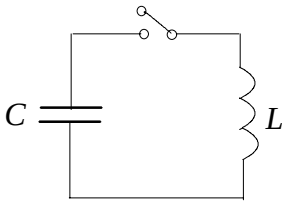
Idealni električni nihajni krog ($R = 0$)

$$\frac{d^2 I}{dt^2} = -\frac{1}{LC} I$$

$$U_C(t=0) = U_0$$



$$U_0 = L I_0 \omega_0$$

Nastavek za rešitev:

$$U_C = U_L = L I_0 \omega_0 \cos(\omega_0 t)$$

$$I = I_0 \sin(\omega_0 t)$$

$$U_L + U_C = 0$$

$$-L\frac{dI}{dt} - \frac{e}{C} = 0 \quad / \frac{d}{dt}$$

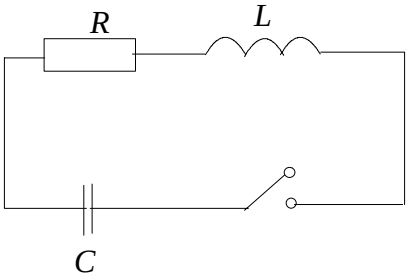
kjer smo upoštevali $I = \frac{de}{dt}$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$-L\frac{d^2 I}{dt^2} - \frac{I}{C} = 0 \ ,$$

$$e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$$

Dušeni električni nihajni krog



$$\beta = \frac{1}{2}\frac{R}{L}$$

Vzamemo: I_0 je realen, U_0 je kompleksen

$$\omega_0'^2 = \sqrt{\omega_0^2 - \beta^2}$$

$$U_L + U_R + U_C = 0 \ ,$$

$$-L\frac{dI}{dt} - RI - \frac{e}{C} = 0 \quad / \frac{d}{dt} \ ,$$

Kompleksne impedance:

$$\omega^2 = \frac{1}{LC} = \omega_0^2 \ ,$$

$$Z = Z_R + Z_L + Z_C$$
$$U_0 = Z I_0$$

$$Z_R = R$$
$$Z_L = i\omega L$$
$$Z_C = -i\frac{1}{\omega C}$$

$$-L\frac{d^2 I}{dt^2} - R\frac{dI}{dt} - \frac{I}{C} = 0$$

$$tg\,\delta = \frac{\mathrm{Im}(I_0)}{\mathrm{Re}(I_0)} = \left(\omega C - \frac{1}{\omega L}\right) R$$

kjer smo upoštevali $I = \frac{de}{dt}$.

$$\bar{j} = \bar{w} c_0 = \varepsilon_0 \overline{E_0^2}\, c_0 = \varepsilon_0 E_0^2 \overline{\cos^2(\omega t - kx)} c_0 = \frac{1}{2}\varepsilon_0 E_0^2 c_0 \ ,$$

$$\bar{P} = \frac{1}{2} I_0 U_0 \cos\delta$$

Nastavek za rešitev:

$$I = I_0 e^{-\beta t} \cos(\omega_0' t)$$

$$E' = \frac{2n}{(n+n')} E$$

$$c = \frac{c_0}{n(\lambda)}$$

$$c = \frac{1}{\sqrt{\varepsilon \ \varepsilon_0 \ \mu_0}} = \frac{c_0}{n}$$

$$E'' = \left(\frac{n-n'}{n+n'}\right) E$$

$$2nh\cos\beta = \frac{\lambda_0}{2n}(2N-1)$$

$$z = \frac{1}{4}(2N-1)\lambda_0$$

$$w = w_E + w_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

$$h = (2N+1)\frac{\lambda_0}{4n} \ ,$$

$$j(x) = j_0 e^{-\mu x}$$

$$x_{1/2} = \frac{\ln 2}{\mu}$$

$$W_p \cong \frac{1}{2}K(r - r_0)^2 - C$$

$$E = j\cos\alpha$$

Infinitezimalni prostorski kot:

$$I = B(\beta) S' = B(\beta) S \cos\beta$$

$$E = \frac{P}{S} = \frac{P}{(S_0/\cos\alpha)} = \left(\frac{P}{S_0}\right)\cos\alpha \ .$$

$$d\Omega = \frac{dS}{r^2}$$

$$dj = \frac{dI}{r^2}$$

enačba leče

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

Brez očal: a > a_0 , da je b normalen

Točkasto svetilo, ki seva enakomerno na vse strani:

$$j = \frac{P}{S} = \frac{P}{4\pi r^2} = \frac{I}{r^2}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

$$dI = \frac{dP}{d\Omega}$$

povečava: $\frac{y}{y'} = \frac{b}{a} = \frac{f}{a-f}$

Z očali: a = a_0.

