Osnovna matematika 02

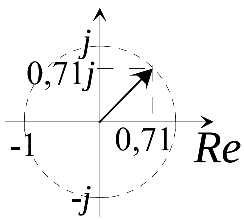
Zapišite po členih in izračunajte:

$$\sum_{i=1}^{3} 2\cdot 5^i = 2\cdot 5^1 + 2\cdot 5^2 + 2\cdot 5^3 = 10+50+250 = 310$$

$$\prod_{i=-1}^{1}\left(2+3\cdot i\right) = \left(2+3\cdot(-1)\right)\cdot\left(2+3\cdot 0\right)\cdot\left(2+3\cdot 1\right) = (-1)\cdot 2\cdot 5 = -10$$

Narišite v kompleksni ravnini in zapišite v drugi obliki:

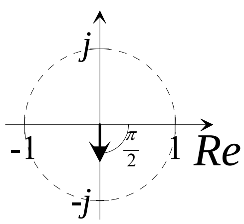
$$\frac{1}{\sqrt{2}}+\frac{j}{\sqrt{2}}$$



$$\frac{1}{\sqrt{2}}+\frac{j}{\sqrt{2}} = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2}\cdot e^{j\arctan\left(\frac{\mathrm{Im}\left(\frac{1}{\sqrt{2}}+\frac{j}{\sqrt{2}}\right)}{\mathrm{Re}\left(\frac{1}{\sqrt{2}}+\frac{j}{\sqrt{2}}\right)}\right)} =$$

$$= \sqrt{\frac{1}{2}+\frac{1}{2}}\cdot e^{j\arctan\left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right)} = 1\cdot e^{j\frac{\pi}{4}} = e^{j\frac{\pi}{4}}$$

$$0,5\cdot e^{-j\frac{\pi}{2}}$$



$$0,5\cdot e^{-j\frac{\pi}{2}} = 0,5\cdot\left(\cos\left(-\frac{\pi}{2}\right)+j\sin\left(-\frac{\pi}{2}\right)\right)$$

$$= 0,5\cdot\left(0-j1\right) = -j0,5$$

Izrazite s funkcijami sinus in cosinus:

$$e^{-j\pi t}+e^{j\pi t} = \cos(-\pi t)+j\sin(-\pi t)+\cos(\pi t)+j\sin(\pi t) =$$

$$= \cos(\pi t)-j\sin(\pi t)+\cos(\pi t)+j\sin(\pi t) =$$

$$= 2\cos(\pi t)$$

$$e^{j2t}-e^{-j2t} = \cos(2t)+j\sin(2t)-\left(\cos(-2t)+j\sin(-2t)\right) =$$

$$= \cos(2t)+j\sin(2t)-\cos(2t)+j\sin(2t) =$$

$$= 2j\sin(2t)$$

Izračunajte:

$$\left(0,5\cdot e^{j\frac{3\pi}{8}}\right)\left(2\cdot e^{j\frac{5\pi}{8}}\right) = 0,5\cdot 2\cdot e^{j\frac{3\pi}{8}+j\frac{5\pi}{8}} = e^{j\pi} = -1$$

$$\sqrt{(1-j)\cdot(1+j)} = \sqrt{1^2-j^2} = \sqrt{1+1} = \sqrt{2} = \sqrt{2\cdot e^{j(0+2k\pi)}} = \left|\sqrt{2}\right|\cdot e^{jk\pi} = \pm\left|\sqrt{2}\right|$$

