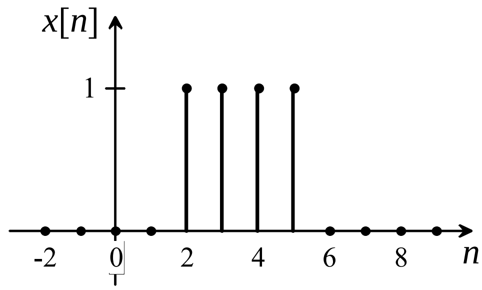
Naloge 2. kolokvija

PROCESIRANJE SIGNALOV

Datum: 14. 01. 2006

1. Izračunajte TDF transform $X(\Omega)$ za podani diskretni aperiodični signal $x[n]$.



Rešitev:

$$x[n] = y[n-2]$$

$$Y(\Omega) = \sum_{n=-\infty}^{\infty} y[n] \cdot e^{-j\Omega n} = \sum_{n=0}^{N-1} e^{-j\Omega n} = \frac{1-e^{-j\Omega N}}{1-e^{-j\Omega}} = \frac{1-e^{-j4\Omega}}{1-e^{-j\Omega}}$$

$$x[n] = y[n-m] \leftrightarrow X(\Omega) = Y(\Omega) e^{-j\Omega m}$$

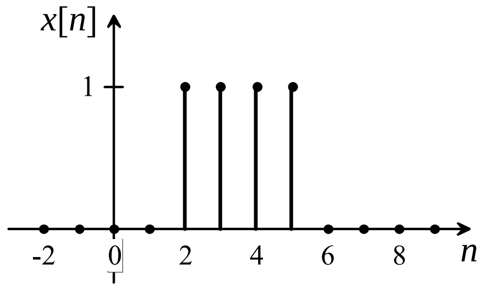
$$X(\Omega) = Y(\Omega) \cdot e^{-j2\Omega}$$

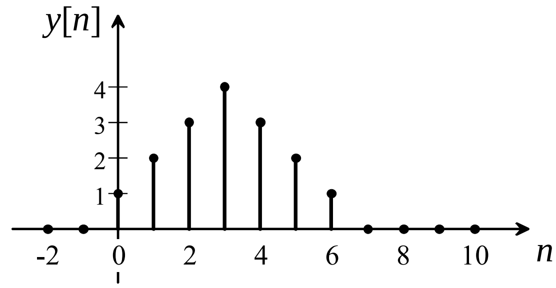
$$X(\Omega) = \frac{1-e^{-j4\Omega}}{1-e^{-j\Omega}} \cdot e^{-j2\Omega}$$

$$X(\Omega) = \frac{e^{-j2\Omega}\left(e^{j2\Omega} - e^{-j2\Omega}\right)}{e^{-j\frac{\Omega}{2}}\left(e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}}\right)} \cdot e^{-j2\Omega} = e^{-j\frac{7\Omega}{2}} \frac{2j\sin(2\Omega)}{2j\sin\left(\frac{\Omega}{2}\right)}$$

$$\underline{\underline{X(\Omega) = \frac{\sin(2\Omega)}{\sin\left(\frac{\Omega}{2}\right)} e^{-j\frac{7\Omega}{2}}}}$$

2. Narišite graf signala $v[n] = x[n] * y[n]$, če sta $x[n]$ in $y[n]$ podana s spodnjima grafoma:



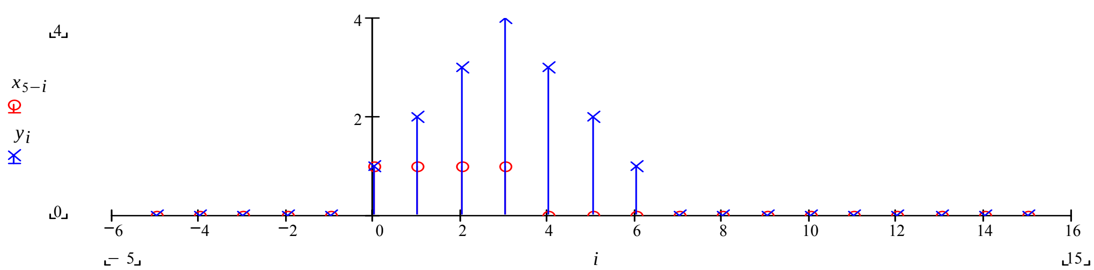


Rešitev:

