FIZIKA I

zapiski z avditornih vaj

Šolsko leto	2007 / 2008
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Avtor dokumenta	Blaž Potočnik
Sodelavci	Blaž Potočnik, Aljoša Praznik

UREJANJE DOKUMENTA

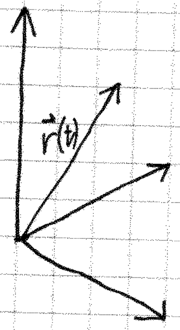
VERZIJA	01	**REVIZIJA**	01
DATUM	24. 2. 2009		
ZADNJI POPRAVLJAL	/		
PREGLEDAL	Blaž Potočnik, Aljoša Praznik		

OPOMBE

POPRAVKI

FIZIKA I - VAJE

2007 / 2008



$$\vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{v} = \int \vec{a}\, dt$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} \qquad \vec{r} = \int \vec{v}\, dt$$

1

- Točka sta telo se giblje premo, hitrost je podana z izrazom $v = v_0 + At + Bt^2$

$v_0 = 1\ m/s$
$A = 0{,}2\ m/s^2$
$B = 0{,}03\ m/s^3$

Kolikšna sta po $t = 10\ s$ v in a telesa, ter kolikšno pot opravi.

$$v = v_0 + At + Bt^2$$
$$= 1 m/s + 0{,}2\ m/s^2 \cdot 10\ s + 0{,}03\ m/s^3 \cdot 100\ s^2$$
$$= 1\ m/s + 2\ m/s + 3\ m/s = \underline{6\ m/s}$$

$$a = \frac{dv}{dt} = 0 + A + 2Bt = 0{,}2\ m/s^2 + 0{,}03\ m/s^{\cancel{3}2} \cdot 10\cancel{s} \cdot 2$$

$$= \underline{0{,}8\ m/s^2}$$

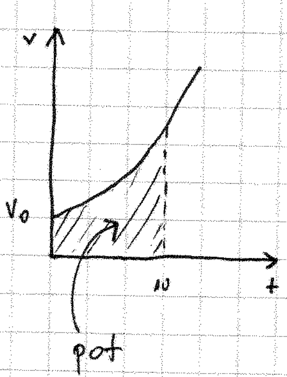
$$s = \int (v_0 + At + Bt^2)\, dt = v_0 \int dt + A\int t\, dt + B\int t^2 dt =$$

$$= v_0 t + \left(A \cdot \frac{t^2}{2}\right) + \left(B\ \frac{t^3}{3}\right) + C \qquad \boxed{\int x^n dx = \frac{x^{n+1}}{n+1}}$$

$$s(t=0) = 0 \rightarrow C = 0$$

$$= v_0 t + A\frac{t^2}{2} + B\frac{t^3}{3}$$

$$= 1 m/s \cdot 10 s + 0{,}2 \frac{m}{\cancel{s^2}} \cdot 10\cancel{s^2} + \frac{0{,}03}{\cancel{s^3}} \cdot 1000\cancel{s^3}\ m = 30\ m$$



določanje poti z integriranjem.

$$\int_0^T v\,dt = v_0 t + A\frac{t^2}{2} + B\frac{t^3}{3}\Bigg|_0^T =$$

$$= v_0 T + A\frac{T^2}{2} + B\frac{T^3}{3} - \left(0+0+0\right)$$

• Kamen spustimo v vodi, da začne padati proti dnu, Njegova hitrost se spreminja po enačbi.

$$v = v_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

$v_0 = 0{,}8\ m/s$
$\tau = 2\ s$

Kolikšna sta po $t = 3\,s$ hitrost in pospešek ter pot.

$$v = 0{,}8\,m/s\left(1 - e^{-1{,}5}\right)$$
$$v = \cancel{0{,}8\,m/s}\ 0{,}6\ m/s$$

$$a = \frac{dv}{dt} = \left(v_0 - v_0 e^{-t/\tau}\right)' = \qquad\qquad (f\cdot g)' = f'g + fg'$$

$$= v_0\left(0 - \left(-\frac{t}{\tau}\right)\cdot e^{-\frac{t}{\tau}}\right) = \frac{v_0 \cdot e^{-t/\tau}}{s\ 2s} = \quad 0{,}09\ m/s^2$$

$$s = \int v\,dt = \int v_0\left(1 - e^{-t/\tau}\right)dt = v_0\int\left(1 - e^{-t/\tau}\right)dt =$$

$$= v_0\cdot t - v_0\cdot e^{-t/\tau}\cdot(-\tau) + C = v_0 t + v_0\tau\cdot e^{-t/\tau} + C = \underline{1{,}16\ m}$$

$$S(t=0) = 0$$
$$v_0\cdot 0 + C + v_0\tau = 0 \quad\rightarrow\quad C = -v_0\tau$$

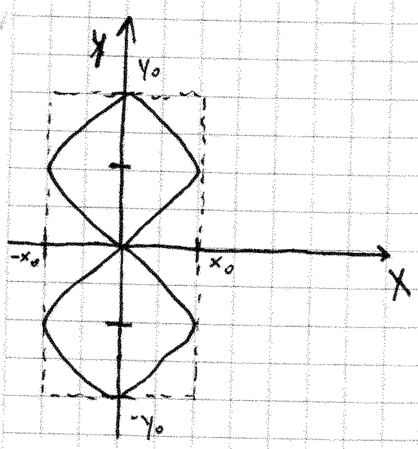
• Točkasto telo se giblje v ravnini xy. Njegovo lego kot f. časa podajata:

$x = x_0 \sin \omega t$
$y = y_0 \cos 2\omega t$

$x_0 = 2cm$
$y_0 = 3cm$

$\omega = 0,5\ s^{-1}$

Kolikšna sta 0,08 s po začetku gibanja v in a po začetku gibanja!



$V_x = \frac{dx}{dt} = x_0 \cdot \cos \omega t \cdot \omega$

$V_y = \frac{dy}{dt} = -2 y_0 \omega \sin 2\omega t$

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{x_0^2 \omega^2 \cos^2 \omega t + 4 y_0^2 \omega^2 \sin^2 2\omega t}$

$=$

kot, ki ga oklepa vektor hitrosti z absciso

$tg\,\alpha = V_y / V_x$

$a_x = \frac{dv_x}{dt} = -x_0 \cdot \sin \omega t \cdot \omega^2$

$a_y = \frac{dv_y}{dt} = -4 y_0 \omega^2 \cos 2\omega t$

$a = \sqrt{a_x^2 + a_y^2}$

Čoln pluje po jezeru s konst. hitrostjo 3m/s. Začne se zaustavljati. Pri tem je njegov trenutni pojemek sorazmeren njegovi trenutni hitrosti, da velja zveza $a = -k \cdot v$

$k = 2\ s^{-1}$

$v_0 = 3 m/s$.

Kolikšna je hitrost čolna 0,5 s po tem in kolikšno pot opravi v tem času.

$a = -k \cdot v$

$$a = \frac{dv}{dt} \rightarrow \boxed{\frac{dv}{dt} = -k \cdot v} \rightarrow v(t)$$

$$\frac{dv}{dt} = -kv \quad / dt$$

$$dv = -kv\,dt \quad /:v$$

$$\frac{dv}{v} = -k\,dt$$

$$\int \frac{dv}{v} = (-k) \int dt$$

$$\ln v = -k \cdot t \quad + C \quad / exp$$

$$V = e^{-kt + c}$$

$$V = e^{-kt} \cdot e^{c} \; K$$

$$V = e^{-kt} \cdot K$$

$$V = K e^{-kt}$$

$$V(t=0) = V_0 = K$$

$$V = V_0 \cdot e^{-kt}$$

$$V = (3\ m/s) \cdot e^{-1} = 1{,}1\ m/s$$

Pot

$$s = \int v\,dt$$

$$s = \int v_0 \cdot e^{-kt}\, dt = v_0 \int e^{-kt}\, dt = v_0 \cdot e^{-kt} \cdot \left(-\frac{1}{k}\right) + C$$

$$s(t=0) = 0 = -\frac{v_0}{k} + C \rightarrow C = v_0/k$$

$$s = \frac{v_0}{k}\left(1 - e^{-kt}\right) = \frac{3\,m\,s}{s\ 2}\left(1 - e^{-1}\right) = 1{,}1\,m$$

- Padalec skoči iz letala, ko doseže hitrost 80 m/s, odpre padalo. Pojemek je sorazmeren: $a = -kv^2$. $k = 0{,}001\, m^{-1}$. Hitrost po $t = 5\,s$ in kolikšno pot opravi.

- Dve točkasti telesi v začetku mirujeta v medsebojni oddaljenosti 2 m. V nekem trenutku eno telo spustimo, da se pod vplivom privlačnosti začne približevati prvemu, pri tem pa je pospešek drugega telesa obratno sorazmeren s kvadratom razdalje med telesoma

$a = -k/x^2$
$k = 0{,}5\, m^3/s^2$

Kolikšna je hitrost, ko je razdalja med telesoma en meter

$v = dx/dt$
$a = d^2x/dt^2$

$$\frac{d^2x}{dt^2} = \frac{-k}{x^2}$$

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} \quad \frac{1}{v}$$

$$v\frac{dv}{dx} = -\frac{k}{x^2} \Big/ dx$$

$$v\, dv = \frac{-k\, dx}{x^2}$$

$$\int v\, dv = (-k)\int \frac{dx}{x^2}$$

$$\frac{v^2}{2} = (-k)\left(-\frac{1}{x}\right) + C$$

$$\frac{v^2}{2} = \frac{k}{x} + c$$

$$v = \sqrt{2\left(\frac{k}{x}\right) + C}$$

$$v(x = r_1) = 0 = \sqrt{2\left(\frac{k}{r_1} + C\right)}$$

$$C = -k/r_1$$

$$V_x = \sqrt{2k\left(\frac{1}{x} - \frac{1}{r_1}\right)}$$

$$V(x = r_2) = \sqrt{2k\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} = \sqrt{\frac{2k(r_1 - r_2)}{r_1 r_2}}$$

Enakomerno pospešeno gibanje

$a =$ konst.

$$\boxed{\begin{aligned} \vec{V} &= \int \vec{a}\, dt = \vec{a}t + \vec{V_0} \\ \vec{r} &= \int \vec{v}\, dt = \frac{\vec{a}t^2}{2} + \vec{V_0}t + \vec{r_0} \end{aligned}}$$

Avto pelje po vodoravni cesti s konstantno hitrostjo $V_0 = 23\ m/s$. Voznik opazi 50 m oddaljeno steno. Zavirati začne z $2,5\ m/s^2$. S kolikšno hitrostjo trči v steno.

Zapišemo pot kot funkcijo časa.

$$s = V_0 t - \frac{at^2}{2}$$

$$\frac{at^2}{2} - V_0 t + s = 0$$

16.10.07

$$at^2 - 2v_0 t + 2s = 0$$

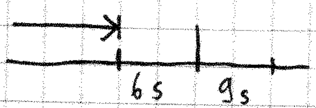
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_{1,2} = \frac{2v_0 \pm \sqrt{4v_0^2 - 8as}}{2a}$$

$$t_{1,2} = \frac{v_0 \pm \sqrt{v_0^2 - 2as}}{a}$$

$$\boxed{v = \sqrt{v_0^2 - 2as}}$$

Lokomotiva se približuje z v_0. V nekem trenutku začne zavirati s konstantnim pojemkom. Prvih 54m prevozi v 6s. Naslednjih pa v 9s. S kolikšnim pojemkom zavira in kolikšna je začetna hitrost.



$$s = v_1 \cdot t_1 - at_1^2/2$$
$$2s = v_1(t_1+t_2) - a(t_1+t_2)^2/2$$

$$s = v_1 t_1 - at_1^2/2 \longrightarrow a$$

$$s - v_1 t_1 = -at_1^2/2$$
$$2s - 2v_1 t_1 = -at_1^2$$
$$\frac{-2s + 2v_1 t_1}{t_1^2} = a$$

$$2s = v_1(t_1+t_2) - \left(\frac{2(v_1 t_1 - s)}{t_1^2}(t_1+t_2)^2\right)/2$$

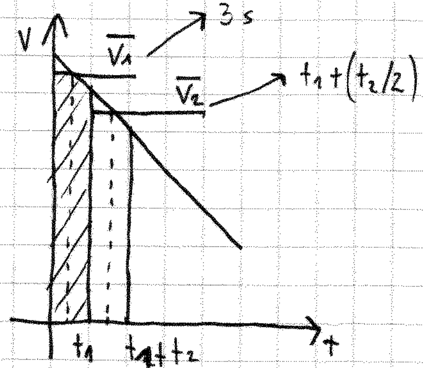
$$2st_1^2 = v_1(t_1+t_2)t_1^2 - 2(v_1 t_1 - s)(t_1+t_2)^2 \cancel{/2}$$
$$2st_1^2 = v_1(t_1+t_2)t_1^2 - 2v_1 t_1(t_1+t_2)^2 + 2s(t_1+t_2)^2 \cancel{/2}$$
$$2st_1^2 - s(t_1+t_2)^2 = v_1((t_1+t_2)t_1^2 - 2t_1(t_1+t_2)^2)$$

$$\frac{s(2t_1^2 - (t_1+t_2)^2)}{t_1(t_1+t_2)(-t_2)} = v_1$$

$$\boxed{v_1 = \frac{s(t_1^2 + 2t_1t_2 - t_2^2)}{t_1 t_2 (t_1+t_2)}}$$

$$a = 2\,\frac{s\left(\frac{t_2^2 + 2t_1t_2 - t_1^2}{t_2(t_1+t_2)}\right) - 1}{t_1^2} = \boxed{\frac{2s(t_2-t_1)}{t_1t_2(t_1+t_2)}}$$

