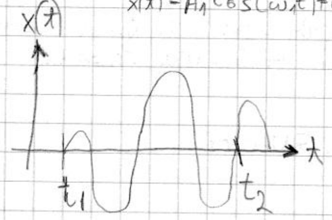
ANTON.UMEK

OTK - OSNOVE TELEK. 1. // štori sistem

SIGNALI IN INFORMATIKA // Bolonc

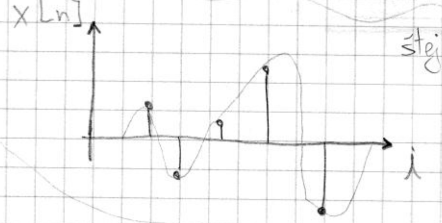
SIGNALI

primer: $x(t) = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)$



Delimo jih:

- → V času zvezni $\overbrace{x(t)}$ (definirana vrednost)
- → V času diskretni $\underbrace{x[i]}$



štej. zap. vrednosti, čas ni def.

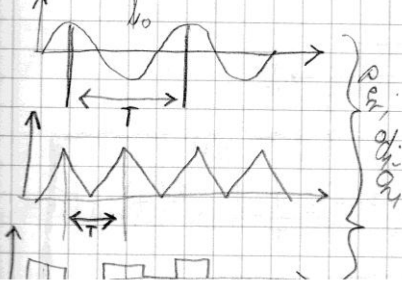
Časovno zvezni signali

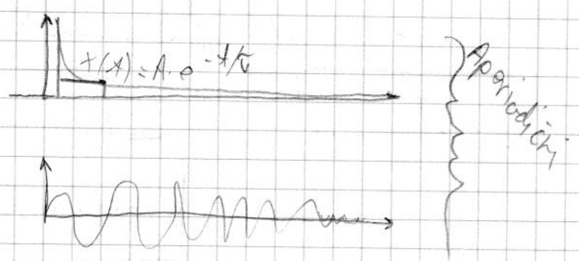
Definiranost

- → Periodični signali (MOČNOSTNI)
- → Aperiodični signali (ENERGIJSKI) → KONČNO TRAJANJE: (IMPULZI)

} Deterministični
poznamo vsak $x(t)$.

- → Naključni (RANDOM) signali } NEdeterministični
Ne poz. vseh $x(t)$





Če imamo harmonično nihanje v obliki cos mat. zapišemo.

$$A \cdot \cos(\underbrace{2\pi f_0}_{\omega_0} \cdot t) \qquad f_0 = \frac{1}{T}$$

Periodičen signal ima ∞ veliko energijo, zato se imenuje tudi MOČNOSTNI SIGNAL.

TRENUTNA MOČ SIGNALA

$$\boxed{p(t) = x^2(t)}$$

$$\boxed{E = \int_{t_1}^{t_2} p(t)\, dt}$$

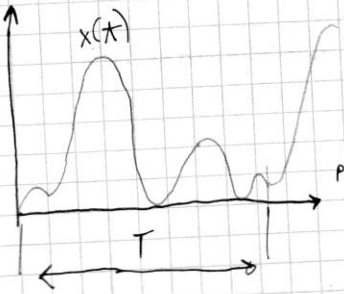
// **Energija** je integral moči po času

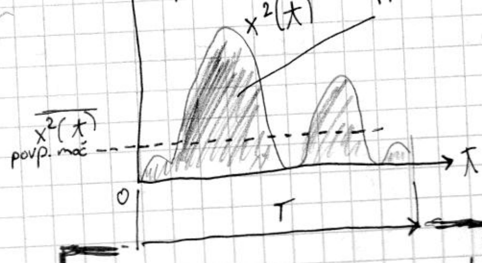
$$E = \int_{-\infty}^{\infty} p(t) \cdot dt$$

Vsak periodiče signal ima povp. moč!!!

povprečna moč

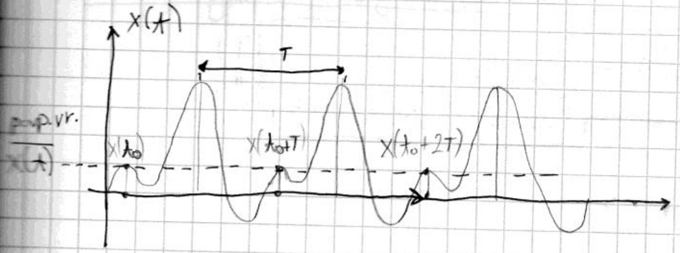
PLOŠČINA = $\int_{t_0}^{t_0+T} x^2(t)\, dt$





Povp. moč: $\overline{x^2(t)} = \frac{1}{T} \int^{t_0+T} x^2(t)\, dt$

... PERIODIČNIH SIGNALOV



$$x(t_0+T) = x(t_0)$$

$$x(t_0+i\cdot T) = x(t_0)$$

$$\boxed{\text{Srednja vrednost per. signala} = \text{ENOSMERNA KOMPONENTA SIGNALA}}$$

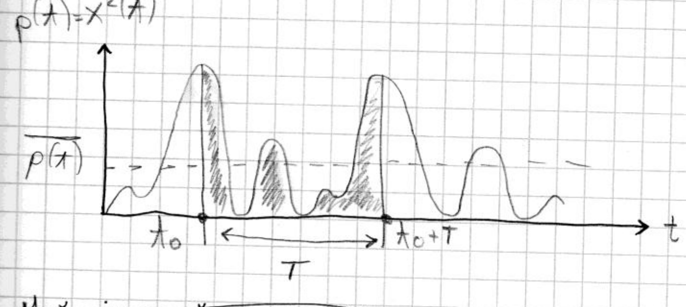
$$\overline{x(t)} = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\,dt$$

$$\overline{x(t)} = \frac{1}{T}\int_{T}^{t_0} x(t)\,dt$$

MOČ SIGNALA : $p(t) = x^2(t)$

POVPREČNA MOČ SIGNALA : $\overline{x^2(t)} = \overline{p(t)}$

$$\overline{x^2(t)} = \overline{p(t)} = \frac{1}{T}\int_T p(t)\cdot dt = \frac{1}{T}\int_T x^2(t)\cdot dt$$



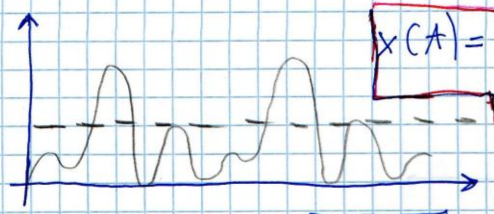
~~MOČ IZMENIČNE KOMPONENTE~~

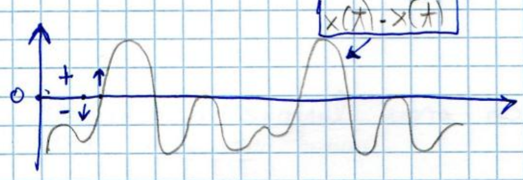
MOČ IZMENIČNE KOMPONENTE ?

Naš signal je sestavljen iz:

$$\boxed{x(t) = \underbrace{\tilde{x}(t)}_{\text{IZMENIČNE}} + \underbrace{\overline{x(t)}}_{\text{ENOSMERNE}}}$$

!!!





izmenična ~~moč~~ KOMPONENTNA SIGNAL

POVP. MOČ IZMENČNE KOMPONENTE:

$$\sigma_x^2 = (x(t) - \overline{x(t)})^2$$

$$= \frac{1}{T} \int_T (x(t) - x(t))^2$$

TRENUTNA MOČ:

$$(x(t) - \overline{x(t)})^2$$

KORELACIJA SIGNALOV = SKALARNI PRODUKT PERIODIČNIH SIGNALOV

Imamo 2 periodična signala

$x(t)$, $y(t)$

Korelacija nam pove kolikšna je izmenjava moči med njima.

Definiramo jo kot povp. produkta. (R_{xy})

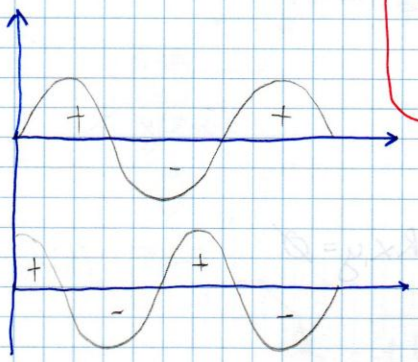
$$R_{x,y} = \overline{x(t) \cdot y(t)} = \frac{1}{T} \int_T x(t) \cdot y(t) \cdot dt$$

Primer:

$$\omega_0 = \frac{2\pi}{T}$$

$$x(t) = \cos(\omega_0 t)$$

$$y(t) = \sin(\omega_0 t)$$



$R_{x,y}$ - KRIŽNA KORELACIJA

AVTOKORELACIJA:

$R_{x,x}$

$$R_{xx} = \frac{1}{T} \int x(t) \cdot x(t)\, dt = \frac{1}{T} \int x^2(t)\, dt$$

$R_{x,x}$ = povp. moč per. signala

$R_{y,y}$

$$R_{x,y} = \frac{1}{T} \int_{t_0} \cos \omega_0 t \cdot \sin(\omega_0 t)\, dt = \emptyset$$

14.10.10

Periodični signali

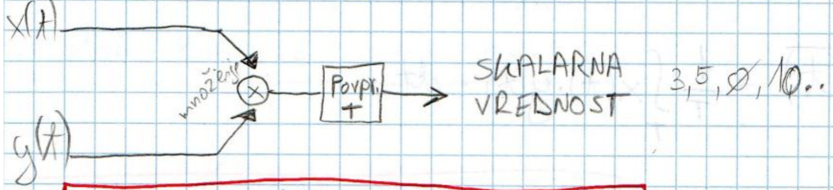
$x_1(t)$, $x_2(t)$ ....