$$v[n] = x[n] * y[n] = \sum_{i=-\infty}^{\infty} x[n-i] \cdot y[i]$$

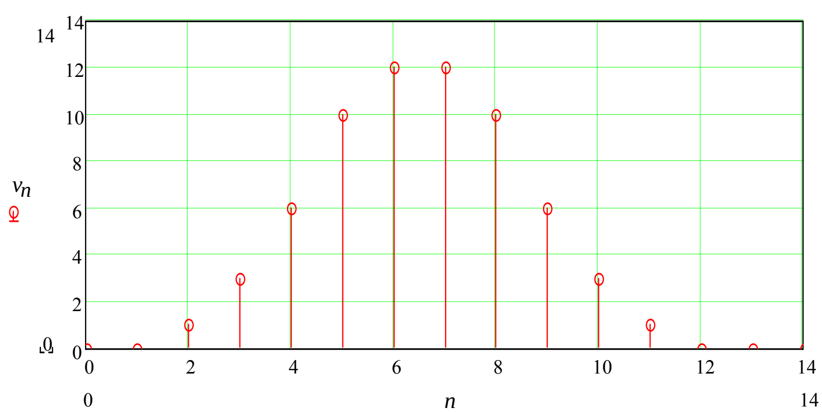
Konvolucijo izračunamo tako, da enega od signalov prezrcalimo okoli ordinatne osi ($x[-i]$) in ga »peljemo« od $-\infty$ proti $+\infty$. V vsakem koraku izračunamo produkte vrednosti istoležnih točk obeh funkcij in vse produkte seštejemo.

Primer za točko n=5:



$v[5] = 1 \cdot 1 + 1 \cdot 2 + 1 \cdot 3 + 1 \cdot 4 = 10$

Končni rezultat:



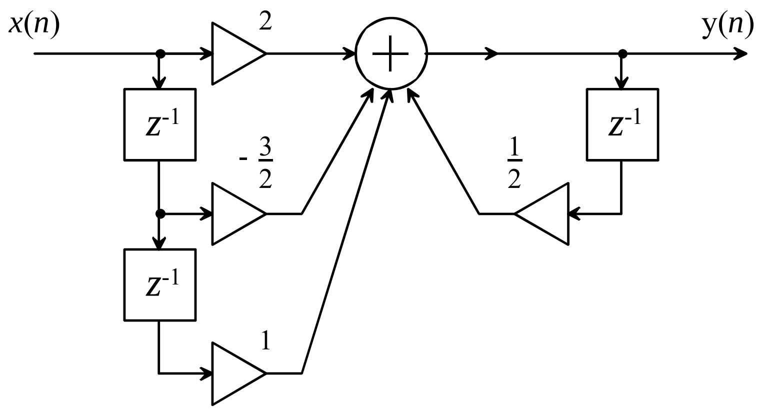
n	v[n]
0	0
1	0
2	1
3	3
4	6
5	10
6	12
7	12
8	10
9	6
10	3
11	1

3. Diskretni časovno nespremenljivi linearni sistem je opisan z diferenčno enačbo

$$y[n] = 2x[n] - \frac{3}{2}x[n-1] + x[n-2] + \frac{1}{2}y[n-1].$$

Narišite shemo vezja, določite njegovo sistemsko funkcijo $H(z)$ ter njene ničle in pole. Ali je podani sistem stabilen?

Rešitev:



$$Y(z) = 2X(z) - \frac{3}{2}X(z)z^{-1} + X(z)z^{-2} + \frac{1}{2}Y(z)z^{-1}$$

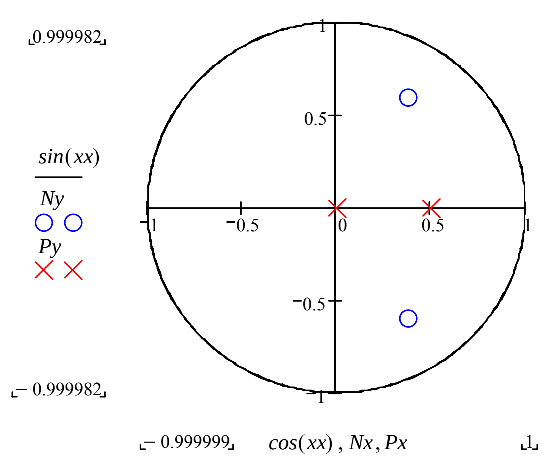
$$Y(z)\cdot\left(1 - \frac{1}{2}z^{-1}\right) = X(z)\cdot\left(2 - \frac{3}{2}z^{-1} + z^{-2}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 - \frac{3}{2}z^{-1} + z^{-2}}{1 - \frac{1}{2}z^{-1}} = \frac{2z^2 - \frac{3}{2}z + 1}{z\cdot\left(z - \frac{1}{2}\right)} = 2\cdot\frac{z^2 - \frac{3}{4}z + \frac{1}{2}}{z\cdot\left(z - \frac{1}{2}\right)}$$

