2. Fouriereva vrsta

UVOD

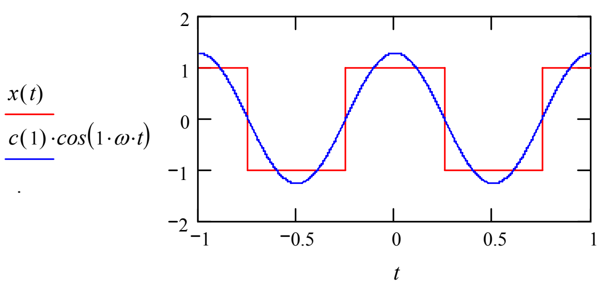
Vsak signal se da zapisati kot vsoto sinusnih ali cosinusnih signalov.

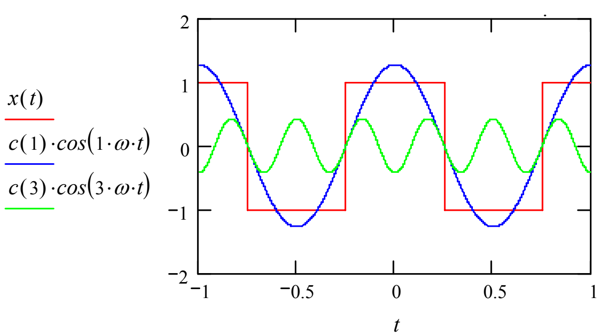
Zgled:
Pravokotni signal lahko zapišemo kot vsoto cosinusnih signalov, ki imajo amplitude:
$c_0 = 0$, $c_1 = 1,27$, $c_2 = 0$, $c_3 = -0,42$, $c_4 = 0$, $c_5 = 0,25$, $c_6 = 0$, $c_7 = -0,18$,... in frekvence
$f_0 = 0$, $f_1 = f_O$, $f_2 = 2f_O$, $f_3 = 3f_O$, $f_4 = 4f_O$, $f_5 = 5f_O$, $f_6 = 6f_O$, $f_7 = 7f_O$,...

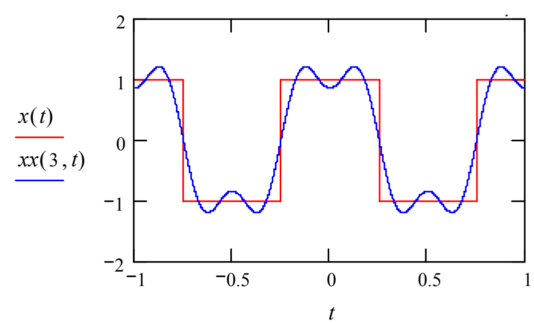
Opomba: f_0 (f nič) je frekvenca enosmernega signala, torej 0 Hz. f_O (f o) je osnovna frekvenca signala – frekvenca osnovne harmonske komponente signala. Torej $f_0 \neq f_O$.

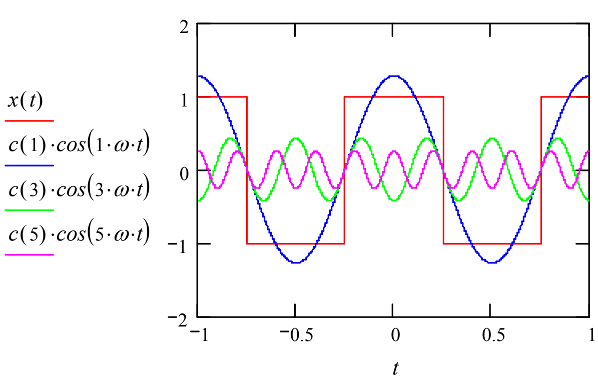
Narišimo posamezne cosinuse in njihove vsote ter jih primerjajmo s pravokotnim signalom. Vsakič dodajmo en cosinus (cosinuse z amplitudo 0 spustimo).

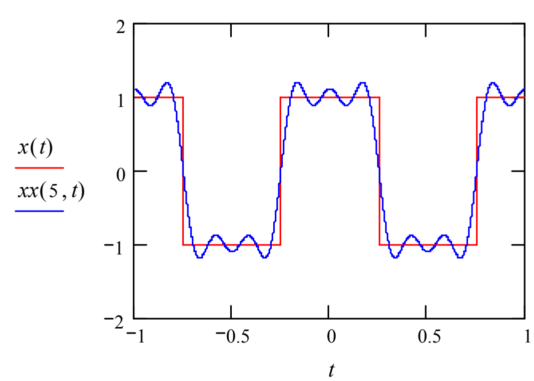
Posamezni signali: Sešteti signali:

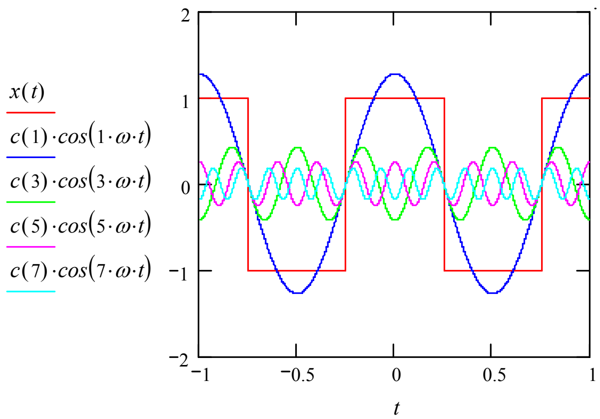


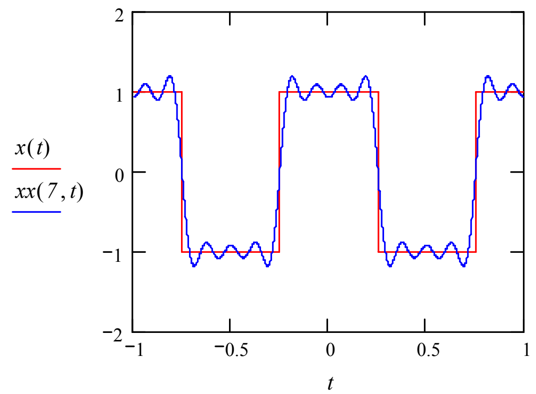






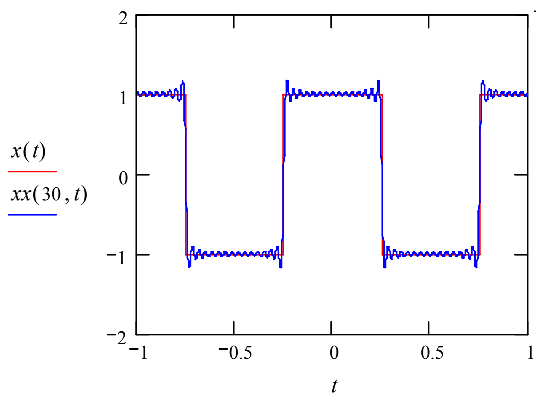




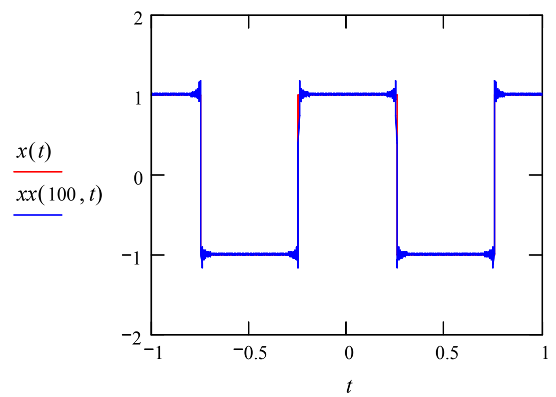


Več komponent upoštevamo, bolj natančen je približek signala:

Upoštevamo prvih 30 komponent:



Upoštevamo prvih 100 komponent:

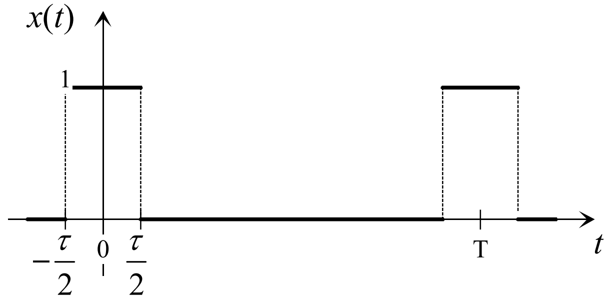


Če bi upoštevali neskončno komponent, bi se neskončno približali pravemu signalu.

1. Naloga

Izračunajte koeficiente kompleksne Fouriereve vrste X_k za podani periodični signal.

$\tau = \frac{T}{5}$



Rešitev:

$$X_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\cdot e^{-jk\omega_0 t}dt \tag{2.1.1}$$

$$X_k = \frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} 1\cdot e^{-jk\omega_0 t}dt = \frac{1}{T}\cdot\frac{1}{-jk\omega_0}\cdot e^{-jk\omega_0 t}\Big|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = -\frac{1}{jkT\omega_0}\cdot\left(e^{-jk\omega_0\frac{\tau}{2}} - e^{jk\omega_0\frac{\tau}{2}}\right) \tag{2.1.2}$$

$$\omega_0 = \frac{2\pi}{T} \tag{2.1.3}$$

$$X_k = -\frac{1}{jkT\frac{2\pi}{T}}\cdot\left(e^{-jk\frac{2\pi}{T}\frac{\tau}{2}} - e^{+jk\frac{2\pi}{T}\frac{\tau}{2}}\right) = -\frac{1}{jk2\pi}\cdot\left(e^{-jk\pi\frac{\tau}{T}} - e^{+jk\pi\frac{\tau}{T}}\right) \tag{2.1.4}$$

$$X_k = -\frac{1}{jk2\pi}\cdot\left(\cos\left(-k\pi\frac{\tau}{T}\right) + j\sin\left(-k\pi\frac{\tau}{T}\right) - \cos\left(k\pi\frac{\tau}{T}\right) - j\sin\left(k\pi\frac{\tau}{T}\right)\right) \tag{2.1.5}$$

$$X_k = -\frac{1}{jk2\pi}\cdot\left(-2j\sin\left(k\pi\frac{\tau}{T}\right)\right) = \frac{\sin\left(k\pi\frac{\tau}{T}\right)}{k\pi} \tag{2.1.6}$$

$$X_k = \frac{\tau}{T}\frac{\sin\left(k\pi\frac{\tau}{T}\right)}{k\pi\frac{\tau}{T}} \tag{2.1.7}$$

$$X_k = \frac{\frac{T}{5}}{T} \frac{\sin\left(k\pi \frac{\frac{T}{5}}{T}\right)}{k\pi \frac{\frac{T}{5}}{T}} = \frac{1}{5} \frac{\sin\left(\frac{k\pi}{5}\right)}{\frac{k\pi}{5}} \tag{2.1.8}$$

Izračunan izraz velja, če je k različen od 0. Za $k = 0$ je potrebno izpeljati svoj izraz. To lahko naredimo tako, da v začetno formulo vstavimo $k = 0$, ali pa dobljeni izraz limitiramo. Naredimo na oba načina:

1. V začetni izraz vstavimo $k = 0$:

$$X_0 = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) \cdot e^{-j0\omega_0 t} dt = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) \cdot 1 \cdot dt = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) dt \tag{2.1.9}$$

   Dobljeni izraz predstavlja povprečno ali srednjo ali enosmerno vrednost signala.

$$X_0 = \frac{1}{T} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} 1 dt = \frac{1}{T} t \Big|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{1}{T} \cdot \left(\frac{\tau}{2} - \left(-\frac{\tau}{2} \right) \right) = \frac{\tau}{T} \tag{2.1.10}$$

$$X_0 = \frac{\frac{T}{5}}{T} = \frac{1}{5} \tag{2.1.11}$$

2. Izraz (2.1.8) limitiramo proti 0:

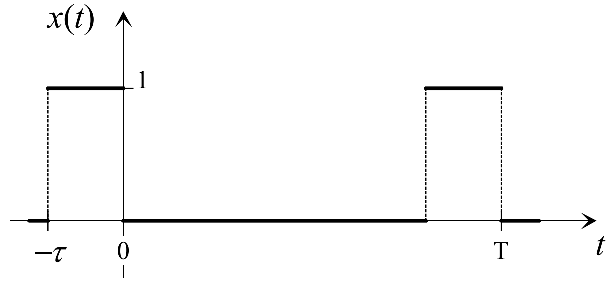
$$X_0 = \lim_{k \to 0} \left(\frac{1}{5} \frac{\sin\left(k \frac{\pi}{5}\right)}{k \frac{\pi}{5}} \right) = \frac{1}{5} \lim_{k \to 0} \left(\frac{\frac{d}{dk}\left(\sin\left(k \frac{\pi}{5}\right)\right)}{\frac{d}{dk}\left(k \frac{\pi}{5}\right)} \right) = \frac{1}{5} \lim_{k \to 0} \left(\frac{\frac{\pi}{5} \cos\left(k \frac{\pi}{5}\right)}{\frac{\pi}{5}} \right) \tag{2.1.12}$$

$$X_0 = \frac{1}{5} \left(\frac{\frac{\pi}{5} \cos(0)}{\frac{\pi}{5}} \right) = \frac{1}{5} \tag{2.1.13}$$

2. Naloga

Izračunajte koeficiente kompleksne Fouriereve vrste X_k za podani periodični signal.

$\tau = \dfrac{T}{5}$



Rešitev:

$$X_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\cdot e^{-jk\omega_0 t}dt \tag{2.2.1}$$

$$X_k = \frac{1}{T}\int_{-\tau}^{0} 1\cdot e^{-jk\omega_0 t}dt = \frac{1}{T}\cdot\frac{1}{-jk\omega_0}\cdot e^{-jk\omega_0 t}\Big|_{-\tau}^{0} = -\frac{1}{jkT\omega_0}\cdot\left(e^{-jk\omega_0 0} - e^{jk\omega_0\tau}\right) \tag{2.2.2}$$

$$X_k = -\frac{1}{jkT\omega_0}\cdot\left(e^{-jk\omega_0\frac{\tau}{2}}\cdot e^{+jk\omega_0\frac{\tau}{2}} - e^{+jk\omega_0\frac{\tau}{2}}\cdot e^{+jk\omega_0\frac{\tau}{2}}\right) = -\frac{1}{jkT\omega_0}\cdot e^{+jk\omega_0\frac{\tau}{2}}\left(e^{-jk\omega_0\frac{\tau}{2}} - e^{+jk\omega_0\frac{\tau}{2}}\right) \tag{2.2.3}$$

$$X_k = -\frac{1}{jkT\omega_0}\cdot e^{+jk\omega_0\frac{\tau}{2}}\cdot\left(\cos\left(-k\omega_0\frac{\tau}{2}\right) + j\sin\left(-k\omega_0\frac{\tau}{2}\right) - \cos\left(k\omega_0\frac{\tau}{2}\right) - j\sin\left(k\omega_0\frac{\tau}{2}\right)\right) \tag{2.2.4}$$

$$X_k = -\frac{1}{jkT\omega_0}\cdot e^{+jk\omega_0\frac{\tau}{2}}\cdot\left(-2j\sin\left(k\omega_0\frac{\tau}{2}\right)\right) = \frac{2\sin\left(k\omega_0\frac{\tau}{2}\right)}{kT\omega_0}\cdot e^{+jk\omega_0\frac{\tau}{2}} \tag{2.2.5}$$

$$\omega_0 = \frac{2\pi}{T} \tag{2.2.6}$$

$$X_k = \frac{2\sin\left(k\frac{2\pi}{T}\frac{\tau}{2}\right)}{kT\frac{2\pi}{T}}\cdot e^{+jk\frac{2\pi}{T}\frac{\tau}{2}} = \frac{\sin\left(k\pi\frac{\tau}{T}\right)}{k\pi}\cdot e^{+jk\pi\frac{\tau}{T}} \tag{2.2.7}$$

$$X_k = \frac{\tau}{T}\frac{\sin\left(k\pi\frac{\tau}{T}\right)}{k\pi\frac{\tau}{T}}\cdot e^{+jk\frac{2\pi}{T}\frac{\tau}{2}} \tag{2.2.8}$$

$$X_k = \frac{\frac{T}{5}}{T} \frac{\sin\left(k\pi \frac{\frac{T}{5}}{T}\right)}{k\pi \frac{\frac{T}{5}}{T}} \cdot e^{+jk\frac{2\pi}{T}\frac{\frac{T}{5}}{2}} = \frac{1}{5} \frac{\sin\left(k\pi \frac{1}{5}\right)}{k\pi \frac{1}{5}} \cdot e^{+jk2\pi\frac{1}{10}} \tag{2.2.9}$$

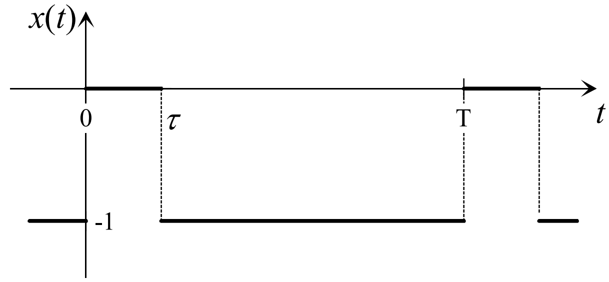
Tudi tokrat velja dobljeni izraz, če je *k* različen od 0. Enosmerno vrednost signala izračunamo posebej:

$$X_0 = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\,dt = \frac{1}{T} \int_{-\tau}^{0} dt = \frac{1}{T} \cdot t\Big|_{-\tau}^{0} = \frac{1}{T}\big(0-(-\tau)\big) = \frac{\tau}{T} = \frac{\frac{T}{5}}{T} = \frac{1}{5} \tag{2.2.10}$$

3. Naloga

Izračunajte koeficiente kompleksne Fourierеve vrste X_k za podani periodični signal.

$\tau = \frac{T}{5}$



Rešitev:

$$X_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\cdot e^{-jk\omega_0 t}dt \tag{2.3.1}$$

$$X_k = \frac{1}{T}\int_{\tau}^{T} -1\cdot e^{-jk\omega_0 t}dt = -\frac{1}{T}\cdot\frac{1}{-jk\omega_0}\cdot e^{-jk\omega_0 t}\Big|_{\tau}^{T} = \frac{1}{jkT\omega_0}\cdot\left(e^{-jk\omega_0 T} - e^{-jk\omega_0 \tau}\right) \tag{2.3.2}$$

$$\omega_0 = \frac{2\pi}{T} \tag{2.3.3}$$

$$X_k = \frac{1}{jkT\frac{2\pi}{T}}\cdot\left(e^{-jk\frac{2\pi}{T}T} - e^{-jk\frac{2\pi}{T}\tau}\right) = \frac{1}{jk2\pi}\cdot\left(e^{-jk2\pi} - e^{-jk2\pi\frac{\tau}{T}}\right) = \frac{1}{jk2\pi}\cdot\left(e^{0} - e^{-jk2\pi\frac{\tau}{T}}\right) \tag{2.3.4}$$

$$X_k = \frac{1}{jk2\pi}\cdot\left(e^{-jk\pi\frac{\tau}{T}}\cdot e^{+jk\pi\frac{\tau}{T}} - e^{-jk\pi\frac{\tau}{T}}\cdot e^{-jk\pi\frac{\tau}{T}}\right) = \frac{1}{jk2\pi}\cdot\left(e^{+jk\pi\frac{\tau}{T}} - e^{-jk\pi\frac{\tau}{T}}\right)\cdot e^{-jk\pi\frac{\tau}{T}} \tag{2.3.5}$$

$$X_k = \frac{1}{jk2\pi}\cdot\left(\cos\left(k\pi\frac{\tau}{T}\right) + j\sin\left(k\pi\frac{\tau}{T}\right) - \cos\left(-k\pi\frac{\tau}{T}\right) + j\sin\left(-k\pi\frac{\tau}{T}\right)\right)\cdot e^{-jk\pi\frac{\tau}{T}} \tag{2.3.6}$$

$$X_k = \frac{1}{jk2\pi}\cdot\left(2j\sin\left(k\pi\frac{\tau}{T}\right)\right)\cdot e^{-jk\pi\frac{\tau}{T}} = \frac{\sin\left(k\pi\frac{\tau}{T}\right)}{k\pi}\cdot e^{-jk\pi\frac{\tau}{T}} \tag{2.3.7}$$

$$X_k = \frac{\tau}{T}\frac{\sin\left(k\pi\frac{\tau}{T}\right)}{k\pi\frac{\tau}{T}}\cdot e^{-jk\frac{2\pi}{T}\frac{\tau}{2}} \tag{2.3.8}$$

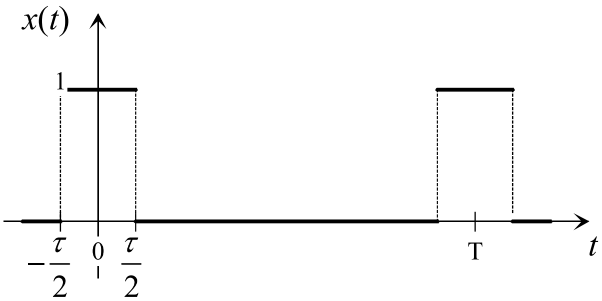
$$X_k = \frac{\frac{T}{5}}{T} \frac{\sin\left(k\pi \frac{\frac{T}{5}}{T}\right)}{k\pi \frac{\frac{T}{5}}{T}} \cdot e^{-jk\frac{2\pi}{T}\frac{\frac{T}{5}}{2}} = \frac{1}{5} \frac{\sin\left(\frac{k\pi}{5}\right)}{\frac{k\pi}{5}} \cdot e^{-jk2\pi\frac{1}{10}} \tag{2.3.9}$$

Tudi tokrat dobljeni izraz velja, če je k različen od 0. Enosmerno vrednost signala izračunamo posebej:

$$X_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{T}\int_\tau^T (-1)\cdot dt = -\frac{1}{T}\cdot t\Big|_\tau^T = -\frac{1}{T}(T-\tau) = -\frac{1}{T}\left(T-\frac{T}{5}\right) = -\frac{1}{T}\frac{4T}{5} = -\frac{4}{5} \tag{2.3.10}$$

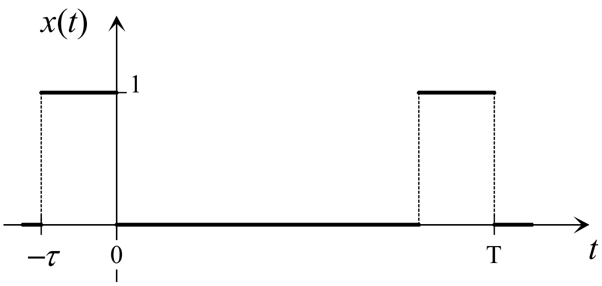
4. Naloga

Primerjajte rezultate prejšnjih treh nalog. Kaj se dogaja s spektrom signala, če signal premikamo po časovni osi? Kaj se dogaja s spektrom, če signal premikamo po navpični osi?



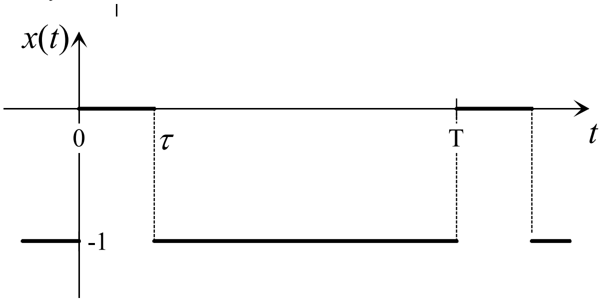
$$X_k = \frac{1}{5}\frac{\sin\left(\frac{k\pi}{5}\right)}{\frac{k\pi}{5}} \quad ; \quad k \neq 0$$

$$X_0 = \frac{1}{5}$$



$$X_k = \frac{1}{5}\frac{\sin\left(\frac{k\pi}{5}\right)}{\frac{k\pi}{5}} \cdot e^{+jk2\pi\frac{1}{10}} \quad ; \quad k \neq 0$$

$$X_0 = \frac{1}{5}$$



$$X_k = \frac{1}{5}\frac{\sin\left(\frac{k\pi}{5}\right)}{\frac{k\pi}{5}} \cdot e^{-jk2\pi\frac{1}{10}} \quad ; \quad k \neq 0$$

$$X_0 = -\frac{4}{5}$$

Če se signal premakne po časovni osi, se spremeni faza spektra – množenje z $e^{j\varphi}$.
Če se signal premakne po navpični osi, se mu spremeni enosmerna vrednost – to je komponenta X_0.

Nauk:
Če je spekter signala težko izračunati za podano obliko, lahko signal premaknemo po časovni in/ali navpični osi, da je spekter laže izračunati. Izračunani spekter na koncu popravimo z ustreznim členom $e^{j\varphi}$, enosmerno komponento pa je treba navadno tako ali tako izračunati posebej.

5. *Naloga*

Za 1. nalogo določite amplitudni in fazni spekter.

$$X_k = \frac{1}{5}\frac{\sin\left(\frac{k\pi}{5}\right)}{\frac{k\pi}{5}} \quad ; \quad k \neq 0 \quad , \quad X_0 = \frac{1}{5}$$

Rešitev:

Kompleksni spekter razdelimo na amplitudni in fazni:

$X_k = |X_k| \cdot e^{j\phi_k}$

$|X_k|$ - amplitudni spekter

ϕ_k - fazni spekter

Kadar ločujemo kompleksni spekter na amplitudnega in faznega, velikokrat poenostavljeno rečemo, da je realna številka pred $e^{j\cdots}$ amplitudni spekter, vse kar je v $e^{j\cdots}$, pa fazni spekter. Vendar zank absolutno odstrani tudi predznak izraza pred $e^{j\cdots}$. Ta predznak zato spada v fazo ($-1 = e^{j\pi} = e^{-j\pi}$).

Primeri:

$$X_2 = 2 \cdot e^{j\frac{2\pi}{7}} \Rightarrow |X_k| = 2 \;,\; \phi_k = \frac{2\pi}{7}$$

$$X_3 = -2 \cdot e^{j\frac{3\pi}{7}} \Rightarrow |X_k| = 2 \;,\; \phi_k = \frac{3\pi}{7} + \pi$$

$$X_{-3} = -2 \cdot e^{j\frac{3\pi}{7}} \Rightarrow |X_k| = 2 \;,\; \phi_k = \frac{3\pi}{7} - \pi$$

Ker mora biti fazni spekter lih, navadno prištejemo π če je $\omega < 0$ oziroma odštejemo π če je $\omega > 0$. Zato za fazo zapišemo:

$$\phi_k = \arctan\left(\frac{\operatorname{Im}(X_k)}{\operatorname{Re}(X_k)}\right) + p \cdot \pi$$

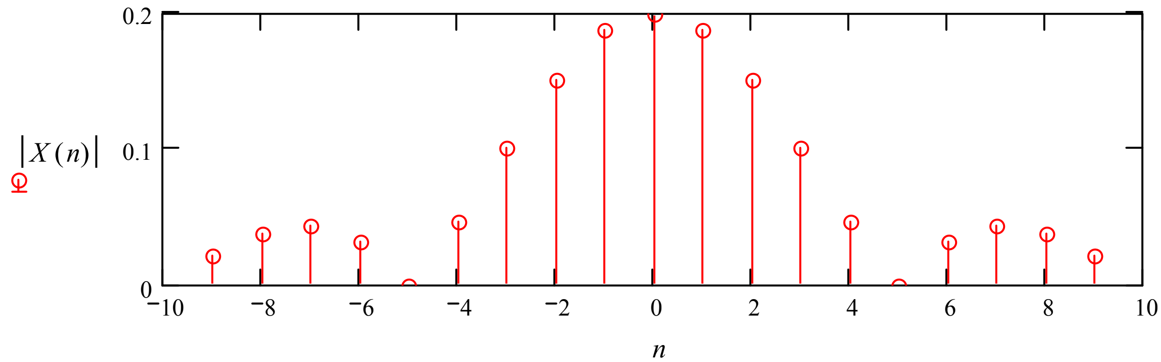
Pri čemer je p lahko -1, 0 ali 1.

$$X_k = \frac{1}{5}\frac{\sin\left(\frac{k\pi}{5}\right)}{\frac{k\pi}{5}} \quad ; \quad k \neq 0 \quad , \quad X_0 = \frac{1}{5}$$

Amplitudni spekter:

$$|X_k| = \left|\frac{1}{5}\frac{\sin\left(\frac{k\pi}{5}\right)}{\frac{k\pi}{5}}\right| = \frac{1}{5}\frac{\left|\sin\left(\frac{k\pi}{5}\right)\right|}{\frac{|k|\pi}{5}}$$

$$|X_0| = 0,2 \ , \ |X_{\pm1}| = 0,187 \ , \ |X_{\pm2}| = 0,151 \ , \ |X_{\pm3}| = 0,101 \ , \ |X_{\pm4}| = 0,047 \ , \ |X_{\pm5}| = 0 \ , \ |X_{\pm6}| = 0,031$$

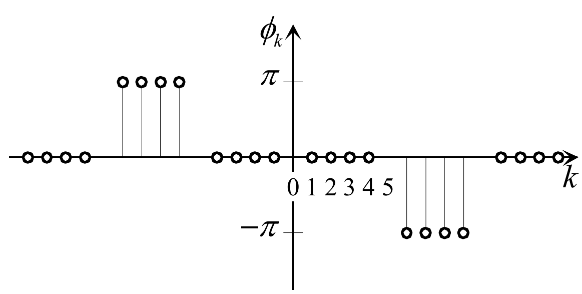


Fazni spekter:

$$\phi_k = \arctan\frac{\operatorname{Im}(X_k)}{\operatorname{Re}(X_k)} + p\cdot\pi = \arctan\left(\frac{0}{\operatorname{Re}(X_k)}\right) + p\cdot\pi = p\cdot\pi \quad ; \quad p = \begin{cases} -1 & ; \quad \sin\left(\frac{k\pi}{5}\right)\Big/\frac{k\pi}{5} < 0, \omega > 0 \\ 0 & ; \quad \sin\left(\frac{k\pi}{5}\right)\Big/\frac{k\pi}{5} \geq 0 \\ 1 & ; \quad \sin\left(\frac{k\pi}{5}\right)\Big/\frac{k\pi}{5} < 0, \omega < 0 \end{cases}$$

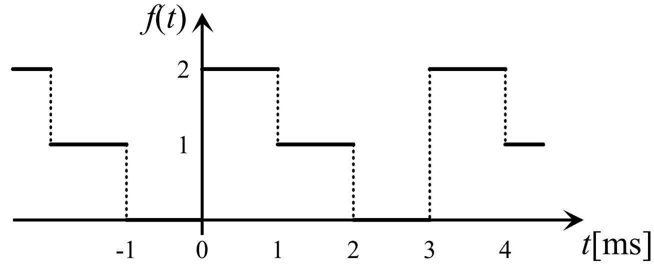
Kadar sta realni in imaginarni del 0, kot ni določen. To je kadar je $k = 0, \pm5, \pm10, ...$

$$\phi_k = \begin{cases} nedoločen & ; \quad k = 0, k = 5\cdot n \\ -\pi & ; \quad k = 6,7,8,9,... \\ 0 & ; \quad k = \pm1,\pm2,\pm3,\pm4,... \\ \pi & ; \quad k = -6,-7,-8,-9,... \end{cases}$$



6. Naloga

Izračunajte koeficiente kompleksne Fouriereve vrste X_k za podani periodični signal.



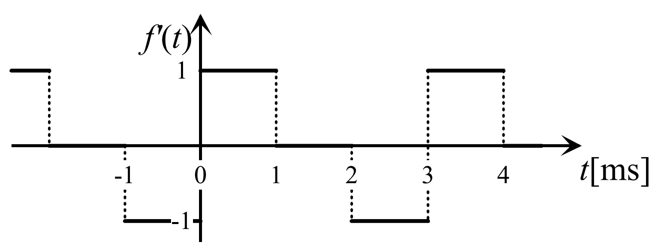
Rešitev:

Najprej določimo periodo T in enosmerno komponento (ali povprečna vrednost) X_0. Oboje lahko določimo kar iz slike:

$$T = 3 \text{ ms} \tag{2.6.1}$$

$$X_0 = 1 \tag{2.6.2}$$

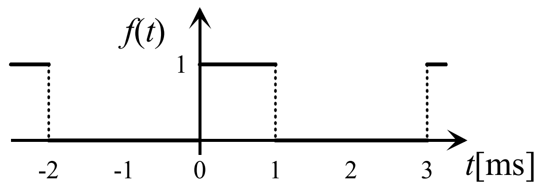
Ko je enosmerna vrednost določena, lahko signal premikamo gor ali dol, saj s tem ne spremenimo nič drugega, kot enosmerno vrednost, ki smo jo že določili.
Če signal premaknemo navzdol za 1, postane lažji za računanje.