2



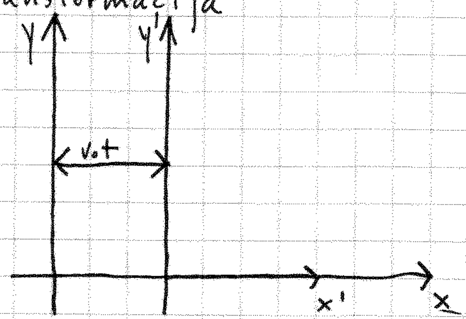
$$s = t_1 \cdot \bar{v}_1 \rightarrow \bar{v}_1 = s/t_1$$
$$s = t_2 \cdot \bar{v}_2 \rightarrow \bar{v}_2 = s/t_2$$

$$a = \frac{\bar{v}_1 - \bar{v}_2}{t_1 + \frac{t_2}{2} - \frac{t_1}{2}} = \frac{\bar{v}_1 - \bar{v}_2}{\frac{t_1+t_2}{2}} = \frac{2s\left(\frac{1}{t_1} - \frac{1}{t_2}\right)}{t_1+t_2} = \frac{2s(t_2-t_1)}{t_1t_2(t_1+t_2)}$$

$$v_1 = \bar{v}_1 + a\frac{t_1}{2} = \frac{s}{t_1} + \frac{s(t_2-t_1)}{t_2(t_1+t_2)} = s\,\frac{t_2(t_1+t_2)+t_1(t_2-t_1)}{t_1t_2(t_1+t_2)} =$$

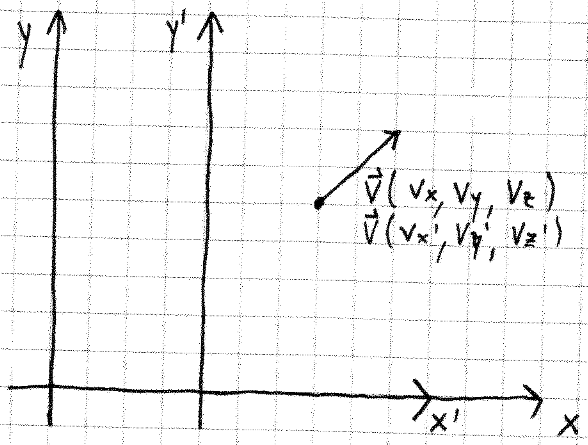
$$= \frac{s(t_1t_2 + t_2^2 + t_1t_2 - t_1^2)}{t_1t_2(t_1+t_2)} = \boxed{\frac{s(t_2^2 + 2t_1t_2 - t_1^2)}{t_1t_2(t_1+t_2)}} = v_1$$

transformacija



(x, y, z)
(x', y', z')

$$x = x' + v_0t \qquad x' = x - v_0t$$
$$y = y' \qquad y' = y$$
$$z = z' \qquad z' = z$$

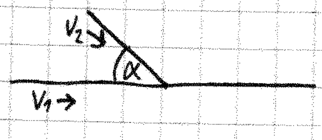


$V_x = V_x' + V_0$
$V_y = V_y'$
$V_z = V_z'$

$V_x' = V_x - V_0$
$V_y' = V_y$
$V_z' = V_z$

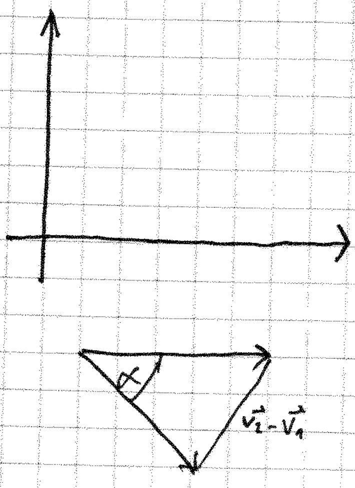
Dve cesti se sekata pod $60°$. Po teh dveh cestah se približujeta 2 avtomobila:

$v_1 = 25\,m/s$
$v_2 = 20\,m/s$



hitrost drugega glede na prvi avtomobil

$\vec{V_{21}} = \vec{V_2} - \vec{V_1}$



$\vec{V_1} = (v_1, 0)$
$\vec{V_2} = (V_2 \cos\alpha, V_2 \sin\alpha)$
$\vec{V_{21}} = (V_2 \cos\alpha - V_1, V_2 \sin\alpha)$

$$V_{21} = \sqrt{(V_2 \cos\alpha - V_1)^2 + (V_2 \sin\alpha)^2}$$

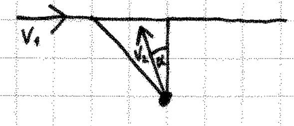
$$V_{21} = \sqrt{V_2^2 \cos^2\alpha - 2V_1V_2\cos\alpha + V_1^2 + V_2^2 \sin^2\alpha}$$

$$V_{21} = \sqrt{V_2^2(\cancel{\cos^2\alpha + \sin^2\alpha}) - 2V_1V_2\cos\alpha + V_1^2}$$

$$V_{21} = \sqrt{V_1^2 - 2V_1V_2\cos\alpha + V_2^2}$$

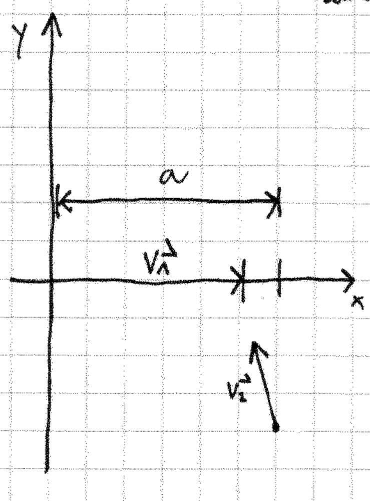
$V_{21} \cong 23\,m/s$

Jezdec jezdi s 5 m/s po vodoravni cesti. Pes miruje 50 m od te steze in zagleda konjenika, ko je ta 200 m oddaljen od njega. Pes steče z 8 m/s, v kateri smeri mora teči, da se srečata.



$$c^2 = a^2 + b^2$$

$$a = \sqrt{c^2 - b^2}$$



$$\vec{v_1} = (v_1, 0)$$

$$\vec{v_2} = (-v_2 \sin\alpha, v_2 \cos\alpha)$$

$$b = v_2 \cdot \cos\alpha \cdot t$$

$$\sqrt{c^2 - b^2} = v_1 \cdot t + v_2 \cdot \sin\alpha \cdot t$$

$$\sqrt{c^2 - b^2} = t(v_1 + v_2 \sin\alpha)$$

$$t = \frac{b}{\cos\alpha \cdot v_2}$$

$$\sqrt{c^2 - b^2} = (v_1 + v_2 \sin\alpha)\left(\frac{b}{\cos\alpha \cdot v_2}\right) \Big/ v_2 \cos\alpha$$

$$\sqrt{c^2 - b^2}\, v_2 \cos x = (v_1 + v_2 \sin\alpha)\, b$$

$$\sqrt{c^2 - b^2} \cdot v_2 \cdot \sqrt{1 - \sin^2\alpha} = b v_1 + b v_2 \sin\alpha$$

$$(c^2 - b^2)\, v_2^2\, (1 - \sin^2\alpha) = b^2 v_1^2 + 2 b^2 v_1 v_2 \sin\alpha + \cancel{b^2 v_2^2 \sin\alpha}\ b^2 v_2^2 \sin^2\alpha$$

$$(c^2 - b^2) v_2^2 - (c^2 - b^2) v_2^2 \sin^2\alpha = b^2 v_1^2 + 2 b^2 v_1 v_2 \sin\alpha + b^2 v_2^2 \sin^2\alpha$$

$$\boxed{\sin^2\alpha\, v_2^2 c^2 + 2 b^2 v_1 v_2 \sin\alpha + b^2 v_1^2 - (c^2 - b^2) v_2^2 = 0}$$

$$64 \cdot 200^2 \sin^2\alpha + 2 \cdot 50^2 \cdot 5 \cdot 8 \cdot \sin\alpha + 50^2 \cdot 5^2 - (200^2 - 50^2) \cdot 64 = 0$$

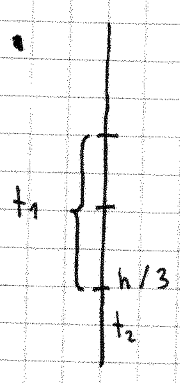
$$25600\cancel{00} \sin^2\alpha + 2000\cancel{00} \sin\alpha - 233750\cancel{0} = 0$$

$$\sin\alpha_{1,2} = \frac{-200000 \pm \sqrt{200000^2 + 4 \cdot 2560000 \cdot 2337500}}{2 \cdot 2560000}$$

66,5°   -81°

$g = 9{,}81\ m/s^2$

Kamen prosto pade. Zadnjo tretjino poti opravi v 0,8 s. Iz kolikšne višine smo spustili kamen



$\cancel{x = (x_0 + t, 0, 0)}$

$$h = g(t_1 + t_2)^2 / 2$$

$$\frac{h}{3} = g t_2^2 / 2$$

$\cancel{h = \frac{3}{2} g t_2^2 / 4}$

$$\frac{2h}{3} = \frac{g t_1^2}{2} \qquad t_1 = \sqrt{\frac{4h}{3g}}$$

$$h = \frac{g\left(\sqrt{\frac{4h}{3g}} + t_2\right)^2}{2} = \frac{g\left(\frac{4h}{3g} + 2 t_2 \sqrt{\frac{4h}{3g}} + t_2^2\right)}{2}$$

$$\frac{2h}{3} - g t_2^2 = 2 g t_2 \sqrt{\frac{4h}{3g}} \quad /^2$$

$$\frac{4h^2}{9} - \frac{g t_2^2 4h}{3} + g^2 t_2^4 = 4 g^{\cancel{2}} t_2^2 \frac{4h}{3\cancel{g}}$$

$$\boxed{4h^2 - 6 g t_2^2 h + 9 g^2 t_2^4 = 0}$$

$h_{1,2}$ ...

Prvi kamen vržemo v smeri navpično navzgor z $v_1 = 25 m/s$. Po $t = 1s$ vržemo še en kamen ~~z~~ $v_2 = 30 m/s$.

Na kolikšni višini trčita.

$\uparrow \vec{v_1}$

$$h = v_1 t - g t^2/2$$

$\uparrow \vec{v_2}$

$$h = v_2 (t - t_0) - g (t - t_0)^2/2$$

$$h = h$$

$$v_1 t - g t^2/2 = v_2 (t - t_0) - g (t - t_0)^2 /2$$

$$2 v_1 t - g t^2 = 2 v_2 (t - t_0) - g (t - t_0)^2$$

$$(t^2 - 2 t t_0 + t_0^2)$$

$$2 v_1 t = 2 v_2 t - 2 v_2 t_0 + g 2 t t_0 - g t_0^2$$

$$2 v_1 t - 2 v_2 t - 2 g t t_0 = -2 v_2 t_0 - g t_0^2$$

$$2 t (v_1 - v_2 - g t_0) = t_0 (-2 v_2 - g t_0)$$

$$2 t (v_2 + g t_0 - v_1) = t_0 (2 v_2 + g t_0)$$

$$t = \frac{t_0 (2 v_2 + g t_0)}{2 (v_2 + g t_0 - v_1)}$$

$$h = \frac{v_1 \frac{t_0 (2 v_2 + g t_0)}{2 (v_2 + g t_0 - v_1)}}{1} - \frac{g \left(\frac{t_0 (2 v_2 + g t_0)}{2 (v_2 + g t_0 - v_1)} \right)^2}{2}$$