kjer je svetilnost točkastega svetila:

$$I = \frac{P}{4\pi}$$

$$\frac{1}{a_0} + \frac{1}{b} = \frac{1}{f} + \frac{1}{f'}$$

povečava $N = \frac{tg\,\alpha'}{tg\,\alpha} = \frac{a_0}{f'}$

$$dE = \frac{dI}{r^2}\cos\alpha$$

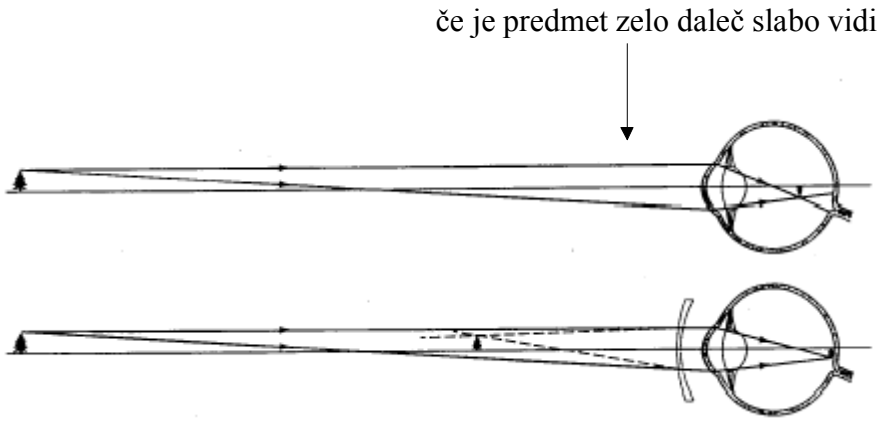
$$dI = B\,dS\cos\beta$$

Odštejemo obe enačbi in dobimo:

dioptrija: $D = \frac{1}{f'} = \frac{a - a_0}{a_0 a} > 0$ zbiralna leča

- **Kratkovidno oko**

Dobro vidi samo bližnje predmete.



Brez očal: $a < a_0$, da je b normalen

$$\boxed{\frac{1}{a}+\frac{1}{b}=\frac{1}{f}}$$

Z očali: $a >> a_0$:

$$\boxed{\frac{1}{\infty}+\frac{1}{b}=\frac{1}{f}+\frac{1}{f'}}$$

Odštejemo obe enačbi in dobimo:

$$-\frac{1}{a}=\frac{1}{f'} \quad \Rightarrow \quad \boxed{D=\frac{1}{f'}=-\frac{1}{a}<0}$$

Meglična celica, selektor hitrosti, masni spektrometer…

- **Meglična celica**

Mehanizem delovanja

zraku z vodno paro sunkovito povečamo prostornino, da se ohladi

$\Downarrow$

z vodno paro nasičeni zrak postane prenasičen

$\Downarrow$

vodna para se useda na gibajočih se ionih in tvori vodne kapljice (kondenzacija)

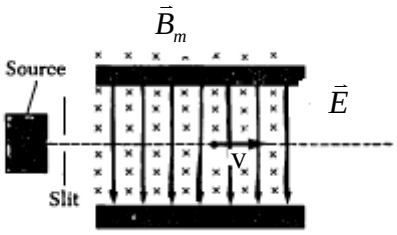
$\Downarrow$

sledovi nabitih delcev lahko postanejo **vidni**

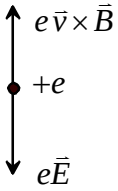
$\odot$
$\vec{B}$ magnetno polje

Bubble chamber photograph. The spiral tracks at the bottom of the photograph are an electron – positron pair (left an right respectively), formed by a gamma ray interacting with a hydrogen nucleus. An applied magnetic field causes the electron and the positron to be defected in opposite directions. The track leaving from the cusp between the two spirals is an additional electron knocked out of a hydrogen atom during this interaction. (G. Holton, F.J. Rutherford, F.G. Watston, Project Physics, New York: HRW, 1981.)

- **Selektor hitrosti** (izbira delcev z enako hitrostjo)



a)



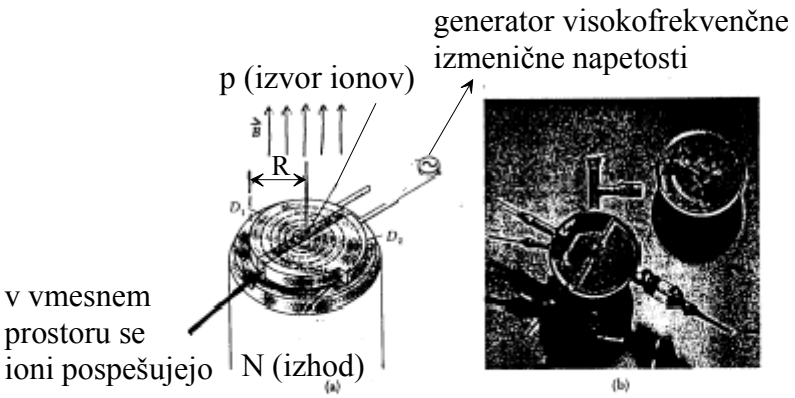
$$e\,v\,B = e\,E \;\Rightarrow\; \boxed{v = \frac{E}{B}}$$

b)

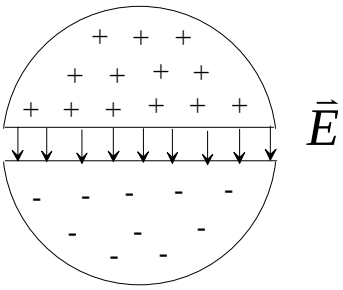
(a) A velocity selector. When a positively charged particle is in the presence of both an inward magnetic field as indicated by the blue crosses, and a downward electric field indicated by the red arrows, it experiences both an electric force $e\vec{E}$ downward and a magnetic force $e\vec{v}\times\vec{B}$ upward. (b) When these forces balance each other as shown here, the particle moves in a horizontal line through the fields.

- **Ciklotron**

2 votli elektrodi D_1, D_2 v evakuiranem prostoru. V elektrodah ni električnega polja!



