Osnovna matematika 01
Rešitve

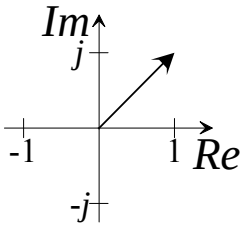
Zapišite po členih in izračunajte:

$$\sum_{i=2}^{5} 3\cdot 2^i = 3\cdot 2^2 + 3\cdot 2^3 + 3\cdot 2^4 + 3\cdot 2^5 = 12 + 24 + 48 + 96 = 180$$

$$\prod_{i=1}^{4} (2+i) = (2+1)\cdot(2+2)\cdot(2+3)\cdot(2+4) = 3\cdot 4\cdot 5\cdot 6 = 360$$

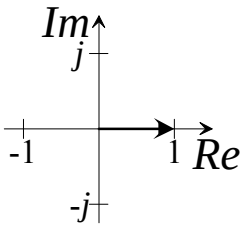
Narišite v kompleksni ravnini in zapišite v drugi obliki:

$1+j$

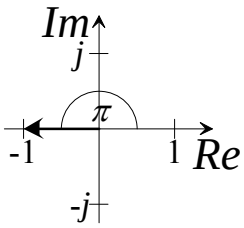


$$1+j = \sqrt{1^2+1^2}\cdot e^{j\arctan\frac{1}{1}} = \sqrt{2}\cdot e^{j\frac{\pi}{4}}$$

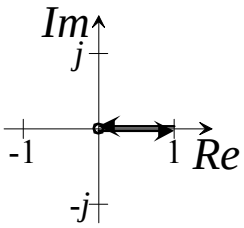
$1+e^{j\pi}$ 1:



$e^{j\pi}$:



$1+e^{j\pi} = 1-1 = 0$:



Izrazite s funkcijami sinus in cosinus:

$$\begin{aligned} e^{j\omega t} + e^{-j\omega t} &= \cos(\omega t) + j\sin(\omega t) + \cos(-\omega t) + j\sin(-\omega t) = \\ &= \cos(\omega t) + j\sin(\omega t) + \cos(\omega t) - j\sin(\omega t) = \\ &= 2\cos(\omega t) \end{aligned}$$

$$\begin{aligned} e^{-j3t} - e^{j3t} &= \cos(-3t) + j\sin(-3t) - (\cos(3t) + j\sin(3t)) = \\ &= \cos(3t) - j\sin(3t) - \cos(3t) - j\sin(3t) = \\ &= -2j\sin(3t) \end{aligned}$$

Izračunajte:

$$(1+j)^2 = 1^2 + 2j + j^2 = 1 + 2j - 1 = 2j$$

ali

$$(1+j)^2 = \left(\sqrt{2}\cdot e^{j\frac{\pi}{4}}\right)^2 = \sqrt{2}^2 \cdot e^{j\frac{\pi}{4}\cdot 2} = 2e^{j\frac{\pi}{2}} = 2j$$

$$\sqrt{\left(\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right)\cdot e^{-j\frac{\pi}{4}}} = \sqrt{e^{j\frac{\pi}{4}}\cdot e^{-j\frac{\pi}{4}}} = \sqrt{e^{j\frac{\pi}{4}-j\frac{\pi}{4}}} = \sqrt{e^{j0}} = \left(e^{j0}\right)^{\frac{1}{2}} = \left(e^{j0+j2k\pi}\right)^{\frac{1}{2}} = e^{j0+jk\pi} = \pm 1$$

Poenostavite ulomka:

$$\frac{x + \frac{2}{(x+1)}\cdot\frac{(x+1)}{(x+2)}}{\frac{(x+3)}{(x+2)}\cdot\frac{(x^2+3x+2)}{(x+1)} + 2} = \frac{x + \frac{2}{\cancel{(x+1)}}\cdot\frac{\cancel{(x+1)}}{(x+2)}}{\frac{(x+3)}{\cancel{(x+2)}}\cdot\frac{\cancel{(x+1)}\,\cancel{(x+2)}}{\cancel{(x+1)}} + 2} = \frac{x + \frac{2}{(x+2)}}{\frac{(x+3)}{1} + 2} = \frac{x + \frac{2}{(x+2)}}{(x+3)^2 + 2}\cdot\frac{(x+2)}{(x+2)} =$$

$$= \frac{x(x+2)+2}{\left((x+3)^2+2\right)(x+2)} = \frac{x^2+2x+2}{(x+2)\left(x^2+6x+9+2\right)} = \frac{x^2+2x+2}{(x+2)\left(x^2+6x+11\right)}$$

$$\frac{3+x}{x + \frac{1}{3x} + \frac{\frac{1}{3x}}{\frac{1}{3x+2}}} = \frac{3+x}{x + \frac{1}{3x} + \frac{3x+2}{3x}} = \frac{3+x}{x + \frac{1+3x+2}{3x}} = \frac{3+x}{x + \frac{3x+3}{3x}} = \frac{3+x}{x + \frac{x+1}{x}}\cdot\frac{x}{x} = \frac{x(x+3)}{x^2+x+1}$$

Izračunajte ničle kvadratne enačbe:

$$x^2 + x + 5 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad a = 1\ ,\ b = 1\ ,\ c = 5$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4\cdot 1\cdot 5}}{2\cdot 1} = -\frac{1}{2} \pm \frac{\sqrt{-19}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{19}}{2}$$

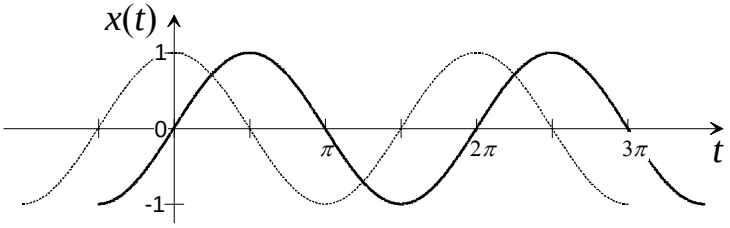
Izračunajte integrala:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(4t)\,dt = \frac{\sin(4t)}{4}\Bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{1}{4}\cdot\left(\sin\left(4\frac{\pi}{2}\right)-\sin\left(-4\frac{\pi}{2}\right)\right) = \frac{1}{4}(0-0) = 0$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 3\cdot e^{-j2t}dt = \frac{3}{-j2}\cdot e^{-j2t}\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{3}{-j2}\cdot\left(e^{-j2\frac{\pi}{4}}-e^{+j2\frac{\pi}{4}}\right) = \frac{3}{-j2}\cdot\left(e^{-j\frac{\pi}{2}}-e^{+j\frac{\pi}{2}}\right) = \frac{3}{-j2}\cdot\left(-j-(+j)\right) = 3$$

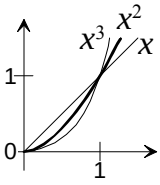
Skicirajte funkcije:

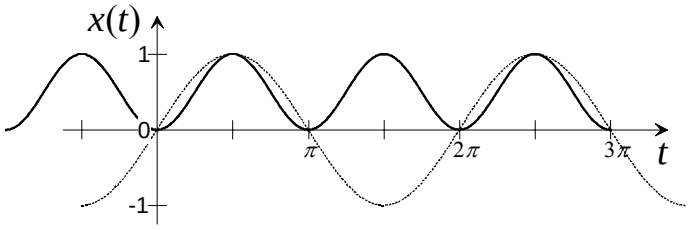
$\cos\left(t-\frac{\pi}{2}\right)$



$\sin^2(t)$

Začnemo s sin(*t*), od tu si pomagamo z grafom kvadratne funkcije:





$2-4e^{-t}$