Kamen vržemo z $v = 25 m/s$ v smeri navpično navzgor. Koliko časa porabi kamen od višine 9 do višine 12 m

$$v = v_0 - g t_0$$

$h_1 = 9 m$
$h_2 = 12 m$

$$h_2 = v t_2 - g t_2^2/2$$
$$h_1 = v t_1 - g t_1^2/2$$

$h_2 - v t_2 + g t_2^2/2 = 0$

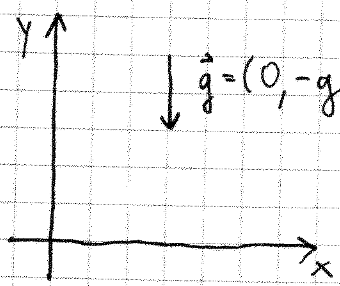
$$t_{2_{1,2}} = \frac{v \pm \sqrt{v^2 - 2gh_2}}{g}$$

$$t_{1_{1,2}} = \frac{v \pm \sqrt{v^2 - 2gh_1}}{g}$$

$$t_2 - t_1 = \frac{v - \sqrt{v^2 - 2gh_2} - v + \sqrt{v^2 - 2hg}}{g} =$$

$$= \boxed{\frac{\sqrt{v^2 - 2gh_1} - \sqrt{v^2 - 2gh_2}}{g}}$$

Poševni met

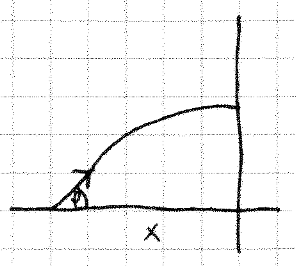


$v_x = v_{0x}$ $\quad (v_0 \cos\varphi)$
$v_y = v_{y0} - gt$ $\quad (v_0 \cdot \sin\varphi)$

$x = v_{x0} \cdot t + x_0$
$y = v_{y0} \cdot t - gt^2/2 + y_0$

kamen 20 m/s $\varphi = 40°$ glede na vodoravnico
proti 8 m oddaljeni steni

- višina, ko zadene steno
- kot glede na vod, da bi zadel steno na največji višini
- max višina



$$x = V_0 t \cos\varphi$$
$$y = V_0 t \sin\varphi - g t^2/2$$

$$\cancel{2x = 2 V_0 t \cos\varphi - g t^2}$$
$$\cancel{g t^2 - 2 V_0 t \cos\varphi + 2x} = 0$$

$$t = \frac{x}{V_0 \cos\varphi} \qquad y = V_0 \frac{x}{V_0 \cos\varphi} \cdot \sin\varphi - \frac{g}{2} \cdot \frac{x^2}{V_0^2 \cos^2\varphi}$$

$$y = \frac{x}{\cos\varphi}\left(\sin\varphi - \frac{gx}{2v_0^2 \cos\varphi}\right)$$

$$\frac{dy}{d\varphi} = 0$$

$$y = x\left(tg\varphi - \frac{gx}{2v_0^2 \cos^2\varphi}\right)$$

$$0 = \frac{1}{\cos^2\varphi} - \frac{gx}{2v_0^2} \cdot (-2) \cos^{-3}\varphi(-\sin\varphi)$$

$$0 = \frac{1}{\cos^2\varphi} - \frac{gx}{V_0^2} \cdot tg\varphi \cdot \frac{1}{\cos^2\varphi} \qquad / \cos^2\varphi$$

$$0 = 1 - \frac{gx}{V_0^2} \cdot tg\varphi$$

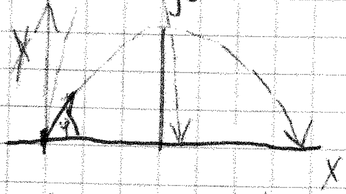
$$\boxed{tg\varphi = V_0^2/gx}$$ najvišji kot

h max : $y = x\left(\frac{V_0^2}{gx} - \frac{gx}{2v_0^2}\left(1 + \frac{V_0^4}{g^2 x^2}\right)\right)$

$y =$

23. 10. 07

Grajsko obzidje je visoko 30 m. Zaradi obrambnega jarka se ne da približati na manj kot 25 m. Streljajo puščice z $v = 80\,m/s$. Določite območje na drugi strani, ki ga lahko zadenejo.



$x = v_0 \cos\varphi t$

$y = v_0 \sin\varphi t - gt^2/2$

① $(25\,m, 30\,m)$

$x = v_0 \cos\varphi t$

$t = x / v_0 \cos\varphi$

$y = v_0 \sin\varphi t - gt^2/2$

$y = v_0 \sin\varphi \cancel{t}\, \frac{x}{v_0 \cos\varphi} - \frac{g\,x^2}{2v_0^2 \cos^2\varphi}$

$y = \frac{x \cdot \sin\varphi}{\cos\varphi} - \frac{g x^2}{2v_0^2 \cos^2\varphi} \quad / : x \cdot \cos^2\varphi$

$\sin\varphi\cos\varphi - \frac{g x}{2v_0^2} = \frac{h}{x} \cos^2\varphi$

$A = \frac{gx}{2v_0^2} = 0{,}02$

$B = h/x = 1{,}2$

$\sin\varphi\cos\varphi - A = B\cos^2\varphi$

$\cos\varphi \sqrt{1 - \cos^2\varphi} = B\cos^2\varphi + A$

$\cos^2\varphi - \cos^4\varphi = B^2\cos^4\varphi + 2AB\cos^2\varphi + A^2$ $\qquad c = \cos^2\varphi$

$c - c^2 = B^2 c^2 + 2ABc + A^2$

$c^2(B^2+1) + c(2AB-1) + A^2 = 0$

$2{,}44\, c^2 - 0{,}952\, c + 0{,}0004 = 0$

$c_1 = 4{,}2 \cdot 10^{-4}$ $\qquad c_2 = 0{,}39$

$\cos\varphi_1 = 0{,}02 \rightarrow 88{,}8^\circ$ $\qquad \cos\varphi_2 = 0{,}62 \rightarrow 51{,}4^\circ$

Akta

$\varphi_1 = 51{,}4^0$
$\varphi_2 = 88{,}8^0$

$y = V_0 \sin\varphi\, t - gt^2/2 = 0$

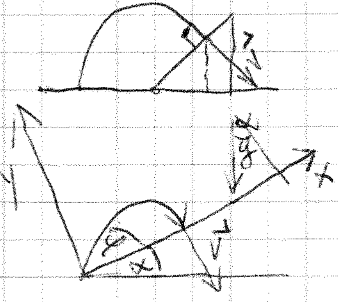
$t = \frac{2v_0}{g} \cdot \sin\varphi$

$t_1 = \frac{2v_0}{g} \sin\varphi_1 \Rightarrow x_1 = V_0 \cos\varphi_1 t_1 = \frac{2v_0^2}{g} \cancel{tg\alpha} \sin\varphi_1 \cos\varphi_1$

$x_2 = \frac{2v_0^2}{g} \cancel{tg\alpha} \sin\varphi_1 \cos\varphi_1$

območja: $x_1 - 25\,m$
$x_2 - 25\,m$

- Klanec je nagnjen za $10°$ navzgor glede na vodoravnico. Pod kolikšnim kotom glede na vodoravnico je treba vreči kamen, da bo zadel strmino pod pravim kotom



$\alpha = 10°$

$V = (V_0 \cos\varphi,\ V_0 \sin\varphi)$

$g_x = -g \sin\alpha$
$g_y = -g \cos\alpha$

$V_x = -gt\sin\alpha + V_0\cos\varphi$

$V_y = -gt\cos\alpha + V_0 \sin\varphi$

V_x mora biti 0

$0 = -gt\sin\alpha + V_0 \cos\varphi \rightarrow t = V_0\cos\alpha / g\sin\alpha$

$x = \frac{-gt^2}{2} \sin\alpha + v_0 t \cos\alpha$

$y = \frac{-gt^2}{2} \cos\alpha + v_0 t \sin\alpha$ y je v tem sistemu 0

$\to 0 \Rightarrow t = 2v_0 \sin\varphi / g\cos\alpha$

$$\frac{2\cancel{v_0}\sin\varphi}{\cancel{g}\cos\alpha} = \frac{\cancel{v_0}\cos\varphi}{\cancel{g}\sin\alpha}$$

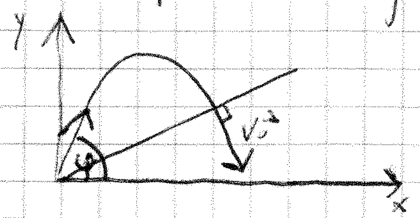
$2\,tg\,\varphi = ctg\,\alpha$

$tg\,\varphi = 1/2\,ctg\,10°$

$\varphi = 70{,}6°$

glede na vodoravnico $80{,}6°$

- še po običajnem k. sistemu (ne glej)



$V_x = V_0 \cos\varphi$

$V_y = V_0 \sin\varphi - gt$

$-tg\,\alpha = V_x / V_y$ (ker se kamen obrne)

$$-tg\,\alpha = \frac{\cancel{V_0}\cos\varphi}{\cancel{V_0}\sin\varphi - gt}$$

$-tg\,\alpha =$

$-tg\,\alpha\, V_0 \sin\varphi + g t\, tg\,\alpha = V_0 \cos\varphi$

$$t = \frac{tg\,\alpha\, V_0 \sin\varphi + V_0\cos\varphi}{g\,tg\,\alpha}$$

$$t = \frac{V_0 \cos\alpha + V_0 \sin\varphi\, tg\,\alpha}{g\,tg\,\alpha}$$

$x = V_0 t \cos\varphi$

$y = V_0 t \sin\varphi - gt^2/2$

$$\operatorname{tg}\alpha = \frac{y}{x} = \frac{V_0 t \sin\varphi - \frac{gt^2}{2}}{V_0 t \cos\varphi} = \operatorname{tg}\alpha$$

$$V_0 \sin\varphi - \frac{gt}{2} = \operatorname{tg}\alpha\, V_0 \cos\varphi$$

$$t = \frac{-2\operatorname{tg}\alpha\, V_0 \cos\varphi + 2 v_0 \sin\varphi}{g}$$

$$t = \frac{2 V_0}{g}\left(\sin\varphi - \operatorname{tg}\alpha \cos\varphi\right)$$

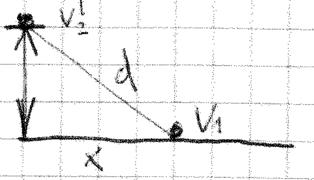
$$\frac{\cos\varphi + \operatorname{tg}\alpha \sin\varphi}{\operatorname{tg}\alpha} = \frac{2}{\cancel{g}}\left(\sin\varphi - \operatorname{tg}\alpha\cos\varphi\right)^2$$

$$\cos\varphi + \operatorname{tg}\alpha \sin\varphi = \operatorname{tg}\alpha \sin\varphi - 2\operatorname{tg}^2\alpha \cancel{\cos\varphi}$$

$$\cos\varphi\,(1 + 2\operatorname{tg}^2\alpha) = \sin\varphi\, \operatorname{tg}\alpha$$

$$\boxed{\operatorname{tg}\varphi = \frac{1 + 2\operatorname{tg}^2\alpha}{\operatorname{tg}\alpha}}$$

Lokomotiva vozi s hitrostjo 20 m/s. Bombnik leti z 80 m/s na 50 m, kolikšna mora biti razdalja med lokomotivo in bombnikom, da jo bo zadel



$d = \sqrt{h^2 + x^2}$

$x = (v_2 - v_1) \cdot t$

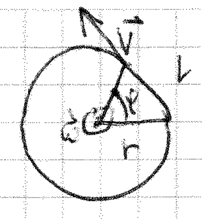
$s = a t^2/2$

$t^2 = 2h/g$

$t = \sqrt{2h/g}$

$X = (v_2 - v_1) \cdot \sqrt{2h/g}$

$d = \sqrt{h^2 + (v_2 - v_1)^2 \frac{2h}{g}}$



Kroženje

$\varphi = l / r$

$\omega = d\varphi / dt$ [rad/s] [s^{-1}]

$\alpha = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt}$ [rad/s^2] [s^{-2}]

$\vec{v} = \vec{\omega} \times r$ $\quad$ ($\sin\varphi = 1$)

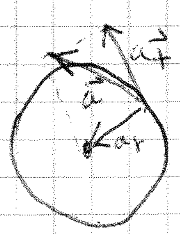
$v = \omega r$

$\vec{a_r} = \vec{\omega} \times \vec{v} = \vec{\omega} \times (\vec{\omega} \times \vec{r})$

$a_r = \omega^2 r = v^2 / r$

$\vec{a_t} = \vec{\alpha} \times \vec{r}$

$a_t = \alpha \cdot r$



$\vec{a} = \vec{a_r} + \vec{a_t}$

$a = \sqrt{a_r^2 + a_t^2}$

$\omega = \alpha t + \omega_0$

$\varphi = \frac{\alpha t^2}{2} + \omega_0 t + \varphi_0$

Vrtiljak se vrti z $\omega_0 = 6\ s^{-1}$, začne zavirati in se popolnoma zaustavi po 15 vrtljajih. Po začetku zaviranja, koliko časa porabi za prvih pet vrtljajev.

