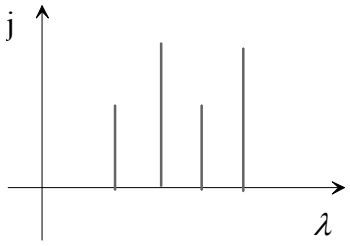
Ker $n = n(\lambda)$ je odklon (δ) posameznih komponent bele svetlobe na prizmi različen:



Fotometrija

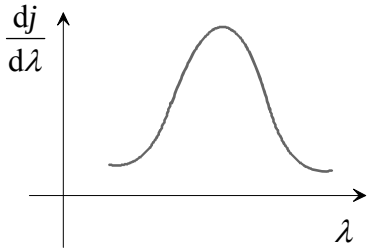
Vrste spektrov:

- **emisijski** spekter: spekter, ki ga svetila (izvori EM valovanja) sevajo,
- **absorpcijski** spekter,
- **odbojni** spekter,
- **črtasti** spektri (v plinih)



$j \equiv$ gostota energijskega toka
$\lambda \equiv$ valovna dolžina

- **zvezni** spektri



$\mathrm{d}j$ je delež j v intervalu $\left[\lambda - \frac{1}{2}\mathrm{d}\lambda\ ,\ \lambda + \frac{1}{2}\mathrm{d}\lambda\right]$

Skupna gostota energijskega (svetlobnega) toka:

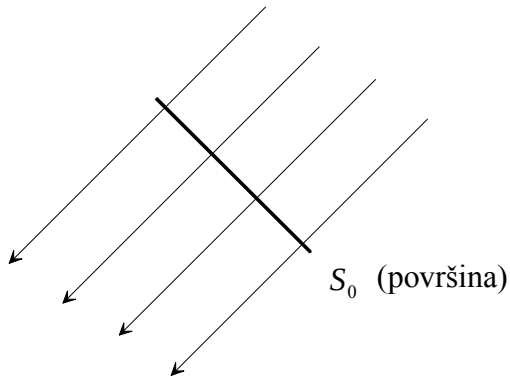
$$j = \int \left(\frac{\mathrm{d}j}{\mathrm{d}\lambda}\right)\mathrm{d}\lambda$$

- **Osvetljenost (E)**

Gostota energijskega (svetlobnega) toka:

$$j = \frac{P}{S_0} \tag{1}$$

kjer je P svetlobni (energijski) tok

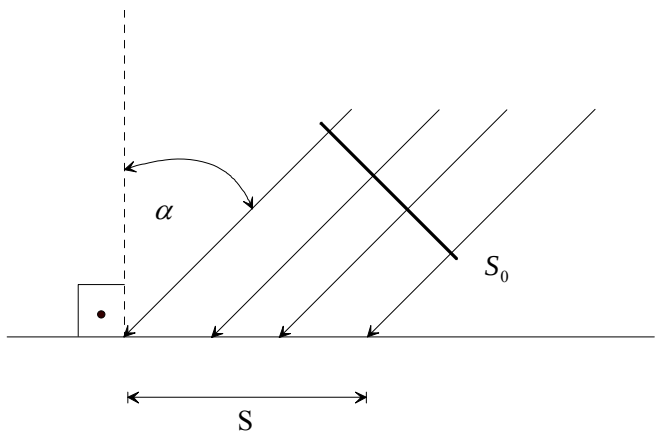


Osvetljenost dela ravne ploskve s površino S je (glejte sliko):

$$E = \frac{P}{S} = \frac{P}{\left(S_0/\cos\alpha\right)} = \left(\frac{P}{S_0}\right)\cos\alpha \ . \tag{2}$$

Ob upoštevanju enačbe (1) iz enačbe (2) sledi:

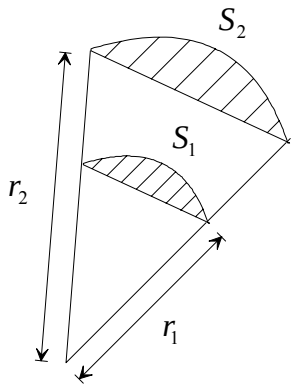
$$\boxed{E = j\cos\alpha} \tag{3}$$



$$\cos\alpha = \frac{S_0}{S} \Rightarrow S = \frac{S_0}{\cos\alpha}$$

- **Svetilnost točkastega svetila**

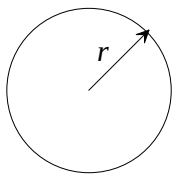
Definicija prostorskega kota



$$\frac{S_1}{r_1^2} = \frac{S_2}{r_2^2}$$

prostorski kot $\Omega = \frac{S}{r^2}$

Poln prostorski kot je 4π :



$$\Omega_{\text{poln}} = \frac{S_{\text{krogle}}}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi$$

Infinitezimalni prostorski kot:

$$\boxed{\mathrm{d}\Omega = \frac{\mathrm{d}S}{r^2}} \quad . \tag{4}$$

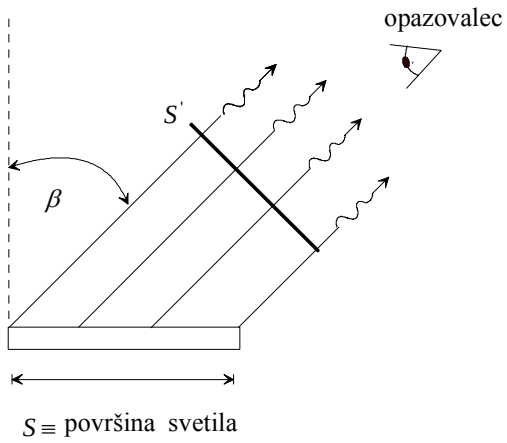
Točkasto svetilo, ki seva enakomerno na vse strani:

$$\boxed{j = \frac{P}{S} = \frac{P}{4\pi r^2} = \frac{I}{r^2}} \quad , \tag{5}$$

kjer je svetilnost točkastega svetila:

$$\boxed{I = \frac{P}{4\pi}} \quad . \tag{6}$$

Svetilnost in svetlost ploskovnega svetila



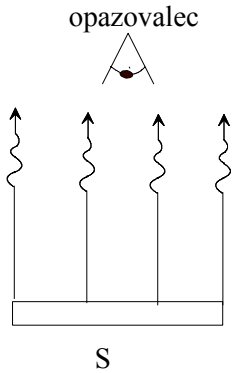
$$\cos\beta = \frac{S'}{S} \Rightarrow S' = S\cos\beta$$

- svetilnost $\boxed{I = B(\beta)S' = B(\beta)S\cos\beta}$ (7)

  B ≡ svetlost svetila

- poseben primer: opazovanje pod pravim kotom:

  $\boxed{I = B(\beta = 0)\, S}$ (8)



- posplošitev:

  $\boxed{\mathrm{d}I = B(\beta)\cos\beta\,\mathrm{d}S}$ če svetilo ni ravna ploskev in sevanje ni izotropno (9)

- Lambertov zakon:

  $B(\beta) = B_0 = \text{konst.}$ (10)

  $\mathrm{d}I = B_0\cos\beta\,\mathrm{d}S$ (11)

Zaključek:

$$\boxed{\mathrm{d}j = \frac{\mathrm{d}I}{r^2}}$$

$$\boxed{\mathrm{d}I = \frac{\mathrm{d}P}{\mathrm{d}\Omega}}$$

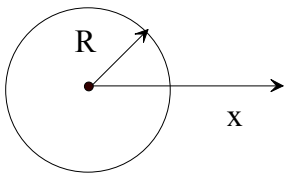
$$\boxed{\mathrm{d}E = \frac{\mathrm{d}I}{r^2}\cos\alpha}$$

$$\boxed{\mathrm{d}I = B\,\mathrm{d}S\cos\beta}$$

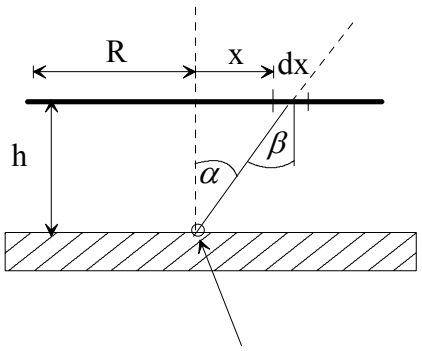
Primer:

Okroglo ploskovno svetilo s polmerom R seva po Lambertovem zakonu na spodaj ležečo površino.

