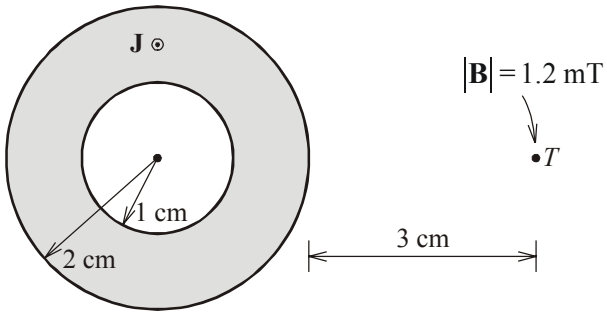
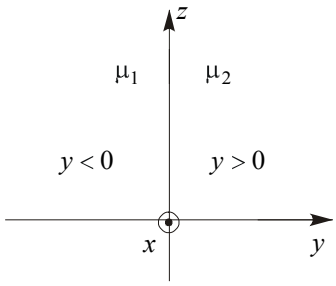
OSNOVE ELEKTROTEHNIKE II (VSP)
izpit, 6. februar 2003

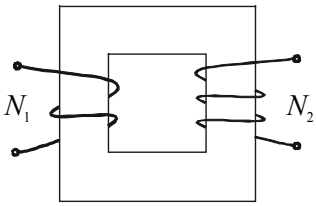
1. Tri centimetre stran od površine ravnega bakrenega cevastega vodnika smo s Hallovo sondo izmerili gostoto magnetnega pretoka $|\vec{B}| = 1.2\ \mathrm{mT}$. Kolikšna je gostota enosmernega toka v vodniku?



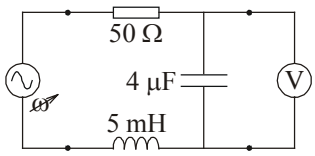
2. Ravnina $y = 0$ je meja dveh linearnih feromagnetikov. V območju $y > 0$, kjer je permeabilnost $\mu_2 = 3 \cdot 10^{-3}\ \mathrm{V \cdot s/A \cdot m}$, je vektor gostote magnetnega pretoka $\vec{B}_2 = (9, 5, 3)\ \mathrm{mT}$. Vektor tokovne obloge na meji je $\vec{K} = (3, 0, -3)\ \mathrm{A/m}$. Določite vektor gostote magnetnega pretoka $\vec{B}_1$ v območju $y < 0$, kjer je permeabilnost $\mu_1 = 4 \cdot 10^{-3}\ \mathrm{V \cdot s/A \cdot m}$!



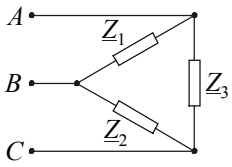
3. Na feromagnetnem jedru sta popolnoma sklopljeni navitji z $N_1 = 20$ ovoji ter $N_2 = 40$ ovoji. Določite medsebojno induktivnost *M*, če je induktivnost $L_1 = 0.3\ \mathrm{H}$!



4. Zaporedni nihajni krog je vzbujan s frekvenčnim generatorjem (spremenljive kotne frekvence ω) efektivne napetosti 30 V. Izrazite frekvenčno odvisnost napetosti na kondenzatorju, ki bi jo meril idealni voltmeter!



5. Trifazno breme ($\underline{Z}_1 = 400\,\Omega$, $\underline{Z}_2 = \mathrm{j}200\,\Omega$, $\underline{Z}_3 = (100 + \mathrm{j}100)\,\Omega$) priključimo na simetričen trifazni sistem napetosti ($3 \times 400\ \mathrm{V_{ef}}$). Določite delovno moč, ki se sprošča na tem bremenu!



Rešitve so objavljene na: http://torina.fe.uni-lj.si/oe.

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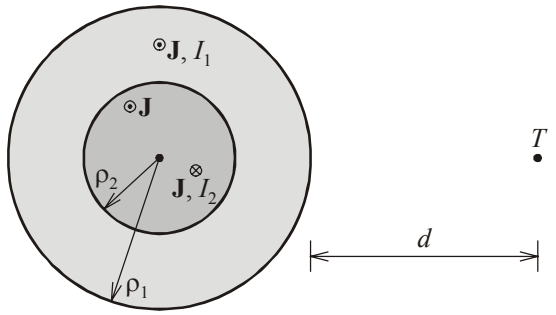
Rešitve

1. Lahko si predstavljamo, da sta v luknji dve enako veliki in nasprotni tokovni gostoti:



Zdaj "imamo" dva polna vodnika, enega s polmerom ρ_1 in tokom $I_1 = J\rho_1^2\pi$ in drugega s polmerom ρ_2 in tokom $I_2 = J\rho_2^2\pi$, ki teče v nasprotni smeri kot I_1. Drugi vodnik je "vstavljen" koncentrično znotraj prvega.