$\omega_0 = 6\ s^{-1}$
$\varphi_1 = 30\pi$
$\varphi_2 = 10\pi$

$$\varphi_1 = \omega_0 t_1 - \frac{\alpha t_1^2}{2}$$

$$\varphi_2 = \omega_0 t_2 - \frac{\alpha t_2^2}{2}$$

$$\omega_0 - \alpha \cdot t_1 = 0$$
$$\omega_0 = \alpha t_1$$
$$t_1 = \frac{\omega_0}{\alpha}$$

$$\varphi_1 = \frac{\omega_0^2}{\alpha} - \frac{\alpha\, \omega_0^2}{2\alpha^2}$$

$$0 = \frac{\omega_0^2}{\alpha} - \frac{\omega_0^2}{2\alpha} - \varphi_1$$

$$0 = 2\omega_0^2 - \omega_0^2 - 2\alpha\varphi_1$$

$$0 = \omega_0^2 - 2\alpha\varphi_1$$
$$\alpha = \frac{\omega_0^2}{2\varphi_1}$$

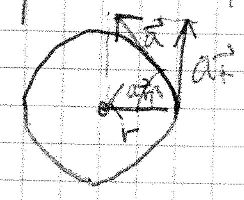
$$\varphi_2 = \omega_0 t_2 - \frac{\alpha t_2^2}{2}$$

$$\varphi_2 = \omega_0 t_2 - \frac{\omega_0^2 t_2^2}{4\varphi_1}$$

$$0 = \frac{\omega_0^2 t_2^2}{4\varphi_1} - \omega_0 t_2 + \varphi_2$$

t_1, t_2 ...

- Točkasto telo začne kroži ti $r = 20\,cm$ $a_t = 5\,cm/s^2$. Kolikšen je kot med vektorjem pospeška telesa in polmerom po $t = 2\,s$.



$$tg\,\beta = \frac{a_t}{a_r}$$

$$a_r = \omega^2 r = v^2/r$$

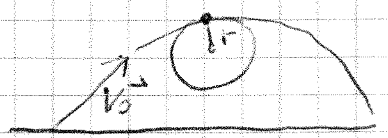
$$v^2 = (a_t \cdot t)^2$$

$$a_r = a_t^2\, t^2 \,/ r$$
$$a_r = \omega^2 r$$

$$tg\,\beta = \frac{a_t \cdot r}{a_t^2 \cdot t^2}$$
$$tg\,\beta = \frac{r}{a_t \cdot t^2}$$

Kamen vržemo z $v_0 = 25\,m/s$ $\varphi = 65^\circ$ glede na vodoravnico.

Kolikšni so po $t = 2s$ a_r, a_t, r.



$$V_x = V_0 \cos\varphi$$
$$V_y = V_0 \sin\varphi - g \cdot t$$
$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{V_0^2 \cos^2\varphi + (V_0 \sin\varphi - gt)^2} =$$

$$= \sqrt{V_0^2 - 2v_0 g t \sin\varphi + g^2 t^2}$$

$$a_t = dv/dt = \frac{1}{2}\left(V_0^2 - 2V_0 g t \sin\varphi + g^2 t^2\right)^{-1/2}\left(2g t - 2v_0 g \sin\varphi\right) =$$

$$= \frac{g(gt - V_0 \sin\varphi)}{\sqrt{V_0^2 - 2v_0 g t \sin\varphi + g^2 t^2}}$$

6.11.07

$a_r = \sqrt{a^2 - a_t^2}$ $\qquad$ $a = g$!

- Sani mirujejo

$m = 6\,kg$
$k_{tr} = 0{,}1$

V nekem trenutku začnemo vleči s silo $F = 8\,N$.
Hitrost po poti $s = 5\,m$

$v = ?$

$$F - F_{tr} = m \cdot a$$

$$F - m \cdot g \cdot k = m \cdot a$$

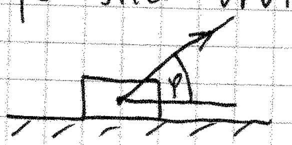
$$a = \frac{F - mgk}{m}$$

$$s = at^2/2$$

$$t = \sqrt{2s/a} \qquad v = a \cdot t = a\sqrt{2s/a}$$

$$v = \sqrt{\frac{2s(F - mgk)}{m}}$$

- Klado z maso 5kg vlečemo po vodoravni podlagi s konstantno hitrostjo. Koeficient trenja je 0,3. Pri kolikšnem kotu med klado in podlago je sila vrvice najmanjša, kolikšna je sila



$$\Sigma F = 0$$

$$\vec{F_v} - \vec{F_{tr}} = 0$$
$$\vec{F_v} \cos\varphi - F_{tr} = 0$$
$$F_v\cos\varphi - k(mg - F\sin\varphi) = 0 \qquad !$$

$$F_v\cos\varphi = kmg - F_v \cdot k \cdot \sin\varphi$$

$$F_v\cos\varphi + F_v k \sin\varphi = kmg$$
$$F_v(\cos\varphi + k\sin\varphi) = kmg$$

$$F = \frac{kmg}{\cos\varphi + k\sin\varphi}$$

$$\frac{dF}{d\varphi} = 0$$

$$F = mgk(\cos\varphi + k\sin\varphi)^{-1}$$

$$\frac{dF}{d\varphi} = (-1)(\cos\varphi + k\sin\varphi)^{-2}(-\sin\varphi + k\cdot\cos\varphi)$$

$$\frac{dF}{d\varphi} = \frac{\sin\varphi - k\cos\varphi}{(\cos\varphi + k\sin\varphi)^2} = 0$$

$$\sin\varphi - k\cos\varphi = 0$$

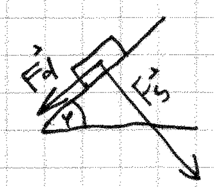
$\boxed{k = tg\varphi}$ ← kot!

$$\cos\varphi = \frac{1}{\sqrt{1+tg\varphi}} \qquad \sin\varphi = \sqrt{1-\cos^2\varphi} =$$

Zaledeneli klanec ima nagib 10°. Sani sunemo v smeri proti vrhu klanca. Sani se nekaj časa gibljejo navzgor, se v neki točki ustavijo in se vrnejo.

Za pot navzgor porabijo 2x krajši čas kot za pot navzdol. Kolikšen je k_{tr} med sanmi in klancem

$\varphi = 10°$
$t_d = 2t_g$
$k = ?$



$F_d = F_g \sin\varphi$
$F_s = F_g \cos\varphi$

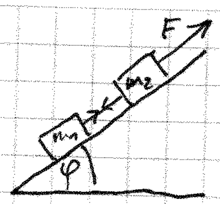
$k = 3/5\ tg\varphi$

Dve kladi z $m_1 = 3kg$ in $m_2 = 5kg$ sta povezani. Vlečemo ju navzgor po klancu z nagibom 20° in silo $F = 70N$

$k_{tr1} = 0{,}4 \qquad k_{tr2} = 0{,}3$

S kolikšnim pospeškom se gibljeta kladi in kolikšna sila napenja vrvico med kladama

$m_1 = 2\,kg$
$m_2 = 3\,kg$
$\varphi = 20°$
$F = 70\,N$
$k_1 = 0{,}4$
$k_2 = 0{,}3$



$$F - F_v - m_2 g \sin\varphi - m_2 g\, k_2 \cos\varphi = m_2 a$$
$$F_v - m_1 g \sin\varphi - m_1 g k_1 \cos\varphi = m_1 a$$

$$F - \cancel{F_v} - m_2 g \sin\varphi - m_2 g k_2 \cos\varphi + \cancel{F_v} - m_1 g \sin\varphi - m_1 g k_1 \cos\varphi = m_2 a + m_1 a$$
$$F - (m_2 g \sin\varphi + m_2 g k_2 \cos\varphi + m_1 g \sin\varphi + m_1 g k_1 \cos\varphi = a\,(m_1 + m_2)$$
$$F - g[\sin\varphi (m_1 + m_2) + \cos\varphi (m_1 k_1 + m_2 k_2)] = a\,(m_1 + m_2)$$
$$a = \frac{F - g[\sin\varphi(m_1 + m_2) + \cos\varphi\,(m_1 k_1 + m_2 k_2)]}{(m_1 + m_2)}$$

$$F - F_v - m_2 g \sin\varphi - m_2 g k_2 \cos\varphi = m_2 a$$
$$F_v - m_1 g \sin\varphi - m_1 g k_1 \cos\varphi = m_1 a \quad / \frac{m_2}{m_1}$$

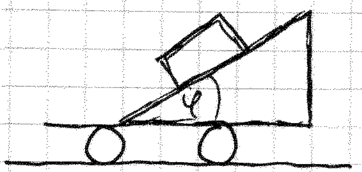
$$\begin{cases} F_v \frac{m_2}{m_1} - m_2 g \sin\varphi - m_2 g k_1 \cos\varphi = m_2 a \\ F_v \frac{m_2}{m_1} - m_2 g \sin\varphi - m_2 g k_1 \cos\varphi = F - F_v - m_2 g \sin\varphi - m_2 g k_2 \cos\varphi \end{cases}$$

$$F_v \frac{m_2}{m_1} + F_v = F - m_2 g k_2 \cos\varphi + m_2 g k_1 \cos\varphi$$

$$F_v\,(m_2/m_1 + 1) = F - m_2 g \cos\varphi\,(k_2 - k_1)$$

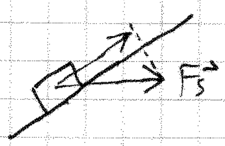
$$F_v = [F - m_2 g \cos\varphi\,(k_2 - k_1)] \,/\, [m_2/m_1 + 1]$$

Tovornjak vozi s konst. pospeškom. V prostoru za tovor ima dve kladi (slika). Koeficient lepenja 0,1, nagib večje klade je $15°$



a_{max} in a_{min} pri kateren manjša klada še miruje na večji kladi.

$g\cdot(\sin\varphi - k\cdot\cos\varphi)$ max:



statična dinamična lepenje

$$0 = m\cdot a\cos\varphi - mg\sin\varphi - k(ma\sin\varphi + mg\cos\varphi)$$

$$a\cos\varphi - g\sin\varphi - ka\sin\varphi - kg\cos\varphi = 0$$

$$a\cos\varphi - ka\sin\varphi = kg\cos\varphi + g\sin\varphi$$

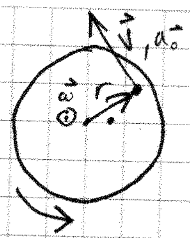
$$a_{max} = \frac{g(\sin\varphi + k\cos\varphi)}{\cos\varphi - k\sin\varphi}$$

min:

$$mg\sin\varphi - ma_{min}\cos\varphi - k(ma_{min}\sin\varphi + mg\cos\varphi) = 0$$

$$g\sin\varphi - a\cos\varphi - ka\sin\varphi - kg\cos\varphi = 0$$

$$a = \frac{g(\sin\varphi - k\cos\varphi)}{\cos\varphi + k\sin\varphi}$$

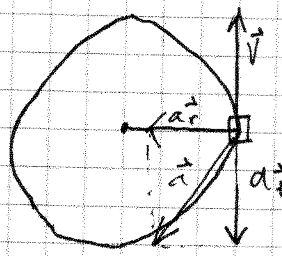


$$\vec{a} = \vec{a_0} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \vec{\alpha} \times \vec{r} + 2\,\vec{\omega} \times \vec{v}$$

njegov radialni

Tovornjak vozi z 8m/s po ovinku s krivinskim polmerom 40 m. Nenadoma začne zavirati konstantno. Kolikšen sme biti pojemek, da ne zdrsne klada v prostoru za tovor.

$v = 8\,m/s$
$r = 40\,m$
$k = 0{,}92$



$a = g \cdot k$

$a = \sqrt{a_r^2 + a_t^2} = g k$

$a_r = v^2/r$

$$\sqrt{\left(\frac{v^2}{r}\right)^2 + a_t^2} = g \cdot k$$

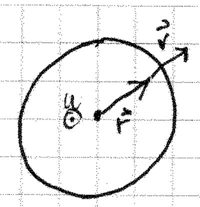
$$\left(\frac{v^2}{r}\right)^2 + a_t^2 = g^2 k^2$$

$$\frac{v^4}{r^2} + a_t^2 = g^2 k^2$$

$$\boxed{a_t = \sqrt{g^2 k^2 - \frac{v^4}{r^2}}}$$

Zelo velika okrogla plošča se vrti okoli osi z $\omega = 5\,rad/s$. Majhna miška na začetku miruje v središču plošče, v nekem trenutku pa steče v radialni smeri proti robu z $v = 45\,cm/s$ glede na ploščo.

Pri kolikšni razdalji od središča začne drseti, če je k med miško in ploščo 0,9



$$\vec{a_c} = 2\vec{\omega} \times \vec{v}$$

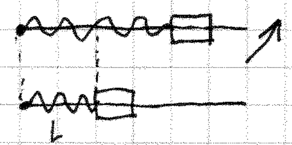
$$a = \sqrt{a_r^2 + a_c^2} = g \cdot k$$

$$a = \sqrt{(\omega^2 r)^2 + (2\omega v)^2}$$

$$g^2 \cdot k^2 = \omega^4 r^2 + 4\omega^2 v^2$$

$$\sqrt{\frac{g^2 k^2 - 4\omega^2 v^2}{\omega^4}} = r$$

Vrtiljak sestavimo iz majhne uteži z $m = 0{,}4\,kg$, zelo lahke ravne prečke in vijačne vzmeti s $k = 2\,N/cm$. Vzmet - neobremenjena 20 cm. Kako daleč od osi vrtenja je utež, ko se vrtiljak vrti z $\omega = 5/s$



$m = 0{,}4\,kg$
$k = 2\,N/cm$
$L = 20\,cm$
$\omega = 5\,s^{-1}$

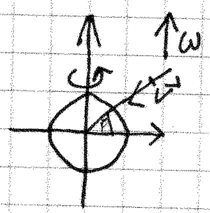
$$F_{vzmeti} = F_{sist}$$

$$kx = m a_r$$
$$kx = m\omega^2 (L + x)$$
$$kx = m\omega^2 L + m\omega^2 x$$
$$kx - m\omega^2 x = m\omega^2 L$$
$$x(k - m\omega^2) = m\omega^2 L$$
$$\boxed{x = m\omega^2 L / (k - m\omega^2)}$$

Kamen spustimo z višine 200 m. To storimo v kraju z geo. širino 37° severno. Za koliko cm proti V zanese kamen

$h = 200\,m$
$\varphi = 37°$

$\omega = \frac{2\pi}{24 \cdot 3600\ s}$



$\vec{a_c} = 2\vec{\omega} \times \vec{v}$

$\vec{\omega} = \omega(0, 1, 0)$
$\vec{v} = v(-\cos\varphi, -\sin\varphi, 0)$

$\vec{v} = (-v)(\cos\varphi, \sin\varphi, 0)$

$2\vec{\omega} \times \vec{v} = -2v\omega(0, 0, \cos\varphi)$

0	1	0
cos	sin	0

$a_c = 2\omega v \cos\varphi$

$a_c = 2\omega v \cos\varphi = 2\omega g t \cos\varphi$

$V = \int a_c dt = 2\omega g \cos\varphi \int t\,dt = \omega g \cos\varphi\, t^2$

$z = \int V dt = \omega g \cos\varphi \int t^2 dt = \frac{1}{3} \omega g \cos\varphi\, t^3 = (1/3)\omega g \cos\varphi \left(\frac{2h}{g}\right)^{3/2}$

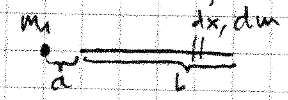
Gravitacija

$m_1 \overset{r}{\bullet\!\!-\!\!-\!\!\bullet} m_2$

$G = 6{,}67 \cdot 10^{-11} \frac{N\,m^2}{kg^2} = \frac{kg\,m}{s^2}\frac{m^2}{kg^2} = \frac{m^3}{kg\,s^2}$

$\vec{F} = G \frac{m_1 m_2}{r^2} \frac{\vec{r}}{r}$

S kolikšno Fg se privlačita 5m dolga ravna homogena palica z m = 30 kg in majhna utež z m = 1 kg, ki se nahaja na isti premici kot palica in je oddaljena 0,7 m



$dm = \frac{m_1}{L} dx$

$l = 5\,m$
$m_1 = 30\,kg$
$m_2 = 1\,kg$
$a = 0{,}7\,m$

$dF = G \frac{m_2 m_1}{L} \frac{dx}{x^2}$

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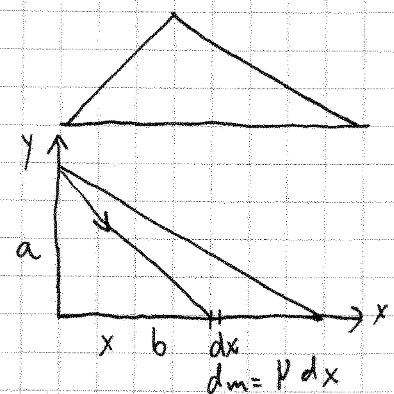
$$F = \int_a^{a+l} \frac{G m_1 m_2}{l} \frac{dx}{x^2} = \frac{G m_1 m_2}{l} \int_a^{a+l} \frac{dx}{x^2} = \frac{G m_1 m_2}{l} \left(-\frac{1}{x}\right)\Big|_a^{a+l} =$$

$$= \frac{G m_1 m_2}{l}\left(-\frac{1}{a+l} + \frac{1}{a}\right) = \frac{G m_1 m_2}{l}\left(\frac{\cancel{a} + l}{a(a+l)}\right) = \frac{G m_1 m_2}{a(a+l)}$$

S kolikšno gravitacijsko silo se privlačita 4m dolga palica z $\mu = 0{,}02\,kg/cm$ in majhna utež z $m = 0{,}8\,kg$, ki je oddaljena 3m od levega in 5m od desnega krajišča palice. Kot, ki ga oklepa vektor sile s palico.



$b = 4m$
$\mu = 0{,}02\,kg/cm$
$m = 0{,}8\,kg$
$a = 3m$
$c = 5m$

$$dF_x = \frac{G m \mu \cdot dx}{r^2} \sin\vartheta \longrightarrow F_x$$

$$dF_y = \frac{G m \mu\, dx}{r^2} \cos\vartheta \longrightarrow F_y$$

$$F = \sqrt{F_x^2 + F_y^2}$$

$$tg\alpha = F_y / F_x$$

$$F_x = G m \mu \int\left(\frac{\sin\vartheta\, dx}{r^2}\right)$$

$$r = \sqrt{a^2 + x^2}$$

$$\sin\vartheta = \frac{x}{\sqrt{a^2+x^2}}$$

$$\cos\vartheta = \frac{a}{\sqrt{a^2+x^2}}$$

$$F_x = Gm\mu \int_0^b \frac{\cancel{a}\, x \, dx}{\sqrt{(a^2+x^2)}^{3/2}} =$$

$$a^2 + x^2 = z$$

$$\frac{dz}{dx} = 2x$$

$$dz = 2x\,dx$$

$$= \frac{Gm\mu}{2} \int \frac{2x\,dx}{(a^2+x^2)^{3/2}} = \frac{Gm\mu}{2} \int_{a^2}^{a^2+b^2=c^2} \frac{dz}{z^{3/2}} = \frac{Gm\mu}{2} \left\{ -2/\sqrt{z} \right\} \Big|_{a^2}^{c^2} =$$

$$= \frac{-Gm\mu}{\sqrt{z}} \Big|_{a^2}^{c^2} = \underline{\underline{\frac{Gm\mu\,(c-a)}{ac}}}$$

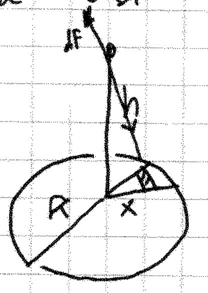
$$F_y = Gm\mu \int_0^b \frac{a\,dx}{(a^2+x^2)^{3/2}} = Gm\mu \frac{x}{a^2\sqrt{a^2+x^2}} \Big|_0^b =$$

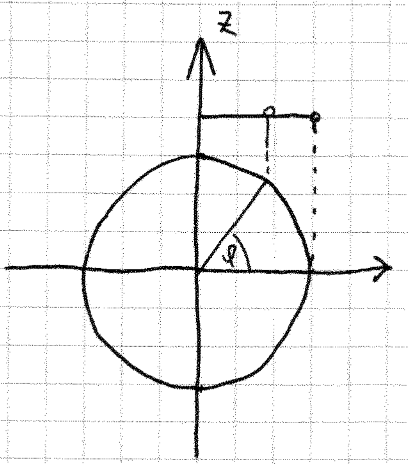
$$= \frac{Gm\mu}{a} \frac{x}{\sqrt{a^2+x^2}} \Big|_0^b = \frac{Gm\mu}{a} \left(\frac{b}{c} - 0 \right)$$

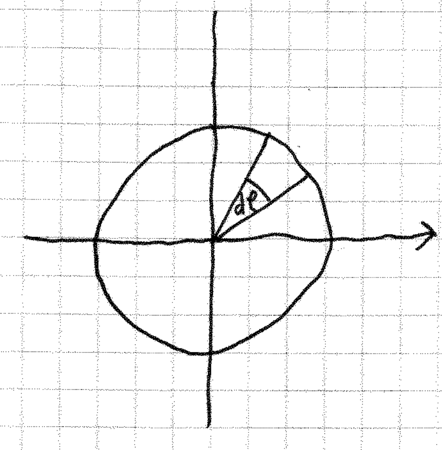
$$\underline{F_y = \frac{Gm\mu b}{ac}}$$

S kolikšno F_g se privlačita utež z $m = 0{,}8\,kg$ ter okrogla plošča z $r = 2\,m$ ter $\sigma = 0{,}01\,kg/cm^2$.

Na osi in je $0{,}7\,m$ oddaljena.







$$dm = \sigma \overbrace{x \cdot dx \; d\varphi}^{ds}$$

$$dF_z = \frac{G \cdot m \cdot \sigma x \; dx \; d\varphi \cos\gamma}{r^2}$$

$$F_z = G m \sigma \int_0^{2\pi} d\varphi \int \frac{x \, dx \cos\gamma}{r^2}$$

$$F_z = 2\pi G m \sigma h \int_0^R \frac{x \, dx}{(h^2 + x^2)^{3/2}}$$

$u = h^2 + x^2$

$du = 2x\,dx$

$$F_z = 2\pi G m \sigma h \int_{h^2}^{h^2+R^2} \frac{du}{u^{3/2}} = -2\pi G m \sigma h \left. \frac{1}{\sqrt{u}} \right|_{h^2}^{h^2+R^2} =$$

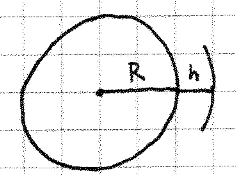
$$= 2\pi G m \sigma h \left(\frac{1}{h} - \frac{1}{\sqrt{h^2+R^2}} \right) = 2\pi G m \sigma h \left(\frac{\sqrt{h^2+r^2} - h}{h\sqrt{h^2+R^2}} \right) =$$

$$\boxed{= 2\pi G m \sigma \left(1 - \frac{h}{\sqrt{h^2+R^2}} \right)}$$

gravitacijsko polje

Na kolikšni višini kroži okoli zemlje satelit, ki zemljo obkroži v 8h urah.

$R = 6400\ km$
$g_0 = 9{,}81\ m/s^2$



$$\cancel{m} \cdot g(h) = \cancel{m} \cdot a_s \qquad \omega^2 r$$

$$g_0 \left(\frac{R}{(R+h)} \right)^2 = \left(\frac{2\pi}{t} \right)^2 (R+h)$$

$$g_0\, R^2 t^2 = 4\pi^2 (R+h)^3$$

$$\frac{g_0 R^2 t^2}{4\pi^2} = (R+h)^3 \quad /\sqrt[3]{\ }$$

$$\sqrt[3]{\frac{g_0 R^2 t^2}{4\pi^2}} = R + h$$

$$h = \sqrt[3]{\frac{g_0 R^2 t^2}{4\pi^2}} - R$$

$$h = \sqrt[3]{\frac{0{,}00981\ km}{\cancel{s^2}} \cdot \frac{6400^2\ km^2\ (8 \cdot 3600)^2\ \cancel{s^2}}{4\pi^2}} - 6400\ km$$

$$h = \ldots$$

Na kolikšni h bi krožil satelit okoli planeta z $R = 8000$ km in $g_0 = 12$ m/s, če bi krožil z $V = 5$ km/s

$$g_0\left(\frac{R}{R+h}\right)^2 = \frac{V^2}{R+h}$$

Gibalna količina

$\vec{G} = m\cdot\vec{v}$

$\Delta\vec{G} = \vec{F}\Delta t$

$d\vec{G} = \vec{F}dt$

$\vec{F} = d\vec{G}/dt = \frac{d}{dt}\left(m\vec{v}\right) = \frac{dm}{dt}\vec{v} + \frac{d\vec{v}}{mdt}$

Po tiru se gibljeta kladi.

$m_1 = 3$ kg
$v_1 = 2$ m/s

$m_2 = 2$ kg
$v_2 = -1/m$

Kladi se sprimeta. s kolikšno v in v katero smer se gibljeta.

