Naloge pisnega izpita

PROCESIRANJE SIGNALOV

Datum: 02. 02. 2007

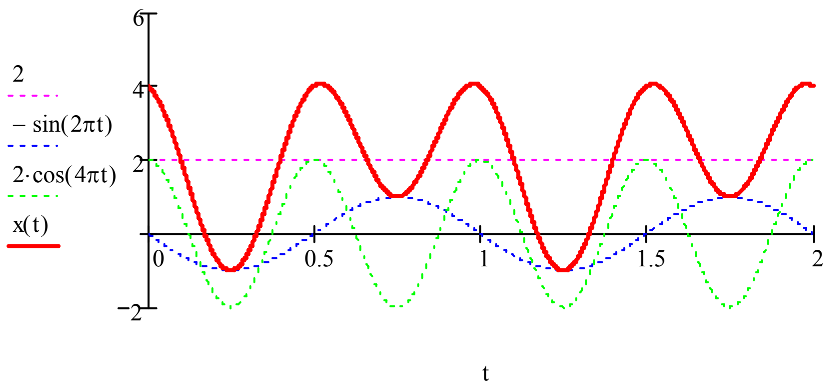
1. Narišite skico in določite Fourierevo vrsto signala

$$x(t) = 2 - \sin(2\pi t) + 2\cos(4\pi t)$$

Koliko znaša X_0? Ali je signal močnostni ali energijski?

Nasvet: Zapišite vrsto a_n, b_n.

Rešitev:



$$x(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos(n\omega_0 t) + b_n \cos(n\omega_0 t)\right) \tag{1.1}$$

$$\omega_0 = 2\pi \tag{1.2}$$

$$\begin{aligned} x(t) = a_0 + a_1 \cos(2\pi t) + b_1 \cos(2\pi t) + a_2 \cos(4\pi t) + \\ + b_2 \cos(4\pi t) + a_3 \cos(6\pi t) + b_3 \cos(6\pi t) + \ldots \end{aligned} \tag{1.3}$$

$$a_0 = X_0 = 2 \tag{1.4}$$

$$b_1 = -1 \tag{1.5}$$

$$a_2 = 2 \tag{1.6}$$

$$a_n = 0 \quad ; \quad n \neq 0, n \neq 2 \tag{1.7}$$

$$b_n = 0 \quad ; \quad n \neq 1 \tag{1.8}$$

2. Podan je odziv linearnega, časovno nespremenljivega sistema:

$$h(t)=\begin{cases}2 & ; \quad 0<t<3 \\ 0 & ; \quad sicer\end{cases}$$

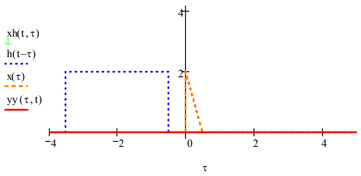
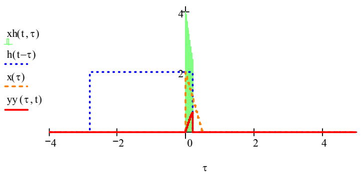
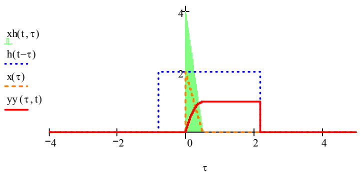
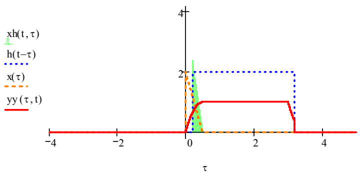
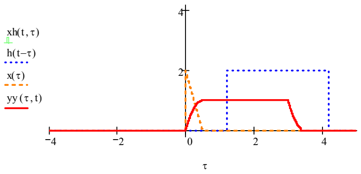
Izračunajte in narišite odziv $y(t)$ takšnega sistema na vzbujanje

$$x(t)=\begin{cases}2-4t & ; \quad 0<t<0,5 \\ 0 & ; \quad sicer\end{cases}$$

Rešitev:

$$y(t)=\int_0^t x(\tau)h(t-\tau)d\tau=\int_0^t x(t-\tau)h(\tau)d\tau \qquad (2.1)$$

