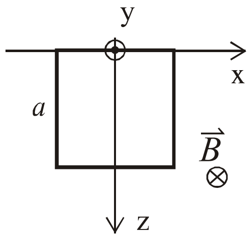
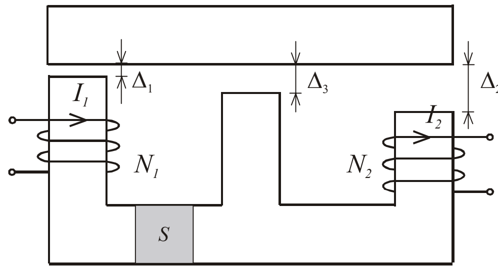
OSNOVE ELEKTROTEHNIKE II (UNI)
2. kolokvij, 12. 6. 2003

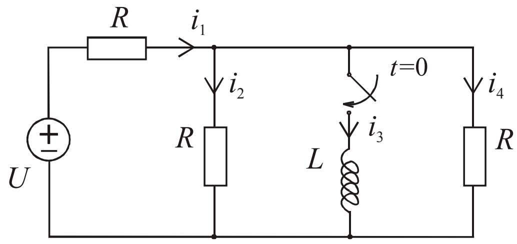
1. Izračunajte inducirano napetost v kvadratni tuljavici s 100 ovoji in stranico $a = 1$ cm, ki se nahaja v izmeničnem magnetnem polju gostote $B_y = B_0 \sin(\pi z / a)\cos(\omega t)$; $B_0 = 0{,}1$ T, $\omega = 10^3$ s^{-1}.



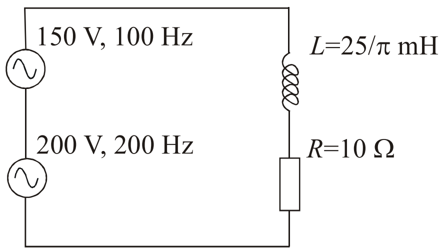
2. Izračunajte medsebojno induktivnost navitij na železnem jedru! Upornost železa zanemarite!
   $\Delta_1 = 0{,}1$ mm, $\Delta_2 = 0{,}3$ mm, $\Delta_3 = 0{,}2$ mm, $N_1 = 2000$, $N_2 = 3000$, $\mu_0 = 4\pi.10^{-7}$ Vs/Am, S=10 cm^2.



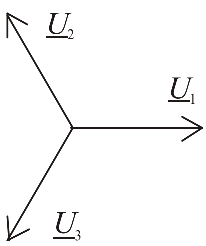
3. Izrazite čas t_1, ob katerem upade tok i_4 na četrtino svoje začetne vrednosti!

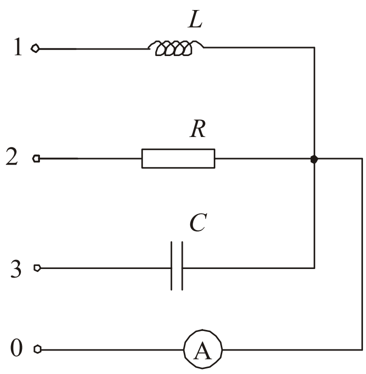


4. Izračunajte povprečno moč na uporniku z upornostjo 10 Ω!



5. Trifazno breme priključimo na negativen simetričen sistem faznih napetosti 3×230 V. Kolikšen tok izmeri A-meter?
   $R = X_L = X_C = 0{,}5$ kΩ.





Rešitve bodo objavljene na http://torina.fe.uni-lj.si/oe

OSNOVE ELEKTROTEHNIKE II (UNI)
2. kolokvij, 12.06.2003
Rešitve

1.

$u_i = -N\dfrac{d\Phi}{dt} = -N\dfrac{d}{dt}\int \vec{B}\cdot d\vec{A}$, kjer je $\vec{B} = \vec{e}_y B_y = \vec{e}_y B_0 \sin\left(\dfrac{\pi z}{a}\right)\cos(\omega t)$ in $d\vec{A} = \vec{e}_y dA_y = \vec{e}_y a\cdot dz$:

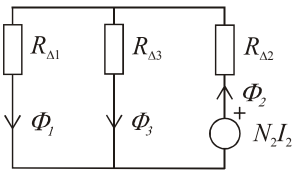
$$u_i = -N\frac{d}{dt}\left[B_0\cos(\omega t)a\int_0^a \sin\left(\frac{\pi}{a}z\right)dz\right] = -N\frac{d}{dt}\left[B_0\cos(\omega t)a\left(-\frac{a}{\pi}\cos\left(\frac{\pi}{a}z\right)\Big|_0^a\right)\right] =$$

$$= -N\frac{d}{dt}\left[B_0\cos(\omega t)\frac{2a^2}{\pi}\right] = -NB_0\frac{2a^2}{\pi}\frac{d\left(\cos(\omega t)\right)}{dt} = -NB_0\frac{2a^2}{\pi}\left(-\sin(\omega t)\omega\right) = NB_0\omega\frac{2a^2}{\pi}\sin(\omega t)$$

$$\boxed{u_i \cong 0.64\sin(\omega t)\ \mathrm{V}}$$

2.

$M_{12} = \dfrac{N_1\Phi_{12}}{I_2}$, kjer je Φ_{12} pretok skozi navitje N_1 zaradi toka v navitju N_2. V modelnem magnetnem vezju je fluks Φ_{12} označen kot Φ_1, to pa je tudi edini podatek, ki ni podan in ga moramo izračunati (z analizo modelnega vezja):



$$\Phi_1 = \Phi_2 - \Phi_3$$

$$\Phi_1 R_{\Delta1} = \Phi_3 R_{\Delta3} \quad\Rightarrow\quad \Phi_3 = \Phi_1\frac{R_{\Delta1}}{R_{\Delta3}} = \frac{\Phi_1}{2};\ \left(R_{\Delta1} = \frac{\Delta_1}{\mu_0 S},\ R_{\Delta3} = \frac{\Delta_3}{\mu_0 S} = 2R_{\Delta1}\right)$$

$$N_2I_2 = \Phi_2 R_{\Delta2} + \Phi_3 R_{\Delta3} \quad\Rightarrow\quad \Phi_2 = \frac{N_2I_2 - \Phi_3 R_{\Delta3}}{R_{\Delta2}} = \frac{N_2I_2}{R_{\Delta2}} - \frac{\Phi_3 2R_{\Delta1}}{3R_{\Delta1}}$$

$$\Phi_1 = \Phi_2 - \Phi_3 = \frac{N_2I_2}{R_{\Delta2}} - \frac{2\Phi_3}{3} - \frac{\Phi_1}{2} = \frac{N_2I_2}{R_{\Delta2}} - \frac{\Phi_1}{3} - \frac{\Phi_1}{2} = \frac{N_2I_2}{R_{\Delta2}} - \frac{\Phi_1}{3} - \frac{\Phi_1}{2} \quad\Rightarrow\quad \Phi_1 = \frac{6}{11}\cdot\frac{N_2I_2}{R_{\Delta2}}$$

$$\boxed{M_{12} = \frac{N_1\Phi_1}{I_2} = \frac{N_1}{I_2}\cdot\frac{6}{11}\cdot\frac{N_2I_2}{R_{\Delta2}} \cong 13.7\ \mathrm{H}}$$

3.

Začetna vrednost (pred vklopom stikala) toka i_4: $i_4 = I_4 = \dfrac{I_1}{2} = \dfrac{U}{3R}$.

Opišemo vezje in pogledamo, kje nastopa tok i_4: $i_4 R = L\dfrac{di_3}{dt} \quad\Rightarrow\quad i_4 = \dfrac{L}{R}\dfrac{di_3}{dt}$; poskusimo zapisati tok i_3 z uporabo Theveninovega teorema (vejo s tuljavo iz vezja izvzamemo in preostanek vezja nadomestimo s Theveninovim nadomestnim virom). Rezultat je vezje z zaporedno vezanimi: Theveninovim virom U_T, Theveninovo upornostjo R_T ter tuljavo.

$$R_T = \frac{R}{3},\ U_T = \frac{U}{3}$$

Zapišemo zančno enačbo: $\frac{U}{3}=\frac{R}{3}i_3+L\frac{di_3}{dt}$, ki je hkrati tudi diferencialna enačba za tok i_3: $\frac{di_3}{dt}+\frac{R}{3L}i_3=\frac{U}{3L}$, rešimo pa jo z nastavkom: $i_3(t)=Ae^{\lambda t}+B$. Z vstavitvijo nastavka v diferencialno enačbo ter uporabo začetnega pogoja izračunamo manjkajoče konstante:

$\lambda=-\frac{R}{3L},\ B=\frac{U}{R}$

zač. pogoj $\Rightarrow A$: $i_3\left(t=0^+\right)=0=Ae^0+B \Rightarrow A=-B=-\frac{U}{R}$

Zapišemo izraz za tok i_3: $i_3(t)=\frac{U}{R}\left(1-e^{-\frac{R}{3L}t}\right)$ in ga vstavimo v enačbo za tok i_4:

$$i_4=\frac{L}{R}\frac{di_3}{dt}=\frac{L}{R}\frac{U}{3L}e^{-\frac{R}{3L}t}=\frac{U}{3R}e^{-\frac{R}{3L}t}.$$

Uporabimo še pogoj iz naloge: $i_4(t=t_1)=\frac{1}{4}\frac{U}{3R}=\frac{U}{3R}e^{-\frac{R}{3L}t_1} \quad\Rightarrow\quad \boxed{t_1=\frac{3L}{R}\ln 4}$

4.

Vira sta nekoherentna, zato uporabimo teorem o superpoziciji moči:

$P=P_1+P_2$

$$P_1=\frac{1}{2}RI_1^2=\frac{1}{2}R\left(\frac{U_1}{Z_1}\right)^2=\frac{1}{2}R\left(\frac{U_1}{\sqrt{R^2+(\omega_1 L)^2}}\right)^2=\frac{1}{2}10\ \Omega\left(\frac{150\ \mathrm{V}}{5\sqrt{5}\ \Omega}\right)^2=900\ \mathrm{W}$$

$$P_2=\frac{1}{2}RI_2^2=\frac{1}{2}R\left(\frac{U_2}{Z_2}\right)^2=\frac{1}{2}R\left(\frac{U_2}{\sqrt{R^2+(\omega_2 L)^2}}\right)^2=\frac{1}{2}10\ \Omega\left(\frac{200\ \mathrm{V}}{10\sqrt{2}\ \Omega}\right)^2=1000\ \mathrm{W}$$

$\boxed{P=1900\ \mathrm{W}}$

5.

Ker je A-meter idealen, velja: $\underline{Z}_\mathrm{A}=0\ \Omega \quad\Rightarrow\quad \underline{V}_\mathrm{ZV}=\underline{U}_0=0\ \mathrm{V}$

$$\underline{I}_\mathrm{A}=\underline{I}_0=\underline{I}_1+\underline{I}_2+\underline{I}_3=\frac{\underline{U}_1}{\underline{Z}_1}+\frac{\underline{U}_2}{\underline{Z}_2}+\frac{\underline{U}_3}{\underline{Z}_3}=\frac{U_f}{j\omega L}+\frac{U_f e^{j120^\circ}}{R}+\frac{U_f e^{-j120^\circ}}{1/j\omega C}=$$

$$=U_f\left(\frac{1}{j\frac{1}{2}\cdot 10^3}+\frac{\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}\cdot 10^3}+\frac{\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)}{\frac{1}{j2}\cdot 10^3}\right)\mathrm{S}=U_f\left(\left(\sqrt{3}-1\right)+j\left(\sqrt{3}-3\right)\right)\cdot 10^{-3}\ \mathrm{S}$$

$$\boxed{I_\mathrm{A,ef}=230\cdot 10^{-3}\cdot\sqrt{\left(\sqrt{3}-1\right)^2+\left(\sqrt{3}-3\right)^2}\ \mathrm{A}\cong 0.337\ \mathrm{A}}$$