$$n_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{\frac{3}{4} \pm \sqrt{\frac{9}{16} - 4\cdot\frac{1}{2}}}{2} = \frac{1}{8}\cdot\left(3 \pm j\sqrt{23}\right)$$

$$H(z) = 2\cdot\frac{\left(z - \frac{1}{8}\cdot\left(3 + j\sqrt{23}\right)\right)\left(z - \frac{1}{8}\cdot\left(3 - j\sqrt{23}\right)\right)}{z\cdot\left(z - \frac{1}{2}\right)}$$

$$p_1 = 0 \qquad p_2 = \frac{1}{2}$$



Vsi poli so znotraj enotske krožnice, torej je sistem stabilen.

4. Sistemska funkcija $H(z)$ diskretnega LTI sistema s končnim impulznim odzivom (FIR) ima ničle: $z_1 = 1 + j$; $z_2 = 1 - j$; $z_3 = -0,5$.

a) določite manjkajoče ničle, da bo frekvenčni odziv vezja imel linearno fazo $\Phi_H(\Omega) = k\cdot\Omega$
b) za dopolnjen sistem določite lego in minimalno število polov, da je sistem fizikalno uresničljiv (vzročen)
c) določite impulzni odziv $h[n]$
d) narišite lego ničel in polov v z – ravnini

Rešitev:

a)

$$z_{1,2} = 1 \pm j = \sqrt{1^2 + 1^2} \cdot e^{\pm j\frac{\pi}{4}}$$

$$z_{4,5} = \left(z_{1,2}\right)^{-1} = \frac{1}{\sqrt{2}} \cdot e^{\mp j\frac{\pi}{4}} = \frac{1}{\sqrt{2}} \cdot \left(\cos\left(\frac{\pi}{4}\right) \mp j\sin\left(\frac{\pi}{4}\right)\right) = \frac{1}{\sqrt{2}} \cdot \left(\frac{1}{\sqrt{2}} \mp j\frac{1}{\sqrt{2}}\right) = \frac{1}{2} \mp \frac{j}{2}$$

$$z_6 = \frac{1}{z_3} = \frac{1}{-0.5} = -2$$

b) Polov mora biti vsaj toliko kolikor je ničel. Da ne spremenijo frekvenčnega poteka morajo biti v izhodišču. Torej dobimo 6-kratni pol v izhodišču: $p_{1,2,3,4,5,6} = 0$.

c)

$$H(z) = \frac{(z-1-j)\cdot(z-1+j)\cdot\left(z-\frac{1}{2}-\frac{j}{2}\right)\cdot\left(z-\frac{1}{2}+\frac{j}{2}\right)\cdot(z+0,5)\cdot(z+2)}{z^6}$$

$$H(z) = \frac{\left((z-1)^2+1^2\right)\cdot\left(\left(z-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2\right)\cdot\left(z^2+2,5z+1\right)}{z^6}$$

$$H(z) = \frac{\left(z^2-2z+2\right)\cdot\left(z^2-z+0,5\right)\cdot\left(z^2+2,5z+1\right)}{z^6}$$

$$H(z) = \frac{\left(z^4-3z^3+4,5z^2-3z+1\right)\cdot\left(z^2+2,5z+1\right)}{z^6}$$

$$H(z) = \frac{z^6-3z^5+4,5z^4-3z^3+z^2+2,5z^5-7,5z^4+11,25z^3-7,5z^2+2,5z+z^4-3z^3+4,5z^2-3z+1}{z^6}$$

$$H(z) = \frac{z^6-0,5z^5-2z^4+5,25z^3-2z^2-0,5z+1}{z^6}$$

$$H(z) = 1-0,5z^{-1}-2z^{-2}+5,25z^{-3}-2z^{-4}-0,5z^{-5}+z^{-6}$$

$$y[n] = x[n] - 0,5x[n-1] - 2x[n-2] + 5,25x[n-3] - 2x[n-4] - 0,5x[n-5] + x[n-6]$$

n	h[n]
0	1
1	-0,5
2	-2
3	5,25
4	-2
5	-0,5
6	1
7	0
8	0

d) Ničle in poli: