Naloge 1. kolokvija

PROCESIRANJE SIGNALOV

Datum: 8. 12. 2005

1. Skicirajte signal $x(t)$ in izračunajte njegovo normalizirano moč ($R = 1\ \Omega$) ter koeficiente X_k kompleksne Fouriereve vrste!

$$x(t) = \sin^2(\omega_0 t), \quad \omega_0 = 5\pi$$

Rešitev:

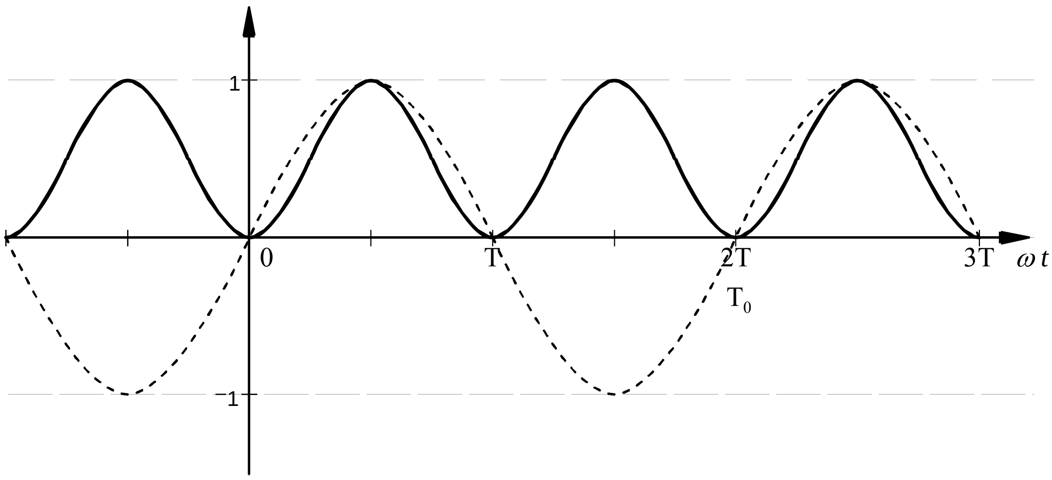
1. Način – brutalni napad:

$$\sin^2(\omega_0 t) = \sin(\omega_0 t)\cdot\sin(\omega_0 t) = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\cdot\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} =$$

$$= \frac{e^{j\omega_0 t}\cdot e^{j\omega_0 t} - e^{-j\omega_0 t}\cdot e^{j\omega_0 t} - e^{j\omega_0 t}\cdot e^{-j\omega_0 t} + e^{-j\omega_0 t}\cdot e^{-j\omega_0 t}}{4\cdot j^2} =$$

$$= \frac{e^{j2\omega_0 t} - 1 - 1 + e^{-j2\omega_0 t}}{4\cdot(-1)} = \frac{-2 + e^{j2\omega_0 t} + e^{-j2\omega_0 t}}{-4} =$$

$$= \frac{-2 + 2\cdot\cos(2\omega_0 t)}{-4} = \frac{1-\cos(2\omega_0 t)}{2}$$



$$T_0 = \frac{2\pi}{\omega_0}, \quad T = \frac{2\pi}{\omega}$$

$$T_0 = 2\cdot T, \quad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T} = \frac{2\pi}{2\cdot\frac{2\pi}{\omega}} = \frac{\omega}{2}$$

$$\omega = 2\omega_0$$

$$X_k = \frac{1}{T}\int_0^T x(t)\cdot e^{-jk\omega t}dt = \frac{1}{T}\int_0^T \frac{1}{2}\cdot\left(1-\cos\left(2\omega_0 t\right)\right)e^{-jk\omega t}dt$$

$$X_k = \frac{1}{2T}\int_0^T \left(1-\cos\left(\omega t\right)\right)e^{-jk\omega t}dt = \frac{1}{2T}\int_0^T e^{-jk\omega t}dt - \frac{1}{2T}\int_0^T \cos\left(\omega t\right)e^{-jk\omega t}dt$$

$$X_k = \frac{1}{2T}\cdot\frac{1}{-jk\omega}\cdot e^{-jk\omega t}\Big|_0^T - \frac{1}{2T}\int_0^T\left(\frac{e^{j\omega t}+e^{-j\omega t}}{2}\right)\cdot e^{-jk\omega t}dt$$