$x(t)$

$R_{x,x} = \overline{x(t) \cdot x(t)} = \overline{x^2(t)}$ = moč signala

Autokorelacija SIGNALA $x(t)$

Primerjava signala samega s seboj

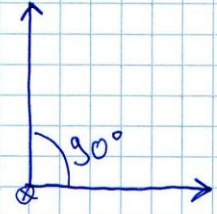


$$\boxed{R_{x,y} = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\cdot y(t)\cdot dt}$$ // skalarni produkt

$$R_{y,x} = \frac{1}{T}\int_{t_0}^{t_0+T} y(t)\cdot x(t)\, dt = R_{x,y}$$

križna korelacija

ORTOGONALNA SIGNALA : $R_{x,y} = \emptyset$



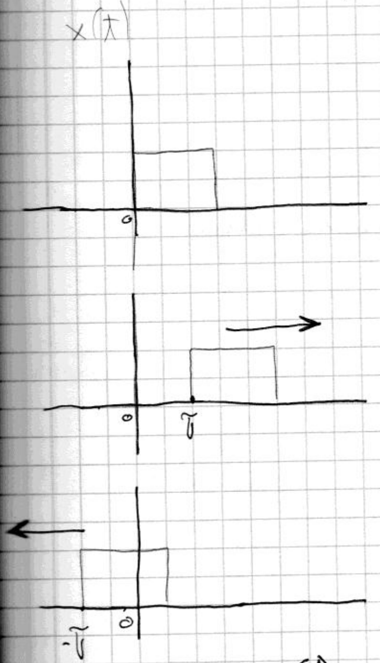
grafična predstavitev s vektorji

KORELACIJSKA FUNKCIJA

zakasnitev

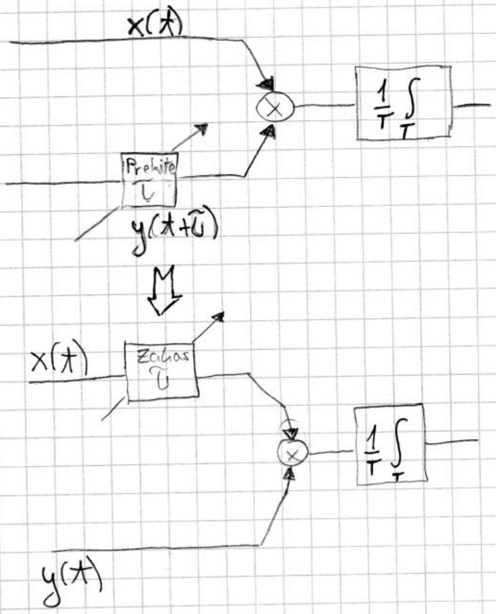
$$R_{x,y}(\tau) = \frac{1}{T}\int x(t)\cdot y(t+\tau)\,dt$$

$R_{x,x}(\tau)$ Autokorelacijska funkcija



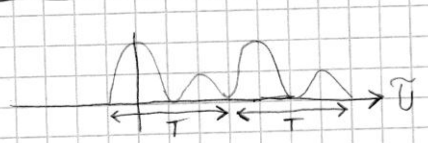
// Zakasnitev (premik v desno) $x(t-\tau)$

// Prehitevanje (premik v levo) $x(t+\tau)$

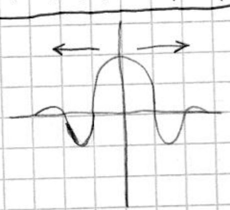


$R_{x,y}(0)$ = KORELACIJA

$$\boxed{R_{x,x}(\tau) = R_{x,x}(\tau \pm n \cdot T)}$$ //Periodična funkcija



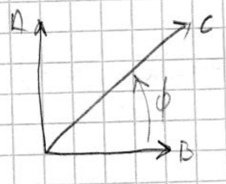
$$\boxed{R_{x,x}(-\tau) = R_{x,x}(\tau)}$$ //Soda funkcija



$$\boxed{\max(R_{xx}) = R_{x,x}(\tau=0) = R_{x,x} = \text{Moč signala}}$$ //Maksimalna vrednost

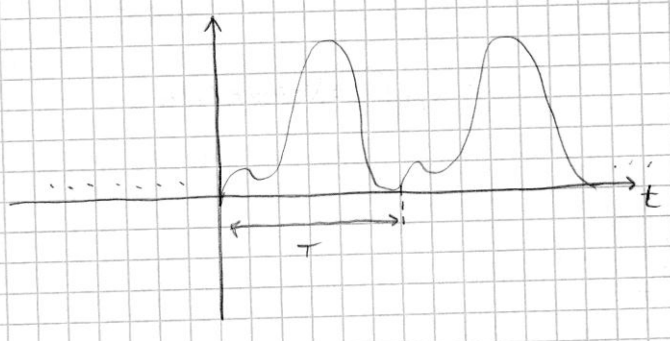
če seštejem 2 nihajna signala dobibo 3nihajni signal

$$A \cos(\omega_0 t) + B \cdot \sin(\omega_0 t) = C \cdot \cos(\omega_0 t + \phi)$$



Analiza signala

$$x(t+T) = x(t)$$



Razvoj funkcije $x(t)$ v Fourierovo vrsto:

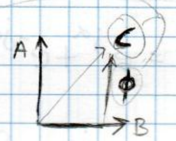
Fourierova vrsta

ENOSMERNA KOMPONENTA

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(k \cdot \omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k \cdot \omega_0 t)$$

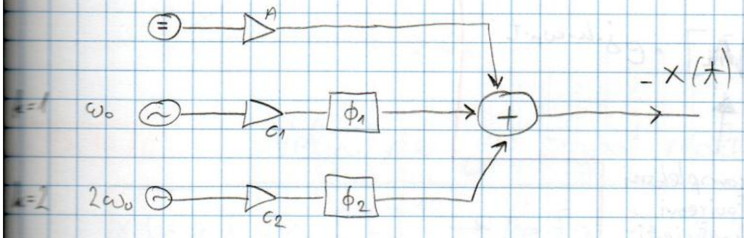
$$\boxed{\omega_0 = \frac{2\pi}{T}}$$ osnovna frekvenca

$$\frac{a_0}{2} + \sum_{k=1}^{\infty} c_k \cdot \cos(k \cdot \omega_0 t + \phi_k)$$



Analiza: Pove iz katerih signalov je funkcija sestavljena

SINTEZA:



$$a_k = 2 \cdot \frac{1}{T} \int x(t) \cdot \cos(k \cdot \omega_0 t)\, dt$$

$$a_1 = 2 \cdot \frac{1}{T} \int x(t) \cos(\omega_0 t)\, dt \qquad b_1 = 2 \cdot \frac{1}{T} \int x(t) \cdot \sin(\omega_0 t)\, dt$$

Kompleksni zapis Fourierove vrste

$\cos\alpha = ?$

$\sin\alpha = ?$

$$e^{j\alpha} = \cos\alpha + j\sin\alpha$$
$$e^{-j\alpha} = \cos\alpha - j\sin\alpha$$

$$\Sigma \qquad = 2\cos\alpha = e^{j\alpha} + e^{-j\alpha}$$

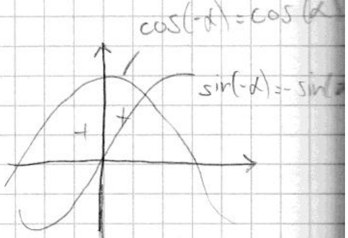
$$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2} \longrightarrow \cos(k\omega_0 t) = \frac{e^{jk\omega_0 t} + e^{-jk\omega_0 t}}{2}$$

$$\sin\alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2}$$

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} X[k] \cdot e^{j \cdot k \cdot \omega_0 \cdot t}}$$

kompleksni Fouireovi koeficienti

$$X[k] = \begin{cases} \dfrac{a_k + jb_k}{2} & k > 0 \\ \dfrac{a_k - jb_k}{2} & k < 0 \\ \dfrac{a_0}{2} & k = 0 \end{cases}$$



$$e^{jk\omega_0 t} = \cos(k \cdot \omega_0 \cdot t) + j\sin(k\omega_0 t)$$

Lastnosti:

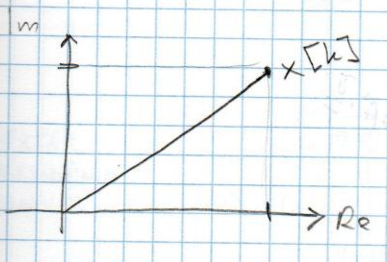
$x(t)$ ... realni signal $\quad Im\{x(t)\} = \emptyset$

$X[-k] = X[k]^*$

$X[k]$ = kompleksni koeficient

$|X[k]| \equiv$ amplituda

$$\arg\{X[k]\} = a\tan \frac{Im\{X[k]\}}{Re\{X[k]\}}$$



25.10.10 Velja, če je signal realen!

$X[-k] = X[k]^* \rightarrow Re\{X[-k]\} = Re\{X[k]\}$ soda F.

$Im\{X[-k]\} = -Im\{X[k]\}$ Liha F.

signal:

sodi $x(-t) = x(t) \rightarrow Im\{X[k]\} \equiv \emptyset$

lihi $x(-t) = -x(t) \rightarrow Re\{X[k]\} \equiv \emptyset$

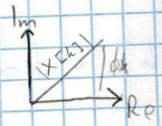
Posebni primeri!

$$X[k] = Re\{X[k]\} + j\, Im\{X[k]\}$$

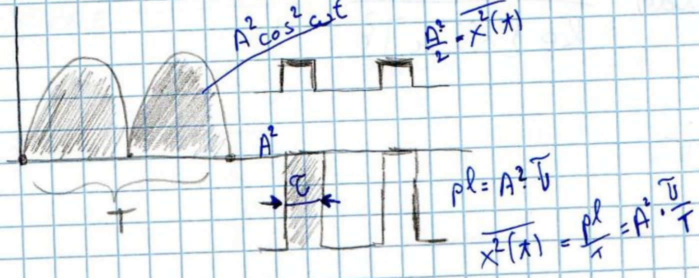
$$A_x[k] = |X[k]|$$ AMPLITUDNI SP.

$$A_x[k]^2 = |X[k]|^2$$ MOČNOSTNI SP.

$$\phi_x[k] = arg(X[k]) = atan\frac{Im\{X[k]\}}{Re\{X[k]\}}$$ FAZNI SP.



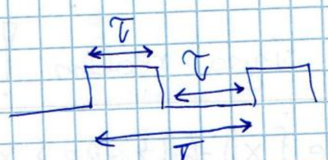
$$\boxed{\overline{x^2(t)} = \frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)\,dt = \sum_k |X[k]|^2}$$ Parseval-ov izrek



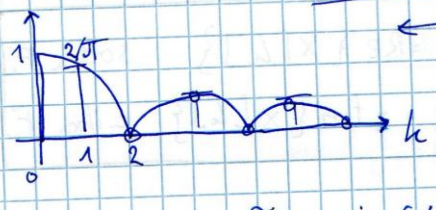
$$p\ell = A^2 \cdot \tau$$

$$\overline{x^2(t)} = \frac{p\ell}{T} = A^2 \cdot \frac{\tau}{T}$$

Primer iz vaj!



če je τ enak pri pozi. in negativni delu je $\tau = \frac{T}{2}$



$$X[k] = A \cdot \frac{\tau}{T} \cdot \frac{\sin(k \cdot \omega_0 \frac{\tau}{2})}{k \cdot \omega_0 \frac{\tau}{2}}$$

$X[1] = \frac{2}{\pi} \cdot \frac{A}{2}$

$X[2] = 0 \cdot \frac{A}{2}$

$X[3] = \frac{2}{3\pi} \cdot \frac{A}{2}$

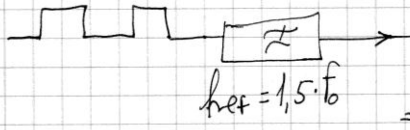
$X[4] = 0$

$X[5] = \frac{2}{5\pi} \cdot \frac{A}{2}$

$$\tau = \frac{T}{2}, \quad \omega_0 = \frac{2\pi}{T}, \quad \omega_0 \cdot \frac{\tau}{2} = \frac{\pi \cdot \tau}{T}$$

$$X[k] = \frac{A}{2}\, \frac{\sin(k\frac{\pi}{2})}{k\frac{\pi}{2}}$$

$\cancel{X[1] = 2/\pi;\ X[2] = 0;}$ $X[3] =$

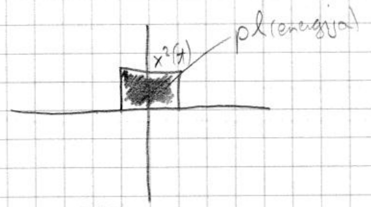


$$|x[0]|^2 + \left(|x[-1]|^2 + \cancel{|x[-2]|^2}\right) \cdot 2 =$$

$$= \frac{A^2}{4}\left(1 + 2 \cdot \frac{4}{\pi^2}\right) = \frac{A^2}{4} + \frac{2A^2}{\pi^2}$$

APERIODIČNI SIGNALI

ENERGIJA SIGNALA $\quad x(t) = \displaystyle\int_{t=-\infty}^{\infty} x^2(t)\,dt = E_x$



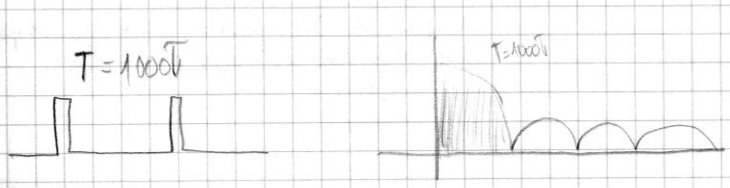
SPEKTER APERIODIČNIH SIGNALOV

Fourierov transform $\quad$ sig. $x(t)$

$$X_a(\omega) = \int_{t=-\infty}^{\infty} x_a(t) \cdot e^{-j\omega t} \cdot dt$$

čas. prostor $\quad x(t) \xleftrightarrow{\text{Fourierova transformacija}} X(\omega) \quad$ frekv. prostor

Fourierov par: če poznaš enega, poznaš oba



$$|X(\omega)|^2$$ = Gostota energi. SPEKTRA

INVERZNA TRANSFORMACIJA

$$\begin{tikzcd} x(t) \arrow[r, bend left, "F"] & X(\omega) \arrow[l, bend left, "F^{-1}"] \end{tikzcd}$$

$$\boxed{x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot e^{j\omega t} \cdot d\omega}$$

Povezava:

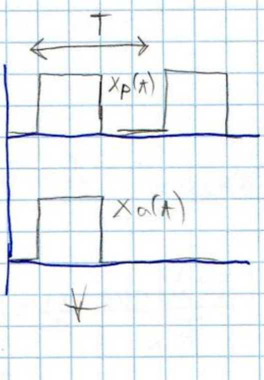
Periodičen signal

$$X_p[k] = \frac{1}{T} \int_{t=t_0}^{t_0+T} x_p(t)\, e^{-jk\omega_0 t} \cdot dt$$

$$x(t) = \sum_{k=-\infty}^{\infty} X[k] \cdot e^{jk\omega_0 t}$$

Aperiodičen signal

$$X_a(\omega) = \int_{-\infty}^{\infty} x_a(t) \cdot e^{-j\omega t}\, dt$$



$$\boxed{X_p[k] = \frac{X_a(\omega = k\omega_0)}{T}}$$

Povezava med signaloma !

Ni enak pri vseh primerih.

(velja le če lahko iz enega impulza aper. signala dobimo periodični)

$X(\omega)$ → Fourierova trans.

$|X(\omega)|$ → Gos. amp. spektra

$\arg(X(\omega)) = a\tan \frac{\mathrm{Im}\, X(\omega)}{\mathrm{Re}\, X(\omega)}$ → gos. faznega spektra

Energija signala

čas. prostor — t

frekv. prostor — f, ω

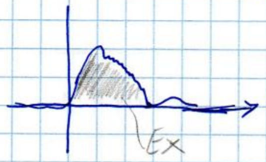
$x(t) \longrightarrow X(\omega)$

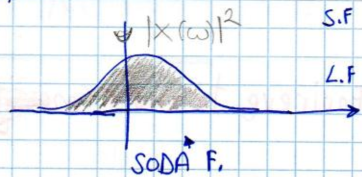
$|X(-\omega)| = |X(\omega)|$

$|X(-\omega) = X(\omega)^*$

S.F $\mathrm{Re}\{X(-\omega)\} = \mathrm{Re}\{X(\omega)\}$

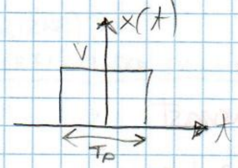
L.F $\mathrm{Im}\{X(\omega)\} = -\mathrm{Im}\{X(-\omega)\}$





$$E_x = \int x^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$$

$\boxed{8.11.2010}$



→ Imamo nek aperiodičen signal.

L

S L

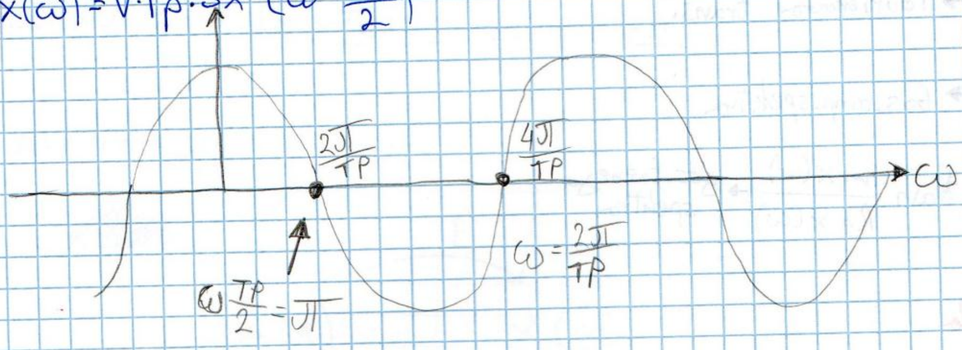
$$X(\omega) = \int_t x(t)\cdot e^{-j\omega t}\,dt = \underbrace{\int_t x(t)\cdot \cos(\omega t)\cdot dt}_{\neq 0} - j\underbrace{\int_t x(t)\cdot \sin(\omega t)\cdot dt}_{\overset{!!}{0}}$$

$$e^{-j\omega t} = \cos\omega t - j\sin\omega t$$

$$X(\omega) = 2\cdot\int_0^{T_p/2} V\cdot\cos(\omega t)\cdot dt = 2V\,\frac{\sin\omega t}{\omega}\Bigg|_0^{T_p/2} = \cancel{2}V\,\frac{\sin(\omega \frac{T_p}{2})}{\omega\, T_p/2}\,\frac{T_p}{\cancel{2}}$$

$$\boxed{X(\omega) = V\cdot T_p\cdot Sx\left(\omega\frac{T_p}{2}\right)}$$

$X(\omega) = V \cdot T_p \cdot S_x\left(\omega \cdot \frac{T_p}{2}\right)$



$$X(0) = \int_{-\infty}^{\infty} x(t) \cdot 1 \cdot dt = V \cdot T_p$$

Lastnosti Fourireve transformacije

signal / spekter

$$x(t) \xrightarrow{\mathcal{F}} X(\omega)$$

$$x(t) \xleftarrow{\mathcal{F}^{-1}} X(\omega)$$

signal	spekter	
$a \cdot x_1(t) + b \cdot x_2(t)$	$a \cdot X_1(\omega) + b \cdot X_2(\omega)$	Linearnost
$x(a \cdot t)$	$X\left(\frac{\omega}{a}\right) \cdot \frac{1}{\lvert a \rvert}$	Sprememba merila
$x(t \pm t_0)$	$X(\omega) \cdot e^{\pm j\omega t_0}$	premik v času
$x(t) \cdot e^{\mp j\omega t_0}$	$X(\omega \mp \omega_0)$	premik v frekvenci (ni veliko x uporabljen v praksi)
$x(t) \cdot \cos(\omega_0 t)$	$\frac{1}{2} X(\omega + \omega_0) + \frac{1}{2} X(\omega - \omega_0)$	Teorem o modulaciji
$\frac{dx(t)}{dt}$	$X(\omega) \cdot j\omega$	
$\int^{t} x(\tau)\, d\tau$	$\frac{X(\omega)}{j\omega}$	

$x(t)$	$X(\omega)$
$E = \int_t x^2(t)\, dt$	$\frac{1}{2\pi} \int \lvert X(\omega)\rvert^2 d\omega$
$y(t)$	$Y(\omega)$
$r_{xy}(t)$	$X(\omega)^* \cdot Y(\omega)$
$r_{xx}(t)$	$X(\omega)^* \cdot X(\omega) = \lvert X(\omega)\rvert^2$ ges.ene.spektra
$h(t)$	$H(\omega)$
$x(t) * h(t)$	$X(\omega) \cdot H(\omega)$

avtokorelacijske funkcije

KORELACIJA APERIODNIH SIGNALOV

(KRIŽNA)
KORELACIJSKA FUNKCIJA

$$r_{x,y}(\tau) = \int_t x(t) \cdot y(t+\tau) \cdot dt$$

$$\cancel{r_{x_1,x_2}(\tau) = \int_t x_1(t) \cdot x_2(t}$$

(KRIŽNA) KORELACIJA

$r_{x,y}(\tau = 0)$

Merilo podobnosti

Signala sta ORTOGONALNA velja $r_{x,y}(0) \neq 0$

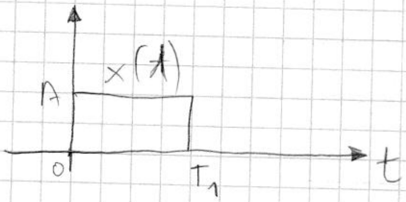
Avtokorelacijska funkcija:

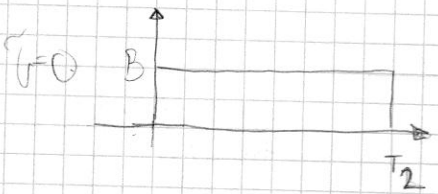
$$r_{x,x}(\tau) = \int_t x(t) \cdot x(t+\tau) \cdot dt$$

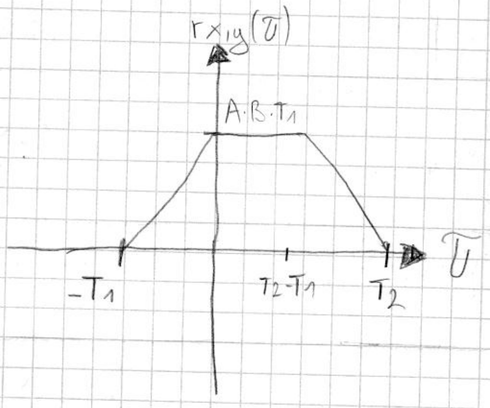
Avtokorelacija

$$r_{xx}(\tau = 0) = \int_t x^2(t)\, dt = E_x$$

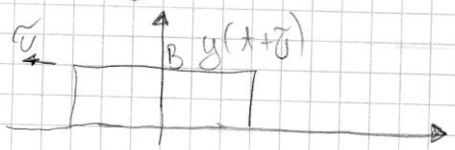
energija

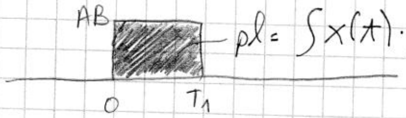






$r_{x,y}(\tau) = ?$

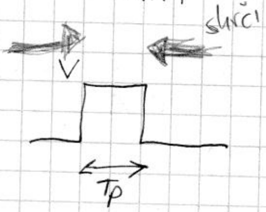


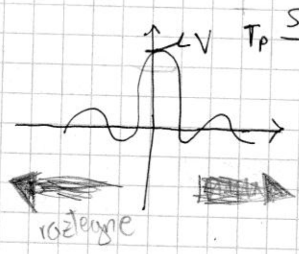


$pl = \int x(t) \cdot y(t+\tau) \cdot d\tau = r_{x,y}(\tau) = A \cdot B \cdot T_1$

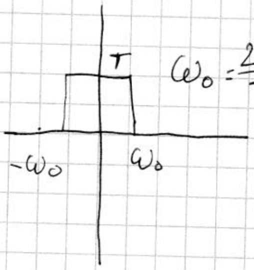
$\boxed{15.11.2010}$

$x(t) \longleftrightarrow X(\omega)$



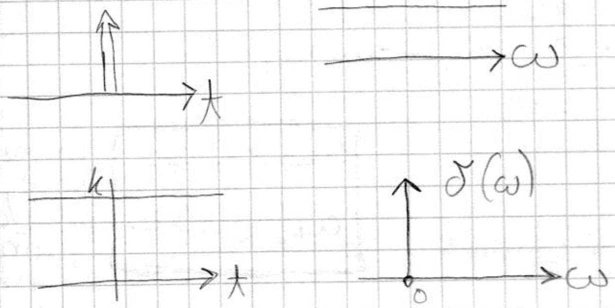


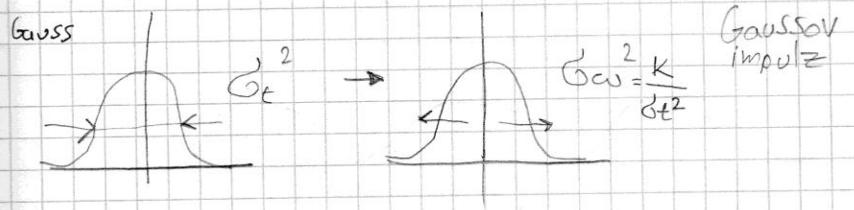
$$T_p \frac{\sin \frac{\omega \cdot T_p}{2}}{\omega_0 \frac{T_p}{2}}$$



$$\omega_0 = \frac{2\pi}{T}$$

$x(t) \longleftrightarrow X(\omega)$

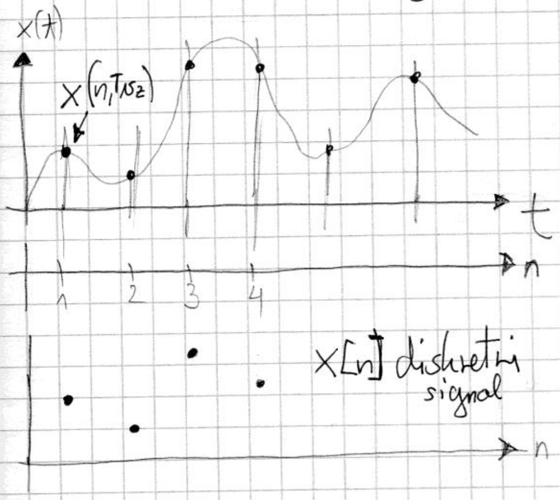


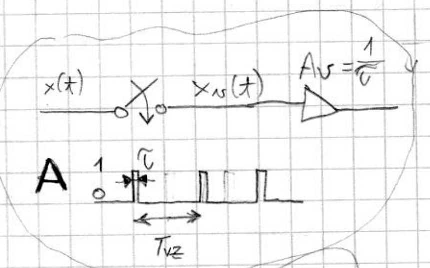


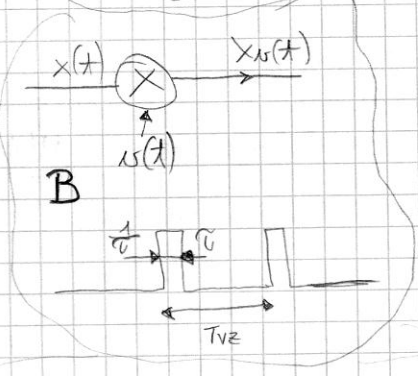
$$x(t) = e^{-at^2} \longleftrightarrow \sqrt{\frac{\pi}{a}} \cdot e^{-\frac{\omega^2}{4a}}$$

PROCES VZORČEVANJA

Točkice so vzorci signala







C idealno vzorčenje

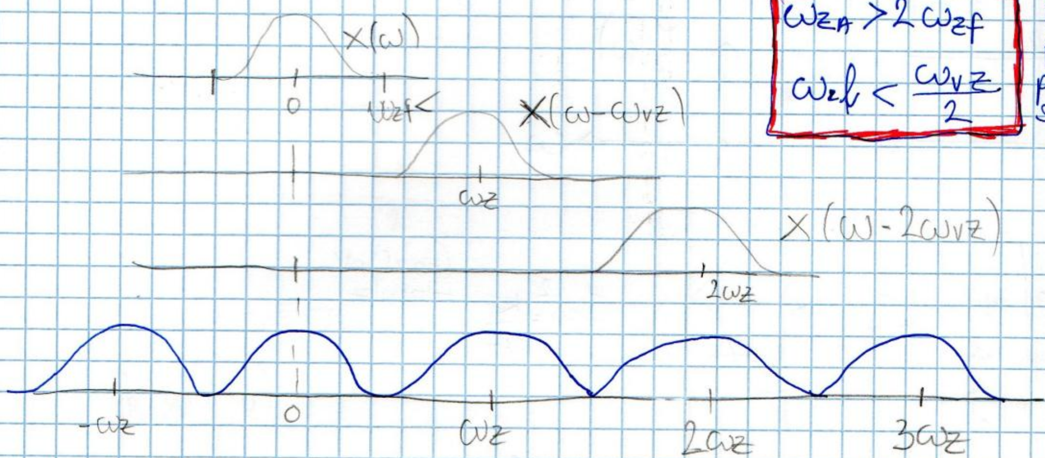
$x(t)$ ⊗ $x_\delta(t) = x(t) \cdot \sum_n \delta(t - n \cdot t_{vz}$

$x_\delta(t) = \sum x(n t_{vz}) \cdot \delta(t - n t_{vz})$ $\quad v(t) = \sum_n \delta(t - n \cdot t_{vz})$

vzorci

$x(t) \longleftrightarrow X(\omega)$

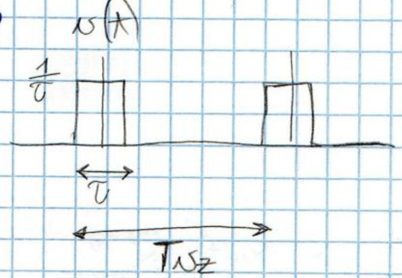
$$x_\delta(t) \longleftrightarrow \frac{1}{T_{vz}} \cdot \sum_{k=-\infty}^{\infty} X(\omega - k \cdot \omega_{vz})$$



$$\boxed{\begin{aligned} &\omega_{zA} > 2\omega_{zf} \\ &\omega_{zf} < \frac{\omega_{vz}}{2} \end{aligned}}$$

Formula pove, da prekrivanja spektru signala

(B)

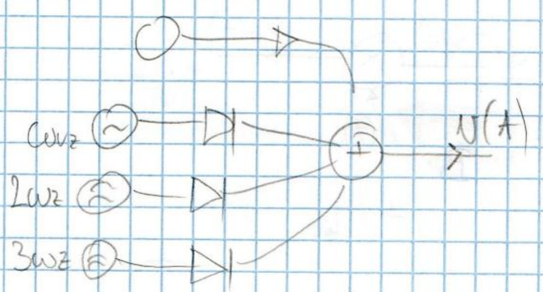


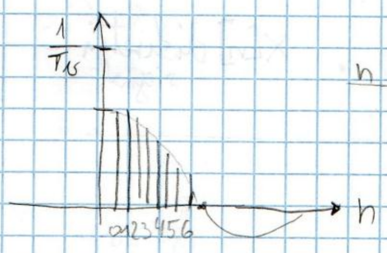
$$\boxed{V[n] = \frac{1}{T_{vz}} \cdot \frac{\sin\left(n \cdot \omega_{vz} \cdot \frac{\tau}{2}\right)}{n \cdot \omega_{vz} \cdot \frac{\tau}{2}}}$$

$V = \frac{1}{\tau}$

$T = T_{vz}$

$\omega_0 = \omega_{vz} = \frac{2\pi}{T_{vz}}$





$n \cdot \omega_{vz} \cdot \frac{\tau}{2} = \pi$

22.11.2010 (Teoremi vzorčenja)

iz spektra računamo pulz!

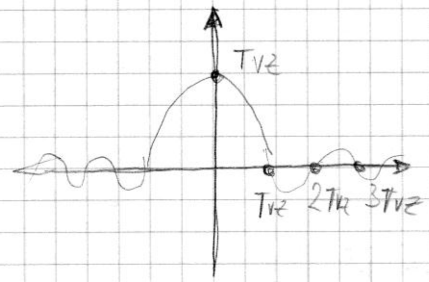
odziv idealnega nizkega sita z frekvenco ω_{zg}.

$$h(t) = \frac{1}{2\pi} \int H(\omega) \cdot e^{-j\omega t} d\omega = \frac{1}{2\pi} \cdot 2 \int_0^{\omega_{zg}} 1 \cdot \cos(\omega t) \cdot d\omega =$$

$$h(t) = \frac{1}{\pi} \cdot \frac{\sin \omega t}{t} \Big|_0^{\omega_{zg}} = \frac{\omega_p}{\pi} \cdot \frac{\sin(\omega_{zg} t)}{\omega_{zg} \cdot t}$$

$$\omega_{zg} = \frac{\omega_{vz}}{2} = \frac{2\pi}{T_{vz} \cdot 2} = \frac{\pi}{T_{vz}}$$

$$h(t) = \underline{\underline{T_{vz}}} \cdot \frac{\sin\left(\pi \cdot \frac{t}{T_{vz}}\right)}{\pi \cdot \frac{t}{T_{vz}}}$$



NAKLJUČNI SIGNALI

$$\boxed{\overline{x^2(t)} = \lim_{T \to \infty} \cdot \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) \cdot dt}$$

povp. moč

aperiodični signali

$$r_{xx}(\tau) = \int_{-\infty}^{\infty} x(t) \cdot x(t+\tau) \cdot dt$$

$$\boxed{r_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \cdot \int_{-T/2}^{T/2} x(t) \cdot x(t+\tau) \cdot dt}$$

autokorelacija

$$\boxed{r_{xx}(\tau) \longleftrightarrow S_x(\omega) = \int_{-\infty}^{\infty} r_{xx}(t) \cdot e^{-j\omega t} \cdot dt = 2\int_{0}^{\infty} r_{xx}(t) \cdot \cos(\omega t)\, dt}$$

gostota močnostnega spektra ($S_x(\omega)$)

realen del

AK. funkcija ($r_{xx}(\tau)$)

soda funkcija

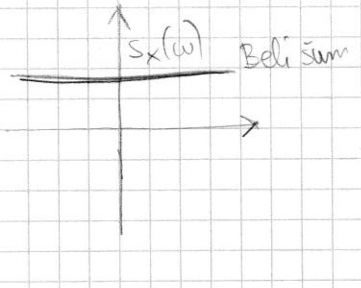
(Največja vrednost je v izhodišču)

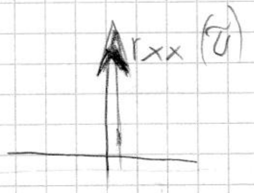
$r_{xx}(0)$ = Moč signala

$$r_{xx}(\tau) = \frac{1}{2\pi} \int S(\omega) \cdot e^{j\omega t} \cdot d\omega$$

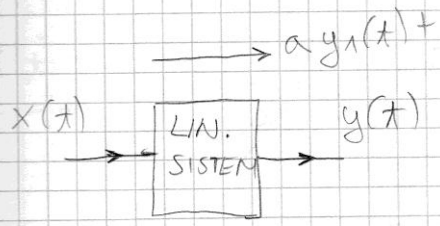
$$\boxed{r_{xx}(0) = \frac{1}{2\pi} \int S_x(\omega) \cdot d\omega = \overline{x^2(t)}}$$

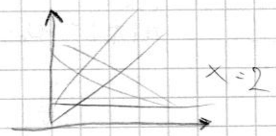
Beli šum naključni signal v ravnem spektru.

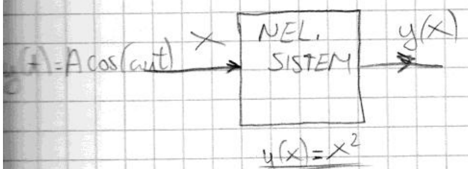


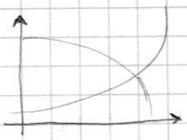


PRENOS







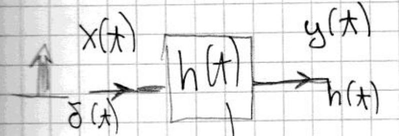


$y(x)=x^2$

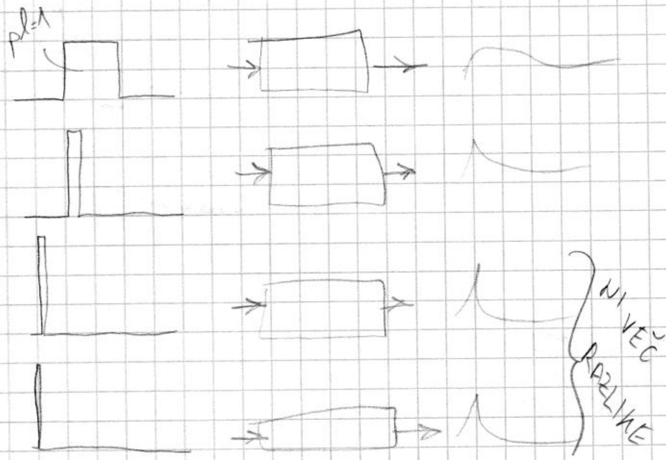
$y(t)=A^2\cdot\cos^2(\omega_1 t)$

$\hookrightarrow \frac{1}{2}(1+\cos\cdot 2\omega_1 t)$

SISTEMSKA FUNKCIJA



LINEARNI SIST.
= SITO (SLO)
= FILTER (ANG)



Sistemska funkcija je odziv lin. sistema na enotin impulz. !!!
$\delta(t)$

ODZIV LINEARNEGA SISTEMA

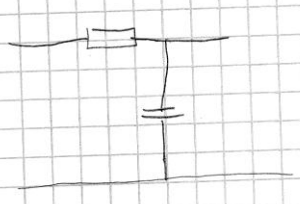
odziv na enotin impulz

$y(t) = x(t) * h(t) = h(t) * x(t)$

KONVOLUCIJA

$$\boxed{y(t)=\int_{-\infty}^{\infty} x(\tau)\cdot h(t-\tau)\cdot d\tau}$$

1. Primer:

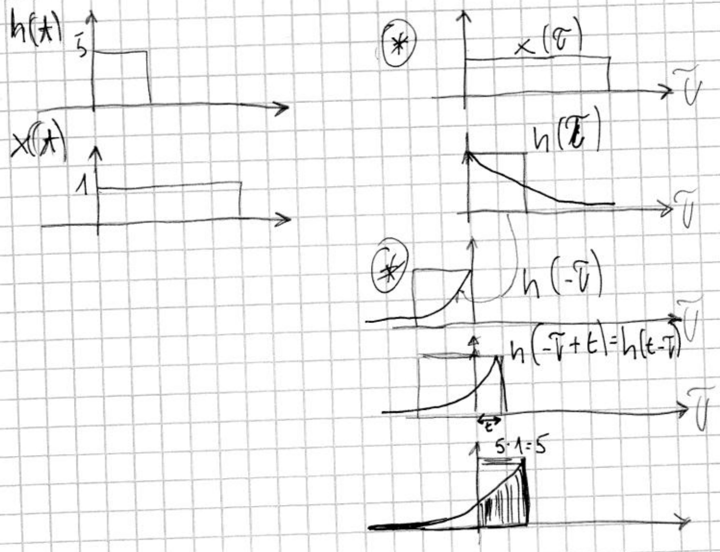


$$h(t) = \omega_0 \cdot e^{-\omega_0 t}, \quad \omega_0 = \frac{1}{RC}$$

$$\Updownarrow$$

$$H(\omega) = \frac{1}{1 + j\frac{\omega}{\omega_0}}$$

2. Primer:



$\mathcal{F}$

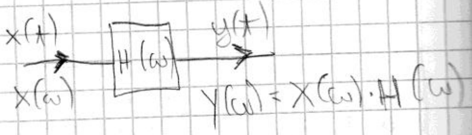
$$x(t) * h(t) \longleftrightarrow X(\omega) \cdot H(\omega)$$

sistemska funkcija

sistemska funkcija

$$\boxed{h(t) \longleftrightarrow H(\omega) = \int h(t) e^{-j\omega t} \cdot dt}$$

Prenosna funkcija sistema



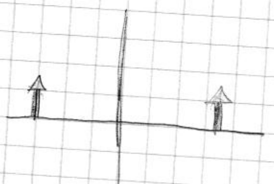
29.11.2010

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$Y(\omega) = X(\omega) \cdot H(\omega)$$

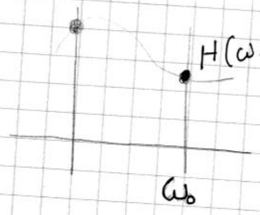
$$h(t) \longleftrightarrow H(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$



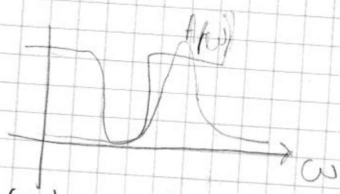
$\rightarrow x(t) = \cos(\omega_0 t)$

$$X(\omega) = \frac{1}{2}\,\delta(\omega + \omega_0) + \frac{1}{2}\,\delta(\omega - \omega_0)$$

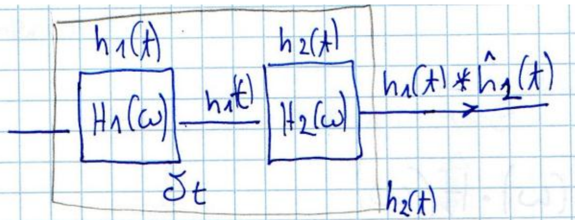


$$H(\omega = \omega_0) = \frac{\hat{Y}(\omega_0)}{\hat{X}(\omega_0)}$$

$$H(\omega) = \underbrace{|H(\omega)|}_{A(\omega)} \cdot e^{j\phi(\omega)}$$



$$\alpha(\omega) = 20 \cdot \log A(\omega)$$
$$= 20 \cdot \log |H(\omega)|$$

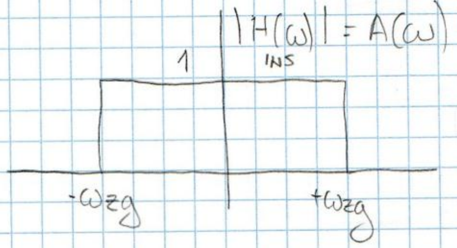


$$H_{12}(\omega) = H_1(\omega)\cdot H_2(\omega) = |H_1(\omega)|\cdot|H_2(\omega)|\cdot e^{j\phi_1(\omega)}\cdot e^{j\phi_2(\omega)}$$

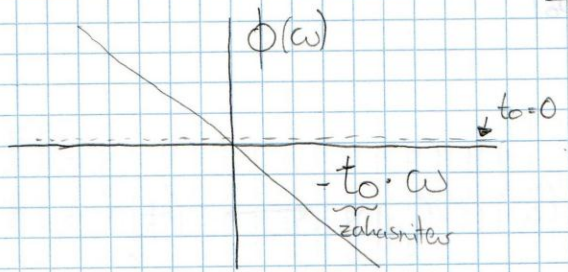
$$|H_{12}(\omega)| = |H_1(\omega)|\cdot|H_2(\omega)|$$

$$\phi_{12}(\omega) = \phi_1(\omega) + \phi_2(\omega)$$ } fazni del se seštewa

Odziv idealnega nizkega sita!



→ karakteristika



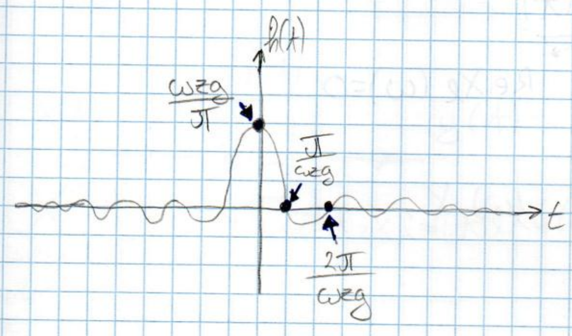
če je $\boxed{t_0=0}$

$H(\omega) = A(\omega)\cdot e^{j0}$

$h(t) = \frac{1}{2\pi}\int H(\omega)\cdot e^{j\omega t}\,dt$

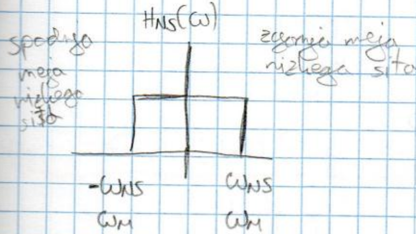
$h(t) = \frac{1}{2\pi}\int H(\omega)\cdot\cos(\omega t)\cdot d\omega = \frac{1}{2\pi}\cdot 2\cdot\int_0^{\omega_{zg}} 1\cdot\cos(\omega t)\cdot d\omega =$

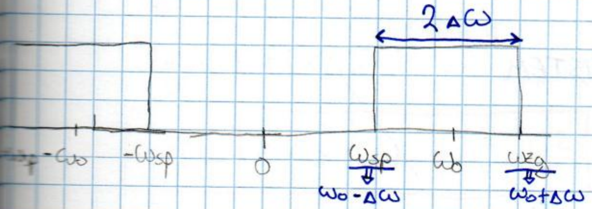
$$h(t) = \frac{1}{\pi} \frac{\sin(\omega t)}{t} \Big|_0^{\omega_{zg}} = \frac{1}{\pi} \frac{\sin(\omega_{zg} \cdot t)^{\circ}}{t} = \frac{\omega_{zg}}{\pi} \cdot \frac{\sin(\omega_{zg} t)}{\omega_{zg} \cdot t}$$



$$\omega_{zg} \cdot t = \pi$$
$$t = \frac{\pi}{\omega_{zg}}$$

KARAKTERISTIKA PASOVNEGA SITA!





$\frac{1}{2} H_{NS}(\omega + \omega_0)$ $\qquad$ $\frac{1}{2} \cdot H_{NS}(\omega - \omega_0)$

teorem o modulaciji

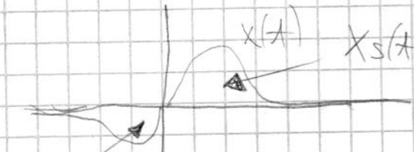
$x(t)$	$X(\omega)$
$x(t)\cos(\omega_0 t)$	$\frac{1}{2} X(\omega + \omega_0) + \frac{1}{2} X(\omega - \omega_0)$

$$h_{ps}(t) = h_{NS}(t) \cdot \cos(\omega_0 t)$$

pasovna sita

$$h_{ps}(t) = \frac{\Delta\omega}{\pi} \cdot \frac{\sin \Delta\omega t}{\Delta\omega t} \cdot \cos(\omega_0 t)$$

$$x(t) = \underbrace{x_s(t)}_{\operatorname{Im} X_s(\omega)=0} + \underbrace{x_\ell(t)}_{\operatorname{Re} X_\ell(\omega)=0}$$



$x_s(t) + x_\ell(t) = 2x_s(t) = 2x_\ell(t)$

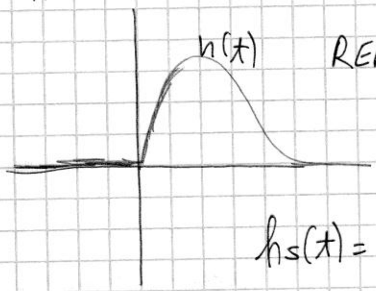
$x_s(t) - x_\ell(t) = 0$

$x_\ell(t) = x_s(t)$

$$x_s(t) = \operatorname{sign}(t) \cdot x_\ell(t)$$

$$\operatorname{sign}(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$$

$h(t) = 0$ za $t < 0$



REALNI SISTEM

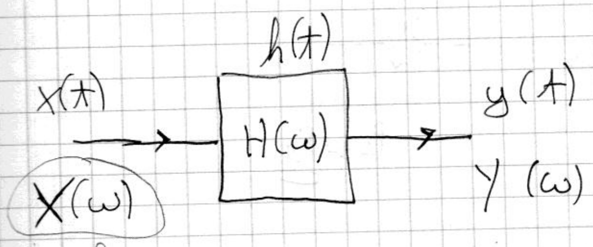
$$h_s(t) = \operatorname{sign}(t) \cdot h_\ell(t)$$

$$h(t) = h_s(t) + h_\ell(t)$$

Amplitudna poteka sta med seboj povezana !!!

$$\boxed{A(\omega) \Longleftrightarrow \phi(\omega)}$$

MOČNOSTNI SPEKTER



$Y(\omega) = X(\omega) \cdot H(\omega)$

signalni
naključeni,
poznamo signal na Iz.

$r_{xx}(t) \longleftrightarrow S_x(\omega) = |X(\omega)|^2$

gos.
moč.
spektra

za deterministične signale velja:

$$S_y(\omega) = |Y(\omega)|^2$$

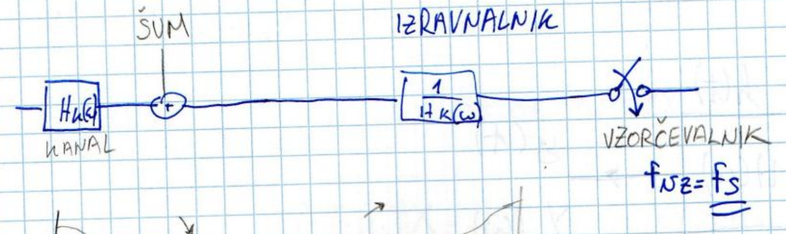
$$S_y(\omega) = |X(\omega) \cdot H(\omega)|^2 = |X(\omega)|^2 \cdot |H(\omega)|^2 = S_x(\omega) \cdot |H(\omega)|^2$$

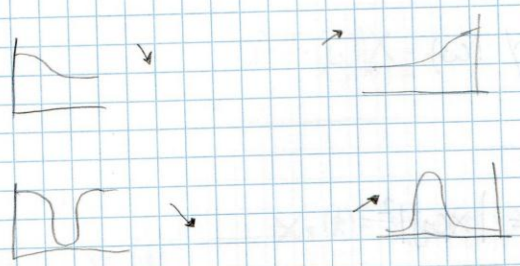
$$\boxed{S_y(\omega) = S_x(\omega) \cdot |H(\omega)|^2}$$

za naključne signale velja enako:

$$\boxed{S_y(\omega) = S_x(\omega) \cdot |H(\omega)|^2}$$

16.12.2010





matematično izvedljivo, praktično pa ne.

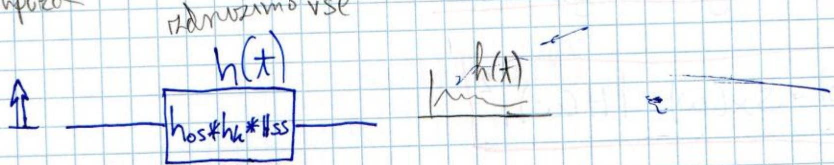
Npr. Telegram.

Pošiljajo se signali, prihaja do popačenj.

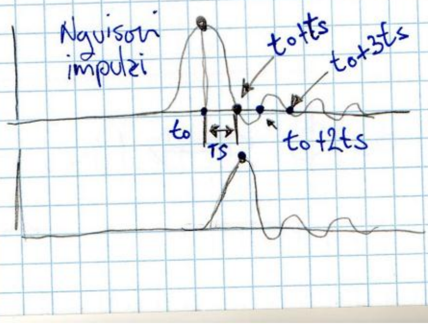
Rešitev: Nquist je ugotovil, da vse to združimo in .... pogledamo vzorčevalnik

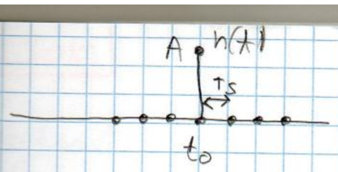
ta sito določa oblike impulza

združimo vse



$$h(t_0 + i \cdot T_s) = \begin{cases} A & i = 0 \\ \emptyset & \text{sin} \end{cases}$$





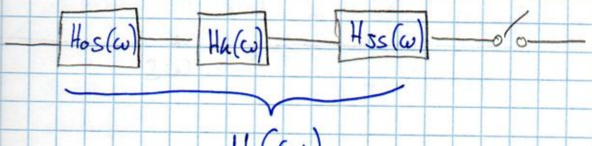
$$h(t) = h_{os}(t) * h_k(t) * h_{ss}(t)$$

pogoj za časovni prostor

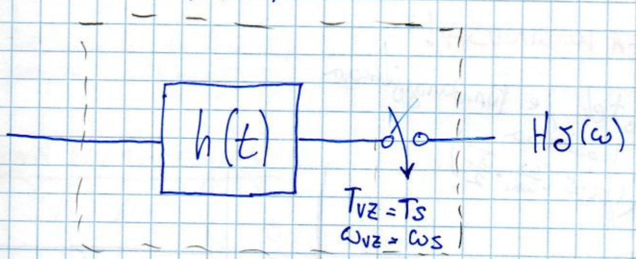
$$\boxed{h(t_0 + i\cdot T_s) = \delta[i]\cdot A}$$

$$\begin{cases} 1 & i=0 \\ 0 & \text{sin} \end{cases}$$

$$\boxed{H(\omega) = H_{os}(\omega)\cdot H_k(\omega)\cdot H_{ss}(\omega)}$$



$H(\omega)$



$$h(t_0 + i\cdot T_s) = A\cdot\delta[i]$$

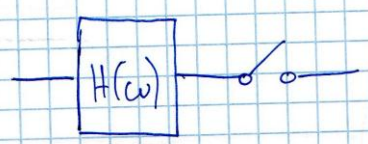
pogoj za frekvenčni prostor

$$\boxed{H_\delta(\omega) = \frac{1}{T_s}\sum_k H(\omega + k\cdot\omega_s)}$$

vzorčena skupna prevajalna funkcija

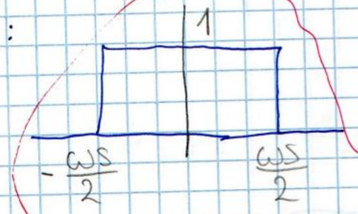
$$\boxed{\sum_k H(\omega + k\cdot\omega_s) = A\cdot T_s}$$

konstanta



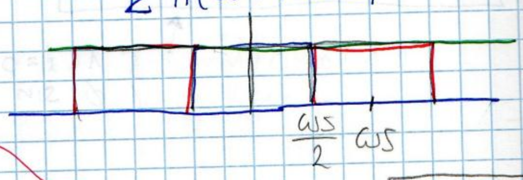
1. PRIMER:

$H(\omega)$:



ko združimo kvadratke, dobimo konstanto

$\Sigma H(\omega + n\omega_s)$



Teoretično najožji pas karakteristika sita

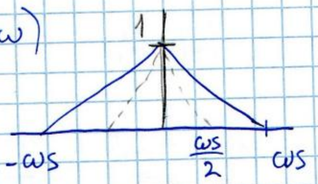
$\boxed{\omega_{zg} = \frac{\omega_s}{2}}$

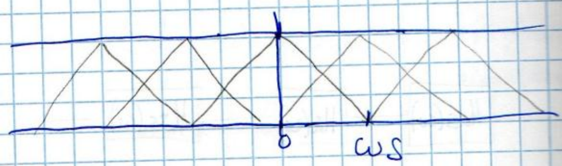
$f_s = 10^6$ 2zraka/s

$f_{zg} = \frac{f_s}{2} = 500\,kHz$

2. Primer:

$H(\omega)$

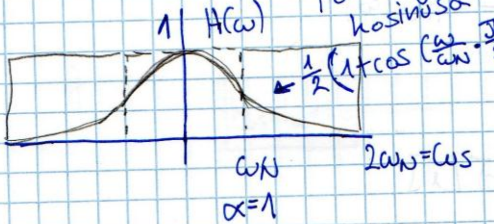


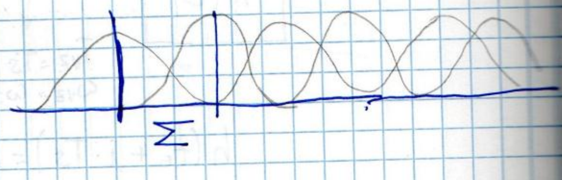


3 PRIMER:

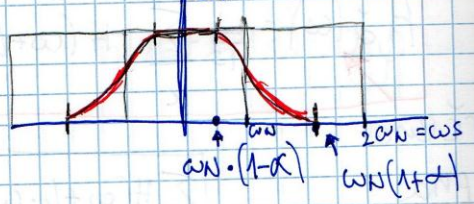
FUNKCIJA DVIGNJENEGA KOSINUSA!

Potek je fun. dvignjenega kosinusa





Družina sit funkcije dvignjenega kosinusa



13.12.2010

Uvod, da lahko na naslednji strani mat. zapišemo informacijo!

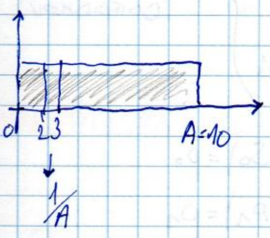
• Verjetnost

• Naključne spremenljivke

$p_x(X)$

↓

GOSTOTA VERJETNOSTNE PORAZDELITVE



$P(X=x_1)=p_i$

↓

VERJETNOST

$P(x=1)=p_1=\frac{1}{6}$

⋮

$P(x=6)=p_1=\frac{1}{6}$

$\Sigma=1$

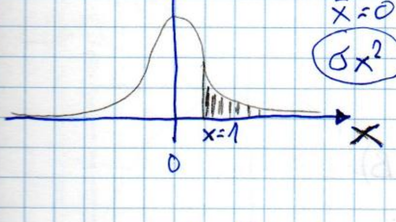
metanje kocke

$\int p(x)\,dx=1$ varianca → $\sigma_x^2=\overline{(x-\bar{x})^2}$

Srednja vrednost → $\bar{x}$

Srednja kvadratna ~~povpreč~~ vrednost → $\overline{x^2}$

(1) GAUSSOVA PORAZDELITEV (normalna porazdelitev)



Kaj je sploh to Gaussova porazdelitev?

$P(x>x_1)=\int_{x_1} p_x(x)\,dx$

$$p_x(x)=\frac{1}{\sqrt{2\pi\sigma_x^2}}\cdot e^{-\frac{x^2}{2\sigma_x^2}}$$

...ke šumijo. (vsaka drugače

neusklajeni pogovori (klepetanje sošolcev...)

INFORMACIJA

Dogodek $\leftrightarrow$ verjetnost dogodka

$S = \{ s_0, s_1, s_2 \dots s_{M-1} \} \equiv M$ znakov

I $\longrightarrow$

Dogodki :

$$\left.\begin{array}{l} S = s_0 \\ S = s_1 \\ S = s_2 \\ \vdots \\ S = s_{M-1} \end{array}\right\} M \text{ RAZLIČNIH DOGODKOV}$$

$P(S = s_0) = p_0$

$P(S = s_1) = p_1$

$\vdots$

$P(S = s_{M-1}) = p_{M-1}$

Bolj verjetni dogodki nosijo več informacije

$S = s_i \qquad P(S = s_i) = p_i$

$$\boxed{I(S = s_i) = \log \cdot \frac{1}{P(S = s_i)}}$$

$I(A) = \log_2 \left(\frac{1}{P(A)} \right)$ [bit] ✓✓

$\ln \left(\frac{1}{P(A)} \right)$ [nat]

M.P

NEODVISNI DOGODEK :

$A \;,\; B \qquad P(A \cap B) = P(A) \cdot P(B)$

$A \downarrow \quad B \downarrow$

$I(A) \qquad I(B) \Rightarrow I(A \cdot B) = I(A) + I(B)$

Imamo 4 dogodke

$I(S=s_0)$

$I(S=s_1)$ povprečje je?

$I(S=s_2)$ $\bar{I}=?$

$I(S=s_3)$

ENTROPIJA INFORMACIJSKEGA IZVORA Nam pove povp. informacijo na izhodu

$$\bar{I}=\sum I(S=s_i)\cdot P(S=s_i)$$

$$\downarrow$$

$$H=\bar{I}=\sum_i p_i\cdot I(S=s_i)=\sum_i p_i\cdot \log_2\frac{1}{p_i}=\boxed{-\sum_{i=0}^{M-1} p_i\log_2(p_i)}$$

ENTROPIJSKO KODIRANJE

1, 0

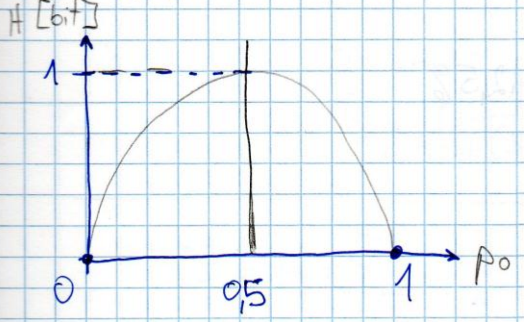
$s_0 \rightarrow$ "0"

$s_1 \rightarrow$ "1"

