Robotika (3. letnik, avtomatika, FE)

$$Rot(x,\alpha)=\begin{bmatrix}1&0&0&0\\0&\cos\alpha&-\sin\alpha&0\\0&\sin\alpha&\cos\alpha&0\\0&0&0&1\end{bmatrix},\qquad Rot(y,\beta)=\begin{bmatrix}\cos\beta&0&\sin\beta&0\\0&1&0&0\\-\sin\beta&0&\cos\beta&0\\0&0&0&1\end{bmatrix}$$

$$Rot(z,\chi)=\begin{bmatrix}\cos\chi&-\sin\chi&0&0\\\sin\chi&\cos\chi&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

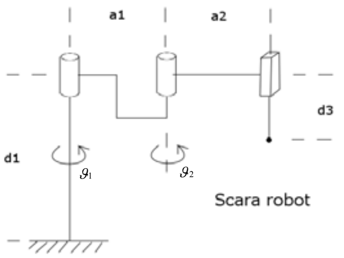
Homogena transformacija:

- Lega
  - Orientacija (0)
  - Pozicija (X)
- Premik
  - Rotacija (0)
  - Translacija (X)

$$\begin{matrix}x'&y'&z'&&\\0&0&0&X&x\\0&0&0&X&y\\0&0&0&X&z\\0&0&0&1&\end{matrix}$$

$\underline{\underline{H}}^{IV}=\underline{\underline{H}}\cdot\underline{\underline{P}}$ Vedno najprej lega, nato premik!

Scara

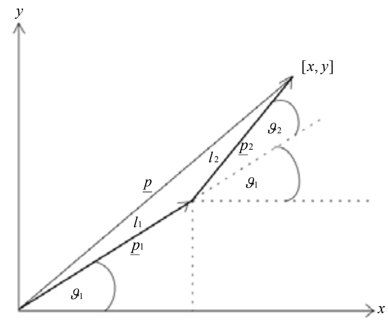


$$\underline{\underline{T}}={}^{0}\underline{\underline{H}}_{1}\cdot{}^{1}\underline{\underline{H}}_{2}\cdot{}^{2}\underline{\underline{H}}_{3}$$

$$c_{12}=\cos(\vartheta_1+\vartheta_2)=c_1c_2-s_1s_2$$
$$s_{12}=\sin(\vartheta_1+\vartheta_2)=s_1c_2+c_1s_2$$

K.S damo v vsak sklep, razen če je zadnji sklep translacijski!

Dvosegmentni robotski manipulator - Kinematika



I. pot – direktna kinematika

$$\underline{p}_1=L_1\cdot\begin{bmatrix}\cos\vartheta_1\\\sin\vartheta_1\end{bmatrix}\qquad \underline{p}_2=L_2\cdot\begin{bmatrix}\cos(\vartheta_1+\vartheta_2)\\\sin(\vartheta_1+\vartheta_2)\end{bmatrix}$$

$$\underline{p}=\underline{p}_1+\underline{p}_2=\begin{bmatrix}L_1\cos\vartheta_1+L_2\cos(\vartheta_1+\vartheta_2)\\L_1\sin\vartheta_1+L_2\sin(\vartheta_1+\vartheta_2)\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

II. hitrost – direktna kinematika (ena rešitev)

$$x=f_1(\vartheta_1(t),\vartheta_2(t))$$
$$y=f_2(\vartheta_1(t),\vartheta_2(t))$$

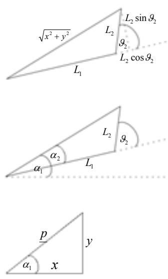
$$\dot{x}=\frac{\partial f_1}{\partial\vartheta_1}\cdot\dot{\vartheta}_1+\frac{\partial f_1}{\partial\vartheta_2}\cdot\dot{\vartheta}_2\qquad \dot{y}=\frac{\partial f_2}{\partial\vartheta_1}\cdot\dot{\vartheta}_1+\frac{\partial f_2}{\partial\vartheta_2}\cdot\dot{\vartheta}_2$$

Jacobijeva matrika $(\underline{\underline{J}})$

$$\begin{bmatrix}\dot{x}\\\dot{y}\end{bmatrix}=\begin{bmatrix}\frac{\partial f_1}{\partial\vartheta_1}&\frac{\partial f_1}{\partial\vartheta_2}\\\frac{\partial f_2}{\partial\vartheta_2}&\frac{\partial f_2}{\partial\vartheta_2}\end{bmatrix}\qquad \dot{\underline{p}}=\begin{bmatrix}\dot{x}\\\dot{y}\end{bmatrix}=\begin{bmatrix}-L_1\cdot\sin\vartheta_1-L_2\cdot\sin(\vartheta_1+\vartheta_2)&-L_2\cdot\sin(\vartheta_1+\vartheta_2)\\L_1\cdot\cos\vartheta_1+L_2\cdot\cos(\vartheta_1+\vartheta_2)&L_2\cdot\cos(\vartheta_1+\vartheta_2)\end{bmatrix}\begin{bmatrix}\dot{\vartheta}_1\\\dot{\vartheta}_2\end{bmatrix}$$

$$\dot{\vartheta}=[\dot{\vartheta}_1\cdot\dot{\vartheta}_2]\qquad \dot{p}=\underline{\underline{J}}\cdot\underline{\dot{\vartheta}}$$

III. pot – inverzna kinematika (več rešitev)



$$x^2+y^2=L_1^2+L_2^2-2L_1L_2\cos(180°-\vartheta_2)$$

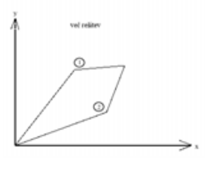
$$\cos(180°-\vartheta_2)=-\cos\vartheta_2$$

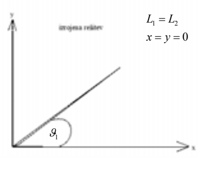
$$\cos\vartheta_2=\frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}$$

$$\vartheta_1=\alpha_1-\alpha_2$$

$$\alpha_2=arctg\frac{L_2\sin\vartheta_2}{L_1+L_2\cos\vartheta_2}$$

$$\alpha_1=arctg\frac{y}{x}$$





$L_1=L_2$
$x=y=0$

IV. hitrost – inverzna kinematika

$$\dot{\underline{p}}=\underline{\underline{J}}\cdot\dot{\underline{\vartheta}}\,/\cdot\underline{\underline{J}}^{-1}$$
$$\dot{\underline{\vartheta}}=\underline{\underline{J}}^{-1}\,\dot{\underline{p}}$$

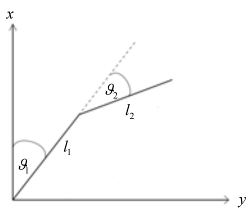
$$\underline{\underline{A}}=\begin{bmatrix}a&b\\c&d\end{bmatrix}\qquad \underline{\underline{A}}^{-1}=\frac{1}{ad-cb}\begin{bmatrix}d&-b\\-c&a\end{bmatrix}$$;Pravilo za inv. matriko 2x2

Statika

$$\begin{bmatrix}M_1\\M_2\end{bmatrix}=\begin{bmatrix}-L_1\sin\vartheta_1-L_2\sin(\vartheta_1+\vartheta_2)&L_1\cos\vartheta_1+L_2\cos(\vartheta_1+\vartheta_2)\\-L_2\sin(\vartheta_1+\vartheta_2)&L_2\cos(\vartheta_1+\vartheta_2)\end{bmatrix}\begin{bmatrix}F_1\\F_2\end{bmatrix}$$

$$\underline{M}=\underline{\underline{J}}^{T}\cdot\underline{F}\qquad A^{T}=\begin{bmatrix}a&c\\b&d\end{bmatrix}$$

Delovni prostor



$$(x-l_1\sin\vartheta_1)^2+(y-l_1\cos\vartheta_1)^2=l_2^2$$

$$x^2+y^2=l_1^2+l_2^2+2l_1l_2\cos\vartheta_2$$

Površina kolobarja

$$S=\frac{\vartheta_1\cdot\pi}{360°}\cdot(r_1^2-r_2^2)[°]$$

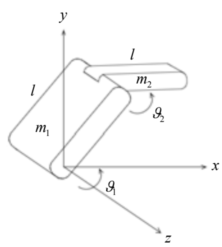
$r_1>r_2,\ \vartheta_1=\vartheta_{1\max}-\vartheta_{1\min}$

$l_1=l_2=1$

$30°\le\vartheta_1\le 60°$	S
$0°\le\vartheta_2\le 30°$	0,07
$30°\le\vartheta_2\le 60°$	0,19
$60°\le\vartheta_2\le 90°$	0,26
$90°\le\vartheta_2\le 120°$	0,26
$120°\le\vartheta_2\le 150°$	0,19
$150°\le\vartheta_2\le 180°$	0,07

$$\vartheta_1=30°=(60°-30°)$$
$$r_1^2=1+1+2\cdot 0{,}5=3$$
$$r_2^2=1+1+20=2$$
$$S=\frac{30\cdot\pi}{360}(3+2)=\underline{\underline{0{,}2618}}$$

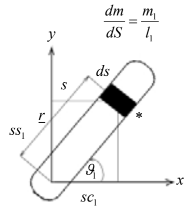
Inverzna dinamika



$$\underline{\vartheta}=\begin{bmatrix}\vartheta_1\\\vartheta_2\end{bmatrix}\qquad \underline{r}=\begin{bmatrix}x\\y\end{bmatrix}\qquad \underline{r}=S\begin{bmatrix}c_1\\s_1\end{bmatrix}$$

položaj delca(*)

$$x=S\cos\vartheta_1(t)$$
$$\frac{dm}{dS}=\frac{m_1}{l_1}$$



M_{11} …moment, ki ga čuti motor v prvem sklepu, zaradi prvega segmenta

$$M_{11}=\frac{d}{dt}\left(\frac{\partial K_1}{\partial\dot{\vartheta}_1}\right)-\frac{\partial K}{\partial\vartheta_1}$$

$M_1=$ vztrajnostni vpliv + gravitacijski vpliv