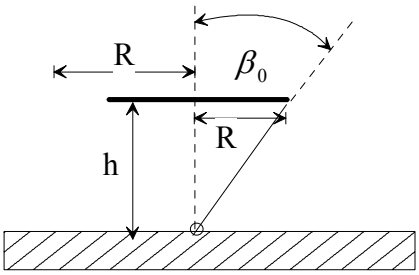
Tloris:



stranski ris:



računamo osvetljenost E v točki



$$\cos\beta = \frac{h}{r} \;\Rightarrow\; r = \frac{h}{\cos\beta}$$

$$tg\,\beta = \frac{x}{h}$$

$$\frac{1}{\cos^2\beta}\,\mathrm{d}\beta = \frac{\mathrm{d}x}{h}$$

$$\mathrm{d}x = \frac{h}{\cos^2\beta}\,\mathrm{d}\beta$$

Velja:

$$\mathrm{d}I = B_0 \cos\beta \; 2\pi \, x \, \mathrm{d}x \; , \tag{12}$$

$$\mathrm{d}E = \frac{\mathrm{d}I}{r^2} \cos\alpha = \frac{B_0}{r^2} \cos^2\beta \; 2\pi x \, \mathrm{d}x \quad . \tag{13}$$

S slike razberemo, da je v obravnavanem primeru $\alpha = \beta$. Torej:

$$E = \int \mathrm{d}E = \int \frac{B_o \cos^4\beta 2\pi h \sin\beta h \mathrm{d}\beta}{\mathrm{h}^2 \cos\beta \cos^2\beta} = \int 2\pi B_0 \sin\beta \cos\beta \mathrm{d}\beta = \pi B_0 \int \sin(2\beta)\mathrm{d}\beta =$$

$$= \frac{\pi B_0}{2} \int_0^{2\beta_0} \sin\omega \mathrm{d}\omega = \frac{\pi B_0}{2}\left(-\cos\omega\right)\Big|_0^{2\beta} = \frac{\pi B_0}{2}\left(-\cos^2 2\beta_0 + 1\right) = \frac{\pi B_0}{2}\left(1 - \cos^2\beta_0\right). \tag{14}$$

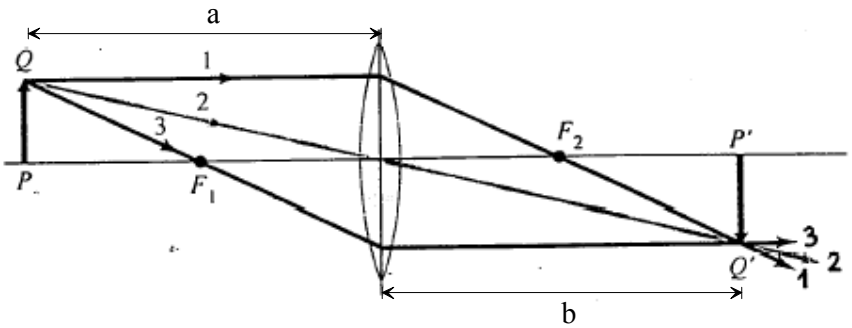
Če je ploščato svetilo zelo veliko je $\beta_0 \rightarrow \frac{\pi}{2}$ in iz enačbe (14) sledi:

$$\boxed{E \rightarrow \pi B_0} \tag{15}$$

Tanke leče

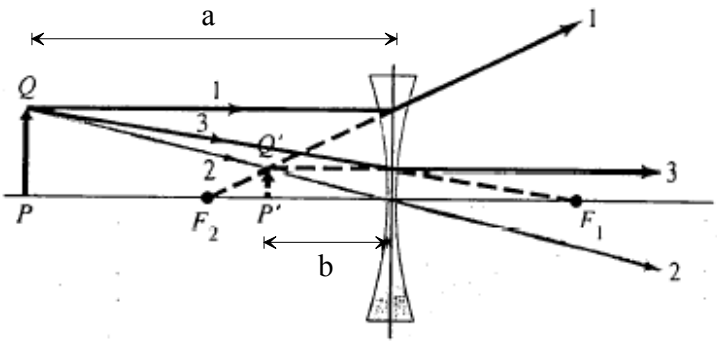
- **Bikonveksna leča:**

f > 0
a > 0
b > 0

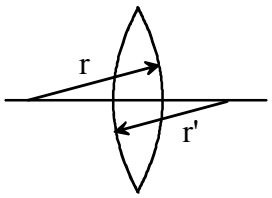


- **Bikonkavna leča:**

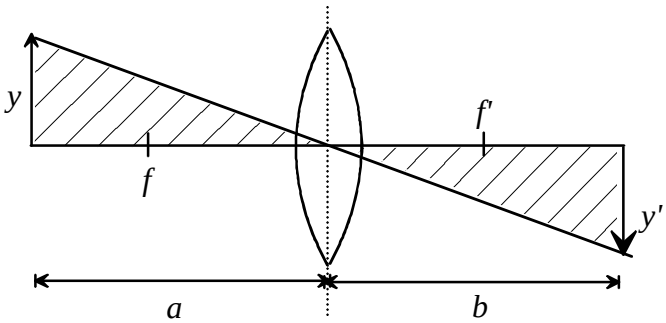
f < 0
a < 0
b < 0



$$\frac{1}{f}=(n-1)\left(\frac{1}{r}+\frac{1}{r'}\right)$$

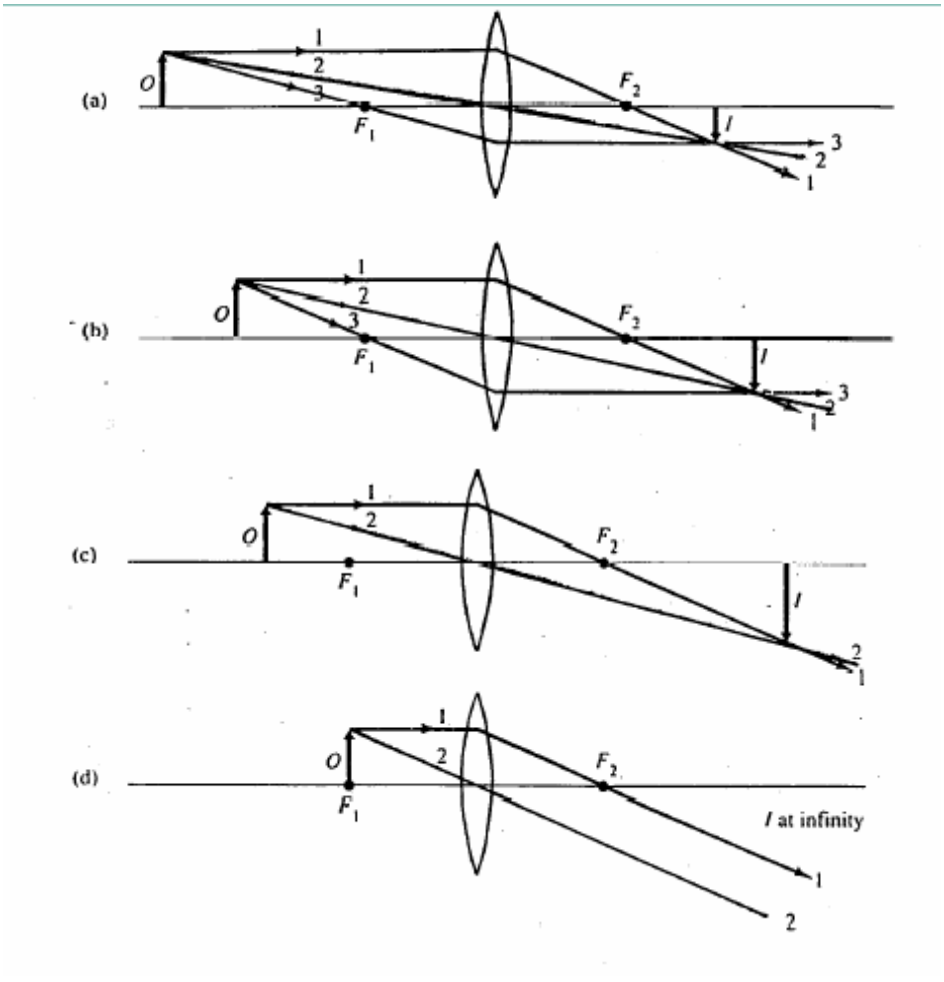


enačba leče $\boxed{\frac{1}{a}+\frac{1}{b}=\frac{1}{f}}$

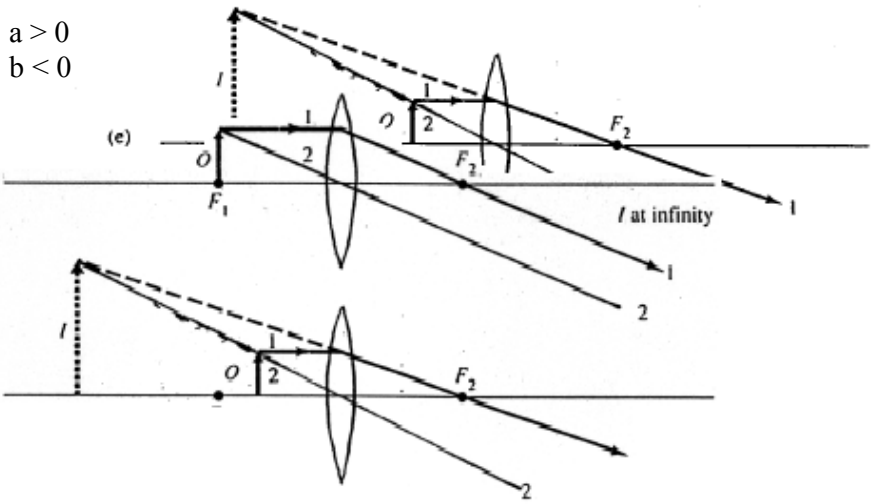


povečava: $\frac{y}{y'}=\frac{b}{a}=\frac{f}{a-f}$

bikonveksna leča



a > 0
b < 0



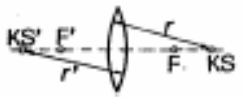
Sistem dveh leč:

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{s}{f_1 f_2}$$

$s \equiv$ razdalja med lečama

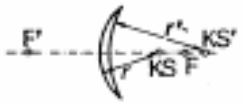
f > 0

bikonveksna: $r > 0, r' > 0$
$1/f = (n-1)(1/r + 1/r')$

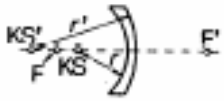


plankonveksna: $r > 0, r' = \infty$
$1/f = (n-1)/r$

konkavno konveksna: $r > 0, r' < 0$
$1/f = (n-1)(1/r - 1/|r'|)$

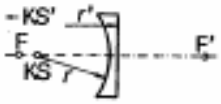


konveksno konkavna: $r < 0, r' > 0$
$1/f = -(n-1)(1/|r| - 1/r')$



f < 0

plankonkavna: $r < 0, r' = \infty$
$1/f = -(n-1)/|r|$



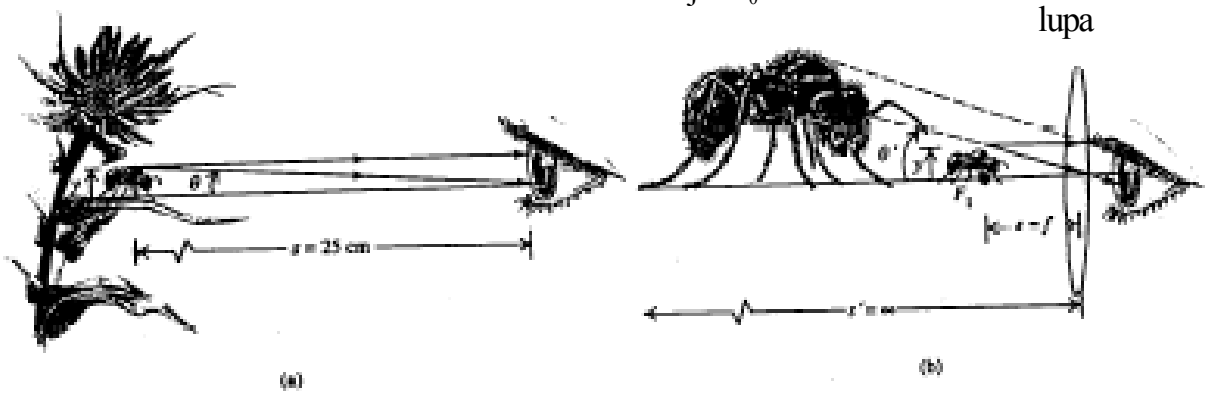
bikonkavna: $r < 0, r' < 0$
$1/f = -(n-1)(1/|r| + 1/|r'|)$

Slike: J. Strnad: Fizika II

Lupa

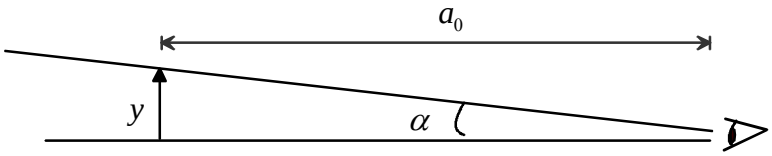
normalna zorna razdalja $a_0 = 25\,\text{cm}$

lupa



Predmet postaviš v gorišče $\Rightarrow$ povečaš zorni kot

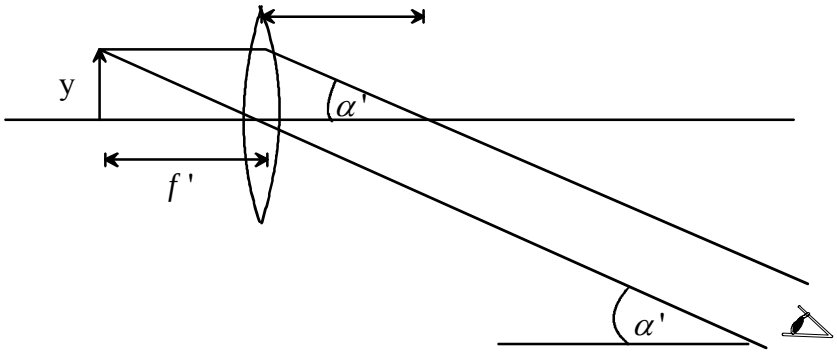
brez lupe:



z lupo:

$$tg\,\alpha = \frac{y}{a_0}$$

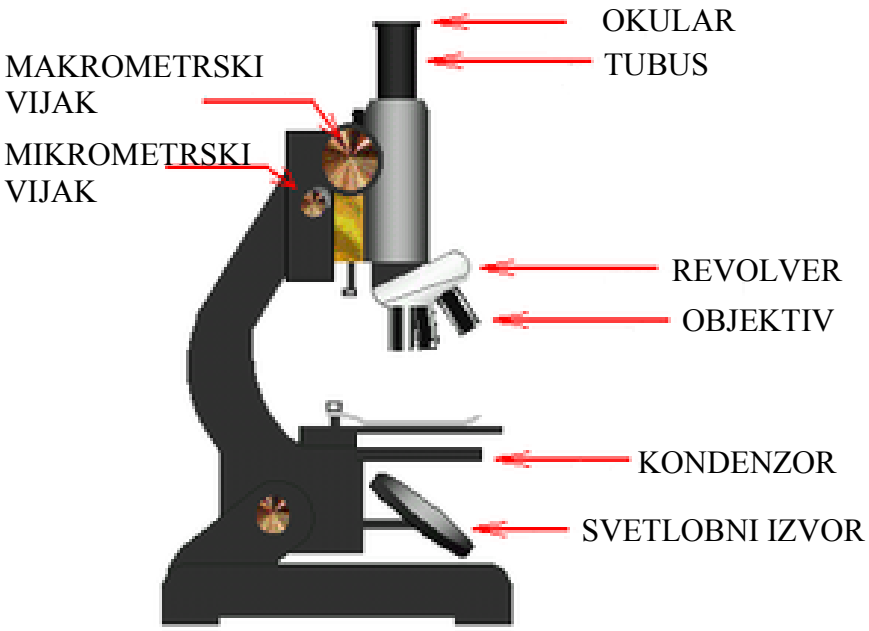
$$tg\,\alpha' = \frac{y}{f'}$$

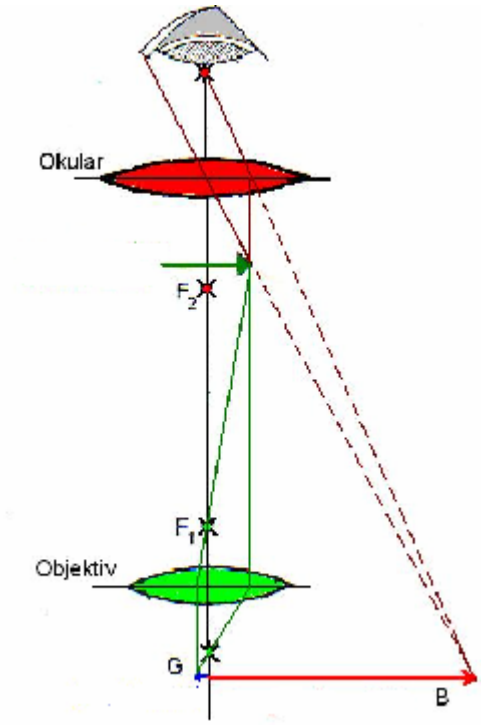


$$\boxed{\text{povečava} \quad N = \frac{tg\,\alpha'}{tg\,\alpha} = \frac{a_0}{f'}}$$