$$X_k = \frac{1}{2T}\cdot\frac{1}{-jk\omega}\cdot\left(e^{-jk\omega T}-e^0\right) - \frac{1}{4T}\int_0^T e^{j\omega t}\cdot e^{-jk\omega t}dt - \frac{1}{4T}\int_0^T e^{-j\omega t}\cdot e^{-jk\omega t}dt$$

$$X_k = \frac{1}{2T}\cdot\frac{1}{-jk\omega}\cdot\left(e^{-jk\frac{\omega T}{2}-jk\frac{\omega T}{2}} - e^{-jk\frac{\omega T}{2}+jk\frac{\omega T}{2}}\right) - \frac{1}{4T}\int_0^T e^{j(1-k)\omega t}dt - \frac{1}{4T}\int_0^T e^{j(1+k)\omega t}dt$$

$$X_k = \frac{1}{2T}\cdot\frac{e^{-jk\frac{\omega T}{2}}}{-jk\omega}\cdot\left(e^{-jk\frac{\omega T}{2}} - e^{+jk\frac{\omega T}{2}}\right) - \frac{1}{4T}\cdot\frac{1}{j(1-k)\omega}\cdot e^{j(1-k)\omega t}\Big|_0^T - \frac{1}{4T}\cdot\frac{1}{j(1+k)\omega}\cdot e^{j(1+k)\omega t}\Big|_0^T$$

$$X_k = \frac{1}{2T}\cdot\frac{e^{-jk\frac{\omega T}{2}}}{-jk\omega}\cdot\left(-2j\cdot\sin\left(k\frac{\omega T}{2}\right)\right) - \frac{1}{4T}\cdot\frac{1}{j(1-k)\omega}\cdot\left(e^{j(1-k)\omega T}-e^0\right) - \frac{1}{4T}\cdot\frac{1}{j(1+k)\omega}\cdot\left(e^{j(1+k)\omega T}-e^0\right)$$

$$X_k = \frac{1}{2T}\cdot\frac{e^{-jk\frac{\omega T}{2}}}{-jk\omega}\cdot\left(-2j\cdot\sin\left(k\frac{\omega T}{2}\right)\right) - \frac{1}{4T}\cdot\frac{1}{j(1-k)\omega}\cdot\left(e^{j\omega T}e^{-jk\omega T}-e^0\right) - \frac{1}{4T}\cdot\frac{1}{j(1+k)\omega}\cdot\left(e^{j\omega T}e^{+jk\omega T}-e^0\right)$$

$$\omega\cdot T = \frac{2\pi}{T}\cdot T = 2\pi$$

$$X_k = \frac{1}{2T}\cdot\frac{e^{-jk\frac{2\pi}{2}}}{-jk\omega}\cdot\left(-2j\cdot\sin\left(k\frac{2\pi}{2}\right)\right) - \frac{1}{4T}\cdot\frac{1}{j(1-k)\omega}\cdot\left(e^{j2\pi}e^{-jk2\pi}-e^0\right) - \frac{1}{4T}\cdot\frac{1}{j(1+k)\omega}\cdot\left(e^{j2\pi}e^{+jk2\pi}-e^0\right)$$

$$X_k = \frac{1}{T}\cdot\frac{e^{-jk}}{k\omega}\cdot\sin\left(k\pi\right) - \frac{1}{4T}\cdot\frac{1}{j(1-k)\omega}\cdot\left(e^{-jk2\pi}-e^0\right) - \frac{1}{4T}\cdot\frac{1}{j(1+k)\omega}\cdot\left(e^{+jk2\pi}-e^0\right)$$

$$X_k = \frac{1}{T}\cdot\frac{e^{-jk}}{k\omega}\cdot\sin\left(k\pi\right) - \frac{1}{4T}\cdot\frac{1}{j(1-k)\omega}\cdot\left(e^{-jk\pi-jk\pi}-e^{-jk\pi+jk\pi}\right) - \frac{1}{4T}\cdot\frac{1}{j(1+k)\omega}\cdot\left(e^{+jk\pi+jk\pi}-e^{+jk\pi-jk\pi}\right)$$