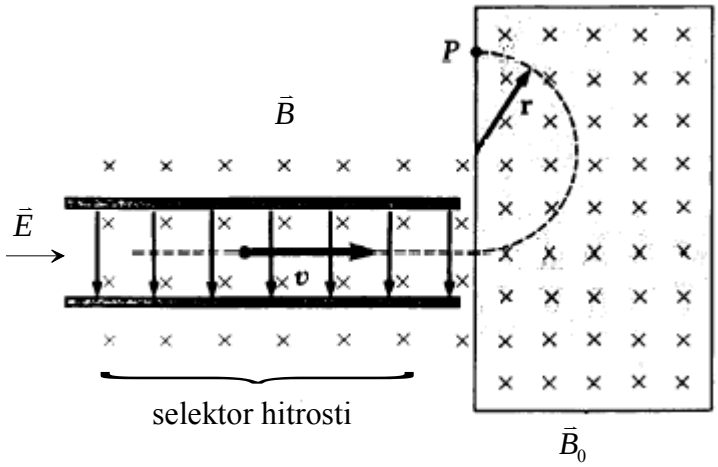
(a) The cyclotron consists of an ion source, two dees across which an alternating voltage is applied, and a uniform magnetic field provided by an electromagnet. (The south pole of the magnet is not shown.) (b) The first cyclotron, invented by E.O. Lawrence and M.S. Livingston in 1934. (Courtesy of Lawrence Berkeley Laboratory, University of California)



$$v \perp \vec{B} \;\Rightarrow\; \text{gibanje po krogu}$$

$$\left(\vec{F} = e\,\vec{v}\times\vec{B}\right) \perp \mathrm{d}\vec{s} \;\Rightarrow\; \mathrm{d}A = \vec{F}\cdot d\vec{s} = 0 \;\Rightarrow W_k = \text{konst.}$$

- **Masni spektrometer**



A mass spectrometer. Charged particles are first sent through a velocity selector. They then enter a region where the magnetic field $\boldsymbol{B}_0$ (inward) causes positive ions to move in a semicircular path and strike a photographic film at P.

Selektor hitrosti: $e\,v\,B = e\,E \quad \Rightarrow \quad \boxed{v = \frac{E}{B}}$ (1)

Newtonov zakon:

$$m\,a_r = e\,v\,B_0$$

$$m\frac{v^2}{r} = e\,v\,B_0$$

$$\Downarrow$$

$$\boxed{r = \frac{m\,v}{e\,B_0}} \qquad (2)$$

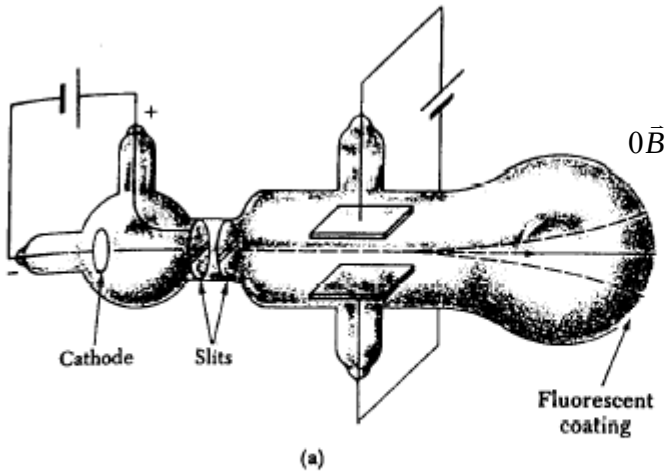
$$r = \frac{m\,v}{e\,B_0} = \frac{m\,E}{e\,B_0\,B} \quad \Rightarrow \quad \boxed{\frac{m}{e} = \frac{r\,B_0\,B}{E}}$$, kjer smo upoštevali enačbo (1)

- **Thomsonov aparat za merjenje razmerja** $\frac{e}{m}$ (specifični naboj)

katodni žarki $\equiv$ elektroni

Thomson: $\frac{e}{m} \sim 0.77 \cdot 10^{11} \frac{\text{As}}{\text{kg}}$

odkritelj elektrona
J.J. Thomson (1856 - 1940)



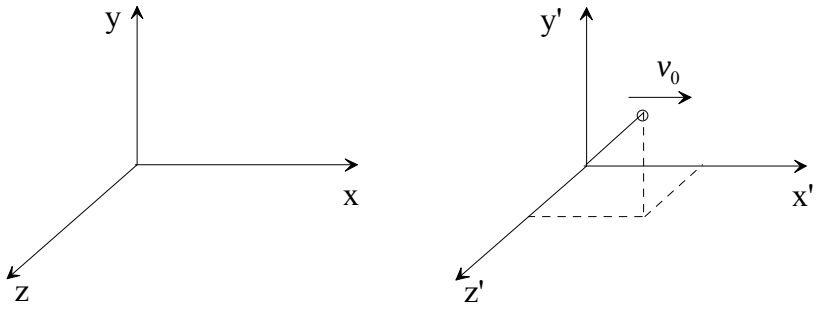
(a) Thomson's apparatus for measuring e/m. Electrons are accelerated from the cathode, pass through two slits, and are deflected by both an electric field and a magnetic field (not shown, but directed into the paper). The deflected beam then strikes a phosphorescent screen. (b) J.J. Thomson in the Cavendish Laboratory, University of Cambridge.