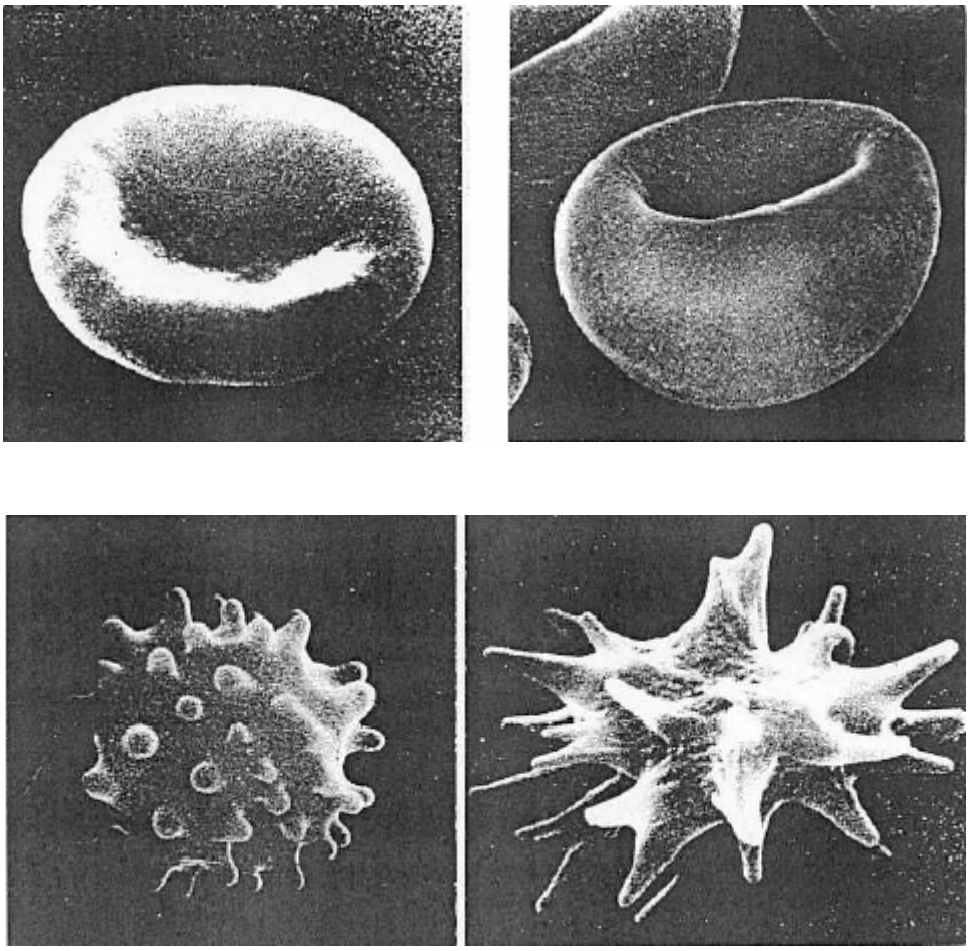
Mikroskop

- Okular uporabimo kot lupo





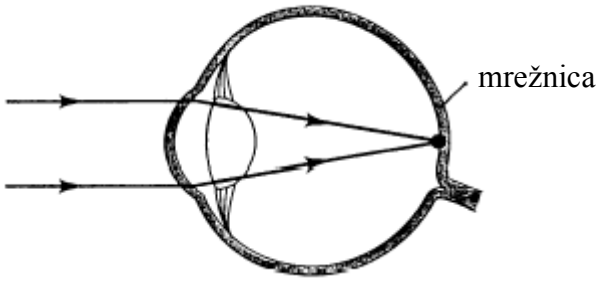
eritrocit: d ~ 6 μm



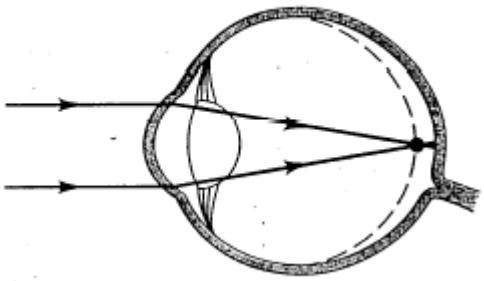
Oko

$f \approx 2\,\mathrm{cm}$

NORMALNO OKO

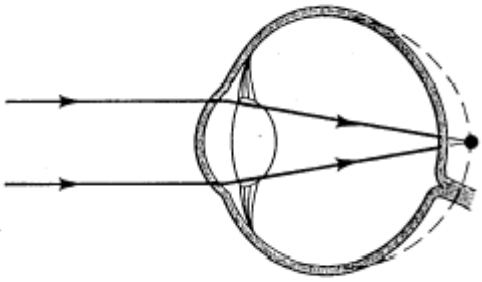


KRATKOVIDNO OKO



Slika ODDALJENIH PREDMETOV nastane pred mrežnico, ker očesna leča ne more imeti dovolj velike goriščne razdalje

DALJNOVIDNO OKO



Slika BLJIŽNIH PREDMETOV nastane za mrežnico, ker očesna leča ne more imeti dovolj majhne goriščne razdalje

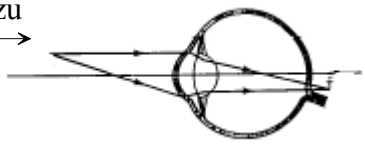
KOREKCIJA Z OČALI

- **Daljnovidno oko**

  Dobro vidi samo oddaljene predmete

  $f \equiv$ goriščna razdalja očesne leče

če je predmet blizu
slabo vidi →



Normalna vidna razdalja $a_0 \approx 25\,\text{cm}$.

Brez očal: a > a_0 , da je b normalen

$$\boxed{\frac{1}{a}+\frac{1}{b}=\frac{1}{f}}$$

Z očali: a = a_0.

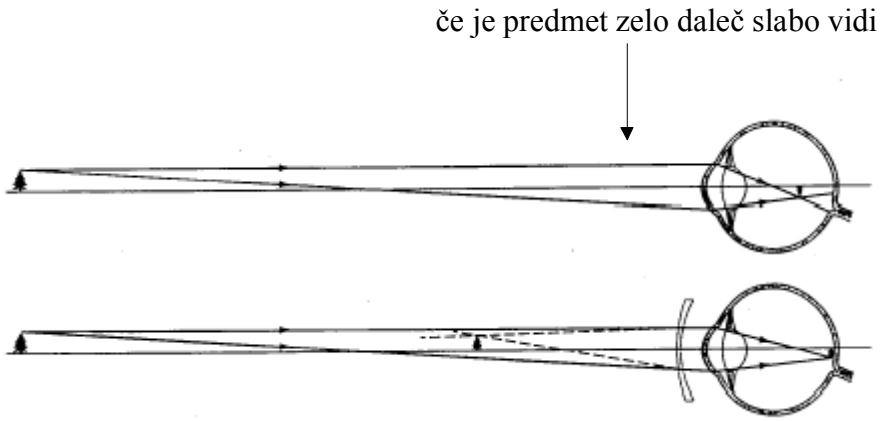
$$\boxed{\frac{1}{a_0}+\frac{1}{b}=\frac{1}{f}+\frac{1}{f'}}$$

Odštejemo obe enačbi in dobimo:

dioptrija: $\boxed{D=\frac{1}{f'}=\frac{a-a_0}{a_0\,a}>0}$ zbiralna leča

- **Kratkovidno oko**

Dobro vidi samo bližnje predmete.



Brez očal: $a < a_0$, da je b normalen

$$\boxed{\frac{1}{a}+\frac{1}{b}=\frac{1}{f}}$$

Z očali: $a >> a_0$:

$$\boxed{\frac{1}{\infty}+\frac{1}{b}=\frac{1}{f}+\frac{1}{f'}}$$

Odštejemo obe enačbi in dobimo:

$$-\frac{1}{a}=\frac{1}{f'} \quad \Rightarrow \quad \boxed{D=\frac{1}{f'}=-\frac{1}{a}<0}$$

Meglična celica, selektor hitrosti, masni spektrometer…

- **Meglična celica**

Mehanizem delovanja

zraku z vodno paro sunkovito povečamo prostornino, da se ohladi

$\Downarrow$

z vodno paro nasičeni zrak postane prenasičen

$\Downarrow$

vodna para se useda na gibajočih se ionih in tvori vodne kapljice (kondenzacija)

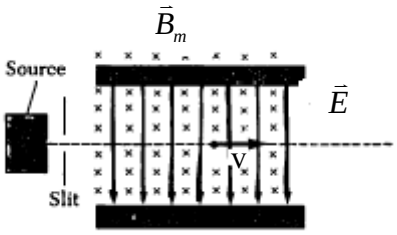
$\Downarrow$

sledovi nabitih delcev lahko postanejo **vidni**

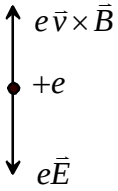
$\odot$
$\vec{B}$ magnetno polje

Bubble chamber photograph. The spiral tracks at the bottom of the photograph are an electron – positron pair (left an right respectively), formed by a gamma ray interacting with a hydrogen nucleus. An applied magnetic field causes the electron and the positron to be defected in opposite directions. The track leaving from the cusp between the two spirals is an additional electron knocked out of a hydrogen atom during this interaction. (G. Holton, F.J. Rutherford, F.G. Watston, Project Physics, New York: HRW, 1981.)