$$X_k = \frac{1}{T}\cdot\frac{e^{-jk}}{k\omega}\cdot\sin\left(k\pi\right) - \frac{1}{4T}\cdot\frac{e^{-jk\pi}}{j(1-k)\omega}\cdot\left(e^{-jk\pi}-e^{+jk\pi}\right) - \frac{1}{4T}\cdot\frac{e^{+jk\pi}}{j(1+k)\omega}\cdot\left(e^{+jk\pi}-e^{-jk\pi}\right)$$

$$X_k = \frac{1}{T}\cdot\frac{e^{-jk}}{k\omega}\cdot\sin\left(k\pi\right) - \frac{1}{4T}\cdot\frac{e^{-jk\pi}}{j(1-k)\omega}\cdot\left(-2j\cdot\sin\left(k\pi\right)\right) - \frac{1}{4T}\cdot\frac{e^{+jk\pi}}{j(1+k)\omega}\cdot\left(2j\cdot\sin\left(k\pi\right)\right)$$

$$X_k = \frac{1}{T}\sin\left(k\pi\right)\left(\frac{e^{-jk}}{k\omega}\cdot -\frac{1}{4}\cdot\frac{e^{-jk\pi}}{j(1-k)\omega}\cdot(-2j) - \frac{1}{4}\cdot\frac{e^{+jk\pi}}{j(1+k)\omega}\cdot(2j)\right)$$

$$\sin\left(k\pi\right)=0, \quad k\in\mathbb{Z}$$

$$X_k = 0, \qquad k\in\mathbb{Z},\ \ k\neq 0,\ \ k\neq\pm 1$$

$$X_0 = \frac{1}{T}\int_0^T \frac{1}{2}\cdot\left(1-\cos\left(2\omega_0 t\right)\right)\cdot e^0 dt = \frac{1}{2T}\int_0^T dt - \frac{1}{2T}\int_0^T \cos\left(2\omega_0 t\right) dt$$

$$X_0 = \frac{1}{2T}\cdot t\Big|_0^T - \frac{1}{2T}\cdot\frac{\sin\left(2\omega_0 t\right)}{2\omega_0}\Bigg|_0^T = \frac{1}{2T}T - \frac{1}{2T}\cdot\frac{\left(\sin\left(2\omega_0 T\right)-\sin\left(0\right)\right)}{2\omega_0}$$

$$2\omega_0 T = \omega T = 2\pi, \quad \sin\left(2\pi\right)=0, \quad \sin\left(0\right)=0$$

$$X_0 = \frac{1}{2}$$

$$X_1 = \frac{1}{T}\int_0^T \frac{1}{2}\cdot\left(1-\cos\left(2\omega_0 t\right)\right)\cdot e^{-j\omega t} dt = \frac{1}{2T}\int_0^T e^{-j\omega t} dt - \frac{1}{2T}\int_0^T \cos\left(2\omega_0 t\right) e^{-j\omega t} dt$$

$$X_1 = \frac{1}{2T}\frac{e^{-j\omega t}}{-j\omega}\Bigg|_0^T - \frac{1}{2T}\int_0^T \cos\left(\omega t\right) e^{-j\omega t} dt = \frac{1}{2T}\cdot\frac{1}{-j\omega}\left(e^{-j\omega T}-e^0\right) - \frac{1}{2T}\int_0^T \frac{e^{j\omega t}+e^{-j\omega t}}{2} e^{-j\omega t} dt$$

$$X_1 = \frac{1}{2T}\cdot\frac{1}{-j\omega}\left(e^{-j2\pi}-e^0\right) - \frac{1}{4T}\int_0^T \left(e^{j\omega t}+e^{-j\omega t}\right) e^{-j\omega t} dt = \frac{1}{2T}\cdot\frac{1}{-j\omega}\left(1-1\right) - \frac{1}{4T}\int_0^T \left(e^{j\omega t}e^{-j\omega t}+e^{-j\omega t}e^{-j\omega t}\right) dt$$

$$X_1 = 0 - \frac{1}{4T}\int_0^T dt - \frac{1}{4T}\int_0^T e^{-j2\omega t} dt = -\frac{1}{4T}\cdot\left(T-0\right) - \frac{1}{4T}\cdot\frac{1}{-j2\omega}\cdot\left(e^{-j2\omega T}-e^0\right)$$

$$X_1 = -\frac{1}{4} - \frac{1}{4T}\cdot\frac{1}{-j2\omega}\cdot\left(e^{-j2\cdot 2\pi}-e^0\right) = -\frac{1}{4} - \frac{1}{4T}\cdot\frac{1}{-j2\omega}\cdot\left(1-1\right)$$

