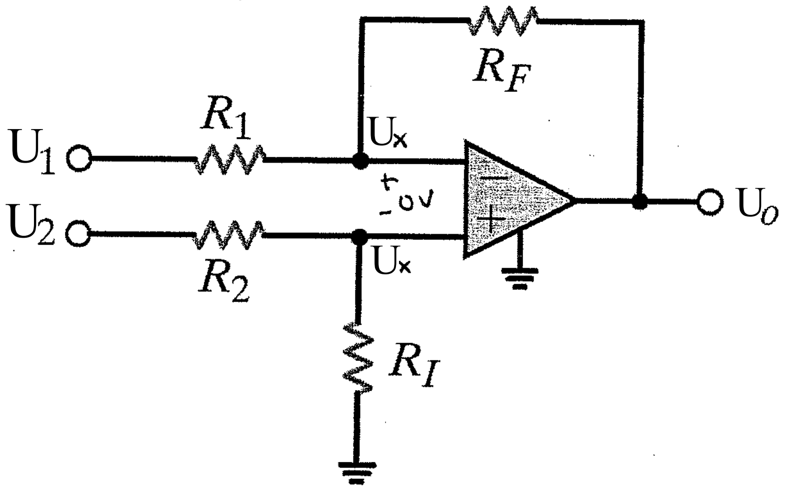
3.1 In the network in the figure derive the expression for U_o in terms of the inputs U_1 and U_2.



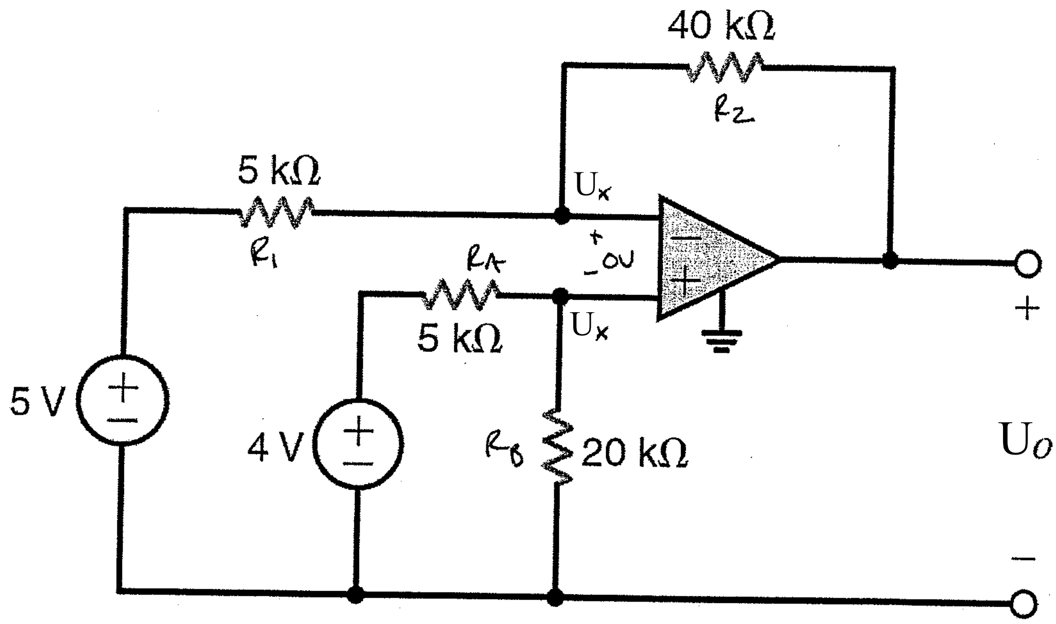
SOLUTION:

KCL at U_+ input: $\frac{U_2 - U_x}{R_2} = \frac{U_x}{R_I} \Rightarrow U_x = U_2\left(\frac{R_I}{R_I + R_2}\right)$

KCL at U_- input: $\frac{U_1 - U_x}{R_1} = \frac{U_x - U_o}{R_F} \Rightarrow U_o = U_x\left(1 + \frac{R_F}{R_1}\right) - \frac{R_F}{R_1} U_1$

$$\boxed{U_o = U_2\left(\frac{R_I}{R_I + R_2}\right)\left(\frac{R_1 + R_F}{R_1}\right) - \frac{R_F}{R_1} U_1}$$

3.2 Find U_o in the circuit



SOLUTION:

KCL at U− input: $\frac{5-U_x}{R_1} = \frac{U_x - U_o}{R_2} \Rightarrow U_x = 5\left(\frac{R_2}{R_1+R_2}\right) + U_o\left(\frac{R_1}{R_1+R_2}\right)$

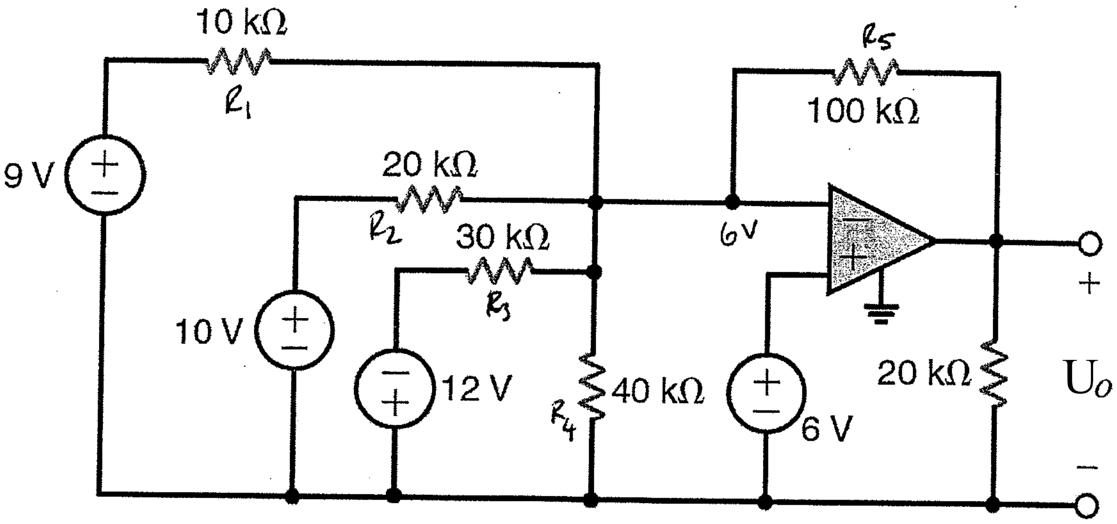
KCL @ U+ input: $\frac{4-U_x}{R_A} = \frac{U_x}{R_B} \Rightarrow U_x = \frac{4R_B}{R_A+R_B}$

$$5\left(\frac{R_2}{R_1+R_2}\right) + U_o\left(\frac{R_1}{R_1+R_2}\right) = \frac{4R_B}{R_A+R_B}$$

$$U_o = \left(\frac{4R_B}{R_A+R_B} - \frac{5R_2}{R_1+R_2}\right)\frac{R_1+R_2}{R_1}$$

$$\boxed{U_o = -11.2\text{V}}$$

3.3 Find U_o in the circuit



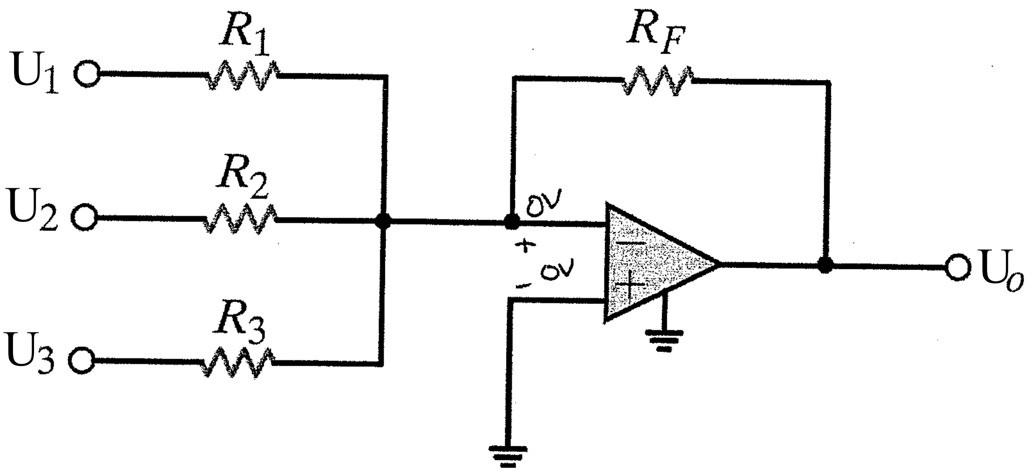
SOLUTION:

KCL at U- input: $\frac{9-6}{R_1} + \frac{10-6}{R_2} + \frac{-12-6}{R_3} = \frac{6}{R_4} + \frac{6-U_o}{R_5}$

$$\frac{3}{10^4} + \frac{4}{2\times10^4} - \frac{18}{3\times10^4} = \frac{6}{4\times10^4} + \frac{6}{10^5} - \frac{U_o}{10^5}$$

$$\boxed{U_o = 31V}$$

3.4 Determine the expression for the output voltage, U_o, of the inverting summer circuit

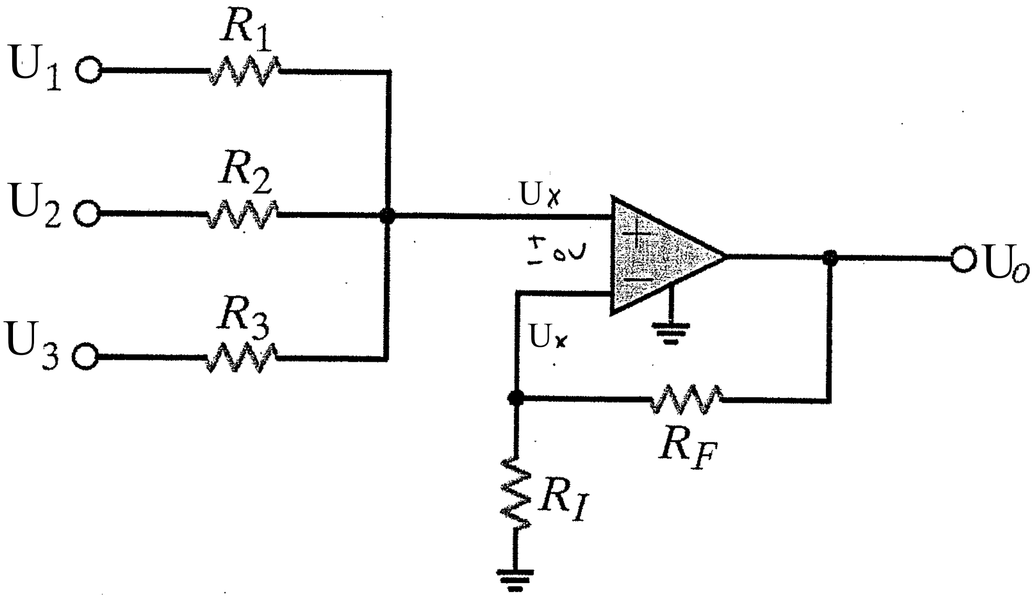


SOLUTION:

KCL at U- input:
$$\frac{U_1 - 0}{R_1} + \frac{U_2 - 0}{R_2} + \frac{U_3 - 0}{R_3} = \frac{0 - U_o}{R_F}$$

$$\boxed{U_o = -\frac{R_F}{R_1}U_1 - \frac{R_F}{R_2}U_2 - \frac{R_F}{R_3}U_3}$$

3.5 Determine the output voltage, U_o, of the noninverting averaging circuit



SOLUTION:

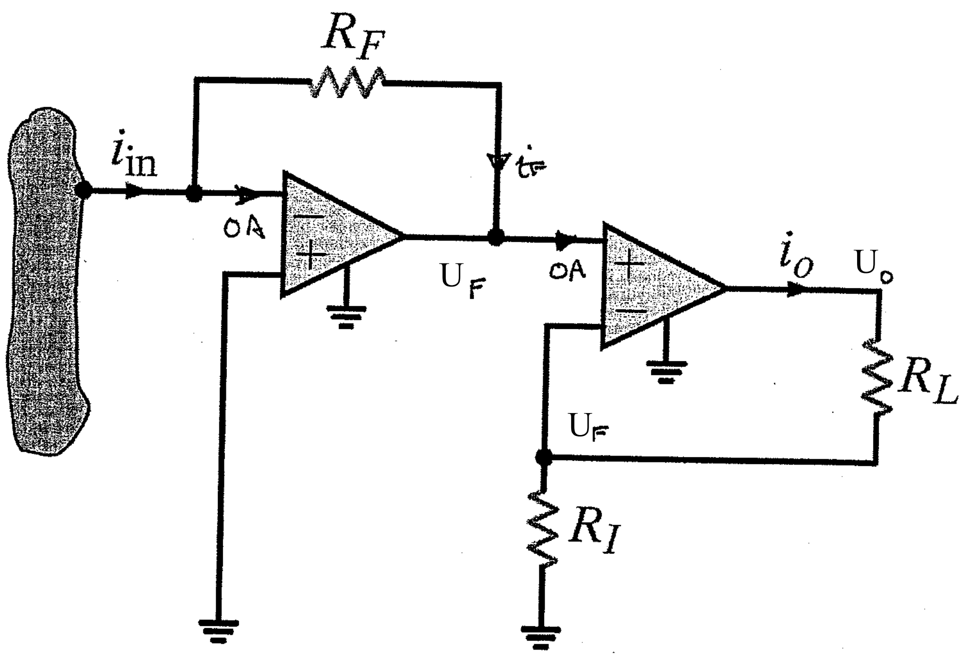
KCL at U+ input: $\frac{U_1 - U_x}{R_1} + \frac{U_2 - U_x}{R_2} + \frac{U_3 - U_x}{R_3} = 0$

KCL at U− input: $\frac{U_0 - U_x}{R_F} = \frac{U_x}{R_I}$ $\quad U_x = U_0 \left(\frac{R_I}{R_I + R_F}\right)$

Eliminate U_x,

$$\boxed{U_0 = \left(\frac{R_I + R_F}{R_I}\right)\left[\frac{R_2 R_3 U_1 + R_1 R_3 U_2 + R_1 R_2 U_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}\right]}$$

3.6 Find the input/output relationship for the current amplifier



SOLUTION: KCL at U_- input of 1st op amp.

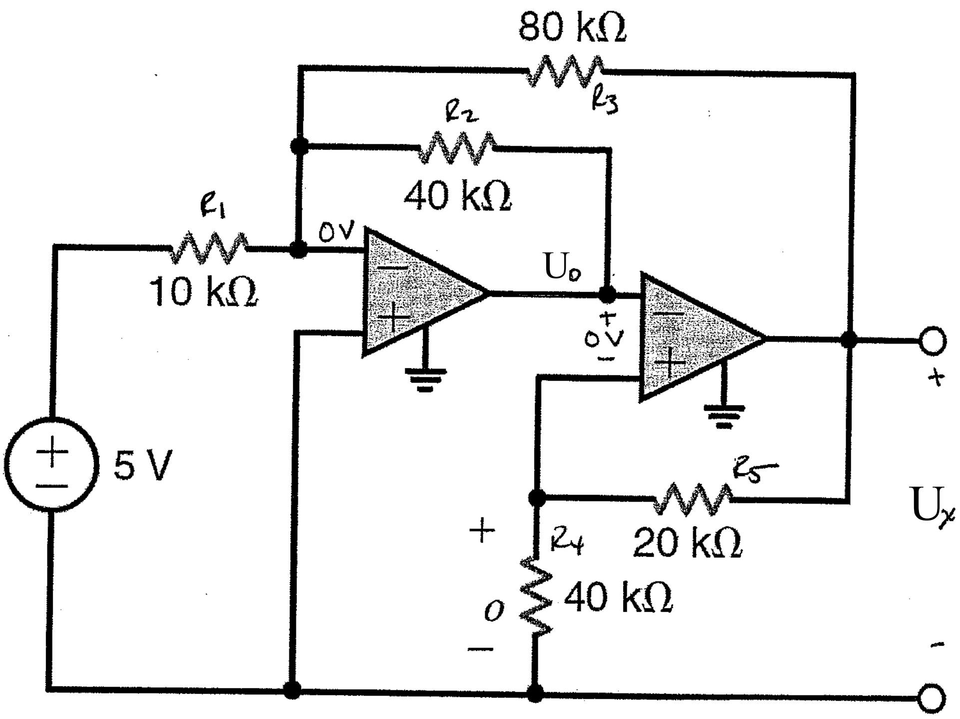
$$i_{in} = \frac{0 - U_F}{R_F} \qquad U_F = -R_F\, i_{in}$$