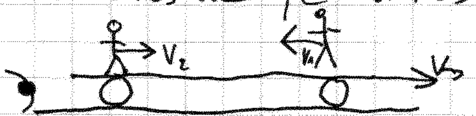
$m_1V_1 + m_2V_2 = m_sV_s$

6 kg m/s $- 2$ kg m/s $= 3$ kg m/s

$v = \frac{G_s}{m_s} = \frac{4}{5}$ m/s $= 0{,}8$ m/s

√ Na tiru miruje vagon m=100kg, na njem stojita moža z masama 90 in 60 kg.
V začetku vsi mirujejo. začneta hoditi po vagonu
- prvi = $V_1' = 1{,}6$ m/s levo
- drugi = $V_2' = 1{,}2$ m/s desno

S kolikšno $\vec{v}$ in v katero smer se premika vagon in kolikšna je hitrost obeh mož glede na tir



$$0 = m_2 V_2 \quad - \quad m_1 V_1 \quad + \quad m_3 V_3$$

$$V_2 = (V_2' + V_0)$$

$$V_1 = -V_1' + V_0$$

$$0 = m_2 (V_2' + V_0) + m_1(\overset{-V_1}{\cancel{V_1'}} + V_0) + m_3 V_0$$

$$0 = m_2 V_2' + m_2 V_0 - m_1 V_1' + m_1 V_0 + m_3 V_0$$

$$0 = m_2 V_2' - m_1 V_1' + V_0 (m_3 + m_2 + m_1)$$

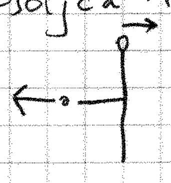
$$\frac{-m_2 V_2' + m_1 V_1'}{(m_3 + m_2 + m_1)} = V_0$$

$$V_0 = 0{,}288 \ m/s$$

$$V_2 = 1{,}488 \ m/s$$
$$V_1 = -1{,}312 \ m/s$$

Vesoljec z vso opremo ima $m = 200kg$. Po breztežnostnem prostoru se premika tako, da iz posebne pištole strelja majhne kroglice v nasprotno smer od željene smeri gibanja. Vsaka kroglica: $m = 0{,}05kg$ in odleti z $v = 50 \ m/s$ glede na vesoljca

Pištola strelja avtomatsko tako, da izstreli eno kroglico vsaki 2 sekundi. Koliko časa porabi vesoljec za pot 10 m, če je na začetku miroval, koliko kroglic je izstreljenih in kolikšna je hitrost vesoljca (ocena)



$s = 10m$
$M = 200kg$
$m = 0{,}05 \ kg$
$V_0 = 50 \ m/s$
$\Delta t = 2s$

$$0 = (M - m) V_1 - m (V_0 - V_1)$$

$$0 = M V_1 - \cancel{m V_1} - m V_0 + \cancel{m V_1}$$

$$0 = M V_1 - m V_0$$
$$V_1 = \frac{m V_0}{M}$$

$(M-m)V_1 = (M-2m)V_2 - m(V_0 - V_2)$

$(M-m)V_1 = MV_2 - \cancel{2}mV_2 - mV_0 + \cancel{mV_2}$

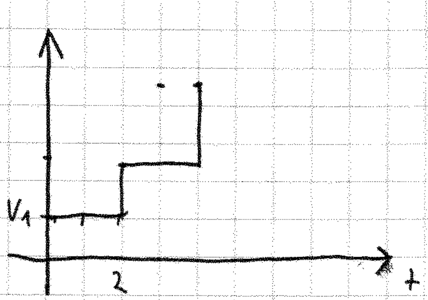
$(M-m)V_1 = MV_2 - mV_2 - mV_0 = V_2(M-m) - mV_0$

$\frac{(M-m)V_1 + mV_0}{(M-m)} = V_2$

$V_2 = V_1 + \frac{mV_0}{M-m}$

$V_3 = \frac{mV_0}{M-3m}$

$a = \Delta V / \Delta t = \frac{mV_0}{M\Delta t}$



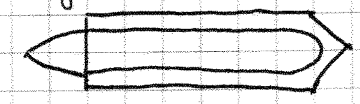
$S = at^2/2$

$t = \sqrt{\frac{2S}{a}} = \sqrt{\frac{2sM\Delta t}{mV_0}} = 56{,}57\ s =$

$N = t/\Delta t = 28$ kroglic

$V = a \cdot t = \sqrt{2as} = \sqrt{\frac{2smV_0}{M\Delta t}}$

Raketa miruje ima z gorivom maso 20 ton, goriva je 18 ton. V nekem trenutku se vžgejo motorji, ki izbruhajo $\phi_m = 500$ kg/s, V_0 plinov je 500 m/s glede na raketo. Kolikšna je hitrost rakete, ko porabi vse gorivo.



$$\phi_m \cdot V_0 = F_c = m \cdot \frac{dv}{dt}$$

$$\phi_m \cdot V_0 = (m_1 - \phi_m \cdot t)\frac{dv}{dt} \qquad / dt$$

$$v(t)$$

$$\phi_m \cdot V_0 \cdot dt = (m_1 - \phi_m \cdot t)\, dv \qquad / (m_1 - \phi_m \cdot t)$$

$$\frac{\phi_m \cdot V_0 \cdot dt}{(m_1 - \phi_m \cdot t)} = dv$$

$$\int dv = \int \frac{\phi_m \cdot V_0 \cdot dt}{(m_1 - \phi_m \cdot t)} = \phi_m \cdot V_0 \cdot \int \frac{dt}{m_1 - \phi_m \cdot t}$$

$$\int_0^V dv = \phi_m V_0 \int_0^t \frac{dt}{m_1 - \phi_m \cdot t}$$

$$V = \phi_m \cdot V_0$$

$$m_1 - \phi_m t = x$$

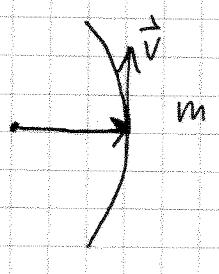
$$-\phi_m dt = dx$$

$$dt = -dx / \phi_m$$

$$V = -V_0 \int_{m_1}^{m_1 - m_2} \frac{dx}{x}$$

20. 11. 07

Vrtilna količina



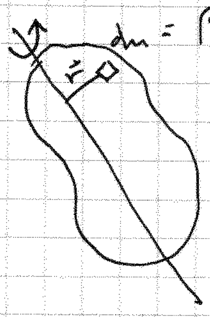
$$G = m \cdot v$$

$$\vec{\Gamma} = \vec{r} \times \vec{G} = m\, \vec{r} \times \vec{v} = m \vec{r} \times (\vec{\omega} \times \vec{r})$$
$$= m r^2 \cdot \vec{\omega}$$

vztrajnostni moment

$$\vec{\Gamma} = J \cdot \vec{\omega}$$

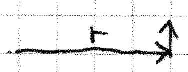
$dm = \rho \cdot dV$



$$dm \cdot r^2 = \rho dV \cdot r^2$$

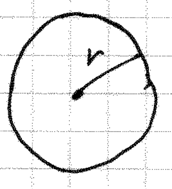
$$\boxed{J = \int \rho(\vec{r}) \cdot r^2 \cdot dV}$$

1.



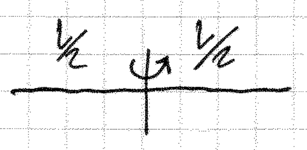
$J = m r^2$ točkasto telo

2.

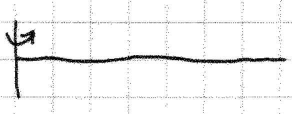


$J = m r^2$

3.

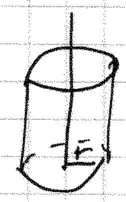


$$J = \frac{m l^2}{12}$$



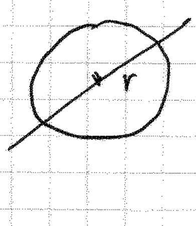
$$J = \frac{m l^2}{3}$$

4.



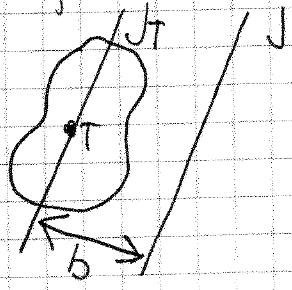
$$J = \frac{m r^2}{2}$$

5.



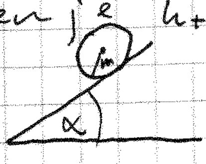
$$J = \frac{2}{5} m r^2$$

Steinerjev izrek



$$J = J_T + mb^2$$

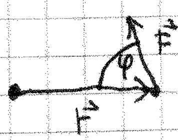
Homogen valj se kotali po klancu $\alpha = 20°$. s kolikšnim a se giblje težišče valja in kolikšen je k trenja med valjem in podlago



$\varphi = 20°$

a, k

$$\vec{\Gamma} = \vec{r} \times G$$
$$\vec{\Gamma} = J \times \omega$$



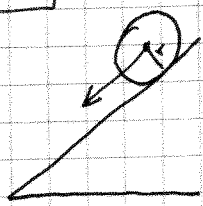
$$M = \vec{r} \times \vec{F}$$
$$M = r F \sin\varphi$$

$$\boxed{\Delta \vec{\Gamma} = \vec{M} \Delta t}$$

J=konst

$$d\Gamma = \vec{M} dt = \vec{M} = \frac{d\vec{\Gamma}}{dt} = \frac{d}{dt}(J \cdot \vec{\omega}) = \cancel{\frac{dJ}{dt}\vec{\omega}} + J\frac{d\vec{\omega}}{dt} = J \cdot \alpha$$

$\boxed{M = J\alpha}$ Newtonov zakon za vrtenje



$$F_g \sin\varphi - F_g \cos\varphi \cdot k = m \cdot a$$

$$M = J \cdot \alpha$$

$$m g \cos\varphi \cdot k \cdot r = \frac{mr^2}{2} \frac{a}{r}$$

$$g \sin\varphi - g \cos\varphi \cdot k = a$$

$$g \cos\varphi \, k \, r = \frac{ra}{2}$$

$$k = \frac{a}{2g\cos\varphi}$$

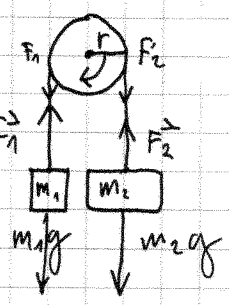
$$a = g \sin\varphi - \cancel{g\cos\varphi} \frac{a}{2g \cancel{\cos\varphi}}$$

$$\frac{3a}{2} = g \sin\varphi$$

$$\boxed{a = \frac{2g\sin\varphi}{3}} \quad k = \frac{1}{3}\operatorname{tg}\varphi$$

Aleta

Dve uteži z $m_1 = 2kg$ in $m_2 = 5kg$ sta povezani z vrvico preko škripca, ki ima obliko homogenega valja z $m = 6kg$. Valj je vrtljiv okoli svoje geometrijske osi, vrvica pa ne spodrsava.
S kolikšnim pospeškom se gibljeta uteži in kolikšni sili napenjata vrvico na vsakem koncu.



$$F_1 - m_1 g = m_1 \cdot a$$

$$m_2 g - F_2 = m_2 a$$

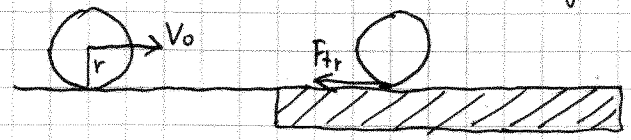
$$M = J \cdot \alpha$$

$$F_2 r - F_1 r = \frac{m_3 r^2}{2} \cdot \frac{a}{r}$$

Seštejemo enačbe

Homogena krogla drsi po vodoravni podlagi, težišče se giblje s 6m/s. Naleti na koeficient trenja $k_{tr} = 0{,}2$. Po kolikšnem t se krogla samo še kotali.

$V_0 = 6 m/s$
$k = 0{,}2$



$F_{tr} = ma$
$mgk = ma$
$gk = a \rightarrow v(t)$
$M = J\alpha$
$\Downarrow$
$\omega(t)$
$\rightarrow v = \omega r$

$V = V_0 - gkt$
$\omega = \alpha t$

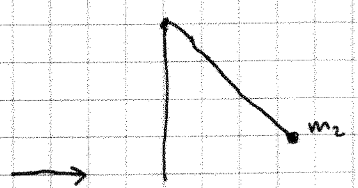
$$mgk \cdot r = \frac{2}{5} mr^2 \alpha \rightarrow \alpha = \frac{5gk}{2r}$$

$\omega = \alpha t$
$\omega = \frac{5gk}{2r} t$

$$V_0 - ght = \frac{5}{2}\, ght$$
$$V_0 = ght + \frac{5}{2} ght_0$$
$$\cancel{V_0}\ t_0 = 2V_0 / 7gh$$

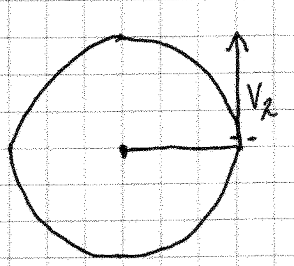
Raven tanek drog z dolžino 1m in $m_1 = 1kg$ je vrtljiv okoli vodoravne osi, ki gre skozi krajišče ~~kroga~~ droga. Majhna kepa z $m_2 = 0{,}4kg$ prileti v vodoravni smeri pravokotno na drog in na os vrtenja s hitrostjo $V = 8m/s$, prileti v spodnje krajišče droga in se nanj prilepi. S kolikšno ω se zavrti drog?