$$X_1 = -\frac{1}{4}$$

$$X_{-1} = \frac{1}{T}\int_0^T \frac{1}{2}\cdot\left(1-\cos\left(2\omega_0 t\right)\right)\cdot e^{+j\omega t} dt = \frac{1}{2T}\int_0^T e^{+j\omega t} dt - \frac{1}{2T}\int_0^T \cos\left(2\omega_0 t\right) e^{+j\omega t} dt$$

$$X_{-1} = \frac{1}{2T}\frac{e^{+j\omega t}}{+j\omega}\Bigg|_0^T - \frac{1}{2T}\int_0^T \cos\left(\omega t\right) e^{+j\omega t} dt = \frac{1}{2T}\cdot\frac{1}{+j\omega}\left(e^{+j\omega T}-e^0\right) - \frac{1}{2T}\int_0^T \frac{e^{j\omega t}+e^{-j\omega t}}{2} e^{+j\omega t} dt$$

$$X_{-1} = \frac{1}{2T}\cdot\frac{1}{+j\omega}\left(e^{+j2\pi}-e^0\right) - \frac{1}{4T}\int_0^T \left(e^{j\omega t}+e^{-j\omega t}\right) e^{+j\omega t} dt = -\frac{1}{4T}\int_0^T \left(e^{j\omega t}e^{j\omega t}+e^{-j\omega t}e^{j\omega t}\right) dt$$

$$X_{-1} = -\frac{1}{4T}\int_0^T e^{j2\omega t} dt - \frac{1}{4T}\int_0^T dt = -\frac{1}{4T}\cdot\frac{1}{j2\omega}\cdot\left(e^{j2\omega T}-e^0\right) - \frac{1}{4T}\cdot\left(T-0\right) = -\frac{1}{4}$$

$$X_k = \begin{cases} \frac{1}{2} & k=0 \\ -\frac{1}{4} & k=\pm 1 \\ 0 & sicer \end{cases}$$

$$P_x = \frac{1}{T}\int_0^T x^2(t)\,dt = \frac{1}{T}\int_0^T \left(\sin^2(\omega_0 t)\right)^2 dt = \frac{1}{T}\int_0^T \left(\frac{1}{2}\cdot\left(1-\cos(2\omega_0 t)\right)\right)^2 dt$$

$$P_x = \frac{1}{4T}\int_0^T \left(1-\cos(2\omega_0 t)\right)^2 dt = \frac{1}{4T}\int_0^T \left(1-2\cos(2\omega_0 t)+\cos^2(2\omega_0 t)\right)dt$$

$$\cos^2(2\omega_0 t) = \left(\frac{e^{j2\omega_0 t}+e^{-2\omega_0 t}}{2}\right)^2 = \frac{e^{j2\omega_0 t}\cdot e^{j2\omega_0 t}+e^{j2\omega_0 t}\cdot e^{-j2\omega_0 t}+e^{-j2\omega_0 t}\cdot e^{j2\omega_0 t}+e^{-j2\omega_0 t}\cdot e^{-j2\omega_0 t}}{4} =$$

$$= \frac{e^{j4\omega_0 t}+1+1+e^{-j4\omega_0 t}}{4} = \frac{2+2\cos(4\omega_0 t)}{4} = \frac{1}{2}+\frac{1}{2}\cdot\cos(4\omega_0 t)$$

$$P_x = \frac{1}{4T}\int_0^T \left(1-2\cos(2\omega_0 t)+\frac{1}{2}+\frac{1}{2}\cdot\cos(4\omega_0 t)\right)dt = \frac{1}{4T}\int_0^T \frac{3}{2}\,dt - \frac{1}{4T}\int_0^T 2\cos(2\omega_0 t)\,dt + \frac{1}{4T}\int_0^T \frac{1}{2}\cdot\cos(4\omega_0 t)\,dt$$

