Naloge pisnega izpita

PROCESIRANJE SIGNALOV

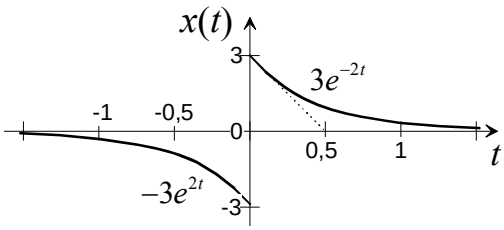
Datum: 23. 01. 2007

1. Narišite skico in določite Fourierev transform signala

$$x(t)=\begin{cases}3e^{-2t} & ; \quad t>0\\ -3e^{2t} & ; \quad t\le 0\end{cases}$$

Koliko znaša $X(0)$? Ali je signal močnostni ali energijski?

Rešitev:



$$X(\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt=\int_{-\infty}^{0}(-3)e^{2t}e^{-j\omega t}dt+\int_{0}^{\infty}3e^{-2t}e^{-j\omega t}dt \tag{1.1}$$

$$X(\omega)=3\left(-\int_{-\infty}^{0}e^{2t-j\omega t}dt+\int_{0}^{\infty}e^{-2t-j\omega t}dt\right)=3\left(-\int_{-\infty}^{0}e^{-(-2+j\omega)t}dt+\int_{0}^{\infty}e^{-(2+j\omega)t}dt\right) \tag{1.2}$$

$$X(\omega)=3\left(-\frac{1}{-(-2+j\omega)}e^{-(-2+j\omega)t}\Big|_{-\infty}^{0}+\frac{1}{-(2+j\omega)}e^{-(2+j\omega)t}\Big|_{0}^{\infty}\right) \tag{1.3}$$

$$X(\omega)=3\left(\frac{1}{(-2+j\omega)}\left(\underbrace{e^{-(-2+j\omega)0}}_{1}-\underbrace{e^{-(-2+j\omega)(-\infty)}}_{0}\right)-\frac{1}{(2+j\omega)}\left(\underbrace{e^{-(2+j\omega)\infty}}_{0}-\underbrace{e^{-(2+j\omega)0}}_{1}\right)\right) \tag{1.4}$$

$$X(\omega)=3\left(\frac{1}{(-2+j\omega)}+\frac{1}{(2+j\omega)}\right)=3\left(\frac{1}{(j\omega-2)}+\frac{1}{(j\omega+2)}\right) \tag{1.5}$$

$$X(\omega)=3\left(\frac{(j\omega+2)}{(j\omega-2)(j\omega+2)}+\frac{(j\omega-2)}{(j\omega+2)(j\omega-2)}\right)=3\frac{j\omega+2+j\omega-2}{j^2\omega^2-4} \tag{1.6}$$

$$X(\omega)=3\frac{2j\omega}{-\omega^2-4}=-j\frac{6\omega}{4+\omega^2} \tag{1.7}$$

$$X(0)=0 \tag{1.8}$$

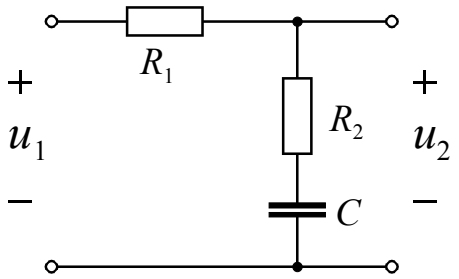
Signal je energijski, ker ima končno energijo, povprečna moč od -∞ do ∞ pa je zaradi tega nič.

2. Določite sistemsko funkcijo $H(s)$ podanega sistema. Narišite lego ničel in polov v s ravnini. Skicirajte frekvenčni odziv $H(\omega)$.

$R_1 = 10\ \Omega$

$R_2 = 10\ \Omega$

$C = 470\ \mu\text{F}$



Rešitev:

$$I(s) = \frac{U_1(s)}{R_1 + R_2 + \frac{1}{Cs}} \tag{2.1}$$

$$U_2(s) = I(s)\left(R_2 + \frac{1}{Cs}\right) = \frac{U_1(s)}{R_1 + R_2 + \frac{1}{Cs}}\left(R_2 + \frac{1}{Cs}\right) = U_1(s)\frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} \tag{2.2}$$

$$U_2(s) = I(s)\left(R_2 + \frac{1}{Cs}\right) = \frac{U_1(s)}{R_1 + R_2 + \frac{1}{Cs}}\left(R_2 + \frac{1}{Cs}\right) = U_1(s)\frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} \tag{2.3}$$

$$H(s) = \frac{U_2(s)}{U_1(s)} = \frac{U_1(s)\frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}}}{U_1(s)} = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} \tag{2.4}$$