Poenostavite ulomka:

$$\frac{1+\frac{1}{x+2}+\frac{\frac{1}{x+2}}{x+3}}{\frac{x+4}{\frac{1}{x+3}}-\frac{2}{\frac{1}{x+3}}+\frac{x(x+3)}{x+5-\frac{1}{\frac{1}{x+2}}}}=\frac{1+\frac{1}{x+2}+\frac{1}{(x+2)(x+3)}}{\frac{x+4-2}{\frac{1}{x+3}}+\frac{x(x+3)}{x+5-(x+2)}}=$$

$$=\frac{\frac{(x+2)(x+3)}{(x+2)(x+3)}+\frac{(x+3)}{(x+2)(x+3)}+\frac{1}{(x+2)(x+3)}}{\frac{x+4-2}{\frac{1}{x+3}}+\frac{x(x+3)}{x+5-(x+2)}}=\frac{\frac{(x+2)(x+3)+(x+3)+1}{(x+2)(x+3)}}{\frac{(x+2)(x+3)}{1}+\frac{x(x+3)}{x+5-x-2}}=$$

$$=\frac{\frac{(x+2)(x+3)+(x+3)+1}{(x+2)(x+3)}}{\frac{3(x+2)(x+3)}{3}+\frac{x(x+3)}{3}}=\frac{3\frac{(x+2)(x+3)+(x+3)+1}{(x+2)(x+3)}}{3(x+2)(x+3)+x(x+3)}=\frac{3\frac{(x+2)(x+3)+(x+3)+1}{(x+2)(x+3)}}{(x+3)(3(x+2)+x)}=$$

$$=\frac{3((x+2)(x+3)+(x+3)+1)}{(x+3)(3(x+2)+x)(x+2)(x+3)}=\frac{3(x^2+5x+6+x+3+1)}{(x+3)(3x+6+x)(x+2)(x+3)}=$$

$$=\frac{3(x^2+6x+10)}{(x+3)(4x+6)(x+2)(x+3)}=\frac{3}{2}\cdot\frac{(x^2+6x+10)}{(x+3)^2(2x+3)(x+2)}$$

$$\frac{\frac{\frac{1}{X_{C1}}}{\frac{X_{C1}+X_{L1}}{X_{C1}}}\cdot X_{C1}-1}{\frac{\frac{1}{X_{C2}+X_{L2}}}{\frac{X_{C1}+X_{L1}}{X_{C1}}}\cdot X_{C1}+\frac{R}{X_{C2}+X_{L2}}+1}=\frac{\frac{X_{C1}}{X_{C1}+X_{L1}}-1}{\frac{X_{C1}{}^{2}}{\left(X_{C1}+X_{L1}\right)\cdot\left(X_{C2}+X_{L2}\right)}+\frac{R}{X_{C2}+X_{L2}}+1}=$$

$$=\frac{\frac{X_{C1}}{X_{C1}+X_{L1}}-\frac{X_{C1}+X_{L1}}{X_{C1}+X_{L1}}}{\frac{X_{C1}{}^{2}}{\left(X_{C1}+X_{L1}\right)\cdot\left(X_{C2}+X_{L2}\right)}+\frac{R\cdot\left(X_{C1}+X_{L1}\right)}{\left(X_{C2}+X_{L2}\right)\cdot\left(X_{C1}+X_{L1}\right)}+\frac{\left(X_{C2}+X_{L2}\right)\cdot\left(X_{C1}+X_{L1}\right)}{\left(X_{C2}+X_{L2}\right)\cdot\left(X_{C1}+X_{L1}\right)}}=$$

$$=\frac{\frac{X_{C1}-X_{C1}-X_{L1}}{X_{C1}+X_{L1}}}{\frac{X_{C1}{}^{2}+R\cdot\left(X_{C1}+X_{L1}\right)+\left(X_{C2}+X_{L2}\right)\cdot\left(X_{C1}+X_{L1}\right)}{\left(X_{C1}+X_{L1}\right)\cdot\left(X_{C2}+X_{L2}\right)}}=$$

$$=-\frac{\frac{X_{L1}}{X_{C1}+X_{L1}}}{\frac{X_{C1}{}^{2}+R\cdot\left(X_{C1}+X_{L1}\right)+\left(X_{C2}+X_{L2}\right)\cdot\left(X_{C1}+X_{L1}\right)}{\left(X_{C1}+X_{L1}\right)\cdot\left(X_{C2}+X_{L2}\right)}}=$$