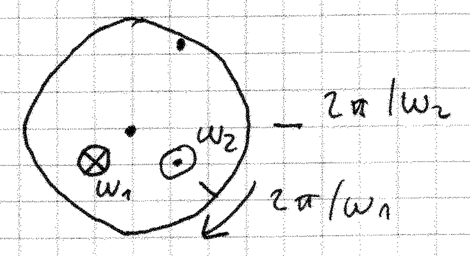
$$\Delta \Gamma = M \Delta t$$
$$\Delta \vec{\Gamma} = \vec{r} \times \vec{F} \Delta t = \vec{r} \times \Delta \vec{G}$$



$$J\omega = \overbrace{m_2 V}^{\Delta G = F\Delta t} \cdot l$$
$$\omega\left(\frac{m_1 l^2}{3} + m_2 l^2\right) = l\ m_2 V$$
$$\omega\, l^2 \left(\frac{m_1 + 3m^2}{3}\right) = 1\, m_2 V$$
$$\omega l \left(\frac{m_1 + 3m^2}{3}\right) = m_2 V$$
$$\omega = \frac{3 m_2 V}{l\,(m_1 + 3m_2)}$$

Na vodoravni ploskvi leži plošča z obliko valja ($m = 40kg$, $r = 3m$). Trenja ni. Na robu plošče stoji človek z $m = 60kg$. V začetku plošča in človek mirujeta. V nekem trenutku pa začne človek hoditi po robu okoli plošče z $v = 1{,}4m/s$ glede na ploščo. S kolikšno ω se vrti plošča in s kolikšno ω kroži človek.





$$\Delta\Gamma = 0$$

$$0 = J_1\omega_1 - J_2\omega_2 \leftarrow (1)$$

$$v_2'' = \omega_2 r = v_2' - \omega_1 r \leftarrow (2)$$

$$0 = \frac{m_1 r^2}{2}\omega_1 - m_2 r^2 \omega_2$$

$$0 = m_1\omega_1 r - 2m_2\omega_2$$

$$0 = m_1\omega_1 r - 2m_2((v_2' - \omega_1 r)/r)$$

$$0 = m_1\omega_1 r - 2m_2 v_2' + 2m_2\omega_1 r$$

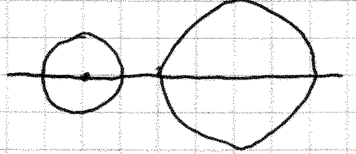
$$0 = \omega_1(m_1 r + 2m_2 r) - 2m_2 v_2'$$

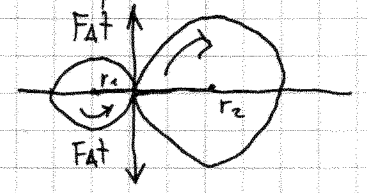
$$\omega_1 = \frac{2m_2 v_2'}{(m_1+2m_2)r}$$

$$\omega_2 \cdot r = v_2' - \frac{2m_2 v_2'}{(m_1+2m_2)r}$$

$$\omega_2 = \frac{v_2'(1 - 2m^2/m_1+2m)}{r^2} = \boxed{\frac{v_2' m_1}{r(m_1+2m_2)}}$$

Dva homogena valja z $m_1 = 3\,kg$, $r_1 = 10\,cm$, $m_2 = 1\,kg$, $r_2 = 20\,cm$ sta vrtljiva. V začetku se prvi vrti z $\omega = 100/s$, drugi pa miruje. Nato osi približamo, da se plašča





stakneta. Kolikšni sta ω_1 in ω_2, ko plašča nehata spodrsavati?

$$J_1(\omega_1 - \omega_0) = F_A t\, r_1$$

$$0 - J_2\omega_2 = F_A t \cdot r_2$$

nehata podrsavat $\omega_1 r_1 = \omega_2 r_2$

$$\frac{-J_2\omega_2}{J_1(\omega_1 - \omega_0)} = \frac{r_2}{r_1} \qquad \omega_2 = \frac{\omega_1 r_1}{r_2}$$

27.11.07

$$\frac{-J_2\omega_1 v_1}{r_2 J_1(\omega_1-\omega_0)} = \frac{r_2}{r_1}$$

$$-J_2\omega_1 r_1^2 = J_1(\omega_1-\omega_0)r_2^2$$

$$m_2 r_2^2 r_1^2 \omega_1 = m_1 r_1^2 r_2 (\omega_0-\omega_1)$$

$$m_2\omega_1 = m_1\omega_0 - m_1\omega_1$$

$$\boxed{\omega_1 = \frac{m_1\omega_0}{m_1+m_2}} \qquad \boxed{\omega_2 = \frac{m_1\omega_0 r_1}{r_2(m_1+m_2)}}$$

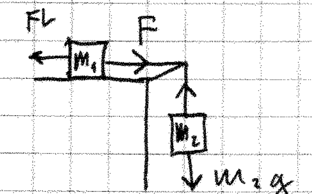
kolokvij - vaje

- Dve kladi

$m_1 = 3\,kg$ sta povezani z zelo lahko vrvico preko škripca.

$k_{t1} = 0{,}3.$

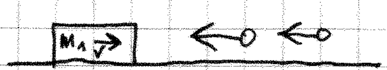
Koliko sme biti največ masa druge klade, da se ne premakneta.



$$F - F_l = 0$$
$$F - mg k_l = 0$$
$$m_2 g - F = 0$$

$$m_2 \cancel{g} - m_1 \cancel{g} k = 0$$
$$m_2 = m_1 k = \boxed{0{,}9\,kg}$$

- Klada z $m = 3\,kg$ se giblje po tiru z $v = 2\,m/s$ proti desni. Iz nasprotne smeri streljamo izstrelke z $m = 0{,}1\,kg$ z $v = 10\,m/s$. Vsi se zapičijo v klado.



S kolikšno v se giblje klada po petem izstrelku.

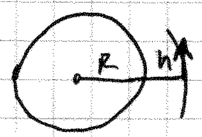
$$M\cdot v_1 - m\cdot v_0 = (M+m)v_2$$

$$v_2 = \frac{Mv_1 - mv_0}{M+m}$$

$$(M+m)v_2 - mv_0 = (M+2m)v_3$$

$$Mv_1 - 5mv_0 = (5m+M)v_k \rightarrow \boxed{v_k = \frac{Mv_1 - 5mv_0}{5m+M}}$$

S kolikšno hitrostjo kroži umetni satelit okoli nekega planeta ki ima $R: 8000\,km$ in $g_0 = 12\,m/s^2$ če kroži na $h = 860\,km = R/10$

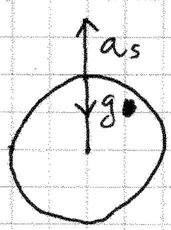


$$a_s = g(h)$$

$$\frac{v^2}{R+h} = g_0\left(\frac{R}{R+h}\right)^2$$

$$v^2 = \frac{g_0 R^2}{R+h}, \quad \boxed{v = \sqrt{\frac{g_0 R^2}{R+h}}}$$

- Letalo leti z $v = 200\,m/s$ po krožnici, ki je pravokotna na površino zemlje. Kolikšen mora biti r krožnice, da se potniki v najvišji točki počutijo kot v breztežnem prostoru.



$$a_s = g$$

$$\frac{v^2}{r} = g$$

$$\boxed{r = v^2/g}$$

$$r = 4\ldots km$$

- dokument, calc, formule (1 list), SVINČNIK!

27.11.07

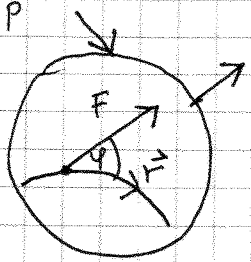
Delo, energija moč

II

← gibalne količine
vrtilne količine

$$A = \int_P \vec{F}\, d\vec{r}$$

cos φ



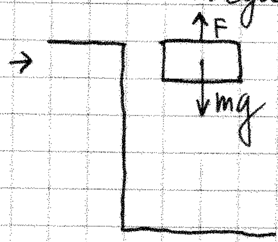
$$A = \int M\, d\varphi$$

- $\boxed{W_k = mv^2/2}$
- $\boxed{W_k = \frac{J\omega^2}{2}}$ pri vrtenju
- $\boxed{W_{pr} = kx^2/2}$
- $\boxed{W_p = mgh}$
- $\boxed{W_p = -\frac{G m_1 m_2}{r} = V_g m_2}$

Moč: $\boxed{P = \frac{dA}{dt}}$

$$A = \int P\, dt$$

- Breme m = 20 kg spustimo z 30 m. S kolikšno silo ga moramo zadrževati, da udari ob tla z 1 m/s. Reši na 2 načina. Z uporabo Newtonovega zakona in z uporabo energije.



$$mg - F = m \cdot a$$

$$\left.\begin{aligned} h &= at^2/2 \\ v &= a \cdot t \end{aligned}\right\} \quad t = \sqrt{2h/a} \qquad v = \sqrt{2ah}$$

$$a = \frac{v^2}{2h} \qquad \boxed{F = m\left(g - \frac{v^2}{2h}\right)}$$

$\rightarrow \quad W_p = W_k + A_F$

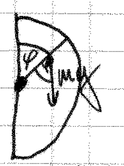
$$mgh = \frac{mv^2}{2} + F \cdot h$$

$$\frac{2mgh - mv^2}{2h} = F \cdot \cancel{h}$$

$$F = \frac{m(2gh - v^2)}{2h} = m\left(g - \frac{v^2}{2h}\right)$$

- Homogena ravna en meter dolga palica je vrtljiva okoli vodoravne osi, ki gre skozi krajišče. V začetku miruje v navpični legi, da gre os skozi ~~spodnje~~ krajišče. ~~zgornje~~ spodnje

S kolikšno $\vec{v}$ gre drugo krajišče palice skozi najnižjo lego, ko palico zavrtimo okoli osi.



$$M = J\alpha$$

$$mg \cdot l/2 \quad \sin\varphi = \frac{ml^2}{3} \quad \frac{d^2\varphi}{dt^2}$$

$v = \omega l$

$$3g \sin\varphi = 2l \quad \frac{d\omega}{dt} \qquad \frac{d\omega}{dt} = \frac{d\omega}{d\varphi}\frac{d\varphi}{dt} = \omega\frac{d\omega}{d\varphi}$$

$$\frac{3g}{2l}\sin\varphi = \omega\frac{d\omega}{d\varphi} \Big/ d\varphi \rightarrow \omega(\varphi)$$

$$\frac{3g}{2l}\int_0^{\pi}\sin\varphi\, d\varphi = \int_0^{\omega}\omega\, d\omega$$

$$\frac{3g}{2l}(-\cos\varphi)\Big|_0^{\pi} = \frac{\omega^2}{2}$$

$$+ \quad \omega = \sqrt{\frac{6g}{l}} \qquad \boxed{v = \omega l = \sqrt{6gl}}$$

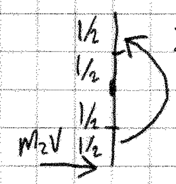
$W_p = W_k$

$mgl = J\omega^2/2$

$$mgl = \frac{ml^2}{3}\frac{\omega^2}{2} \rightarrow \omega \rightarrow v = \omega \cdot l = \sqrt{6gl}$$

- Palica $m = 1kg$ z $l = 1m$ je vrtljiva okoli osi, ki gre skozi zgornje krajišče.

Kepa ilovice $m = 0{,}4kg$ prileti v vodoravni smeri pravokotno na palico in na os in se prilepi na spodnje krajišče. Najmanj kolikšna mora biti v kepe, da se palica zavrti.



> zgornja meja koliko J gre v izgubo.

$$W_p = W_k$$

$$\underbrace{m_2 2gl}_{\text{kepa}} + \underbrace{m_1 gl}_{\text{palica}} = \frac{J\omega^2}{2}$$

$$gl(m_1 + 2m_2) = \frac{\omega^2}{2}\overbrace{\left(\frac{m_1 l^2}{3} + m_2 l^2\right)}^{J}$$

$$\frac{2gl(m_1 + 2m_2)}{\left(\frac{m_1 l^2 + 3m_2 l^2}{3}\right)} = \omega^2$$

$$\sqrt{\frac{6g(m_1 + 2m_2)}{l(m_1 + 3m_2)}} = \omega$$

$$\Delta\Gamma = M\Delta t \qquad M\Delta t = \vec{r} \times \vec{F}\Delta t = \vec{r} \times \Delta\vec{G}$$

$$\downarrow$$

$$J\omega = m_2 v \cdot l$$

$$v = \frac{J\omega}{m_2 l} = \frac{\left(\frac{m_1 l^2}{3} + m_2 l^2\right)\sqrt{\frac{6g(m_1 + 2m_2)}{l(m_1 + 3m_2)}}}{m_2 l} =$$

$$= \boxed{\frac{6gl(m_1 + 2m_2)(m_1 + 3m_2)}{3m_2}} \boxed{= v}$$

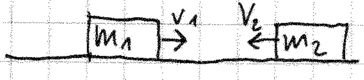
• $\frac{1}{2} m_2 v^2 - gl(m_1+2m_2) = \frac{6gl(m_1+2m_2)(m_1+3m_2)}{18 m_2} - gl(m_1+2m_2)$

$= gl(m_1+2m_2)\left(\frac{m_1+3m_2}{3m_2} - 1\right) = gl(m_1+2m_2)\left(\frac{m_1+3m_2-2m_2}{3m_2}\right) =$

$= \boxed{gl(m_1+2m_2)\left(\frac{m_1}{3m_2}\right)}$

• Po vodoravnem tiru sta gibljiva dva vozička.
$m_1 = 3kg$ $v_1 = 2m/s$ (D) $m_2 = 1kg$ $v_2 = 4m/s$ (L)
Vozička prožno trčita

S kolikšnima hitrostima in v katero smer se po trku gibljeta vozička?