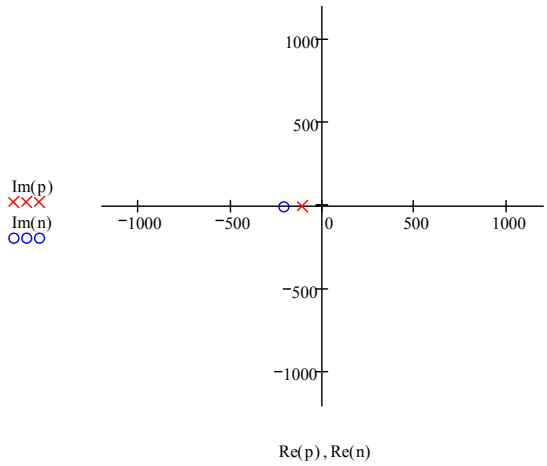
$$H(s) = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} \cdot \frac{Cs}{Cs} = \frac{R_2 Cs + 1}{(R_1 + R_2)Cs + 1} = \frac{R_2 C\left(s + \frac{1}{R_2 C}\right)}{(R_1 + R_2)C\left(s + \frac{1}{(R_1 + R_2)C}\right)} \tag{2.5}$$

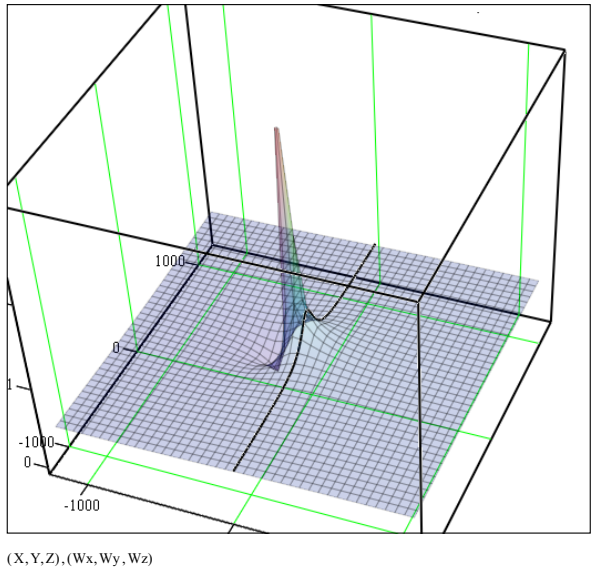
$$H(s) = \frac{R_2}{R_1 + R_2} \cdot \frac{s + \frac{1}{R_2 C}}{s + \frac{1}{(R_1 + R_2)C}} \tag{2.6}$$

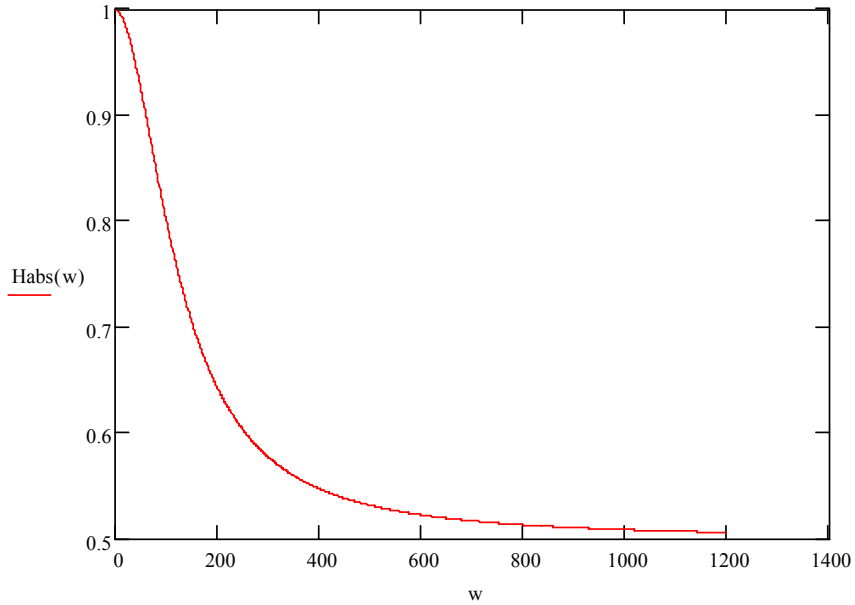
Ničla: $$n_1 = -\frac{1}{R_2 C} = -\frac{1}{10\,\Omega \cdot 470 \cdot 10^{-6}\ \mathrm{F}} = -213\ \mathrm{s}^{-1}$$

Pol: $$p_1 = -\frac{1}{(R_2 + R_1)C} = -\frac{1}{(10\,\Omega + 10\,\Omega) \cdot 470 \cdot 10^{-6}\ \mathrm{F}} = -106\ \mathrm{s}^{-1}$$

Pol in ničla v s-ravnini, 3D slika s-ravnine in skica frekvenčnega odziva:







3. Za podano sistemsko funkcijo $H(z)$ zapišite diferenčno enačbo in narišite shemo vezja. Določite in narišite lego ničel in polov v z-ravnini. Ali je podano časovno diskretno linearno vezje stabilno? Skicirajte potek frekvenčnega odziva $|H(\Omega)|$!

$$H(z)=\frac{z^2-1,414z+1}{z^2-1,343z+0,903}$$

Rešitev:

$$H(z)=\frac{Y(z)}{X(s)}=\frac{z^2-1,414z+1}{z^2-1,343z+0,903}=\frac{1-1,414z^{-1}+z^{-2}}{1-1,343z^{-1}+0,903z^{-2}} \tag{3.1}$$

$$Y(z)\left(1-1,343z^{-1}+0,903z^{-2}\right)=X(s)\left(1-1,414z^{-1}+z^{-2}\right) \tag{3.2}$$