- **Selektor hitrosti** (izbira delcev z enako hitrostjo)



a)



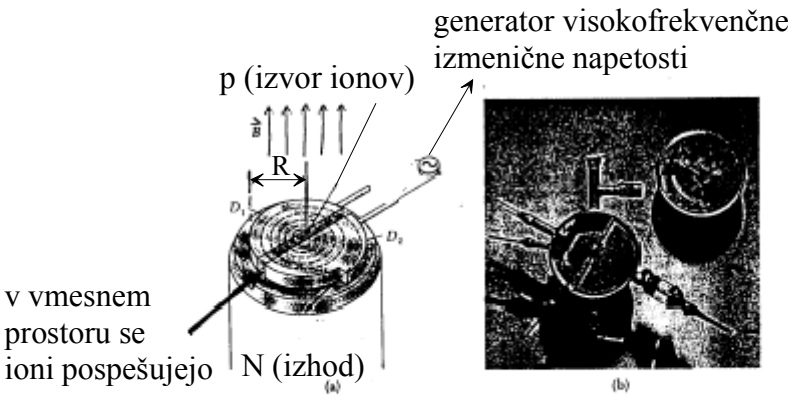
$$e\,v\,B = e\,E \;\Rightarrow\; \boxed{v = \frac{E}{B}}$$

b)

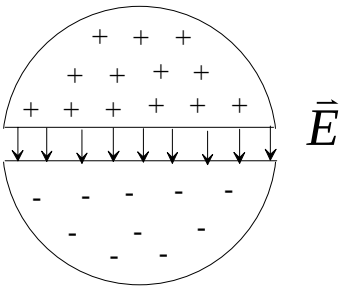
(a) A velocity selector. When a positively charged particle is in the presence of both an inward magnetic field as indicated by the blue crosses, and a downward electric field indicated by the red arrows, it experiences both an electric force $e\vec{E}$ downward and a magnetic force $e\vec{v}\times\vec{B}$ upward. (b) When these forces balance each other as shown here, the particle moves in a horizontal line through the fields.

- **Ciklotron**

2 votli elektrodi D_1, D_2 v evakuiranem prostoru. V elektrodah ni električnega polja!



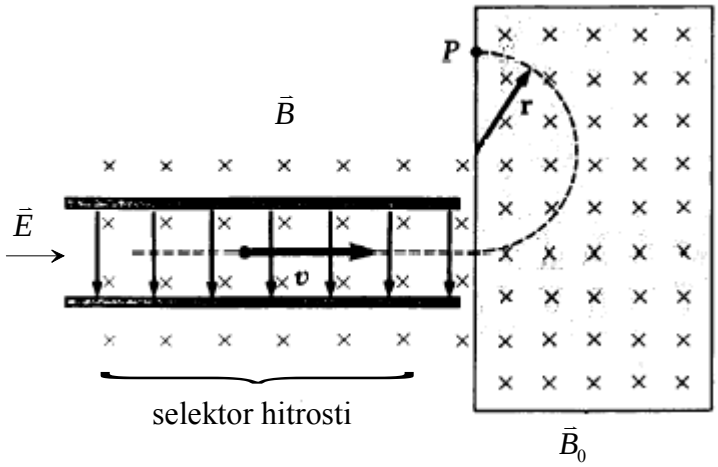
(a) The cyclotron consists of an ion source, two dees across which an alternating voltage is applied, and a uniform magnetic field provided by an electromagnet. (The south pole of the magnet is not shown.) (b) The first cyclotron, invented by E.O. Lawrence and M.S. Livingston in 1934. (Courtesy of Lawrence Berkeley Laboratory, University of California)



$$v \perp \vec{B} \;\Rightarrow\; \text{gibanje po krogu}$$

$$\left(\vec{F} = e\,\vec{v}\times\vec{B}\right) \perp \mathrm{d}\vec{s} \;\Rightarrow\; \mathrm{d}A = \vec{F}\cdot d\vec{s} = 0 \;\Rightarrow W_k = \text{konst.}$$

- **Masni spektrometer**



A mass spectrometer. Charged particles are first sent through a velocity selector. They then enter a region where the magnetic field $\boldsymbol{B}_0$ (inward) causes positive ions to move in a semicircular path and strike a photographic film at P.

Selektor hitrosti: $e\,v\,B = e\,E \quad \Rightarrow \quad \boxed{v = \frac{E}{B}}$ (1)

Newtonov zakon:

$$m\,a_r = e\,v\,B_0$$

$$m\frac{v^2}{r} = e\,v\,B_0$$

$$\Downarrow$$

$$\boxed{r = \frac{m\,v}{e\,B_0}} \qquad (2)$$

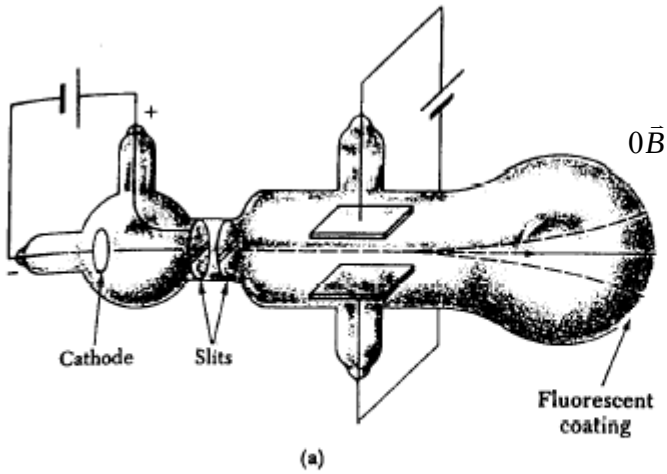
$$r = \frac{m\,v}{e\,B_0} = \frac{m\,E}{e\,B_0\,B} \quad \Rightarrow \quad \boxed{\frac{m}{e} = \frac{r\,B_0\,B}{E}}$$, kjer smo upoštevali enačbo (1)

- **Thomsonov aparat za merjenje razmerja** $\frac{e}{m}$ (specifični naboj)

katodni žarki $\equiv$ elektroni

Thomson: $\frac{e}{m} \sim 0.77 \cdot 10^{11} \frac{\text{As}}{\text{kg}}$

odkritelj elektrona
J.J. Thomson (1856 - 1940)



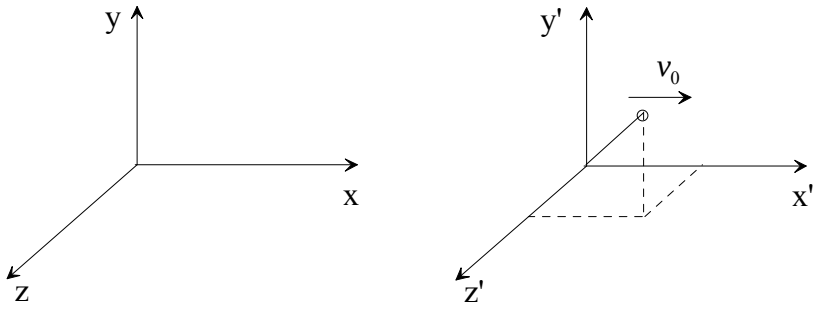
(a) Thomson's apparatus for measuring e/m. Electrons are accelerated from the cathode, pass through two slits, and are deflected by both an electric field and a magnetic field (not shown, but directed into the paper). The deflected beam then strikes a phosphorescent screen. (b) J.J. Thomson in the Cavendish Laboratory, University of Cambridge.

Relativnostna mehanika

- **Klasična fizika**

  - **predpostavka o univerzalnem času**

$$\boxed{t = t'}$$



  - inercialni koordinatni sistem: $\vec{v} = \text{konst.}$
  - **inercialni sistem S:** (x, y, z) ≡ mirujoči koordinatni sistem
  - **inercialni sistem S':** (x', y', z') ≡ gibajoči se koordinatni sistem (hitrost v_0)

GALILEJEVA TRANSFORMACIJA:

$$x' = x - v_0 t$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

$$v' = \frac{\mathrm{d}x'}{\mathrm{d}t} = \frac{\mathrm{d}(x - v_0 t)}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}t} - v_0 = v - v_0$$

$$\boxed{v' = v - v_0}$$

  - hitrosti se seštevajo (ni zgornje meje!)
  - oblika zakonov (Newtonovih zakonov) je **neodvisna** od hitrosti premikanja koordinatnega sistema.