$P(S=s_0)=p_0$

$P(S=s_1)=p_1=1-p_0$

$$H=-\sum_{i=0}^{1} p_i\log_2(p_i)=-p_0\log_2(p_0)-(1-p_0)\log_2(1-p_0)$$



$$\boxed{\begin{array}{l} H_{max}=\log_2(M) \\ \text{nastopi pri} \\ P(S=s_i)=\frac{1}{M}\end{array}}$$

s_0	0
s_1	1

s_0	00
s_1	01
s_2	10
s_3	11

$L = 2$

s_0	000
s_1	001
⋮	⋮
s_7	111

Primer za 4 znake

$M = 4$

$H_{MAX} = 2$

$p_0 = \frac{1}{2}$

$p_1 = \frac{1}{4}$

$p_2 = \frac{1}{8}$

$p_3 = \frac{1}{8}$

$\Sigma\ 1$

$I(s = s_0) = \log_2(2) = 1$

$I(s = s_1) = \log_2(4) = 2$

$I(s = s_2) = \log_2(8) = 3$

$I(s = s_3) = \log_2(8) = 3$

$$H = p_0 \cdot I(s = s_0) + \ldots + p_3 \cdot I(s = s_3) =$$

$$= \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2 + \frac{1}{8} \cdot 3 + \frac{1}{8} \cdot 3 =$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{4} = \underline{1{,}75 \text{ bit/znak}}$$

$$\boxed{R = 1 - \frac{H}{H_{max}}}$$ Redudanca!

$$R = \frac{2 - 1{,}75}{2} = \frac{0{,}25}{2} = 12{,}5\%$$

PRIMER ENTROPIJSKEGA KODIRANJA:

HUFFMANOVA KODA

$S = S_0 \rightarrow p_0 = \frac{1}{8}$

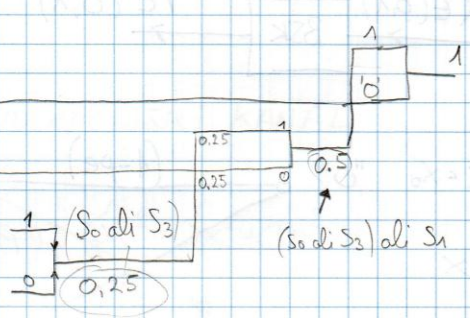
$S = S_1 \rightarrow p_1 = \frac{1}{4}$

$S = S_2 \rightarrow p_2 = \frac{1}{2}$

$S = S_3 \rightarrow p_3 = \frac{1}{8}$

Najbolj verjetna znak damo na vrh

1	0	S_2	$p_2 = 0.5$
2	10	S_1	$p_1 = 0.25$
3	111	S_0	$p_0 = 0.125$
3	110	S_3	$p_3 = 0.125$



$$\bar{L} = \sum p_i \cdot L_i = 0.125 \cdot 3 + 0.25 \cdot 2 + 0.5 \cdot 1 + 0.125 \cdot 3 = 1.75 \text{ bit/znak}$$

20.12.2010

$M = \text{št. simbolov}$

$S \in \{S_0, S_1, \ldots S_{M-1}\}$

$I \rightarrow$ IZVORNI KODIRNIK $\rightarrow$ 1.0,1,1,0,0 (kodirni niz)

$\tilde{L} \geq H$

$P(S = S_i) = p_i \quad I(S = S_0)$

$\vdots$

$I(S = S_{M-1})$

$$I = \log_2\left(\frac{1}{p(S = S_i)}\right)$$

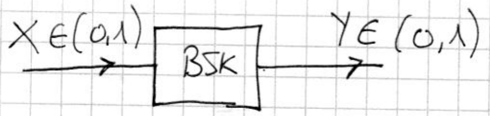
$$\bar{I} = H = \sum_{i=0}^{M-1} p_i \cdot \log \frac{1}{p_i}$$

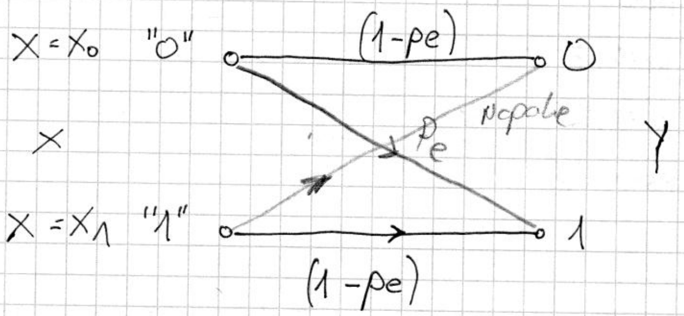
Kako prenašamo informacijo preko kodirnega kanala?

DISKRETNI INFORMACIJSKI KANAL

lastnost: verjetnost napake

1. Primer: BINARNI SIMETRIČNI KANAL





Povprečna informacija na vhodu:

$$H_x = P(x=0)\cdot \log_2 \frac{1}{P(x=0)} + p(x=1)\cdot \log_2 \frac{1}{p(x=1)}$$

$$P(x=1) = 1 - P(x=0)$$

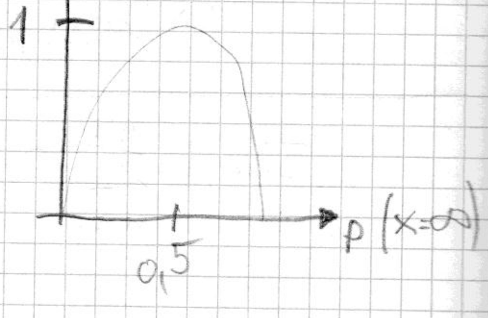
$$H_x \leq 1 \text{ bit/znak}$$

$$P(X=0, Y=0) = 1-pe$$

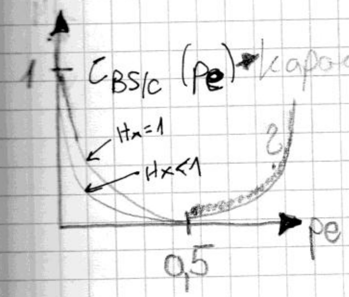
$$P(X=0, Y=1) = pe$$

$$P(X=1, Y=0) = pe$$

$$P(X=1, Y=1) = 1-pe$$



max vzajemno informacijo med izhodom in vhodom !!!



$$\boxed{C_{BS/C} = (1-p_e)\log_2\left(2(1-p_e)\right) + p_e \cdot \log_2(2p_e)}$$

2. Primer

GAUSSOV KOMUNIKACIJSKI KANAL

$$\sigma_y^2 = \sigma_x^2 + \sigma_n^2$$

C = max vzajemno informacijo med I/V kanala!

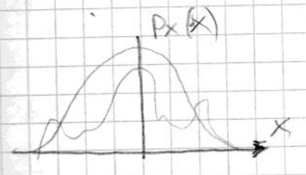
$$\boxed{C = \frac{1}{2}\log_2\left(1 + \frac{\sigma_x^2}{\sigma_n^2}\right)}$$

C → nam pove koliko informacije pošljemo brez izgube

Pogoj: zelo dolgo sporočilo

Gaussova porazdelitev:

pri dani moči ima max. porazdelitev entropije.



$$H_x = \int p_x(x)\log_2\left(\frac{1}{p_x(x)}\right)dx$$

$$\bar{x} = 0$$

$$\sigma_x^2 = \int x^2 p(x)\,dx = \text{povprečna moč}$$

$\max\left(\frac{H_X}{\sigma_x^2}\right) : p_x(x) = ?$ Gaussov porazdeliter !

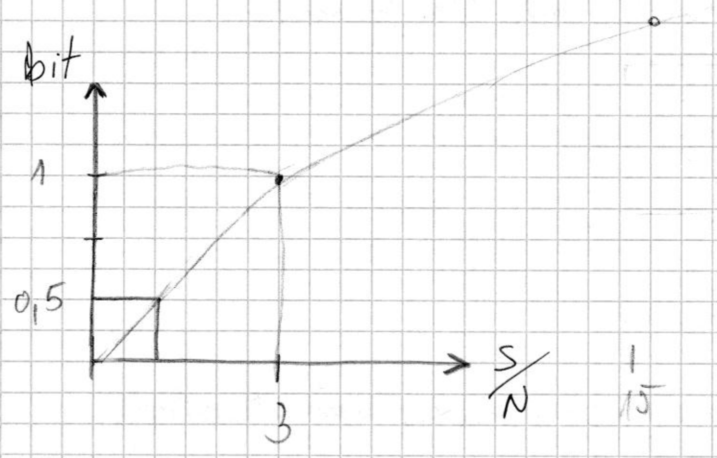
Moč SIGNALA $X : \sigma_x^2 \equiv S$

Moč ŠUMA $n : \sigma_n^2 \equiv N$

Moč SIGNALA $y : \sigma_y^2 = \sigma_x^2 + \sigma_n^2 = S + N$

$$C = \log_2 \sqrt{\frac{S+N}{N}} = \sqrt{1 + \frac{S}{N}}$$

$$C = \frac{1}{2}\log_2\left(1 + \frac{S}{N}\right) = \frac{1}{2}\log_2\left(1 + \frac{\sigma_x^2}{\sigma_n^2}\right)$$



Prehod iz 1 na 3 naslednic gre na 255.

Exponentno narošča. Gaussov šum je najhujši šum.

Uporabimo pasovno širino !!! zgoraj !

C [bit/znak]

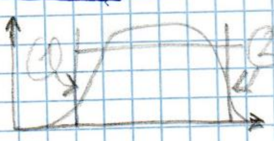
f_s [znak/s] SIMBOLNA FREKVENCA

$\boxed{r = b_s \cdot f_s}$ INFORMACIJSKI PRETOK

Nyquist

$f_{SMAX} = 2 \cdot B$ pasovna širina kanala

BANDWIDTH



1. Nizko sito

2. f. dvignjeni cos

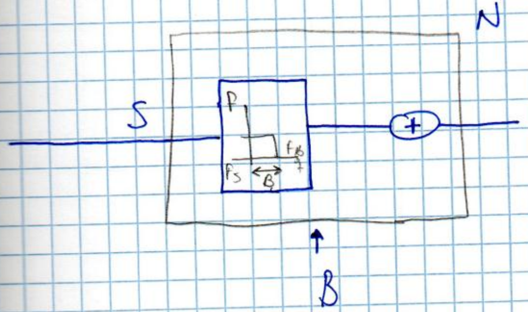
$\max(b_s) = C$

$\max(f_s) = 2 \cdot B$

$r_{max} \left[\frac{bit}{s}\right] = 2 \cdot B \cdot C$

$r_{max} = \cancel{2} B \frac{1}{\cancel{2}} \log_2\left(1 + \frac{S}{N}\right)$

Nam pove koliko bitov na sekundo lahko pošljemo čez gaussov šum.



$SNR = 26\,dB$

$\frac{S}{N} = 10^{\frac{26}{10}}$

Tel. kanal.

$B = 4kHz$

$SNR = 26\,dB$

$r = B \log_2\left(1 + \frac{S}{N}\right) = 8000 \cdot 4{,}3 = 34{,}400\ bit/s$