2nd op-amp in classic inverting configuration

$$U_o = U_F\left(1 + \frac{R_L}{R_I}\right) \qquad i_o = \frac{U_o - U_F}{R_L} = \frac{U_F}{R_I}$$

$$i_o/i_{in} = (U_F/i_{in})\,(i_o/U_F) \qquad \frac{i_o}{i_{in}} = -\frac{R_F}{R_I}$$

3.7 Find U_o in the circuit



SOLUTION:

KCL at U_- of 1st op amp: $\frac{5}{R_1} + \frac{U_o}{R_2} + \frac{U_x}{R_3} = 0 \Rightarrow U_x = -\frac{R_3}{R_1}(5) - \frac{R_3}{R_2}U_o$

KCL at U_+ of 2nd op amp: $\frac{U_o}{R_4} + \frac{U_o - U_x}{R_5} = 0 \Rightarrow U_x = U_o\left(1 + R_5/R_4\right)$

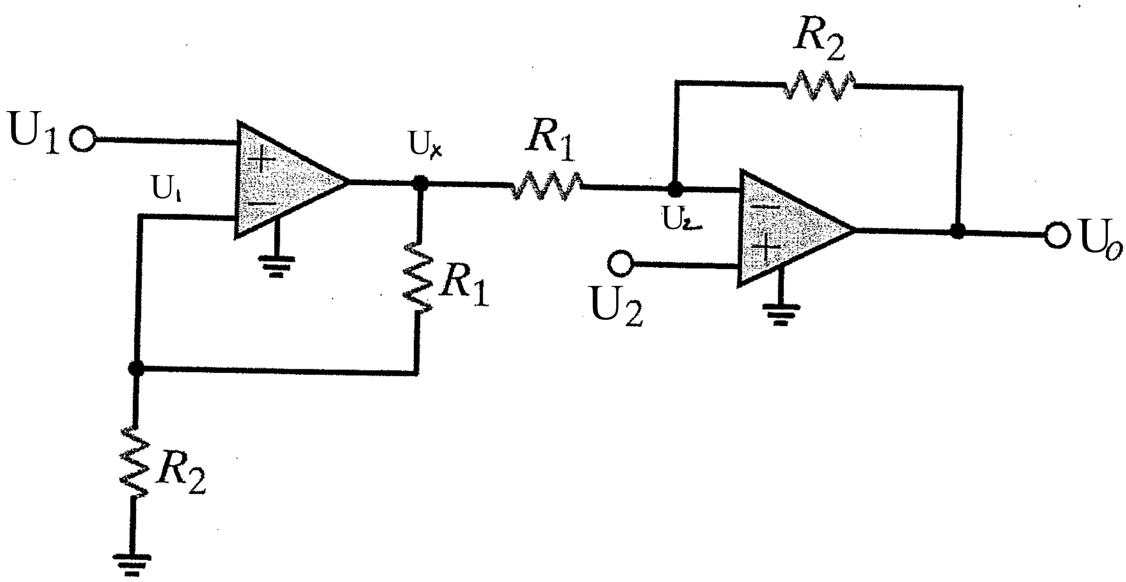
Put in numbers,

$U_x = -40 - 2U_o$ & $U_x = 1.5U_o$

Eliminate Ux,

$\boxed{U_o = -11.43\,V}$

3.8 Find U_o in the circuit



SOLUTION:

1st Op amp in basic non-inverting configuration:

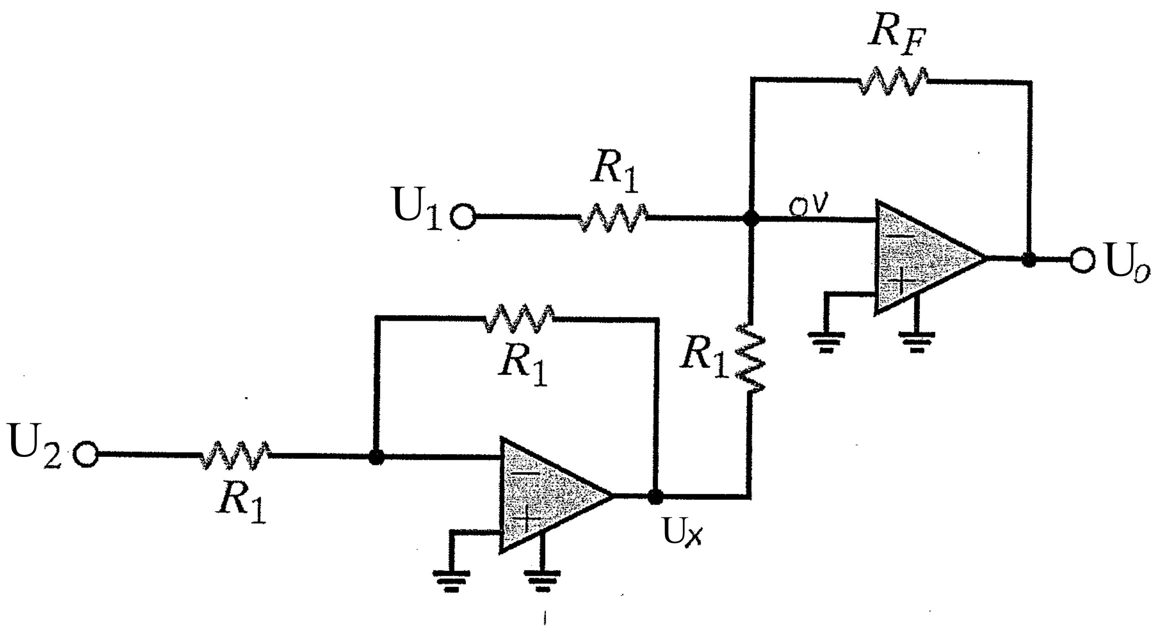
$$\frac{U_x}{U_1} = 1 + \frac{R_1}{R_2} \Rightarrow U_x = U_1\left(\frac{R_2+R_1}{R_2}\right)$$

KCL at U- of 2nd op amp: $\frac{U_x - U_2}{R_1} + \frac{U_o - U_2}{R_2} = 0$

$$U_o = U_2\left(1 + \frac{R_2}{R_1}\right) - \frac{R_2}{R_1}U_x$$

$$\boxed{U_0 = \left(1 + \frac{R_2}{R_1}\right)\left(U_2 - U_1\right)}$$

3.9 Find the expression for U_o in the differential amplifier circuit



SOLUTION:

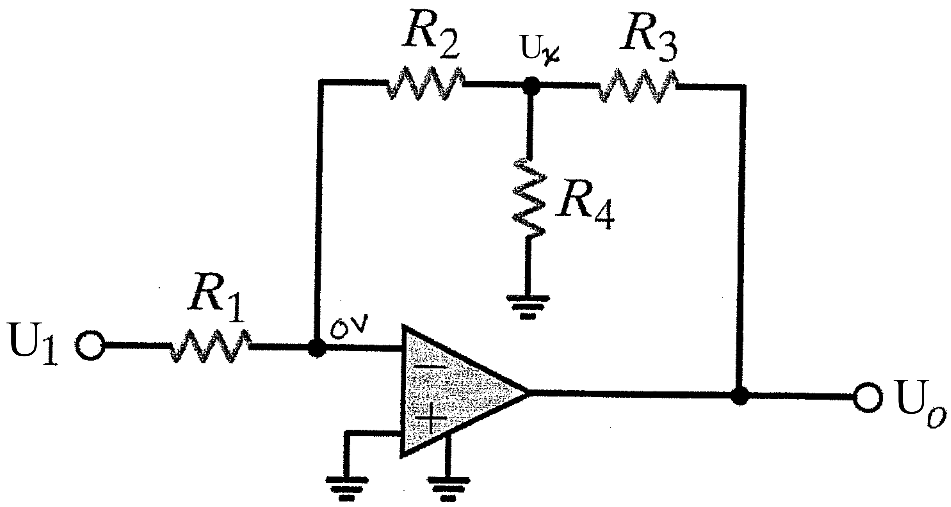
1st Op-amp in classic inverting configuration:

$U_X = -\frac{R_1}{R_1} U_2 \qquad U_X = -U_2$

KCL at U_- of 2nd op amp: $\frac{U_1}{R_1} + \frac{U_X}{R_1} + \frac{U_o}{R_F} = 0$

$$\boxed{U_o = \frac{R_F}{R_1}\left[U_2 - U_1\right]}$$

3.10 Find U_o in the circuit



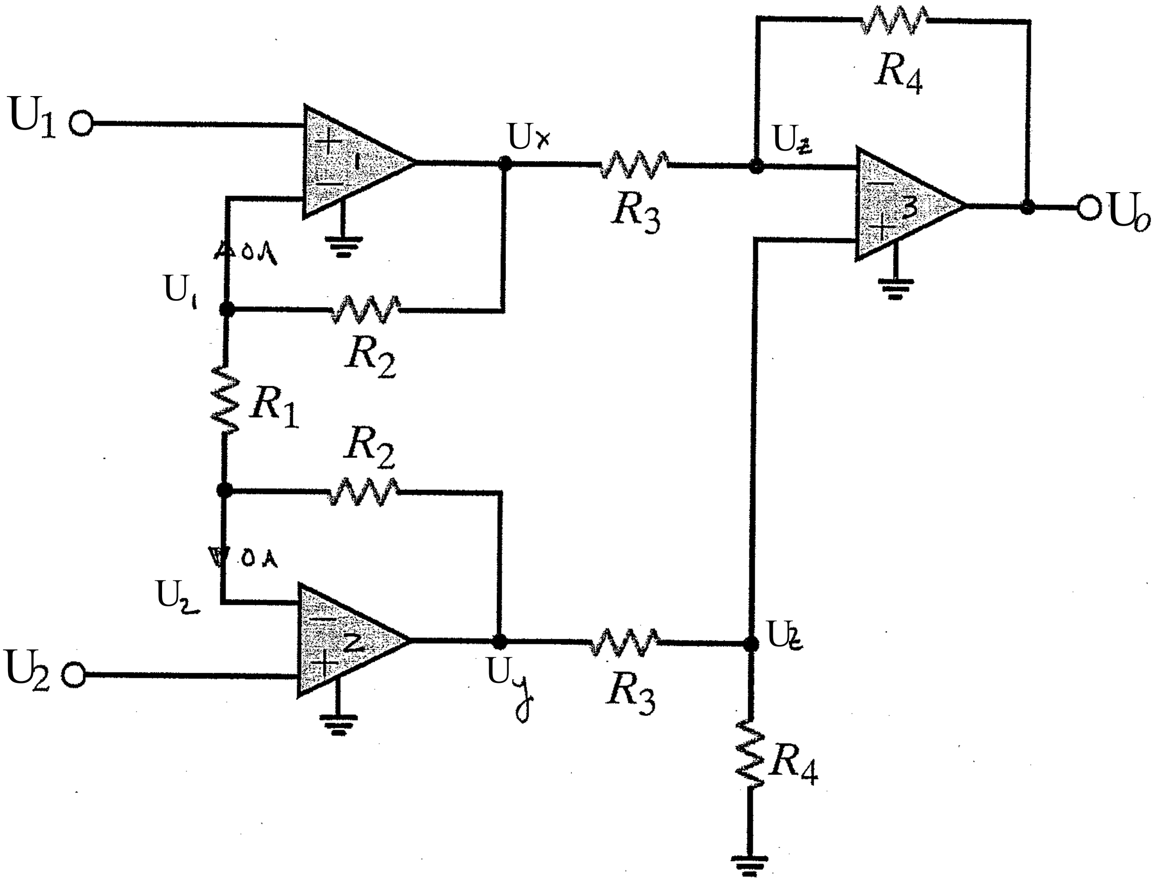
SOLUTION:

KCL at U- input: $\frac{U_1}{R_1} + \frac{U_x}{R_2} = 0$ $U_x = -\frac{R_2}{R_1} U_1$

KCL at U_x node: $\frac{U_x}{R_2} + \frac{U_x}{R_4} + \frac{U_x - U_o}{R_3} = 0$ $U_o = U_x\left(\frac{R_3}{R_2} + \frac{R_3}{R_4} + 1\right)$

$$\boxed{U_o = U_1\left[1 + \frac{R_3}{R_2} + \frac{R_3}{R_4}\right]\left(-\frac{R_2}{R_1}\right)}$$

3.11 Find the output voltage, U_o, in the circuit



SOLUTION:

KCL at U_- of op amp 1: $\frac{U_x - U_1}{R_2} = \frac{U_1 - U_2}{R_1} \Rightarrow U_x = U_1\left(1 + \frac{R_2}{R_1}\right) - \frac{R_2}{R_1}U_2$

KCL at U_- of op amp 2: $\frac{U_y - U_2}{R_2} = \frac{U_2 - U_1}{R_1} \Rightarrow U_y = U_2\left(1 + \frac{R_2}{R_1}\right) - \frac{R_2}{R_1}U_1$

KCL at U_+ of op amp 3: $\frac{U_y - U_z}{R_3} = \frac{U_z}{R_4} \Rightarrow U_z = U_y\left(\frac{R_4}{R_3 + R_4}\right)$

KCL at U_- of op amp 3: $\frac{U_x - U_z}{R_3} + \frac{U_o - U_z}{R_4} = 0 \Rightarrow U_o = U_z\left(1 + R_4/R_3\right) - U_x\frac{R_4}{R_3}$

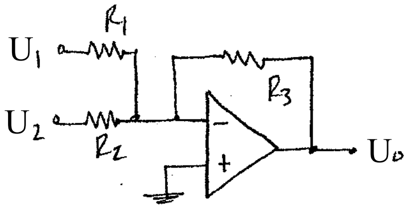
$$\boxed{U_o = \frac{R_4}{R_3}\left(1 + 2\frac{R_2}{R_1}\right)(U_2 - U_1)}$$

3.12 Given a box of 10-kΩ resistors and an op-amp, design a circuit that will have an output voltage of

$$U_o = -2U_1 - 4U_2$$

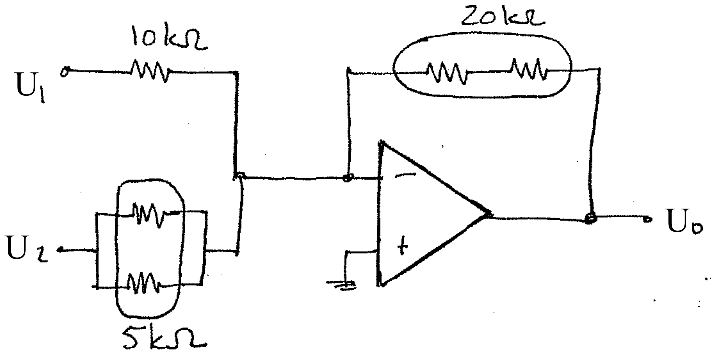
SOLUTION:

Since signs on gains associated with U_1 & U_2 are both negative, a simple summer will suffice.



$$U_o = -\frac{R_3}{R_1}U_1 - \frac{R_3}{R_2}U_2 = -2U_1 - 4U_2$$

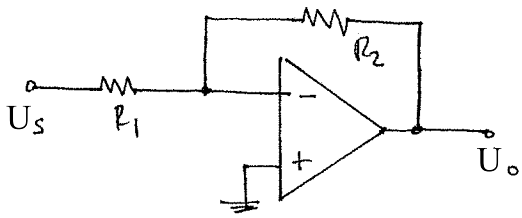
the following circuit with all resistors = 10kΩ works.



3.13 Design an op-amp circuit that has a gain of −50 using resistors no smaller than 1 kΩ.

SOLUTION:

Since gain is negative, use inverting configuration:



$$\frac{U_o}{U_S} = -\frac{R_2}{R_1} = -50$$

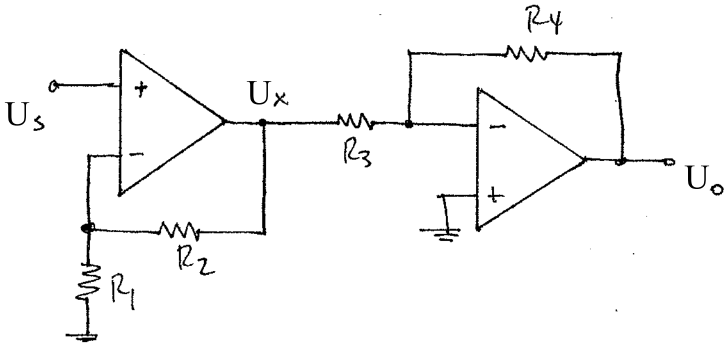
$\boxed{\text{Choose } R_1 = 2k\Omega \Rightarrow R_2 = 100k\Omega}$

3.14 Design a two-stage op-amp network that has a gain of −50,000 while drawing no current into its input terminal. Use no resistors smaller than 1 kΩ.

SOLUTION:

for no input current, a non-inverting configuration is needed

for negative, one inverting stage is needed.



$$\frac{U_x}{U_s} = 1 + \frac{R_2}{R_1} \qquad \frac{U_o}{U_x} = -\frac{R_4}{R_3} \qquad \frac{U_o}{U_s} = -\frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right)$$

Choose $\frac{U_x}{U_s} = 250$ and $\frac{U_o}{U_x} = -200$ $\boxed{R_1 = R_3 = 2k\Omega}$

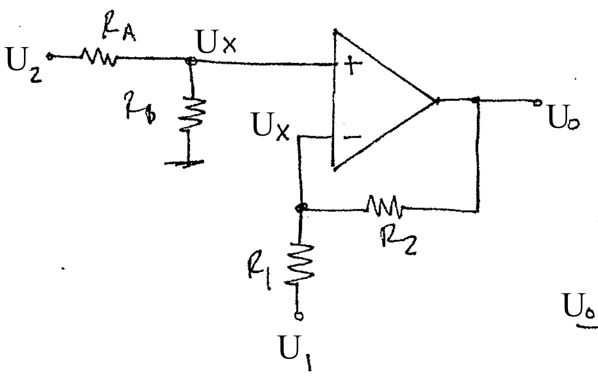
$\boxed{R_2 = 498\ k\Omega}$ $\boxed{R_4 = 400\ k\Omega}$

3.15 Design an op-amp circuit that has the following input/output relationship:

$$U_o = -5U_1 + 0.5U_2$$

SOLUTION:

A single op-amp will do if we use both + & - inputs.



KCL at U_+,

$$\frac{U_2 - U_X}{R_A} = \frac{U_X}{R_B} \Rightarrow \frac{U_X}{U_2} = \frac{R_B}{R_A + R_B}$$

KCL at U_-,

$$\frac{U_o - U_X}{R_2} = \frac{U_X - U_1}{R_1}$$

$$U_o = U_X\left(1 + \frac{R_2}{R_1}\right) - \frac{R_2}{R_1}U_1$$

$$U_o = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_B}{R_A + R_B}\right)U_2 - \frac{R_2}{R_1}U_1$$

So, $R_2/R_1 = 5$

Now, $\frac{6R_B}{R_A + R_B} = \frac{1}{2}$

Choose $R_1 = 1k\Omega \Rightarrow R_2 = 5k\Omega$

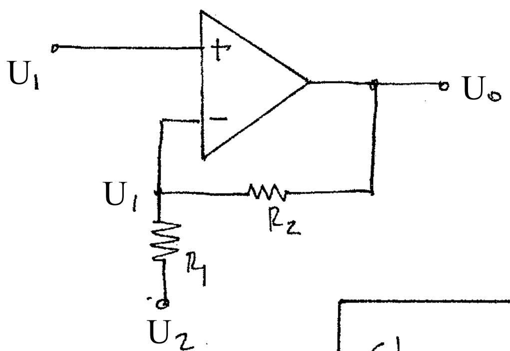
Choose $R_B = 1k\Omega \Rightarrow R_A = 11k\Omega$