Relativnostna mehanika

- **Klasična fizika**

  - **predpostavka o univerzalnem času**

$$\boxed{t = t'}$$



  - inercialni koordinatni sistem: $\vec{v} = \text{konst.}$
  - **inercialni sistem S:** (x, y, z) ≡ mirujoči koordinatni sistem
  - **inercialni sistem S':** (x', y', z') ≡ gibajoči se koordinatni sistem (hitrost v_0)

GALILEJEVA TRANSFORMACIJA:

$$x' = x - v_0 \; t$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

$$v' = \frac{\mathrm{d}x'}{\mathrm{d}t} = \frac{\mathrm{d}(x - v_0 \, t)}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}t} - v_0 = v - v_0$$

$$\boxed{v' = v - v_0}$$

  - hitrosti se seštevajo (ni zgornje meje!)
  - oblika zakonov (Newtonovih zakonov) je **neodvisna** od hitrosti premikanja koordinatnega sistema.

- **Posebna teorija relativnosti**

A. Einstein (1905 – annus mirabilis)

Dve glavni načeli:

1. **Načelo relativnosti:** fizikalni zakoni imajo enako obliko v vseh inercialnih opazovalnih sistemih.
2. **Hitrost svetlobe** v praznem prostoru $c_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ je enaka v vseh inercialnih opazovalnih sistemih.

LORENTZOVA TRANSFORMACIJA:

$$x' = \gamma_0 \left(x - v_0 t \right) = \gamma \left(x - \frac{v_0}{c_0} c_0 t \right) \tag{1}$$

$$y' = y \tag{2}$$

$$z' = z \tag{3}$$

$$c_0 t' = \gamma_0 \left(c_0 t - \frac{v_0}{c_0} x \right) \tag{4}$$

$$\boxed{\gamma_0 = \frac{1}{\sqrt{1 - \frac{v_0^2}{c_0^2}}}} \tag{5}$$

- Vsak inercialni sistem ima svoj **lastni čas.**

OBRATNA LORENTZOVA TRANSFORMACIJA

$$x = \gamma_0 \left(x' + \frac{v_0}{c_0} c_0 t' \right) \tag{6}$$

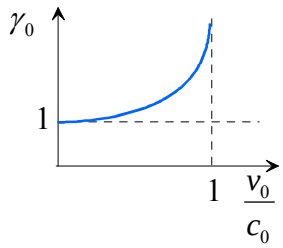
$$y = y' \tag{7}$$

$$z = z' \tag{8}$$

$$c_0 t = \gamma_0 \left(c_0 t' + \frac{v_0}{c_0} x' \right) \tag{9}$$

Zahteva, da preide nova Lorentzova transformacija pri majhnih hitrostih v **GALILEJEVO TRANSFORMACIJO** je izpolnjena:

$$\gamma_0 = \frac{1}{\left(1-\frac{v_0^2}{c_0^2}\right)^{1/2}} \underset{\frac{v_0}{c_0}\to 0}{\cong} 1+\frac{1}{2}\frac{v_0^2}{c_0^2}+.... \to 1$$



DODATEK: Izpeljava γ_0 (enačba (5)) in enačbe (4)

Na osnovi enačb (1) in (6):

$$x' = \gamma_0 \left(x - v_0 t\right), \tag{D1}$$

$$x = \gamma_0 \left(x' + v_0 t'\right), \tag{D2}$$

in načelo o invariantnosti svetlobne hitrosti

$$c_0 = c_0' \tag{D3}$$

lahko izpeljemo vrednost γ_0 kot sledi. Najprej vstavimo x' iz enačbe (D1) v enačbo (D2):

$$x = \gamma_0 \left[\gamma_0 \left(x - v_0 t\right) + v_0 t'\right] = \gamma_0^2 x - \gamma_0^2 v_0 t + \gamma_0 v_0 t' , \tag{D4}$$

torej:

$$\gamma_0 v_0 t' = x\left(1-\gamma_0^2\right) + \gamma_0^2 v_0 t . \tag{D5}$$

Iz enačbe (D5) sledi:

$$\boxed{t' = \gamma_0 \left(t - \frac{\left(\gamma_0^2 - 1\right)}{\gamma_0^2 v_0} x\right).} \tag{D6}$$

Naredimo še diferencial enačbe (D6):

$$\mathrm{d}t' = \gamma_0 \left(\mathrm{d}t - \frac{\left(\gamma_0^2 - 1\right)}{\gamma_0^2 v_0} \mathrm{d}x\right) . \tag{D7}$$

V nadaljevanju upoštevamo načelo invariantnosti svetlobne hitrosti (enačba (D3)) in enačbo (D7). Dobimo:

$$c_0 = c_0^{'} = \frac{\mathrm{d}x'}{\mathrm{d}t'} = \frac{\gamma_0\left(\mathrm{d}x - v_0\mathrm{d}t\right)}{\gamma_0\left(\mathrm{d}t - \frac{\left(\gamma_0^2 - 1\right)}{\gamma_0^2\, v_0}\mathrm{d}x\right)} = \frac{c_0 - v_0}{1 - \frac{\left(\gamma_0^2 - 1\right)}{\gamma_0^2\, v_0}c_0} = \qquad \text{D(8)}$$

$$= \frac{c_0 - v_0}{1 - \frac{\left(\gamma_0^2 - 1\right)}{\gamma_0^2\, v_0}c_0}.$$

Iz enačbe (D8) sledi:

$$1 - \frac{\left(\gamma_0^2 - 1\right)}{\gamma_0^2\, v_0}c_0 = \frac{c_0 - v_0}{c_0} = 1 - \frac{v_0}{c_0} \quad , \qquad \text{(D9)}$$

oziroma:

$$\frac{\left(\gamma_0^2 - 1\right)}{\gamma_0^2\, v_0}c_0 = \frac{v_0}{c_0} \quad . \qquad \text{(D10)}$$