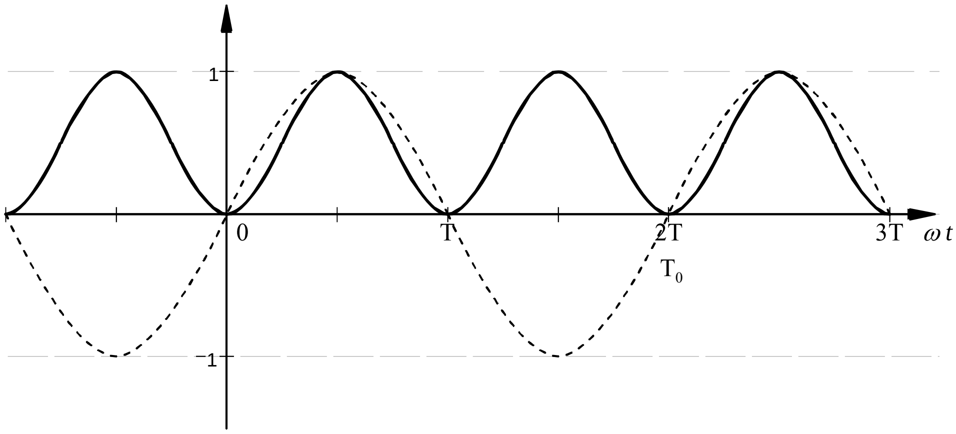
$$P_x = \frac{3}{8T}\int_0^T dt - \frac{1}{2T}\int_0^T \cos(2\omega_0 t)\,dt + \frac{1}{8T}\int_0^T \cos(4\omega_0 t)\,dt = \frac{3}{8T}\cdot(T-0) - \frac{1}{2T}\cdot\left.\frac{\sin(2\omega_0 t)}{2\omega_0}\right|_0^T + \frac{1}{8T}\cdot\left.\frac{\sin(4\omega_0 t)}{4\omega_0}\right|_0^T$$

$$P_x = \frac{3}{8} - \frac{1}{2T}\cdot\frac{1}{2\omega_0}\left(\sin(2\omega_0 T)-\sin(0)\right) + \frac{1}{8T}\cdot\frac{1}{4\omega_0}\left(\sin(4\omega_0 T)-\sin(0)\right)$$

$$2\omega_0 T = \omega T = 2\pi, \quad \sin(2\pi) = \sin(4\pi) = \sin(0) = 0$$

$$\underline{\underline{P_x = \frac{3}{8}}}$$

2. Način – metoda pretkanega pogleda:



$$x(t)=\sin^2(\omega_0 t)=\frac{1}{2}-\frac{1}{2}\cos(2\omega_0 t)=\frac{1}{2}-\frac{1}{2}\cos(\omega t)$$

Enosmerna komponenta: $x_{DC}=\frac{1}{2}\Rightarrow X_0=\frac{1}{2}$

Harmonična komponenta je le ena: $x_{AC}(t)=-\frac{1}{2}\cos(\omega t)\Rightarrow X_{\pm 1}=-\frac{1}{4}$

ali s pravilom o modulaciji: $y(t)=x(t)\cdot\cos(n\omega t)\leftrightarrow Y_k=\frac{1}{2}\left(X_{k-n}+X_{k+n}\right)$

$$-\frac{1}{2}\leftrightarrow X_k=\begin{cases}-\frac{1}{2} & k=0\\ 0 & k\neq 0\end{cases}\Rightarrow Y_k=\begin{cases}-\frac{1}{4} & k=\pm 1\\ 0 & k\neq \pm 1\end{cases}$$

ali tako da cos rzstavimo na dva fazorja:

$$x_{AC}(t)=-\frac{1}{2}\cos(\omega t)=-\frac{1}{2}\cdot\frac{e^{j\omega t}+e^{-j\omega t}}{2}=-\frac{1}{4}\left(e^{j\omega t}+e^{-j\omega t}\right)$$

$$X_{ACk}=\frac{1}{T}\int_0^T\left(-\frac{1}{4}\right)\left(e^{j\omega t}+e^{-j\omega t}\right)e^{-jk\omega t}dt=-\frac{1}{4T}\int_0^T e^{j\omega t}e^{-jk\omega t}dt-\frac{1}{4T}\int_0^T e^{-j\omega t}e^{-jk\omega t}dt$$