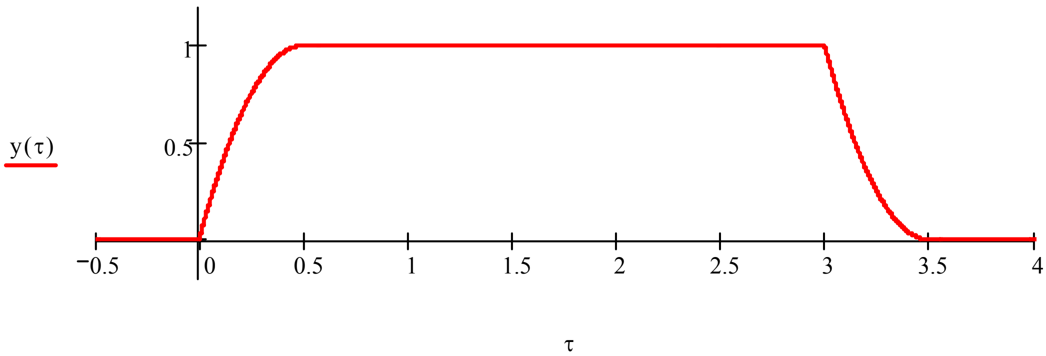
V tem primeru laže obrniti $h(t)$. Zato bomo uporabili prvo enačbo.

$t<0$		$y(t)=0$ (2.2)
$0<t<0,5$		$y(t)=\int_0^t 2\cdot(2-4\tau)d\tau=$ $=4\int_0^t d\tau-8\int_0^t \tau d\tau=$ (2.3) $=4\tau\Big\|_0^t-8\frac{\tau^2}{2}\Big\|_0^t=4t-4t^2$
$0,5<t<3$		$y(t)=\int_0^{0,5} 2\cdot(2-4\tau)d\tau=$ (2.4) $=4\tau\Big\|_0^{0,5}-8\frac{\tau^2}{2}\Big\|_0^{0,5}=2-1=1$
$3<t<3,5$		$y(t)=\int_{t-3}^{0,5} 2\cdot(2-4\tau)d\tau=4\tau\Big\|_{t-3}^{0,5}-8\frac{\tau^2}{2}\Big\|_{t-3}^{0,5}=$ $=4\left(0,5-t+3-\left(0,5^2-t^2+6t-9\right)\right)=$ (2.5) $=14-4t+4t^2-24t+35=$ $=4t^2-28t+49$
$3,5<t$		$y(t)=0$ (2.6)

Združimo vse delne rezultate v končni rezultat:

$$y(t)=\begin{cases} -4t^2+4t & ; \quad 0<t<0,5 \\ 1 & ; \quad 0,5<t<3 \\ 4t^2-28t+49 & ; \quad 3<t<3,5 \\ 0 & ; \quad sicer \end{cases} \tag{2.7}$$

Skica:



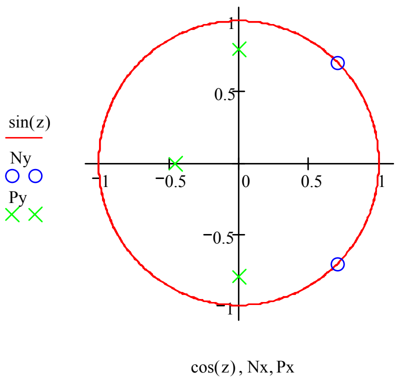
3. Časovno diskretni sistem je podan z ničlami in poli:

$$n_{1,2}=\frac{\sqrt{2}}{2}\pm j\frac{\sqrt{2}}{2} \quad , \quad p_{1,2}=\pm j0,8 \quad , \quad p_3=-0,45$$

Narišite lego ničel in polov v z-ravnini. Ali je podano časovno diskretno linearno vezje stabilno? Skicirajte potek frekvenčnega odziva $|H(\Omega)|$! Določite sistemsko funkcijo $H(z)$, zapišite diferenčno enačbo in narišite shemo vezja.

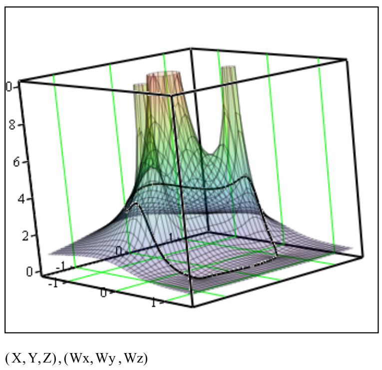
Rešitev:

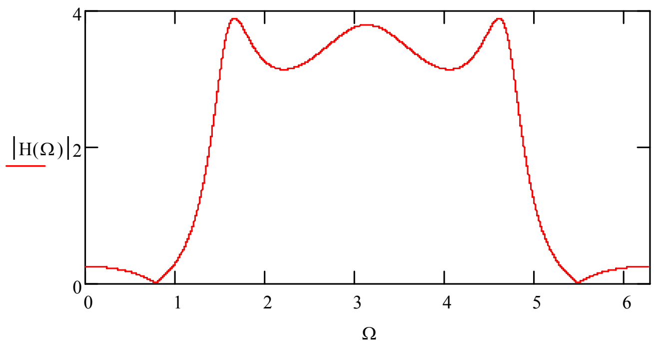
Lega ničel in polov v z-ravnini:



Vezje je stabilno, ker so vsi poli znotraj enotske krožnice.

3D slika z-ravnine in skica frekvenčnega odziva:





Sistemska funkcija $H(z)$:

$$H(z)=\frac{(z-n_1)(z-n_2)}{(z-p_1)(z-p_2)(z-p_3)} \tag{3.1}$$

$$\begin{aligned} H(z)&=\frac{z^2-(n_1+n_2)z+n_1n_2}{\left(z^2-(p_1+p_2)z+p_1p_2\right)(z-p_3)} \\ &=\frac{z^2-(n_1+n_2)z+n_1n_2}{z^3-(p_1+p_2)z^2+p_1p_2z-p_3z^2+p_3(p_1+p_2)z-p_1p_2p_3}= \\ &=\frac{z^2-(n_1+n_2)z+n_1n_2}{z^3-(p_1+p_2+p_3)z^2+(p_1p_2+p_1p_3+p_2p_3)z-p_1p_2p_3} \end{aligned} \tag{3.2}$$

$$-\left(n_1+n_2\right)=-\left(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2}\right)=-\sqrt{2}$$

$$n_1n_2=\left(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2}\right)=\left(\frac{\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2=1$$

$$-\left(p_1+p_2+p_3\right)=-\left(j0,8-j0,8-0,45\right)=0,45 \tag{3.3}$$

$$p_1p_2+p_1p_3+p_2p_3=\left(j0,8\right)\left(-j0,8\right)-0,45\left(j0,8\right)-0,45\left(-j0,8\right)=0,64$$

$$-p_1p_2p_3=-\left(j0,8\right)\left(-j0,8\right)\left(-0,45\right)=0,64\cdot 0,45=0,288$$

$$H\left(z\right)=\frac{z^2-\sqrt{2}z+1}{z^3+0,45z^2+0,64z+0,288}=\frac{z^{-1}-\sqrt{2}z^{-2}+z^{-3}}{1+0,45z^{-1}+0,64z^{-2}+0,288z^{-3}} \tag{3.4}$$

Diferenčna enačba:

$$H\left(z\right)=\frac{Y\left(z\right)}{X\left(z\right)}=\frac{z^{-1}-\sqrt{2}z^{-2}+z^{-3}}{1+0,45z^{-1}+0,64z^{-2}+0,288z^{-3}} \tag{3.5}$$

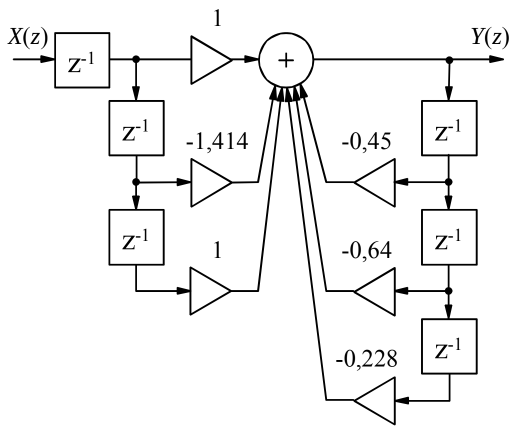
$$Y\left(z\right)\left(1+0,45z^{-1}+0,64z^{-2}+0,288z^{-3}\right)=X\left(z\right)\left(z^{-1}-\sqrt{2}z^{-2}+z^{-3}\right) \tag{3.6}$$

$$\begin{aligned}&Y\left(z\right)+0,45Y\left(z\right)z^{-1}+0,64Y\left(z\right)z^{-2}+0,288Y\left(z\right)z^{-3}=\\&=X\left(z\right)z^{-1}-\sqrt{2}X\left(z\right)z^{-2}+X\left(z\right)z^{-3}\end{aligned} \tag{3.7}$$

$$\begin{aligned}Y\left(z\right)=&\,X\left(z\right)z^{-1}-\sqrt{2}X\left(z\right)z^{-2}+X\left(z\right)z^{-3}-\\&-0,45Y\left(z\right)z^{-1}-0,64Y\left(z\right)z^{-2}-0,288Y\left(z\right)z^{-3}\end{aligned} \tag{3.8}$$

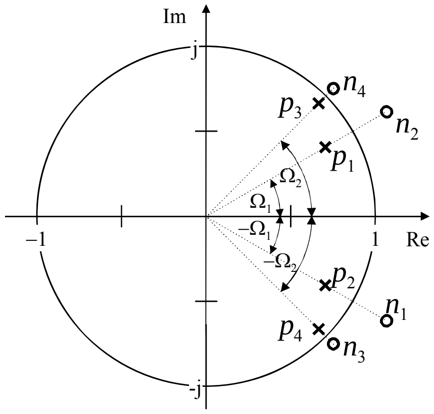
$$\begin{aligned}y[n]=&\,x[n-1]-\sqrt{2}x[n-2]+x[n-3]\\&-0,45y[n-1]-0,64y[n-2]-0,288y[n-3]\end{aligned} \tag{3.9}$$

Shema vezja:



4. Teoretična vprašanja

a) Kakšna mora biti razporeditev polov in ničel časovno diskretnega faznega sukalnika? Ponazorite na primeru sistema 4. reda!



Ničle in poli si morajo biti inverzni.

$$n_i = \frac{1}{p_i}$$

Da je sistem stabilen, morajo biti poli znotraj enotske krožnice. Iz tega sledi, da so ničle na zunanji strani enotske krožnice.

b) Zapišite vse rešitve enačbe $z^3 = 1$!

$$z^3 = 1 = e^{j(0+2k\pi)} \tag{4.1}$$

$$\left(z^3\right)^{\frac{1}{3}} = \left(e^{j2k\pi}\right)^{\frac{1}{3}} \tag{4.2}$$

$$z = e^{j\frac{2k\pi}{3}} \tag{4.3}$$

$$z_1 = e^{j\frac{2\cdot 0\cdot\pi}{3}} = 1 \tag{4.4}$$

$$z_2 = e^{j\frac{2\cdot 1\cdot\pi}{3}} = -0,5 + j0,866 \tag{4.5}$$

$$z_3 = e^{j\frac{2\cdot 2\cdot\pi}{3}} = -0,5 - j0,866 \tag{4.6}$$

c) Kaj mora veljati za lego ničel in polov stabilnega časovno diskretnega sistema z maksimalno fazo?

Poli morajo biti znotraj enotske krožnice. Ničle morajo biti na zunanji strani enotske krožnice.

d) Časovno diskreten sistem je podan z ničlami in poli:

$$n_{1,2} = 1,\ n_{3,4} = e^{\pm j\frac{\pi}{6}},\ n_{5,6} = e^{\pm j\frac{\pi}{3}},\ p_{1,2} = 0,9e^{\pm j\frac{\pi}{6}},\ p_{3,4} = 0,9e^{\pm j\frac{\pi}{3}}$$

Dopolnite ga z ničlami ali poli tako, da bo postal uresničljiv. Pri tem se frekvenčni odziv $|H(\Omega)|$ ne sme spremeniti.

$$p_{5,6} = 0 \tag{4.7}$$

e) Napišite Fouriereve transforme funkcij:

- $x(t) = 5 \quad \leftrightarrow \quad X(\omega) = 5\cdot 2\pi\delta(\omega)$
- $x(t) = 2\delta(t) \quad \leftrightarrow \quad X(\omega) = 2$