$$=-\frac{X_{L1}}{\frac{X_{C1}{}^{2}+R\cdot\left(X_{C1}+X_{L1}\right)+\left(X_{C2}+X_{L2}\right)\cdot\left(X_{C1}+X_{L1}\right)}{\left(X_{C2}+X_{L2}\right)}}=$$

$$=-\frac{X_{L1}\cdot\left(X_{C2}+X_{L2}\right)}{X_{C1}{}^{2}+R\cdot\left(X_{C1}+X_{L1}\right)+\left(X_{C2}+X_{L2}\right)\cdot\left(X_{C1}+X_{L1}\right)}=$$

$$=-\frac{X_{L1}X_{C2}+X_{L1}X_{L2}}{X_{C1}{}^{2}+RX_{C1}+RX_{L1}+X_{C2}X_{C1}+X_{C2}X_{L1}+X_{L2}X_{C1}+X_{L2}X_{L1}}$$

Izračunajte ničle kvadratne enačbe:

$$x^2-3x=-3$$

$$x^2-3x+3=0$$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\quad a=1\ ,\ b=-3\ ,\ c=3$$

$$x_{1,2}=\frac{3\pm\sqrt{\left(-3\right)^2-4\cdot1\cdot3}}{2\cdot1}=\frac{3}{2}\pm\frac{\sqrt{-3}}{2}=-\frac{3}{2}\pm j\frac{\sqrt{3}}{2}$$

Izračunajte integrala:

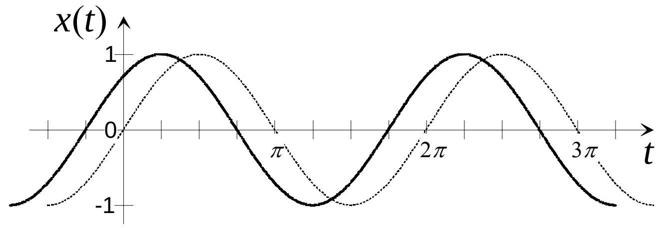
$$\int_0^{\pi} \sin\left(\frac{t}{2}\right) dt = -\frac{1}{\frac{1}{2}} \cdot \cos\left(\frac{t}{2}\right)\Bigg|_0^{\pi} = -2 \cdot \left(\cos\left(\frac{\pi}{2}\right) - \cos\left(\frac{0}{2}\right)\right) = -2 \cdot (0-1) = 2$$

$$\int_{-1}^{1} 2 \cdot e^{-j\frac{\pi}{4}t} dt = 2 \cdot \frac{1}{-j\frac{\pi}{4}} \cdot e^{-j\frac{\pi}{4}t}\Bigg|_{-1}^{1} = -\frac{8}{j\pi} \cdot \left(e^{-j\frac{\pi}{4}} - e^{j\frac{\pi}{4}}\right) = -\frac{8}{j\pi} \cdot \left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} - \left(\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right)\right) =$$

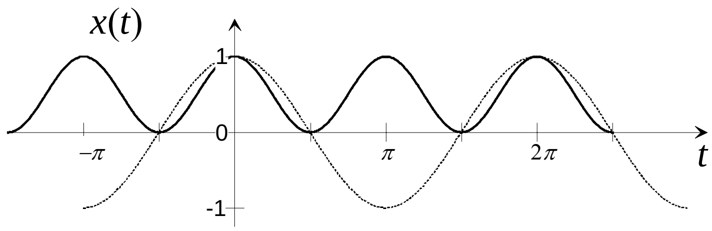
$$= -\frac{8}{j\pi} \cdot \left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = -\frac{8}{j\pi} \cdot \left(-\frac{2j}{\sqrt{2}}\right) = \frac{16}{\pi\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{\pi}$$

Skicirajte funkcije:

$\sin\left(t+\frac{\pi}{4}\right)$



$\cos^2(t)$



$-4-2\cdot e^{-t}$