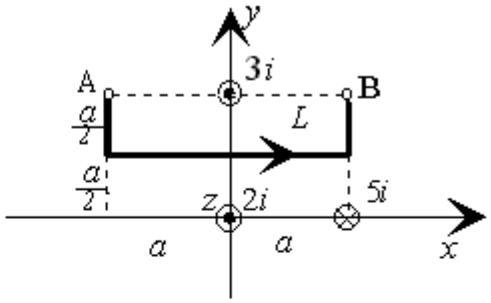
Izračunajte magnetno napetost Θ_{AB} vzdolž krivulje L med točko A in B v okolici treh premih vodnikov s tokovi po sliki! ($i = 15$ A, $a = 0.2$ m)



Rešitev:

$$\Theta_{AB} = \Theta_{AB1} + \Theta_{AB2} + \Theta_{AB3} = \frac{i_1}{2\pi}\varphi_1 + \frac{i_2}{2\pi}\varphi_2 + \frac{i_3}{2\pi}\varphi_3$$

$$\Theta_{AB} = \frac{3i}{2\pi}(\pi) + \frac{2i}{2\pi}\left(-\frac{\pi}{2}\right) + \frac{-5i}{2\pi}\left(-\operatorname{arctg}\frac{2}{1}\right)$$

$$\Theta_{AB} = 1.88i = 1.88 \cdot 15 = 28.20\,\text{A}$$

V valjastem vodniku premera 20 mm je gostota električnega toka $J = 9\cdot 10^6\, r/r_0\ \mathrm{A/m^2}$. ($\mu = \mu_0$) Kolikšen je magnetni fluks v notranjosti vodnika v dolžini 3 m?

Rešitev:

$$\Phi = \int_0^{r_0} \vec{B}\cdot \mathrm{d}\,\vec{A}, \qquad \vec{B} = \vec{e}_\varphi \mu_0 \frac{i(r)}{2\pi r}, \quad \mathrm{d}\vec{A} = \vec{e}_\varphi\, \ell\, \mathrm{d}r$$

$$i(r) = \int_0^{r} \vec{J}\cdot \mathrm{d}\,\vec{A}_1$$

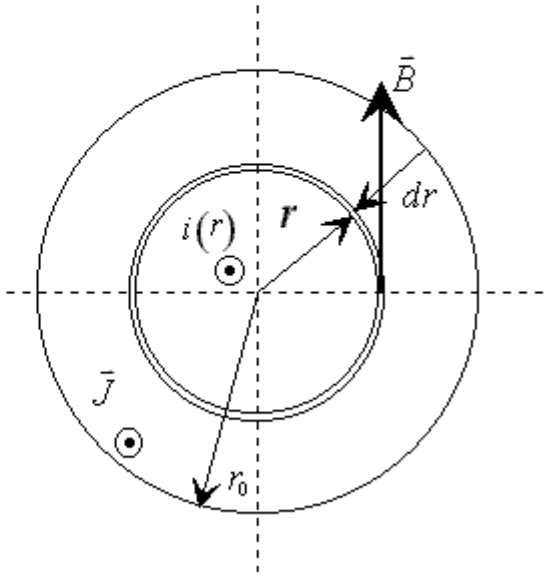
$$\vec{J} = \vec{e}_z \cdot 9\cdot 10^6\, r/r_0$$

$$\mathrm{d}\vec{A}_1 = \vec{e}_z\, 2\pi r\cdot \mathrm{d}r$$

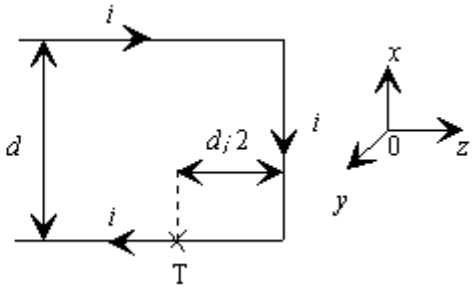
$$i(r) = \int_0^{r} \vec{e}_z \cdot 9\cdot 10^6 \left(r/r_0\right)\cdot \vec{e}_z\, 2\pi r\cdot \mathrm{d}r = 6\pi\cdot 10^6\, r^3/r_0$$

$$\Phi = \int_0^{r_0} \vec{e}_\varphi \cdot \frac{6\pi\cdot 10^6\, r^3}{r_0}\cdot \frac{1}{2\pi r}\cdot \vec{e}_\varphi\, \ell\, \mathrm{d}r = 10^6\, \mu_0\, \ell\, r_0^2$$

$$\Phi = 10^6\cdot 4\pi\cdot 10^{-7}\cdot 0.01^2\cdot 3 = 377\cdot 10^{-4}\ \mathrm{Wb}$$

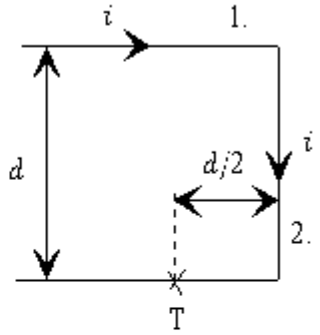


Premi vodnik, ki vodi tok i=25 A tvori tokovno zanko po sliki. Kolikšna je sila na enoto dolžine $\frac{d\vec{F}}{dl}$ v točki T? ($d = 0.2$ m)



Rešitev:

$$d\vec{F} = i\, d\vec{\ell} \times \vec{B}, \qquad d\vec{\ell} = -\vec{e}_z\, d\ell, \qquad \vec{B} = \vec{B}_1 + \vec{B}_2$$



$$\vec{B}_1 = -\vec{e}_y\, \mu_0 \frac{i}{4\pi d}\left(\cos\alpha_1 - \cos\alpha_2\right) =$$

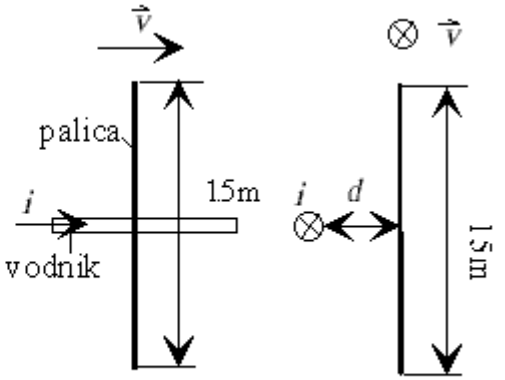
$$= -\vec{e}_y\, \mu_0 \frac{i}{4\pi d}\left(\cos 0 - \left(-\frac{d/2}{\sqrt{d^2 + (d/2)^2}}\right)\right) = -\vec{e}_y\, \mu_0$$

$$\vec{B}_2 = -\vec{e}_y\, \mu_0 \frac{i}{4\pi\left(\frac{d}{2}\right)}\left(\frac{d}{\sqrt{d^2 + (d/2)^2}} - \cos\frac{\pi}{2}\right) = -\vec{e}_y\, \mu$$

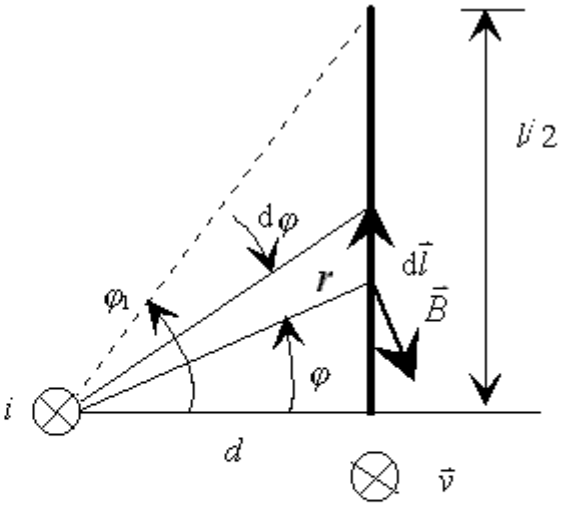
$$\vec{B} = -\vec{e}_y\, \mu_0 \frac{i}{4\pi d} \cdot 3.24 = -\vec{e}_y\, B, \quad d\vec{F} = i\left(-\vec{e}_z\, d\ell\right) \times \left(-\vec{e}_y\right.$$

$$\frac{d\vec{F}}{d\ell} = -\vec{e}_x\, i\, B = -\vec{e}_x\, 25 \cdot 4\pi 10^{-7}\, \frac{25}{4\pi 0.2} \cdot 3.24 = -\vec{e}_x \cdot 1.0$$

Vzporedno z dolgim premim vodnikom se giba 1.5 m dolga prevodna palica z enakomerno hitrostjo 4 m/s. Po vodniku teče tok 7 A. Kolikšna napetost se inducira med sredino in zgornjim koncem palice? ($d = 0.2$ m). Določite polariteto inducirane napetosti!



Rešitev:



$$u_i = \int_0^{\ell/2} \left(\vec{v} \times \vec{B}\right) \mathrm{d}\vec{\ell} = \int_0^{\ell/2} \vec{v}\left(\vec{B} \times \mathrm{d}\vec{\ell}\right)$$

$$\vec{v} = \vec{e}_z v, \qquad \vec{B} = -\vec{e}_\varphi \mu_0 \frac{i}{2\pi r}; \qquad \mathrm{d}\ell = r\,\mathrm{d}\varphi /$$

$$\vec{B} \times \mathrm{d}\vec{\ell} = -\vec{e}_z B\,\mathrm{d}\ell \sin\varphi$$

$$u_i = -\int_0^{\ell/2} v \mu_0 \frac{i}{2\pi} r \frac{\sin\varphi}{\cos\varphi}\mathrm{d}\varphi = v\frac{\mu_0 i}{2\pi} \ln\cos\varphi \Big|_0^{\varphi_1}$$

$$u_i = v\frac{\mu_0 i}{2\pi} \ln\frac{d}{\sqrt{d^2 + (\ell/2)^2}} = 4\cdot 4\,\pi 10^{-7}\ \frac{7}{2\pi} \ln$$

$u_i = -7.6\cdot 10^{-6}$ V; + je na sredini palice.

Toroidni navitji s 500 in 100 ovoji sta tesno naviti na feromagnetno jedro z $\mu_r = 2000$ s srednjo dolžino gostotnice 60 cm in prerezom 1 cm^2. Kolikšna je lastna induktivnost navitij in kolikšna je medsebojna induktivnost navitij?

Rešitev:

$$L = \frac{N \cdot \Phi}{i} = \frac{N B A}{i} = \frac{N \mu H A}{i} = \frac{N \mu i N A}{\ell i}$$

$$L = N^2 \mu \frac{A}{\ell}$$

$$L_1 = 500^2 \cdot 2000 \cdot 4\pi \cdot 10^{-7} \frac{10^{-4}}{0.6} = 0.105\,\mathrm{H}$$

$$L_2 = 100^2 \cdot 2000 \cdot 4\pi \cdot 10^{-7} \frac{10^{-4}}{0.6} = 4.19 \cdot 10^{-3}\,\mathrm{H}$$

$$M = N_1 \cdot N_2 \mu \frac{A}{\ell} = 500 \cdot 100 \cdot 2000 \cdot 4\pi \cdot 10^{-7} \frac{10^{-4}}{0.6} = 20.9 \cdot 10^{-3}\,\mathrm{H}$$