- **Posebna teorija relativnosti**

A. Einstein (1905 – annus mirabilis)

Dve glavni načeli:

1. **Načelo relativnosti:** fizikalni zakoni imajo enako obliko v vseh inercialnih opazovalnih sistemih.
2. **Hitrost svetlobe** v praznem prostoru $c_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ je enaka v vseh inercialnih opazovalnih sistemih.

LORENTZOVA TRANSFORMACIJA:

$$x' = \gamma_0 \left(x - v_0 t \right) = \gamma \left(x - \frac{v_0}{c_0} c_0 t \right) \tag{1}$$

$$y' = y \tag{2}$$

$$z' = z \tag{3}$$

$$c_0 t' = \gamma_0 \left(c_0 t - \frac{v_0}{c_0} x \right) \tag{4}$$

$$\boxed{\gamma_0 = \frac{1}{\sqrt{1 - \frac{v_0^2}{c_0^2}}}} \tag{5}$$

- Vsak inercialni sistem ima svoj **lastni čas.**

OBRATNA LORENTZOVA TRANSFORMACIJA

$$x = \gamma_0 \left(x' + \frac{v_0}{c_0} c_0 t' \right) \tag{6}$$

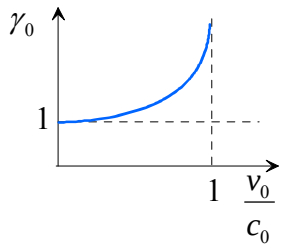
$$y = y' \tag{7}$$

$$z = z' \tag{8}$$

$$c_0 t = \gamma_0 \left(c_0 t' + \frac{v_0}{c_0} x' \right) \tag{9}$$

Zahteva, da preide nova Lorentzova transformacija pri majhnih hitrostih v **GALILEJEVO TRANSFORMACIJO** je izpolnjena:

$$\gamma_0 = \frac{1}{\left(1-\frac{v_0^2}{c_0^2}\right)^{1/2}} \underset{\frac{v_0}{c_0}\to 0}{\cong} 1+\frac{1}{2}\frac{v_0^2}{c_0^2}+.... \to 1$$



DODATEK: Izpeljava γ_0 (enačba (5)) in enačbe (4)

Na osnovi enačb (1) in (6):

$$x' = \gamma_0 \left(x - v_0 t\right), \tag{D1}$$

$$x = \gamma_0 \left(x' + v_0 t'\right), \tag{D2}$$

in načelo o invariantnosti svetlobne hitrosti

$$c_0 = c_0' \tag{D3}$$

lahko izpeljemo vrednost γ_0 kot sledi. Najprej vstavimo x' iz enačbe (D1) v enačbo (D2):

$$x = \gamma_0 \left[\gamma_0 \left(x - v_0 t\right) + v_0 t'\right] = \gamma_0^2 x - \gamma_0^2 v_0 t + \gamma_0 v_0 t' , \tag{D4}$$

torej:

$$\gamma_0 v_0 t' = x\left(1-\gamma_0^2\right) + \gamma_0^2 v_0 t . \tag{D5}$$

Iz enačbe (D5) sledi:

$$\boxed{t' = \gamma_0 \left(t - \frac{\left(\gamma_0^2 - 1\right)}{\gamma_0^2 v_0} x \right).} \tag{D6}$$

Naredimo še diferencial enačbe (D6):

$$\mathrm{d}t' = \gamma_0 \left(\mathrm{d}t - \frac{\left(\gamma_0^2 - 1\right)}{\gamma_0^2 v_0} \mathrm{d}x \right) . \tag{D7}$$

V nadaljevanju upoštevamo načelo invariantnosti svetlobne hitrosti (enačba (D3)) in enačbo (D7). Dobimo:

$$c_0 = c_0^{'} = \frac{\mathrm{d}x'}{\mathrm{d}t'} = \frac{\gamma_0\left(\mathrm{d}x - v_0\mathrm{d}t\right)}{\gamma_0\left(\mathrm{d}t - \frac{\left(\gamma_0^2 - 1\right)}{\gamma_0^2\, v_0}\mathrm{d}x\right)} = \frac{c_0 - v_0}{1 - \frac{\left(\gamma_0^2 - 1\right)}{\gamma_0^2\, v_0}c_0} = \qquad \text{D(8)}$$

$$= \frac{c_0 - v_0}{1 - \frac{\left(\gamma_0^2 - 1\right)}{\gamma_0^2\, v_0}c_0}.$$

Iz enačbe (D8) sledi:

$$1 - \frac{\left(\gamma_0^2 - 1\right)}{\gamma_0^2\, v_0}c_0 = \frac{c_0 - v_0}{c_0} = 1 - \frac{v_0}{c_0} \quad , \qquad \text{(D9)}$$

oziroma:

$$\frac{\left(\gamma_0^2 - 1\right)}{\gamma_0^2\, v_0}c_0 = \frac{v_0}{c_0} \quad . \qquad \text{(D10)}$$

Iz enačbe (D10) lahko izračunamo γ_0:

$$\frac{\gamma_0^2 - 1}{\gamma_0^2} = 1 - \frac{1}{\gamma_0^2} = \frac{v_0^2}{c_0^2} \quad , \qquad \text{(D11)}$$

$$\frac{1}{\gamma_0^2} = 1 - \frac{v_0^2}{c_0^2} \ ,$$

$$\boxed{\gamma_0^2 = \frac{1}{1 - \frac{v_0^2}{c_0^2}} \quad ,} \qquad \text{(D12)}$$

oziroma

$$\boxed{\gamma_0 = \frac{1}{\sqrt{1 - \frac{v_0^2}{c_0^2}}} \quad .} \qquad \text{(D13)}$$

Če vstavimo relacijo (glejte enačbo (D11))

$$\frac{\gamma_0^2 - 1}{\gamma_0^2} = \frac{v_0^2}{c_0^2}$$

v enačbo (D6) dobimo:

$$\boxed{t' = \gamma_0 \left(t - \frac{v_0}{c_0^2} x \right),} \tag{D14}$$

oziroma enačba (4):

$$\boxed{c_0 t' = \gamma_0 \left(c_0 t - \frac{v_0}{c_0} x \right)} \tag{D15}$$

- **Transformacije hitrosti**

Iz enačb (1) – (4) sledi

Infinitezimaln Lorentzova transformacija

$$\mathrm{d}t' = \gamma_0 \left(\mathrm{d}t - \frac{v_0}{c_0^2} \mathrm{d}x \right) \tag{10}$$

$$\mathrm{d}x' = \gamma_0 \left(\mathrm{d}x - v_0 \, \mathrm{d}t \right) \tag{11}$$

$$\mathrm{d}y' = \mathrm{dy} \tag{12}$$

$$\mathrm{d}z' = \mathrm{dz} \tag{13}$$

$$v_x' = \frac{\mathrm{d}x'}{\mathrm{d}t'} \qquad v_y' = \frac{\mathrm{d}y'}{\mathrm{d}t'} \qquad v_z' = \frac{\mathrm{d}z'}{\mathrm{d}t'} \tag{14}$$

$$v_x = \frac{\mathrm{d}x}{\mathrm{d}t} \qquad v_y = \frac{\mathrm{d}y}{\mathrm{d}t} \qquad v_z = \frac{\mathrm{d}z}{\mathrm{d}t} \tag{15}$$

Iz enačb (10) – (15) sledi:

$$v_x' = \frac{\mathrm{d}x'}{\mathrm{d}t'} = \frac{\gamma_0 \left(\mathrm{d}x - v_0 \mathrm{d}t \right)}{\gamma_0 \left(\mathrm{d}t - \frac{v_0}{c_0^2} \mathrm{d}x \right)} =$$

$$= \frac{\mathrm{d}x - v_0 \mathrm{d}t}{\mathrm{d}t - \frac{v_0}{c_0^2} \mathrm{d}x} = \frac{\frac{\mathrm{d}x}{\mathrm{d}t} - v_0}{1 - \frac{v_0}{c_0^2} \frac{\mathrm{d}x}{\mathrm{d}t}} = \frac{v_x - v_0}{1 - \frac{v_0 v_x}{c_0^2}},$$

torej

$$\boxed{v_x = \frac{v_x - v_0}{1 - \frac{v_0 v_x}{c_0^2}}} \tag{16}$$

Podobno lahko dobimo še ostale transformacije za hitrosti**:**

$$v_y' = \frac{v_y}{\gamma_0 \left(1 - \frac{v_0 v_x}{c_0^2} \right)} \tag{17}$$

$$v_z' = \frac{v_z}{\gamma_0 \left(1 - \frac{v_0 v_x}{c_0^2} \right)} \tag{18}$$

Obratna transformacija $\left(v_0 \to -v_0 \right)$:

$$v_x' = \frac{v_x' + v_0}{\left(1 + \frac{v_0 v_x'}{c_0^2} \right)} \tag{19}$$

$$v_y = \frac{v_y'}{\gamma_0 \left(1 + \frac{v_0 v_x'}{c_0^2} \right)} \tag{20}$$

$$v_z = \frac{v_z'}{\gamma_0 \left(1 + \frac{v_0 v_x'}{c_0^2} \right)} \tag{21}$$

- transformacija vsebuje NAČELO O INVARIANTNOSTI SVETLOBNE HITROSTI v praznem prostoru $c_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$:

$$v_x = \frac{c_0 + v_0}{1 + \frac{v_0 c_0}{c_0^2}} = \frac{c_0 + v_0}{1 + \frac{v_0}{c_0}} = \frac{c_0 \left(c_0 + v_0 \right)}{\left(c_0 + v_0 \right)} = c_0$$

FORMALIZEM: 4-razsežni prostor - čas

Svetovni četverec definiran kot:

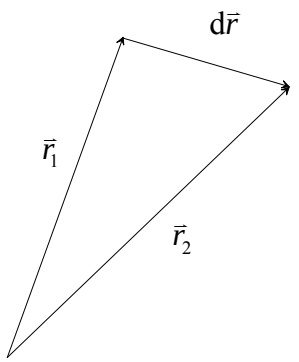
$${}^4 x = \left(c_0 t, x, y, z \right) = \left(c_0 t, \vec{r} \right)$$

Lorentzova transformacija:

$$\boxed{\begin{aligned} c_0 t' &= \gamma_0 \left(c_0 t - \beta x\right) \\ x' &= \gamma_0 \left(x - \beta c_0 t\right) \\ y' &= y \\ z' &= z \end{aligned}}$$

kjer je $\boxed{\beta = \frac{v_0}{c_0}}$

- **Klasična** 3-D slika: Razdalja med dvema točkama $\mathrm{d}r = \sqrt{\mathrm{d}\vec{r} \cdot \mathrm{d}\vec{r}}$ je neodvisna od izbire opazovalnega sistema. Pravimo, da je $\mathrm{d}r^2 = \mathrm{d}\vec{r} \cdot \mathrm{d}\vec{r}$ invarianta.



- **Specialna teorija relativnosti**

Invarianta je:

$$\mathrm{d}s^2 = -c_0^2 \mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 = -c_0 \, \mathrm{d}t'^2 + \mathrm{d}x'^2 + \mathrm{d}y'^2 + \mathrm{d}z'^2 \ . \qquad (22)$$

Novost: negativni znak pred prvim členom, klasično je namreč

$$\mathrm{d}r^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 = \mathrm{d}\vec{r} \cdot \mathrm{d}\vec{r} \ .$$

- **Skrčenje dolžin**

Inercialna opazovalna sistema S in S':

$$\mathrm{S}: \quad \left.\begin{aligned} &\text{dogodek 1}: \ t_1 = 0, \ x_1 = 0 \\ &\text{dogodek 2}: \ t_2 = \frac{v_0 L}{c_0^2}, \ x_2 = L \end{aligned}\right\} \Rightarrow \text{palica miruje v sistemu S}: \quad \mathrm{x}_2 - x_1 \equiv L$$

Lorentzova transformacija:

$$S': \left.\begin{array}{l} \text{dogodek 1: } t_1' = 0,\ x_1' = 0 \\ \text{dogodek 2: } t_2' = 0,\ x_2' = \dfrac{L}{\gamma_0} \end{array}\right\} \Rightarrow x_2' - x_1' = \frac{L}{\gamma_0} = L'$$

ZAKLJUČKI:

- izmerjena dolžina je odvisna od hitrosti gibanja koordinatnega sistema,
- dogodka sočasna samo, če se dogodita v isti točki,
- opazovalcu se zdi gibajoča se palica skrčena:

$$\boxed{L' = \frac{L}{\gamma_0}} \tag{23}$$

- sočasnost dogodkov je relativen pojem

o **Podaljšanje časa** (inercialna sistema S in S')

$$S: \left.\begin{array}{l} \text{prvi dogodek: mesto } x,\ \text{čas } t_1 \\ \text{drugi dogodek: mesto } x,\ \text{čas } t_2 \end{array}\right\} \Delta t = t_2 - t_1$$

Lorentzova transformacija:

$$S': \begin{array}{l} t_1' = \gamma_0 \left(t_1 - \dfrac{v_0 x}{c_0^2} \right) \\ t_2' = \gamma_0 \left(t_2 - \dfrac{v_0 x}{c_0^2} \right) \end{array}$$

torej: $t_2' - t_1' = \gamma_0 \left(t_2 - t_1 \right)$

$$\boxed{\Delta t' = \gamma_0 \Delta t} \tag{24}$$

- $\Delta t' > \Delta t \Rightarrow$ za gibajočega opazovalca dogodki potekajo počasneje kot za opazovalca, ki miruje ob dogodkih $\rightarrow$ **čas je relativna količina.**
- **lastni čas** $\equiv$ čas, ki ga kaže ura v izbranem koordinatnem sistemu.

ZAKLJUČKI:

- **Popolna novost** specialne teorije relativnosti je, da se **tudi čas transformira.**
- **Prostor** in **čas** sta povezana v skupni **prostor - čas.**
- Svetovni četverec popisuje dogodek.

- **Zakoni gibanja**

Na osnovi enačbe (24) naredimo posplošitev in definiramo lastni časovni razmik z enačbo:

$$\boxed{\mathrm{d}t = \gamma\,\mathrm{d}\tau} \quad , \tag{25}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c_0^2}}} \quad , \tag{26}$$

kjer je:

$\mathrm{d}t \equiv$ čas, ki ga meri opazovalec, ki opazuje gibanje točkastega telesa
$\mathrm{d}\tau \equiv$ lastni čas (lastni časovni razmik, ki ga izmeri opazovalec, ki se giblje skupaj z opazovanim gibajočim točkastim telesom)

Novost: za hitrost v izrazu (26) dopuščamo, da **ni** konstantna, kar pomeni, da se telo lahko giblje pospešeno.

- **Četverec hitrosti ^4v**

svetovni četverec $\quad {}^4x = (c_0 t, x, y, z)$ (27)
odvajamo po lastnem času τ :

$$^4v = \frac{\mathrm{d}\left(^4x\right)}{\mathrm{d}\tau} = \gamma\frac{\mathrm{d}\left(^4x\right)}{\mathrm{d}t} = \gamma\left(c_0, v_x, v_y, v_z\right) , \tag{28}$$

kjer so

$$v_x = \frac{\mathrm{d}x}{\mathrm{d}t}, \quad v_y = \frac{\mathrm{d}y}{\mathrm{d}t}, \quad v_z = \frac{\mathrm{d}z}{\mathrm{d}t} \; .$$

Enačbo (28) zapišemo v vektorski obliki:

četverec hitrosti $\quad \boxed{^4v = (\gamma c_0, \gamma\bar{v})} \quad , \tag{29}$

kjer je $\bar{v} = \left(v_x, v_y, v_z\right)$.

Pri majhnih hitrostih $\frac{v}{c_0} \to 0$:

$\gamma \to 1$,

torej:

$$\gamma\bar{v} \to \bar{v} \; , \tag{30}$$
$$\mathrm{d}\tau \to \mathrm{d}t \; . \tag{31}$$

Opomba:

koordinatni čas (t ali dt) ni skalar ampak komponenta 4-D vektorja $(c_0 t,\ x,\ y,\ z)$. Odvajanje po koordinatnem času dt zato **ne** proizvede novega vektorja četverca. Zato odvajamo po lastnem času $(\mathrm{d}\tau)$. Lastni čas (τ) je skalar, ki preide pri majhnih hitrostih v koordinatni čas t.

- **Četverec gibalne količine**

$$\boxed{{}^4P = m_0\,{}^4v = (m_0\,\gamma\,c_0,\ m_0\,\gamma\,\vec{v})} \quad . \tag{32}$$

Izraz $m_0\,\gamma\,\vec{v}$ zapišemo v obliki:

$$\boxed{\vec{P} = m\,\vec{v}} \quad , \tag{33}$$

kjer je

$$\boxed{m = m_0\,\gamma} \quad , \tag{34}$$

$m_0 \equiv$ mirovna ali lastna masa telesa.

Pri majhnih hitrostih $\dfrac{v}{c_0} \to 0$:

$$\begin{aligned} &\gamma \to 1\ , \\ &m \to m_0\ , \\ &P \to m_0 v\ . \end{aligned} \tag{35}$$

Vidimo, da pri majhnih hitrostih P preide v klasično gibalno količino $G = m_0\,v$.

Opomba:

kljub temu, da $v \le c_0$ je lahko relativistična gibalna količina poljubno velika, ker se lahko poljubno veča zaradi naraščajoče vrednosti γ .