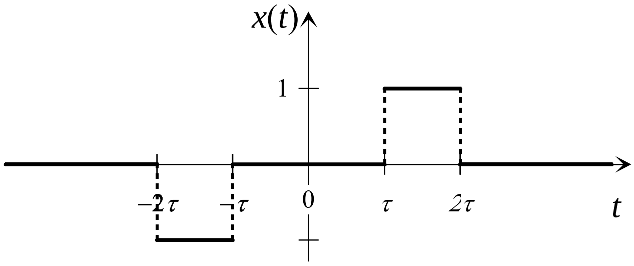
$$X_{ACk}=-\frac{1}{4T}\left(\int_0^T e^{-j(k-1)\omega t}dt+\int_0^T e^{-j(k+1)\omega t}dt\right)=\begin{cases}-\frac{1}{4} & k=\pm 1\\ 0 & k\neq \pm 1\end{cases}$$

$$X_k=\begin{cases}\frac{1}{2} & k=0\\ -\frac{1}{4} & k=\pm 1\\ 0 & sicer\end{cases}$$

Moč signala je vsota kvadratov vseh spektralnih komponent:

$$P_x=\sum_k X_k^{\,2}=\left(-\frac{1}{4}\right)^2+\left(\frac{1}{2}\right)^2+\left(-\frac{1}{4}\right)^2=2\cdot\frac{1}{16}+\frac{1}{4}=\frac{1}{8}+\frac{2}{8}=\frac{3}{8}$$

2. Izračunajte Fourierev transform narisanega aperiodičnega signala.



Katere vrednosti lahko zavzema faza za ta primer?

Rešitev:

1. Način:

$$X(\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt=\int_{-2\tau}^{-\tau}(-1)\cdot e^{-j\omega t}dt+\int_{\tau}^{2\tau}1\cdot e^{-j\omega t}dt$$

$$X(\omega)=-\frac{1}{-j\omega}\cdot e^{-j\omega t}\Big|_{-2\tau}^{-\tau}+\frac{1}{-j\omega}\cdot e^{-j\omega t}\Big|_{\tau}^{2\tau}=\frac{1}{j\omega}\cdot\left(e^{j\omega\tau}-e^{j2\omega\tau}\right)-\frac{1}{j\omega}\cdot\left(e^{-j2\omega\tau}-e^{-j\omega\tau}\right)$$

$$X(\omega)=\frac{e^{j\omega\tau}}{j\omega}\cdot\left(e^{0}-e^{j\omega\tau}\right)-\frac{e^{-j\omega\tau}}{j\omega}\cdot\left(e^{-j\omega\tau}-e^{0}\right)$$

$$X(\omega)=\frac{e^{j\omega\tau}}{j\omega}\cdot\left(e^{j\omega\frac{\tau}{2}-j\omega\frac{\tau}{2}}-e^{j\omega\frac{\tau}{2}+j\omega\frac{\tau}{2}}\right)-\frac{e^{-j\omega\tau}}{j\omega}\cdot\left(e^{-j\omega\frac{\tau}{2}-j\omega\frac{\tau}{2}}-e^{j\omega\frac{\tau}{2}-j\omega\frac{\tau}{2}}\right)$$

$$X(\omega)=\frac{e^{j\omega\tau}\cdot e^{j\omega\frac{\tau}{2}}}{j\omega}\cdot\left(e^{-j\omega\frac{\tau}{2}}-e^{j\omega\frac{\tau}{2}}\right)-\frac{e^{-j\omega\tau}\cdot e^{-j\omega\frac{\tau}{2}}}{j\omega}\cdot\left(e^{-j\omega\frac{\tau}{2}}-e^{j\omega\frac{\tau}{2}}\right)$$

$$X(\omega)=\frac{e^{j\omega\frac{3\tau}{2}}}{j\omega}\cdot\left(-2j\sin\left(\omega\frac{\tau}{2}\right)\right)-\frac{e^{-j\omega\frac{3\tau}{2}}}{j\omega}\cdot\left(-2j\sin\left(\omega\frac{\tau}{2}\right)\right)$$