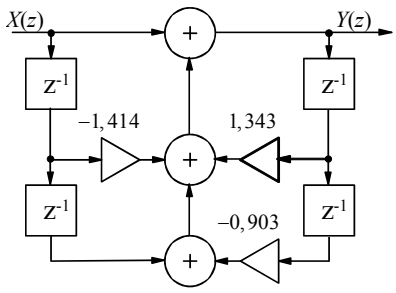
$$Y(z)-1,343Y(z)z^{-1}+0,903Y(z)z^{-2}=X(s)-1,414X(s)z^{-1}+X(s)z^{-2} \tag{3.3}$$

$$Y(z)=X(s)-1,414X(s)z^{-1}+X(s)z^{-2}+1,343Y(z)z^{-1}-0,903Y(z)z^{-2} \tag{3.4}$$

Diferenčna enačba:

$$y[n]=x[n]-1,414\cdot x[n-1]+x[n-2]+1,343\cdot y[n-1]-0,903\cdot y[n-2] \tag{3.5}$$

Shema vezja:



Ničle:

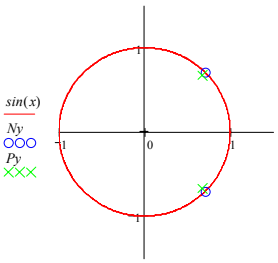
$$n_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{1,414\pm\sqrt{(-1,414)^2-4}}{2}=0,707\pm j0,707=\frac{\sqrt{2}}{2}\pm j\frac{\sqrt{2}}{2} \tag{3.6}$$

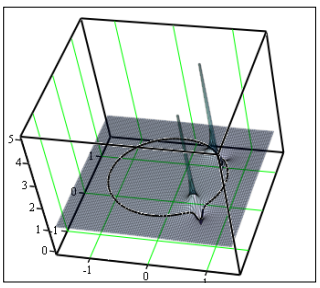
Poli:

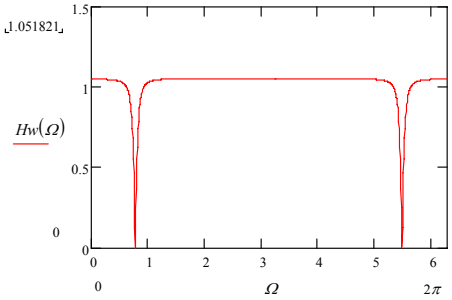
$$p_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{1,343\pm\sqrt{(-1,343)^2-4\cdot 0,903}}{2}=0,672\pm j0,672 \tag{3.7}$$

Sistem je stabilen, ker s poli znotraj enotske krožnice.

Ničle in poli v z-ravnini, 3D-slika z-ravnine in skica frekvenčnega odziva:

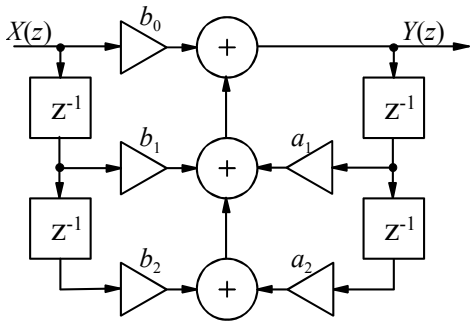






4. Teoretična vprašanja

a) Narišite shemo časovno diskretnega IIR sistema drugega reda (direktna oblika I)!



b) Zapišite vse štiri rešitve enačbe $z^4 - 1 = 0$!

$$z^4 = 1 = e^{j(0+2k\pi)} \quad , \quad k \in \mathbb{Z} \tag{4.1}$$

$$\left(z^4\right)^{\frac{1}{4}} = z = \left(e^{j2k\pi}\right)^{\frac{1}{4}} = e^{j\frac{k\pi}{2}} \quad , \quad k = 0,1,2,3 \tag{4.2}$$

$$z_1 = e^{j0} = 1 \quad , \quad z_2 = e^{j\frac{\pi}{2}} = j \quad , \quad z_3 = e^{j\pi} = -1 \quad , \quad z_4 = e^{j\frac{3\pi}{2}} = -j \tag{4.3}$$

c) Kaj mora veljati za lego ničel in polov stabilnega časovno zveznega sistema z maksimalno fazo?

Poli stabilnega časovno zveznega sistema morajo biti na levi strani s-ravnine. Za maksimalno fazo morajo biti ničle na drugi strani s-ravnine kot poli, torej na desni strani s-ravnine.

d) Napišite Parsevalov izrek za periodične signale.

$$P_x = \frac{1}{T}\int_{\tau}^{\tau+T} x^2(t)\,dt = \sum_{k=-\infty}^{\infty} \left|X_k\right|^2 \tag{4.4}$$

e) Kakšna je zveza med številom ničel in številom polov za kavzalne (fizikalno uresničljive) sisteme.

Polov mora biti enako ali več kot ničel.