- **Polna in lastna energija**

Četverec gibalne količine lahko pišemo tudi kot (glejte enačbo (32)):

$${}^4P = \left(\frac{W}{c_0}, m_0\,\gamma\,\vec{v}\right), \tag{36}$$

kjer je W **polna** energija delca:

$$\boxed{W = m_0\,\gamma\,c_0^2} \quad . \tag{37}$$

Polna energija je odvisna od hitrosti. Najmanjša polna energija ima delec, ki miruje je $(\gamma = 1)$:

$$\boxed{W_0 = m_0\ c_0^2} \quad , \tag{38}$$

$W_0 \equiv$ **lastna** ali **mirovna** energija delca. Sedaj definiramo **kinetično** energijo delca kot:

$$\boxed{W_k = W - W_0 = m_0\,\gamma\, c_0^2 - m_0\ c_0^2 = m_0\ c_0^2\,(\gamma - 1)\ .} \tag{39}$$

Kinetična energija je torej tisti del polne energije, ki je **posledica gibanja.**

Pri majhnih hitrostih $\left(\frac{v}{c_0} \to 0\right)$ velja razvoj:

$$W_k = m_0\ c_0^2\,(\gamma - 1) = m_0\ c_0^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c_0^2}}} - 1 \right) \cong m_0\ c_0^2 \left(1 + \frac{1}{2}\frac{v^2}{c_0^2} + ... - 1 \right) \cong \frac{1}{2} m_0\, v^2 . \tag{40}$$

Vidimo, da relativistični izraz za kinetično energijo telesa $W_k = m_0\ c_0^2\,(\gamma - 1)$ preide pri $\frac{v}{c_0} \to$ v klasični izraz $W_k = \frac{1}{2} m_0\, v^2$, ki velja v Newtonovi mehaniki.

Skalarni produkt četverca gibalne količine ${}^4P = \left(\frac{W}{c_0}, \vec{P} \right)$ s samim seboj je invarianta:

$${}^4P \cdot {}^4P = \frac{-W}{c_0^2} + P^2 \quad = \text{ invarianta .} \tag{41}$$

V **lastnem sistemu** delec miruje torej je $P = 0$. Iz enačbe (41) v tem primeru sledi:

$$-\frac{W}{c_0^2} + \left(P^2 = 0\right) = -\frac{m_0^2 c_0^4}{c_0^2} = -\ m_0^2 c_0^2 . \tag{42}$$

Ker pa je ${}^4P \cdot {}^4P$ invarianta, velja v splošnem:

$$\boxed{-\frac{W}{c_0^2} + P^2 = -m_0^2 c_0^2} \quad , \tag{43}$$

oziroma

$$-\ W^2 + c_0^2 P^2 = -m_0^2 c_0^4 ,$$

ali drugače:

$$\boxed{W^2 = W_0^2 + c_0^2 P^2} \quad . \tag{44}$$

- **Enačbe gibanja**

  - Newtonova mehanika $\boxed{\vec{F} = \frac{\mathrm{d}\vec{G}}{\mathrm{d}t}}$, $\vec{G} = m\vec{v}$

  - Posebna teorija relativnosti $\left({}^4P = \left(\frac{W}{c_0}, \vec{P} \right), \vec{P} = m\gamma\vec{v} \right)$:

$$\boxed{{}^4F = \frac{\mathrm{d}\left({}^4P\right)}{\mathrm{d}\tau} = \gamma \frac{\mathrm{d}\left({}^4P\right)}{\mathrm{d}t} = \gamma \left(\frac{1}{c_0}\frac{\mathrm{d}W}{\mathrm{d}t}, \frac{\mathrm{d}\vec{P}}{\mathrm{d}t} \right)} \quad . \tag{45}$$

Četverec sile ${}^4F$ na točkasti delec z nabojem e v električnem in magnetnem polju:

$$\boxed{{}^4F = \left(\frac{e\gamma\vec{E}\cdot\vec{v}}{c_0}, e\gamma\left(\vec{E} + \vec{v}\times\vec{B}\right) \right)} \quad . \tag{46}$$

Izenačimo ločeno časovni in krajevni del med enačbama (45) in (46):

- časovni del: $\frac{\mathrm{d}W}{\mathrm{d}t} = e\vec{E}\cdot\vec{v} \;\Rightarrow\; \Delta W = e\int \vec{E}\cdot\vec{v}\,\mathrm{d}t = e\int \vec{E}\cdot\mathrm{d}\vec{r} = -e\Delta U,$ (47)

- krajevni del: $\boxed{\frac{\mathrm{d}\vec{P}}{\mathrm{d}t} = e\left(\vec{E} + \vec{v}\times\vec{B}\right)}$, $\vec{P} = m\gamma\vec{v}$, (48)

Ohranitvene enačbe:

$${}^4F = \frac{\mathrm{d}\left({}^4P\right)}{\mathrm{d}\tau} \;\Rightarrow\; \Delta^4P = \int {}^4F\,\mathrm{d}\tau$$

$$\text{če} \int {}^4F\,\mathrm{d}\tau = 0 \;\Rightarrow\; \Delta^4P = 0$$

$\nearrow$ $\boxed{\Delta W = 0}$ OHRANITEV POLNE ENERGIJE (49)

$\searrow$ $\boxed{\Delta\vec{P} = 0}$ OHRANITEV KRAJEVNEGA DELA GIBALNE KOLIČINE (50)

- **Uporaba enačb (48):**

$$\boxed{\frac{\mathrm{d}\vec{P}}{\mathrm{d}t} = e\left(\vec{E} + \vec{v}\times\vec{B}\right), \quad \vec{P} = m\gamma\vec{v}} \quad . \tag{48}$$

Posebna primera: $\vec{E} = 0$ ali $\vec{B} = 0$

- A) **Poseben primer:** če $\vec{E} = 0$ iz enačbe (48) sledi (kroženje v magnetnem polju):

$$\frac{\mathrm{d}\vec{P}}{\mathrm{d}t} = e\vec{v}\times\vec{B},$$

$$\boxed{\frac{\mathrm{d}\left(m_0\,\gamma\,\vec{v}\right)}{\mathrm{d}t} = e\,\vec{v}\times\vec{B}}\ , \tag{51}$$

ker $e\,\vec{v}\times\vec{B}\perp\vec{v},\mathrm{d}\vec{s}\ \Rightarrow\ \mathrm{d}A = \vec{F}\cdot\mathrm{d}\vec{s} = 0 \Rightarrow v = \text{konst.}$

$\gamma = \text{konst.}$

Torej: $\dfrac{\gamma\,\mathrm{d}\left(m_0\,\vec{v}\right)}{\mathrm{d}t} = e\,\vec{v}\times\vec{B}$,

$\gamma\,m_0\,\vec{a}_r = e\,\vec{v}\times\vec{B}$, kjer je $\vec{a}_r$ radialni pospešek,

$$\gamma\,m_0\,\frac{v^2}{r} = e\,v\,B$$

$$\gamma\ m_0\,v = e\,r\,B \Rightarrow \boxed{r = \frac{\gamma\,m_0\,v}{e\,B} = \frac{P}{e\,B}} \tag{52}$$

Obhodni čas: $t_0 = \dfrac{2\pi\,r}{v} = \dfrac{2\pi\,\gamma\,m_0\,v}{e\,B\,v} = \dfrac{2\pi\,\gamma\,m_0}{e\,B}$

Frekvenca: $\boxed{\nu = \dfrac{1}{t_0} = \dfrac{e\,B}{\gamma\,2\pi\,m_0}}$. (53)

B) **Poseben primer:** če $\vec{B} = 0$ iz enačbe (48) sledi:

$$\boxed{\frac{\mathrm{d}\left(m_0\gamma\,\vec{v}\right)}{\mathrm{d}t} = e\,\vec{E}}\ . \tag{54}$$

Enačbo (54) predelamo v obliko:

$$\mathrm{d}\left(m_0\gamma\,\vec{v}\right) = e\,\vec{E}\,\mathrm{d}t. \tag{55}$$

Če je $\vec{E}$ = konst. velja:

$$m_0\gamma\,\vec{v} = \int e\,\vec{E}\,\mathrm{d}t = e\,\vec{E}\,t\ , \tag{56}$$

oziroma

$$\boxed{\frac{m_0\, v}{\sqrt{1-\frac{v^2}{c_0^2}}} = e\, E\, t} \quad . \tag{57}$$

Iz enačbe (57) po krajšem računu dobimo:

$$v = c_0 \frac{\alpha\, t}{\sqrt{1+\alpha^2 t^2}} \quad , \tag{58}$$

kjer je

$$\alpha = \frac{e\, E}{m\, c_0} \quad . \tag{59}$$

Pot pa je:

$$\boxed{s = \int_0^t v\, \mathrm{d}t = \frac{c_0}{\alpha}\left[\left(\alpha^2\, t^2 + 1\right)^{\frac{1}{2}} - 1\right].} \quad . \tag{60}$$