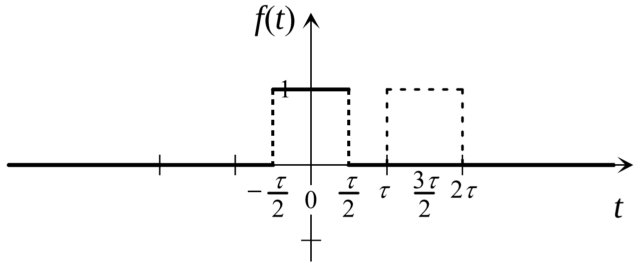
$$X(\omega)=\frac{1}{j\omega}\cdot\left(-2j\sin\left(\omega\frac{\tau}{2}\right)\right)\cdot\left(e^{j\omega\frac{3\tau}{2}}-e^{-j\omega\frac{3\tau}{2}}\right)=\frac{-2}{\omega}\cdot\sin\left(\omega\frac{\tau}{2}\right)\cdot\left(2j\sin\left(\omega\frac{3\tau}{2}\right)\right)$$

$$X(\omega)=\frac{-4j}{\omega}\cdot\sin\left(\omega\frac{\tau}{2}\right)\cdot\sin\left(\omega\frac{3\tau}{2}\right)$$

$$\Phi(\omega)=\arctan\left(\frac{\mathrm{Im}}{\mathrm{Re}}\right)\pm p\cdot\pi=\arctan\left(\frac{\mathrm{Im}}{0}\right)\pm p\cdot\pi=\frac{\pi}{2}\pm p\cdot\pi=\pm\frac{\pi}{2}$$

2. Način:

$$y(t) = x(t - t_0) \leftrightarrow Y(\omega) = X(\omega) \cdot e^{-j\omega t_0}$$



$$X(\omega) = -F(\omega) \cdot e^{j\omega\frac{3\tau}{2}} + F(\omega) \cdot e^{-j\omega\frac{3\tau}{2}}$$

$$X(\omega) = F(\omega) \cdot \left(e^{-j\omega\frac{3\tau}{2}} - e^{j\omega\frac{3\tau}{2}} \right) = F(\omega) \cdot \left(-2j \sin\left(\omega \frac{3\tau}{2} \right) \right)$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} 1 \cdot e^{-j\omega t} dt = \frac{1}{-j\omega} \cdot e^{-j\omega t} \Big|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{1}{-j\omega} \cdot \left(e^{-j\omega\frac{\tau}{2}} - e^{j\omega\frac{\tau}{2}} \right)$$

$$F(\omega) = \frac{1}{-j\omega} \cdot \left(-2j \sin\left(\omega \frac{\tau}{2} \right) \right) = \frac{2}{\omega} \cdot \sin\left(\omega \frac{\tau}{2} \right)$$

$$X(\omega) = \frac{2}{\omega} \cdot \sin\left(\omega \frac{\tau}{2} \right) \cdot \left(-2j \sin\left(\omega \frac{3\tau}{2} \right) \right)$$

$$X(\omega) = \frac{-4j}{\omega} \cdot \sin\left(\omega \frac{\tau}{2} \right) \cdot \sin\left(\omega \frac{3\tau}{2} \right)$$

$$\Phi(\omega) = \arctan\left(\frac{\mathrm{Im}}{\mathrm{Re}} \right) \pm p \cdot \pi = \arctan\left(\frac{\mathrm{Im}}{0} \right) \pm p \cdot \pi = \frac{\pi}{2} \pm p \cdot \pi = \pm \frac{\pi}{2}$$

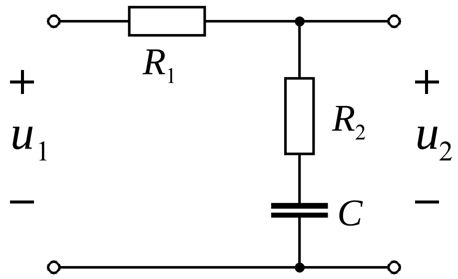
3. Način – s sinusno Fourierevo transformacijo ali s cosinusno Fourierevo transformacijo in premikom, ki da rezultat, ki je na prvi pogled drugačen. Ker ni veliko kandidatov uporabilo to metodo, podajam samo rezultat:

$$X(\omega) = \frac{2j}{\omega} \left(\cos(2\omega\tau) - \cos(\omega\tau) \right)$$

$$\Phi(\omega) = \arctan\left(\frac{\mathrm{Im}}{\mathrm{Re}} \right) \pm p \cdot \pi = \arctan\left(\frac{\mathrm{Im}}{0} \right) \pm p \cdot \pi = \frac{\pi}{2} \pm p \cdot \pi = \pm \frac{\pi}{2}$$

3. Izračunajte sistemsko funkcijo $H(s)$ narisanega vezja in narišite lego ničel in polov v ravnini kompleksne frekvence s!