$$B(T) = 1.2\ \text{mT} = \left|\frac{\mu_0 I_1}{2\pi r} - \frac{\mu_0 I_2}{2\pi r}\right| \quad ; \quad r = 2\ \text{cm} + 3\ \text{cm} = 5\ \text{cm}$$

$$B(T) = \frac{\mu_0 J}{2}\left|\frac{\rho_1^2}{r} - \frac{\rho_2^2}{r}\right| \quad \Rightarrow \quad J = \frac{2B(T)}{\mu_0}\left(\frac{\rho_1^2 - \rho_2^2}{r}\right)^{-1} = \frac{1.2\cdot 10^{-3}\ \text{V}\cdot\text{s/m}^2}{2\pi\cdot 10^{-7}\ \text{V}\cdot\text{s/A}\cdot\text{m}}\left(\frac{4\ \text{cm}^2 - 1\ \text{cm}^2}{5\ \text{cm}}\right)^{-1}$$

$$J \cong \boxed{3.18\cdot 10^5\ \text{A/m}^2}$$

2.

$$\vec{n} = \vec{e}_y \quad , \quad \vec{n}\cdot\left(\vec{B}_2 - \vec{B}_1\right) = 0 \quad \Rightarrow \quad B_{1y} = B_{2y} = 5\ \text{mT}$$

$$\vec{H}_2 = \frac{\vec{B}_2}{\mu_2}$$

$$\vec{n}\times(\vec{H}_2 - \vec{H}_1) = \vec{K} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 1 & 0 \\ H_{2x} - H_{1x} & H_{2y} - H_{1y} & H_{2z} - H_{1z} \end{vmatrix} = \vec{e}_x\left(\frac{B_{2z}}{\mu_2} - H_{1z}\right) - \vec{e}_z\left(\frac{B_{2x}}{\mu_2} - H_{1x}\right)$$

$$\vec{K} = (3,\ 0,\ -3)\ \text{A/m} = \vec{e}_x(1\ \text{A/m} - H_{1z}) + \vec{e}_z(H_{1x} - 3\ \text{A/m}) \quad \Rightarrow \quad H_{1x} = 0\ \text{A/m} \quad , \quad H_{1z} = -2\ \text{A/m}$$

$$\vec{B}_1 = \mu_1\vec{H}_1 \quad \Rightarrow \quad B_{1x} = 0\ \text{T} \quad , \quad B_{1z} = -8\ \text{mT}$$

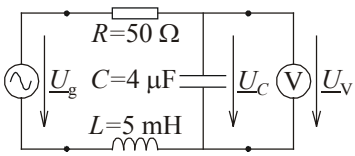
$$\vec{B}_1 = \boxed{(0,\ 5,\ -8)\ \text{mT}}$$

3.

$$\phi_1^{(1)} = \phi_2^{(1)}, \qquad M = N_2\frac{\phi_2^{(1)}}{I_1}, \qquad L_1 = N_1\frac{\phi_1^{(1)}}{I_1} \Rightarrow \frac{\phi_1^{(1)}}{I_1} = \frac{\phi_2^{(1)}}{I_1} = \frac{L_1}{N_1}, \qquad M = N_2\frac{\phi_2^{(1)}}{I_1} = N_2\frac{L_1}{N_1}$$

$$M = 40\frac{0.3\ \text{H}}{20} = \boxed{0.6\ \text{H}}$$

4.



$$\underline{U}_C = \frac{\underline{U}_\mathrm{g}}{R + j\omega L + 1/j\omega C} \cdot \frac{1}{j\omega C} = \frac{\underline{U}_\mathrm{g}}{j\omega CR - \omega^2 LC + 1}$$

$$U_{\mathrm{V,\,ef.}} = \frac{1}{\sqrt{2}}\left|\underline{U}_C\right| = \frac{U_{\mathrm{g,\,ef.}}}{\sqrt{(1-\omega^2 LC)^2 + \left(\omega CR\right)^2}}$$

$$U_{\mathrm{V,\,ef.}} = \frac{30\ \mathrm{V}}{\sqrt{\left(1-\omega^2 \cdot 20\cdot 10^{-9}\ \mathrm{s}^2\right)^2 + \left(\omega \cdot 200\cdot 10^{-6}\ \mathrm{s}\right)^2}} = \boxed{\frac{30\ \mathrm{V}}{\sqrt{1+\omega^4 \cdot 4\cdot 10^{-16}\ \mathrm{s}^4}}}$$

5.

$$\underline{S}_1 = \frac{U_\mathrm{m}^2}{\underline{Z}_1^*} = \frac{(400\ \mathrm{V})^2}{400\ \Omega} = 400\ \mathrm{VA} \quad \Rightarrow \quad P_1 = 400\ \mathrm{W}$$

$$\underline{S}_2 = \frac{U_\mathrm{m}^2}{\underline{Z}_2^*} = \frac{(400\ \mathrm{V})^2}{-\mathrm{j}200\ \Omega} = \mathrm{j}800\ \mathrm{VA} \Rightarrow P_2 = 0$$

$$\underline{S}_3 = \frac{U_\mathrm{m}^2}{\underline{Z}_3^*} = \frac{(400\ \mathrm{V})^2}{(100-\mathrm{j}100)\ \Omega} = (800+\mathrm{j}800)\ \mathrm{VA} \Rightarrow P_3 = 800\ \mathrm{W}$$

$$P = P_1 + P_2 + P_3 = \boxed{1200\ \mathrm{W}}$$