Iz enačbe (D10) lahko izračunamo γ_0:

$$\frac{\gamma_0^2 - 1}{\gamma_0^2} = 1 - \frac{1}{\gamma_0^2} = \frac{v_0^2}{c_0^2} \quad , \qquad \text{(D11)}$$

$$\frac{1}{\gamma_0^2} = 1 - \frac{v_0^2}{c_0^2} \ ,$$

$$\boxed{\gamma_0^2 = \frac{1}{1 - \frac{v_0^2}{c_0^2}} \quad ,} \qquad \text{(D12)}$$

oziroma

$$\boxed{\gamma_0 = \frac{1}{\sqrt{1 - \frac{v_0^2}{c_0^2}}} \quad .} \qquad \text{(D13)}$$

Če vstavimo relacijo (glejte enačbo (D11))

$$\frac{\gamma_0^2 - 1}{\gamma_0^2} = \frac{v_0^2}{c_0^2}$$

v enačbo (D6) dobimo:

$$\boxed{t' = \gamma_0 \left(t - \frac{v_0}{c_0^2} x \right),} \tag{D14}$$

oziroma enačba (4):

$$\boxed{c_0 t' = \gamma_0 \left(c_0 t - \frac{v_0}{c_0} x \right)} \tag{D15}$$

- **Transformacije hitrosti**

Iz enačb (1) – (4) sledi

Infinitezimaln Lorentzova transformacija

$$\mathrm{d}t' = \gamma_0 \left(\mathrm{d}t - \frac{v_0}{c_0^2} \mathrm{d}x \right) \tag{10}$$

$$\mathrm{d}x' = \gamma_0 \left(\mathrm{d}x - v_0 \, \mathrm{d}t \right) \tag{11}$$

$$\mathrm{d}y' = \mathrm{d}y \tag{12}$$

$$\mathrm{d}z' = \mathrm{d}z \tag{13}$$

$$v_x' = \frac{\mathrm{d}x'}{\mathrm{d}t'} \qquad v_y' = \frac{\mathrm{d}y'}{\mathrm{d}t'} \qquad v_z' = \frac{\mathrm{d}z'}{\mathrm{d}t'} \tag{14}$$

$$v_x = \frac{\mathrm{d}x}{\mathrm{d}t} \qquad v_y = \frac{\mathrm{d}y}{\mathrm{d}t} \qquad v_z = \frac{\mathrm{d}z}{\mathrm{d}t} \tag{15}$$

Iz enačb (10) – (15) sledi:

$$v_x' = \frac{\mathrm{d}x'}{\mathrm{d}t'} = \frac{\gamma_0 \left(\mathrm{d}x - v_0 \mathrm{d}t \right)}{\gamma_0 \left(\mathrm{d}t - \frac{v_0}{c_0^2} \mathrm{d}x \right)} =$$

$$= \frac{\mathrm{d}x - v_0 \mathrm{d}t}{\mathrm{d}t - \frac{v_0}{c_0^2} \mathrm{d}x} = \frac{\frac{\mathrm{d}x}{\mathrm{d}t} - v_0}{1 - \frac{v_0}{c_0^2} \frac{\mathrm{d}x}{\mathrm{d}t}} = \frac{v_x - v_0}{1 - \frac{v_0 v_x}{c_0^2}},$$

torej

$$\boxed{v_x = \frac{v_x - v_0}{1 - \frac{v_0 v_x}{c_0^2}}} \tag{16}$$

Podobno lahko dobimo še ostale transformacije za hitrosti**:**

$$v_y' = \frac{v_y}{\gamma_0 \left(1 - \frac{v_0 v_x}{c_0^2} \right)} \tag{17}$$

$$v_z' = \frac{v_z}{\gamma_0 \left(1 - \frac{v_0 v_x}{c_0^2} \right)} \tag{18}$$

Obratna transformacija $\left(v_0 \to -v_0 \right)$:

$$v_x' = \frac{v_x' + v_0}{\left(1 + \frac{v_0 v_x'}{c_0^2} \right)} \tag{19}$$

$$v_y = \frac{v_y'}{\gamma_0 \left(1 + \frac{v_0 v_x'}{c_0^2} \right)} \tag{20}$$

$$v_z = \frac{v_z'}{\gamma_0 \left(1 + \frac{v_0 v_x'}{c_0^2} \right)} \tag{21}$$

- transformacija vsebuje NAČELO O INVARIANTNOSTI SVETLOBNE HITROSTI v praznem prostoru $c_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$:

$$v_x = \frac{c_0 + v_0}{1 + \frac{v_0 c_0}{c_0^2}} = \frac{c_0 + v_0}{1 + \frac{v_0}{c_0}} = \frac{c_0 \left(c_0 + v_0 \right)}{\left(c_0 + v_0 \right)} = c_0$$

FORMALIZEM: 4-razsežni prostor - čas

Svetovni četverec definiran kot:

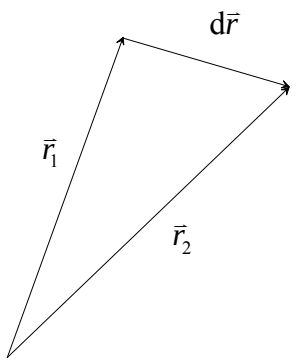
$${}^4 x = \left(c_0 t, x, y, z \right) = \left(c_0 t, \vec{r} \right)$$

Lorentzova transformacija:

$$\boxed{\begin{aligned} c_0 t' &= \gamma_0 \left(c_0 t - \beta x\right) \\ x' &= \gamma_0 \left(x - \beta c_0 t\right) \\ y' &= y \\ z' &= z \end{aligned}}$$

kjer je $\boxed{\beta = \frac{v_0}{c_0}}$

- **Klasična** 3-D slika: Razdalja med dvema točkama $\mathrm{d}r = \sqrt{\mathrm{d}\vec{r} \cdot \mathrm{d}\vec{r}}$ je neodvisna od izbire opazovalnega sistema. Pravimo, da je $\mathrm{d}r^2 = \mathrm{d}\vec{r} \cdot \mathrm{d}\vec{r}$ invarianta.