$R_1 = 1000\ \Omega$ $R_2 = 1\ \Omega$ $C = 10\ \mu\text{F}$



Rešitev:

$$\frac{U_2(s)-U_1(s)}{R_1}+\frac{U_2(s)}{R_2+X_C}=0$$

$$\frac{U_2(s)}{R_1}+\frac{U_2(s)}{R_2+X_C}=\frac{U_1(s)}{R_1}$$

$$U_2(s)=\frac{U_1(s)}{R_1}\cdot\frac{1}{\frac{1}{R_1}+\frac{1}{R_2+X_C}}=U_1(s)\frac{1}{1+\frac{R_1}{R_2+X_C}}\cdot\frac{(R_2+X_C)}{(R_2+X_C)}$$

$$U_2(s)=U_1(s)\frac{R_2+X_C}{R_2+X_C+R_1}$$

$$H(s)=\frac{R_2+X_C}{R_2+X_C+R_1}$$

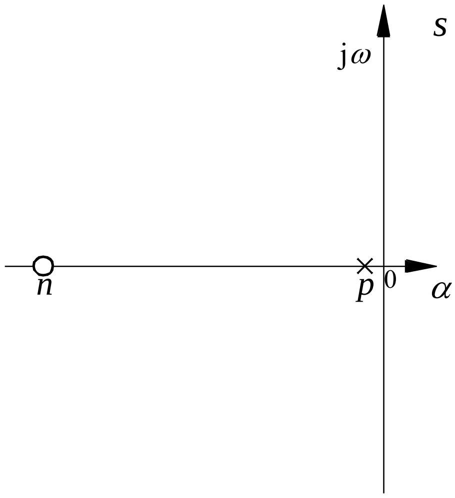
$$X_C=\frac{1}{Cs}$$

$$H(s)=\frac{R_2+\frac{1}{Cs}}{R_2+\frac{1}{Cs}+R_1}\cdot\frac{Cs}{Cs}=\frac{R_2Cs+1}{R_2Cs+1+R_1Cs}=\frac{R_2C\left(s+\frac{1}{R_2C}\right)}{(R_1+R_2)Cs+1}$$

$$H(s)=\frac{R_2C\left(s+\frac{1}{R_2C}\right)}{(R_1+R_2)C\left(s+\frac{1}{(R_1+R_2)C}\right)}=\frac{R_2\left(s+\frac{1}{R_2C}\right)}{(R_1+R_2)\left(s+\frac{1}{(R_1+R_2)C}\right)}$$

$$n=-\frac{1}{R_2C}=-\frac{1}{1\ \text{V/A}\cdot 10\cdot 10^{-6}\ \text{As/V}}=-10^5\ \text{s}^{-1}$$

$$p=-\frac{1}{(R_1+R_2)C}=-\frac{1}{1001\ \text{V/A}\cdot 10\cdot 10^{-6}\ \text{As/V}}\approx -10^2\ \text{s}^{-1}$$



4. Teoretična vprašanja

a) Napišite Parsevalov izrek (teorem) za periodičen signal $x(t)$ in koeficiente kompleksne Fourierove vrste X_k!

$$P_X = \frac{1}{T}\int_{\tau}^{T+\tau} x^2(t)\,dt = \sum_{k=-\infty}^{\infty} \left|X_k\right|^2$$

b) Z besedami opišite pomen (vsebino) Parsevalovega izreka!

Moč periodičnega signala, ki je enaka povprečni moči signala znotraj ene periode, je tudi enaka vsoti kvadratov absolutnih vrednosti vseh spektralnih komponent signala X_k.

Podan je Fourierev par $x(t) \leftrightarrow X(\omega) = \left|X(\omega)\right| \cdot e^{j\Phi_X(\omega)}$. Določite $Y(\omega)$, če je:

c) $y(t) = x(t)\cos\omega_0 t$ $\leftrightarrow$ $Y(\omega) = \frac{1}{2}\left(X(\omega-\omega_0) + X(\omega+\omega_0)\right)$

d) $y(t) = Ax(t - t_0)$ $\leftrightarrow$ $Y(\omega) = AX(\omega)\cdot e^{-j\omega t_0}$

e) $y(t) = -x(-t)$ $\leftrightarrow$ $Y(\omega) = -X(-\omega)$