Sestavljen je iz dveh pravokotnih impulzov. Eden je malo premaknjen v desno, drugi pa malo v levo in obrnjen. Fourierevo vrsto lahko izračunamo za vsakega posebej in ju potem seštejemo, saj velja:

$$x(t) = f(t) + g(t) \quad \rightarrow \quad \begin{aligned} X_k &= \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-jk\omega_0 t} dt = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left(f(t) + g(t)\right) e^{-jk\omega_0 t} dt = \\ &= \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jk\omega_0 t} dt + \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} g(t) e^{-jk\omega_0 t} dt = F_k + G_k \end{aligned} \tag{2.6.3}$$

Najprej izračunajmo Fourierevo vrsto za pozitivno stopnico:



$$F_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_0^{\tau} e^{-jk\omega_0 t}dt = \frac{-1}{jk\omega_0 T}\cdot e^{-jk\omega_0 t}\Big|_0^{\tau} = \frac{-1}{jk2\pi}\left(e^{-jk\omega_0\tau} - e^{-jk\omega_0 0}\right) \quad (2.6.4)$$

$$F_k = \frac{-1}{jk2\pi}\left(e^{-j\frac{k\omega_0\tau}{2}}e^{-j\frac{k\omega_0\tau}{2}} - e^{-j\frac{k\omega_0\tau}{2}}e^{+j\frac{k\omega_0\tau}{2}}\right) = \frac{-1}{jk2\pi}\left(e^{-j\frac{k\omega_0\tau}{2}} - e^{+j\frac{k\omega_0\tau}{2}}\right)e^{-j\frac{k\omega_0\tau}{2}} \quad (2.6.5)$$

$$F_k = \frac{-1}{jk2\pi}\left(-2j\sin\left(\frac{k\omega_0\tau}{2}\right)\right)e^{-j\frac{k\omega_0\tau}{2}} = \frac{\sin\left(\frac{k\omega_0\tau}{2}\right)}{k\pi}e^{-j\frac{k\omega_0\tau}{2}} \quad (2.6.6)$$

$$\tau = \frac{T}{3} \quad (2.6.7)$$

$$F_k = \frac{\sin\left(\frac{k2\pi\cdot T}{T\cdot 2\cdot 3}\right)}{k\pi}e^{-j\frac{k2\pi\cdot T}{T\cdot 2\cdot 3}} = \frac{1}{3}\cdot\frac{\sin\left(\frac{k\pi}{3}\right)}{\frac{k\pi}{3}}e^{-j\frac{k\pi}{3}} \quad (2.6.8)$$

Negativna stopnica bo imela zelo podobno Fourierevo vrsto. Razlika je le v predznaku in smeri zamika, zato bo imel člen $e^{j\varphi}$ drugačen predznak:

$$G_k = -\frac{1}{3}\cdot\frac{\sin\left(\frac{k\pi}{3}\right)}{\frac{k\pi}{3}}e^{+j\frac{k\pi}{3}} \quad (2.6.9)$$

Dobljeni vrsti sedaj seštejemo:

$$X_k = F_k + G_k = \frac{1}{3}\cdot\frac{\sin\left(\frac{k\pi}{3}\right)}{\frac{k\pi}{3}}e^{-j\frac{k\pi}{3}} - \frac{1}{3}\cdot\frac{\sin\left(\frac{k\pi}{3}\right)}{\frac{k\pi}{3}}e^{+j\frac{k\pi}{3}} = \frac{1}{3}\cdot\frac{\sin\left(\frac{k\pi}{3}\right)}{\frac{k\pi}{3}}\cdot\left(e^{-j\frac{k\pi}{3}} - e^{+j\frac{k\pi}{3}}\right) \quad (2.6.10)$$

$$X_k = \frac{1}{3}\cdot\frac{\sin\left(\frac{k\pi}{3}\right)}{\frac{k\pi}{3}}\cdot\left(e^{-j\frac{k\pi}{3}} - e^{+j\frac{k\pi}{3}}\right) = \frac{1}{3}\cdot\frac{\sin\left(\frac{k\pi}{3}\right)}{\frac{k\pi}{3}}\cdot\left(-2j\sin\left(\frac{k\pi}{3}\right)\right) = \frac{-2j}{3}\cdot\frac{\sin^2\left(\frac{k\pi}{3}\right)}{\frac{k\pi}{3}} \quad (2.6.11)$$