- **Specialna teorija relativnosti**

Invarianta je:

$$\mathrm{d}s^2 = -c_0^2 \mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 = -c_0 \, \mathrm{d}t'^2 + \mathrm{d}x'^2 + \mathrm{d}y'^2 + \mathrm{d}z'^2 \ . \qquad (22)$$

Novost: negativni znak pred prvim členom, klasično je namreč

$$\mathrm{d}r^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 = \mathrm{d}\vec{r} \cdot \mathrm{d}\vec{r} \ .$$

- **Skrčenje dolžin**

Inercialna opazovalna sistema S in S':

$$\mathrm{S}: \quad \left.\begin{aligned} &\text{dogodek 1}: \ t_1 = 0, \ x_1 = 0 \\ &\text{dogodek 2}: \ t_2 = \frac{v_0 L}{c_0^2}, \ x_2 = L \end{aligned}\right\} \Rightarrow \text{palica miruje v sistemu S}: \quad \mathrm{x}_2 - x_1 \equiv L$$

Lorentzova transformacija:

$$S': \left.\begin{array}{l} \text{dogodek 1: } t_1' = 0,\ x_1' = 0 \\ \text{dogodek 2: } t_2' = 0,\ x_2' = \dfrac{L}{\gamma_0} \end{array}\right\} \Rightarrow x_2' - x_1' = \frac{L}{\gamma_0} = L'$$

ZAKLJUČKI:

- izmerjena dolžina je odvisna od hitrosti gibanja koordinatnega sistema,
- dogodka sočasna samo, če se dogodita v isti točki,
- opazovalcu se zdi gibajoča se palica skrčena:

$$\boxed{L' = \frac{L}{\gamma_0}} \tag{23}$$

- sočasnost dogodkov je relativen pojem

o **Podaljšanje časa** (inercialna sistema S in S')

$$S: \left.\begin{array}{l} \text{prvi dogodek: mesto } x,\ \text{čas } t_1 \\ \text{drugi dogodek: mesto } x,\ \text{čas } t_2 \end{array}\right\} \Delta t = t_2 - t_1$$

Lorentzova transformacija:

$$S': \begin{array}{l} t_1' = \gamma_0 \left(t_1 - \dfrac{v_0 x}{c_0^2} \right) \\ t_2' = \gamma_0 \left(t_2 - \dfrac{v_0 x}{c_0^2} \right) \end{array}$$

torej: $t_2' - t_1' = \gamma_0 \left(t_2 - t_1 \right)$

$$\boxed{\Delta t' = \gamma_0 \Delta t} \tag{24}$$

- $\Delta t' > \Delta t \Rightarrow$ za gibajočega opazovalca dogodki potekajo počasneje kot za opazovalca, ki miruje ob dogodkih $\rightarrow$ **čas je relativna količina.**
- **lastni čas** $\equiv$ čas, ki ga kaže ura v izbranem koordinatnem sistemu.

ZAKLJUČKI:

- **Popolna novost** specialne teorije relativnosti je, da se **tudi čas transformira.**
- **Prostor** in **čas** sta povezana v skupni **prostor - čas.**
- Svetovni četverec popisuje dogodek.

- **Zakoni gibanja**

Na osnovi enačbe (24) naredimo posplošitev in definiramo lastni časovni razmik z enačbo:

$$\boxed{\mathrm{d}t = \gamma\,\mathrm{d}\tau} \quad , \tag{25}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c_0^2}}} \quad , \tag{26}$$

kjer je:

$\mathrm{d}t \equiv$ čas, ki ga meri opazovalec, ki opazuje gibanje točkastega telesa
$\mathrm{d}\tau \equiv$ lastni čas (lastni časovni razmik, ki ga izmeri opazovalec, ki se giblje skupaj z opazovanim gibajočim točkastim telesom)

Novost: za hitrost v izrazu (26) dopuščamo, da **ni** konstantna, kar pomeni, da se telo lahko giblje pospešeno.

- **Četverec hitrosti ^4v**

svetovni četverec $\quad {}^4x = (c_0 t, x, y, z)$ (27)
odvajamo po lastnem času τ :

$$^4v = \frac{\mathrm{d}\left(^4x\right)}{\mathrm{d}\tau} = \gamma\frac{\mathrm{d}\left(^4x\right)}{\mathrm{d}t} = \gamma\left(c_0, v_x, v_y, v_z\right) , \tag{28}$$

kjer so

$$v_x = \frac{\mathrm{d}x}{\mathrm{d}t}, \quad v_y = \frac{\mathrm{d}y}{\mathrm{d}t}, \quad v_z = \frac{\mathrm{d}z}{\mathrm{d}t} \; .$$

Enačbo (28) zapišemo v vektorski obliki:

četverec hitrosti $\quad \boxed{^4v = (\gamma c_0, \gamma\bar{v})} \quad , \tag{29}$

kjer je $\bar{v} = \left(v_x, v_y, v_z\right)$.

Pri majhnih hitrostih $\frac{v}{c_0} \to 0$:

$\gamma \to 1$,

torej:

$$\gamma\bar{v} \to \bar{v} \; , \tag{30}$$
$$\mathrm{d}\tau \to \mathrm{d}t \; . \tag{31}$$

Opomba:

koordinatni čas (t ali dt) ni skalar ampak komponenta 4-D vektorja $(c_0 t,\ x,\ y,\ z)$. Odvajanje po koordinatnem času dt zato **ne** proizvede novega vektorja četverca. Zato odvajamo po lastnem času $(\mathrm{d}\tau)$. Lastni čas (τ) je skalar, ki preide pri majhnih hitrostih v koordinatni čas t.

- **Četverec gibalne količine**

$$\boxed{{}^4P = m_0\,{}^4v = (m_0\,\gamma\,c_0,\ m_0\,\gamma\,\vec{v})} \quad . \tag{32}$$

Izraz $m_0\,\gamma\,\vec{v}$ zapišemo v obliki:

$$\boxed{\vec{P} = m\,\vec{v}} \quad , \tag{33}$$

kjer je

$$\boxed{m = m_0\,\gamma} \quad , \tag{34}$$

$m_0 \equiv$ mirovna ali lastna masa telesa.

Pri majhnih hitrostih $\dfrac{v}{c_0} \to 0$:

$$\begin{aligned} &\gamma \to 1\ , \\ &m \to m_0\ , \\ &P \to m_0 v\ . \end{aligned} \tag{35}$$

Vidimo, da pri majhnih hitrostih P preide v klasično gibalno količino $G = m_0\,v$.

Opomba:

kljub temu, da $v \le c_0$ je lahko relativistična gibalna količina poljubno velika, ker se lahko poljubno veča zaradi naraščajoče vrednosti γ .

- **Polna in lastna energija**

Četverec gibalne količine lahko pišemo tudi kot (glejte enačbo (32)):

$${}^4P = \left(\frac{W}{c_0}, m_0\,\gamma\,\vec{v}\right), \tag{36}$$

kjer je W **polna** energija delca:

$$\boxed{W = m_0\,\gamma\,c_0^2} \quad . \tag{37}$$

Polna energija je odvisna od hitrosti. Najmanjša polna energija ima delec, ki miruje je $(\gamma = 1)$:

$$\boxed{W_0 = m_0\ c_0^2} \quad , \tag{38}$$

$W_0 \equiv$ **lastna** ali **mirovna** energija delca. Sedaj definiramo **kinetično** energijo delca kot:

$$\boxed{W_k = W - W_0 = m_0\,\gamma\, c_0^2 - m_0\ c_0^2 = m_0\ c_0^2\,(\gamma - 1)\ .} \tag{39}$$

Kinetična energija je torej tisti del polne energije, ki je **posledica gibanja.**

Pri majhnih hitrostih $\left(\frac{v}{c_0} \to 0\right)$ velja razvoj:

$$W_k = m_0\ c_0^2\,(\gamma - 1) = m_0\ c_0^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c_0^2}}} - 1 \right) \cong m_0\ c_0^2 \left(1 + \frac{1}{2}\frac{v^2}{c_0^2} + ... - 1 \right) \cong \frac{1}{2} m_0\, v^2 . \tag{40}$$

Vidimo, da relativistični izraz za kinetično energijo telesa $W_k = m_0\ c_0^2\,(\gamma - 1)$ preide pri $\frac{v}{c_0} \to$ v klasični izraz $W_k = \frac{1}{2} m_0\, v^2$, ki velja v Newtonovi mehaniki.

Skalarni produkt četverca gibalne količine ${}^4P = \left(\frac{W}{c_0}, \vec{P} \right)$ s samim seboj je invarianta:

$${}^4P \cdot {}^4P = \frac{-W}{c_0^2} + P^2 \quad = \text{ invarianta .} \tag{41}$$

V **lastnem sistemu** delec miruje torej je $P = 0$. Iz enačbe (41) v tem primeru sledi:

$$-\frac{W}{c_0^2} + \left(P^2 = 0\right) = -\frac{m_0^2 c_0^4}{c_0^2} = -\ m_0^2 c_0^2 . \tag{42}$$

Ker pa je ${}^4P \cdot {}^4P$ invarianta, velja v splošnem:

$$\boxed{-\frac{W}{c_0^2} + P^2 = -m_0^2 c_0^2} \quad , \tag{43}$$

oziroma

$$-\ W^2 + c_0^2 P^2 = -m_0^2 c_0^4 ,$$

ali drugače:

$$\boxed{W^2 = W_0^2 + c_0^2 P^2} \quad . \tag{44}$$

- **Enačbe gibanja**

  - Newtonova mehanika $\boxed{\vec{F} = \frac{\mathrm{d}\vec{G}}{\mathrm{d}t}}$, $\vec{G} = m\vec{v}$

  - Posebna teorija relativnosti $\left({}^4P = \left(\frac{W}{c_0}, \vec{P} \right), \ \vec{P} = m\gamma\vec{v} \right)$:

$$\boxed{{}^4F = \frac{\mathrm{d}\left({}^4P\right)}{\mathrm{d}\tau} = \gamma \frac{\mathrm{d}\left({}^4P\right)}{\mathrm{d}t} = \gamma \left(\frac{1}{c_0}\frac{\mathrm{d}W}{\mathrm{d}t}, \frac{\mathrm{d}\vec{P}}{\mathrm{d}t} \right)} \quad . \tag{45}$$

Četverec sile ${}^4F$ na točkasti delec z nabojem e v električnem in magnetnem polju:

$$\boxed{{}^4F = \left(\frac{e\gamma\vec{E}\cdot\vec{v}}{c_0}, \ e\gamma\left(\vec{E} + \vec{v}\times\vec{B}\right) \right)} \quad . \tag{46}$$

Izenačimo ločeno časovni in krajevni del med enačbama (45) in (46):

- časovni del: $\frac{\mathrm{d}W}{\mathrm{d}t} = e\vec{E}\cdot\vec{v} \ \Rightarrow \ \Delta W = e\int \vec{E}\cdot\vec{v}\,\mathrm{d}t = e\int \vec{E}\cdot\mathrm{d}\vec{r} = -e\Delta U,$ (47)

- krajevni del: $\boxed{\frac{\mathrm{d}\vec{P}}{\mathrm{d}t} = e\left(\vec{E} + \vec{v}\times\vec{B}\right)}$, $\vec{P} = m\gamma\vec{v}$, (48)

Ohranitvene enačbe:

$${}^4F = \frac{\mathrm{d}\left({}^4P\right)}{\mathrm{d}\tau} \ \Rightarrow \ \Delta^4P = \int {}^4F\,\mathrm{d}\tau$$

$$\text{če} \int {}^4F\,\mathrm{d}\tau = 0 \ \Rightarrow \Delta^4P = 0$$

$\nearrow$ $\boxed{\Delta W = 0}$ OHRANITEV POLNE ENERGIJE (49)

$\searrow$ $\boxed{\Delta\vec{P} = 0}$ OHRANITEV KRAJEVNEGA DELA GIBALNE KOLIČINE (50)

- **Uporaba enačb (48):**

$$\boxed{\frac{\mathrm{d}\vec{P}}{\mathrm{d}t} = e\left(\vec{E} + \vec{v}\times\vec{B}\right), \ \ \vec{P} = m\gamma\vec{v}} \quad . \tag{48}$$

Posebna primera: $\vec{E} = 0$ ali $\vec{B} = 0$

- A) **Poseben primer:** če $\vec{E} = 0$ iz enačbe (48) sledi (kroženje v magnetnem polju):

$$\frac{\mathrm{d}\vec{P}}{\mathrm{d}t} = e\vec{v}\times\vec{B},$$

$$\boxed{\frac{\mathrm{d}\left(m_0\,\gamma\,\vec{v}\right)}{\mathrm{d}t} = e\,\vec{v}\times\vec{B}}\ , \tag{51}$$

ker $e\,\vec{v}\times\vec{B}\perp\vec{v},\mathrm{d}\vec{s}\ \Rightarrow\ \mathrm{d}A = \vec{F}\cdot\mathrm{d}\vec{s} = 0 \Rightarrow v = \text{konst.}$

$\gamma = \text{konst.}$

Torej: $\dfrac{\gamma\,\mathrm{d}\left(m_0\,\vec{v}\right)}{\mathrm{d}t} = e\,\vec{v}\times\vec{B}$,

$\gamma\,m_0\,\vec{a}_r = e\,\vec{v}\times\vec{B}$, kjer je $\vec{a}_r$ radialni pospešek,

$$\gamma\,m_0\,\frac{v^2}{r} = e\,v\,B$$

$$\gamma\ m_0\,v = e\,r\,B \Rightarrow \boxed{r = \frac{\gamma\,m_0\,v}{e\,B} = \frac{P}{e\,B}} \tag{52}$$

Obhodni čas: $t_0 = \dfrac{2\pi\,r}{v} = \dfrac{2\pi\,\gamma\,m_0\,v}{e\,B\,v} = \dfrac{2\pi\,\gamma\,m_0}{e\,B}$

Frekvenca: $\boxed{\nu = \dfrac{1}{t_0} = \dfrac{e\,B}{\gamma\,2\pi\,m_0}}$. (53)

B) **Poseben primer:** če $\vec{B} = 0$ iz enačbe (48) sledi:

$$\boxed{\frac{\mathrm{d}\left(m_0\gamma\,\vec{v}\right)}{\mathrm{d}t} = e\,\vec{E}}\ . \tag{54}$$

Enačbo (54) predelamo v obliko:

$$\mathrm{d}\left(m_0\gamma\,\vec{v}\right) = e\,\vec{E}\,\mathrm{d}t. \tag{55}$$

Če je $\vec{E}$ = konst. velja:

$$m_0\gamma\,\vec{v} = \int e\,\vec{E}\,\mathrm{d}t = e\,\vec{E}\,t\ , \tag{56}$$

oziroma

$$\boxed{\frac{m_0\, v}{\sqrt{1-\frac{v^2}{c_0^2}}} = e\, E\, t} \quad . \tag{57}$$

Iz enačbe (57) po krajšem računu dobimo:

$$v = c_0 \frac{\alpha\, t}{\sqrt{1+\alpha^2 t^2}} \quad , \tag{58}$$

kjer je

$$\alpha = \frac{e\, E}{m\, c_0} \quad . \tag{59}$$

Pot pa je:

$$\boxed{s = \int_0^t v\, \mathrm{d}t = \frac{c_0}{\alpha}\left[\left(\alpha^2\, t^2 + 1\right)^{\frac{1}{2}} - 1\right].} \quad . \tag{60}$$