3.16 Design an op-amp-based circuit to produce the function

$$U_o = 5U_1 - 4U_2$$

SOLUTION:

To get + & - gains, we can use both + & - inputs,



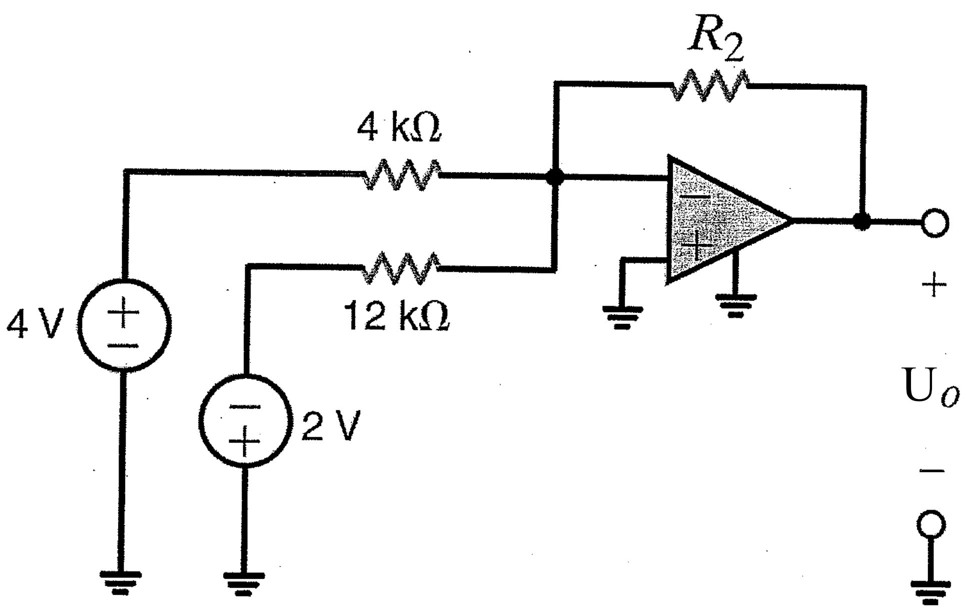
KCL @ U- input,

$$\frac{U_o - U}{R_2} = \frac{U_1 - U_2}{R_1}$$

$$U_o = U_1\left(1 + \frac{R_2}{R_1}\right) - \frac{R_2}{R_1}U_2$$

$$\boxed{\text{Choose } R_1 = 5k\Omega \Rightarrow R_2 = 20k\Omega}$$

3.17 Given the summing amplifier shown in the figure select the values of R_2 that will produce an output voltage of −3 V.

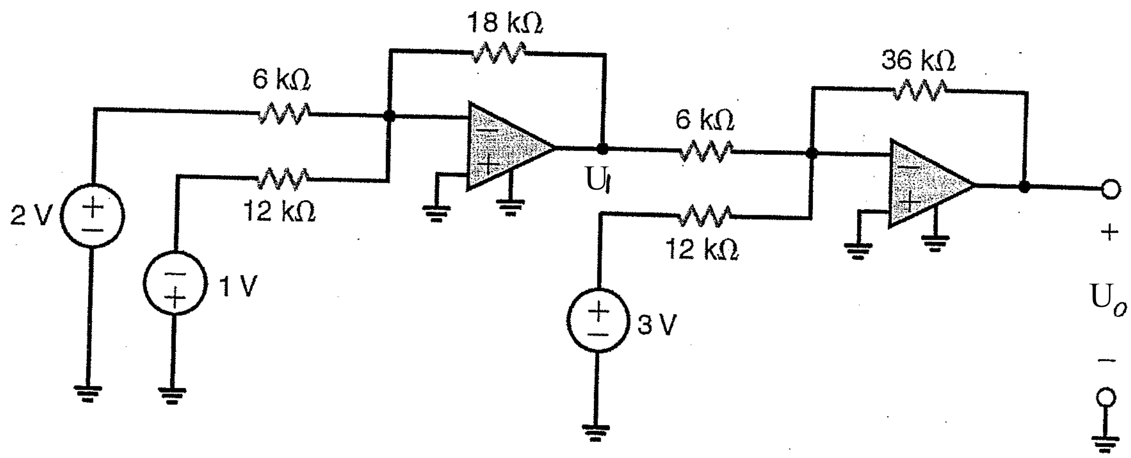


SOLUTION: For summing amp:

$$U_O = -\left(\frac{R_2}{4000}\right)4 - \left(\frac{R_2}{12000}\right)(2) = -3$$

$$\boxed{R_2 = 2.57\,k\Omega}$$

3.18 Determine the output voltage U_o of the summing op-amp circuit



SOLUTION:

$$U_1 = -2\left(\frac{18\times10^3}{6\times10^3}\right) + 1\left(\frac{18\times10^3}{12\times10^3}\right) = -4.5\,V$$

$$U_o = -U_1\left(\frac{36\times10^3}{6\times10^3}\right) - 3\left(\frac{36\times10^3}{12\times10^3}\right) \Rightarrow \boxed{U_o = 18\,V}$$