1) $m_1 v_1 - m_2 v_2 = m_1 v_3 + m_2 v_4$

$m_1 v_1^2 + m_2 v_2^2 = m_1 v_3^2 + m_2 v_4^2$

$\left.\begin{array}{l} m_1(v_1^2 - v_3^2) = m_2(v_4^2 - v_2^2) \\ m_1(v_1 - v_3) = m_2(v_4 + v_2) \end{array}\right\}$ delimo

2) $(v_1 + v_3) = (v_4 - v_2)$

$v_3 = v_4 - v_2 - v_1$

$m_2 v_4 - m_1 v_2 - m_2 v_2 - m_1(v_4 - v_2 - v_1)$

$m_2 v_4 = m_1 v_1 - m_2 v_2 - m_1 v_4 + m_1 v_2 + m_1 v_1$

~~...~~ =

$v_4(m_1 + m_2) = \left| 2m_1 v_1 + v_2(m_1 - m_2)\right.$

$v_4 = \boxed{\frac{2m_1 v_1 + v_2(m_1 - m_2)}{m_1 + m_2}} = 5 m/s$

$v_3 = \frac{2m_1 v_1 + v_2(m_1 - m_2)}{m_1 + m_2} - \frac{v_2(m_1+m_2) + v_1(m_1+m_2)}{m_1+m_2} =$

$= \boxed{\frac{v_1(m_1 - m_2) - 2m_2 v_2}{m_1 + m_2}} = -1 m/s$

posebni primeri

1) $m_1 = m_2$

$$V_4 = V_1$$
$$V_3 = -V_2$$

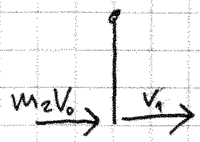
2) $m_1 >> m_2$

$$V_4 = 2V_1 + V_2 = V_1 + \underbrace{V_1 + V_2}_{\text{hitrost lahkega merjena v sistemu težkega}}$$

$$V_3 = V_1$$

• Homogen raven tanek drog z $l = 1m$ in $m_1 = 1kg$ je vrtljiv okoli osi (zgornje krajišče). kroglica prileti pravokotno z $v = 80 m/s$ in zadene spodnje krajišče droga. Trk je idealno prožen. S kolikšno ω se zavrti drog ter s kolikšno hitrostjo in v katero smer se giblje kroglica

$$\frac{1}{2} m_2 V_0^2 = \frac{1}{2} m_2 V_1^2 + \frac{J\omega^2}{2}$$



$$\left.\begin{aligned} \Delta \Gamma &= M \Delta t \\ J\omega &= l\,(V_0 - V_1) \end{aligned}\right\}$$

$$(m_2V_1 - m_2V_0)$$

$$m_2(V_0^2 - V_1^2) = J\omega^2$$
$$m_2\, l\,(V_0 - V_1) = J\omega$$

$$\frac{1}{l}(V_0 + V_1) = \omega$$

$$V_0 + V_1 = \omega l$$

$$V_1 = \omega l - V_0$$
$$J\omega = l m_2 V_0 - l m_2(\omega l - V_0)$$

$$J\omega = l m_2 V_0 - l^2 m_2 \omega + l m_2 V_0$$

$$\frac{m_1 l^2}{3}\omega = 2 l m_2 V_0 - l^2 m_2 \omega$$

$$\omega\left(\frac{m_1 l^2}{3} + l^2 m_2\right) = 2 l m_2 V_0$$

$$\boxed{\omega = \frac{6 m_2 V_0}{l(m_1 + 3m_2)}}$$

$V_1 = \frac{6m_2V_0}{m_1+3m_2} - V_0$

$V_1 = V_0\left(\frac{6m_2 \cancel{V_0} - m_1 - 3m_2}{m_1+3m_2}\right) = \boxed{V_0\left(\frac{3m_2-m_1}{m_1+3m_2}\right)}$

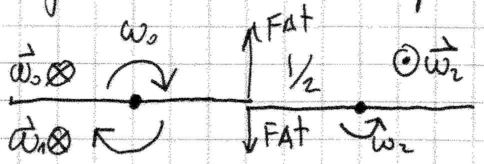
Dva drogova sta vrtljiva okoli vzporednih vrtljivih osi, ki greste skozi težišči

$m_1 = 0{,}7\,kg$
$l_1 = 1\,m$
$m_2 = 1\,kg$
$l_2 = 80\,cm$

Osi sta razmaknjeni $< 90\,cm$, da se krajišči lahko dotakneta.

$\omega_0 = 200\,s^{-1}$.

S kolikšnimi ω se vrtita drogova, ko krajišči idealno prožno trčita



$J_1\omega_0^2 = J_1\omega_1^2 + J_2\omega_2^2$ $\quad J = \frac{ml^2}{12}$

$J_2\omega_2 = F\Delta t\,\frac{l_2}{2} \rightarrow \boxed{F\Delta t = \frac{2J_2\omega_2}{l_2}}$

$J_1(-\omega_1+\omega_0) = F\Delta t\,\frac{l_1}{2} \rightarrow \cancel{F\Delta t = \frac{2J_1(-\omega_1+\omega_0)}{l_1}}$

$J_1(-\omega_1+\omega_0) = \frac{\cancel{2}J_2\omega_2}{l_2}\frac{l_1}{\cancel{2}}$

$J_1(\omega_0^2-\omega_1^2) = J_2\omega_2^2$

2) $J_1(\omega_0-\omega_1) = \frac{J_2\omega_2 l_1}{l_2} \rightarrow m_1l_1^2(\omega_0-\omega_1) = \frac{l_1}{l_2}\, m_2l_2^2\, \frac{l_1}{l_2}(\omega_0+\omega_1)$

1) $\omega_0+\omega_1 = \frac{\omega_2 l_2}{l_1}$ $\qquad m_1(\omega_0-\omega_1) = m_2(\omega_0+\omega_1)$

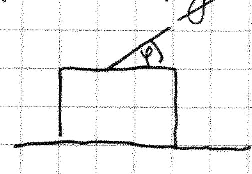
$\omega_2 = \frac{l_1(\omega_0+\omega_1)}{l_2} \rightarrow \boxed{\omega_1 = \frac{\omega_0(m_1-m_2)}{m_1+m_2}}$ $\qquad \omega_2 = \frac{\omega_0 l_1}{l_2}\left(1+\frac{m_1-m_2}{m_1+m_2}\right) = \boxed{\frac{\omega_0 l_1}{l_2}\left(\frac{2m_1}{m_1+m_2}\right)}$

$m = 30t$
$\varphi = 10°$
4 izstrelki /s
$m_0 = 4kg$
$V = 1600 \, m/s$

5. naloga knjiga

hitrost po 25 s -

$\phi_m = 16 kg/s$



$F = m \cdot a$
$\phi_m V_0 \cos\varphi = (m_1 - \phi_m t) \frac{dv}{dt}$

$dv = \frac{\phi_m V_0 \cos\varphi \, dt}{(m_1 - \phi_m t)}$

$\int_0^v dv = \phi_m V_0 \cos\varphi \int_0^t \frac{dt}{m_1 - \phi_m t}$

$m_1 - \phi_m \cdot t = x$
$-\phi_m dt = dx$
$dt = -\frac{dx}{\phi_m}$

$V = -V_0 \cos\varphi \cdot \int_{m_1}^{m_1 - \phi_m \cdot t} \frac{dx}{x}$

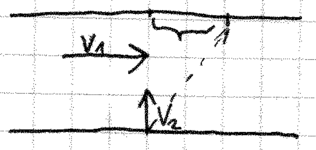
$$\boxed{V = V_0 \cos\varphi \ln \frac{m_1}{m_1 - \phi_m \cdot t}} = 1600 \frac{m}{s} \cdot \cos 10 \cdot$$

$$\cdot \ln \frac{30000}{30000 - 16 \cdot 25} = 21 \, m/s$$

Akta®

29. 11. 07 vaje

50m široka reka teče z 3m/s. Čoln odrine s hitrostjo 5m/s v smeri pravokotno na breg. Kako daleč navzdol po toku ga zanese, preden doseže drugi breg in pod kolikšnim kotom bi moral usmeriti čoln, da ga ne bi zaneslo.



$t = s/V_2$

$\boxed{x = V_1 t = 30\,m}$

$V_x = V_1$

$V_2 \sin\varphi = V_1$

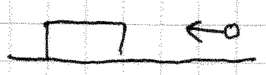
$\boxed{\sin\varphi = V_1/V_2}$

Koliko časa rabi v tem primeru

$$\frac{s}{V_2\cos\varphi} = \frac{s}{V_2\sqrt{1-V_1^2/V_2^2}} = \frac{50}{4} = 12{,}5\,s$$

Neelastični trk

Klada z $m = 5kg$ se giblje po tiru z $V = 5m/s$ (D). V nasprotni smeri streljamo majhne izstrelke z $m_2 = 0.1kg$ $V_2 = 50m/s$. Vsak izstrelek se tapiči. Kolikšna je hitrost, ko jo zadenejo trije izstrelki.



$m_{s1}V_{s1} = m_1V_1 - m_2V_2 \rightarrow V_1$

$m_{s2}V_{s2} = m_{s1}V_{s1} - m_2V_2 \rightarrow V_2$

$m_{s3}V_{s3} = m_{s2}V_{s2} - m_2V_2 \rightarrow V_3$

$Mv - mv_0 = (M+m)V_1$ $\quad (M+m)V_1 - mv_0 = (M+2m)V_0$

$V_1 = \frac{Mv - mv_0}{(M+m)}$ $\quad V_2 = \frac{(M+m)V_1 - mv_0}{M+2m} =$

$(M+2m)V_2 - mv_0 = (M+3m)V_3$

$V_3 = \frac{(M+2m)V_2 - mv_0}{M+3m}$

$\boxed{\text{klade, GK, poševni met}}$

4.12.2007

moč $P = \frac{dA}{dt}$

$A = \int P\,dt$

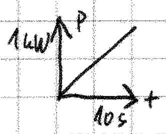
Moč motorja v vozilu z maso 950 kg narašča premo sorazmerno s časom in po 10s doseže 1kW, v začetku je bila 0. Kolikšna je po 10s hitrost, pospešek in kolikšno pot prevozi vozilo.

Predpostavi, da gre vse delo v Wk.

$A = W_k$

$$\int P\,dt = \frac{mv^2}{2}$$

$$P(t) = \frac{P_0}{t_0} \cdot t$$



$W = [J/s] = [kg\,m/s^3]$

$$\frac{P_0}{t_0}\int_0^{t_0} t\,dt = \frac{mv^2}{2}$$

$$\frac{P_0}{\cancel{t_0}} \cdot \frac{t_0^{\cancel{2}}}{\cancel{2}} = \frac{mv^2}{\cancel{2}} \rightarrow v = \sqrt{\frac{P_0 t_0}{m}} = \boxed{\sqrt{\frac{1000\,W \cdot 10s}{950\,kg}}}$$

v splošnem

$$V = \underbrace{\sqrt{\frac{P_0\, t}{\cdot t_0}}}_{a} \cdot t \rightarrow a = \frac{dv}{dt} = \boxed{\sqrt{\frac{P_0}{m t_0}}}$$

$$s = \int v\,dt = \boxed{\sqrt{\frac{P_0}{m t_0}} \cdot \frac{t^2}{2}} \quad \left(\frac{at^2}{2}\right)$$

Motor s konst. močjo 1kW poganja dvigalo. To dvigalo dviguje maso 100kg, ki je mirovalo na tleh. V kolikšnem času dvigalo dvigne breme na 20 m in kolikšna je takrat hitrost.

$P = 1000 W$
$m = 100 kg$
$h = 20 m$

$t = ?$, $v = ?$

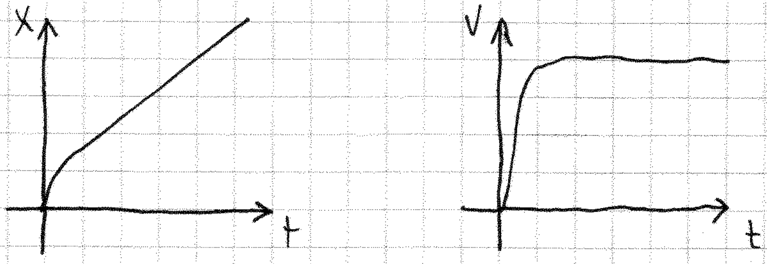
$A = W_k + W_p$

$\int P dt$, ker je konst, $P \int dt \rightarrow P \cdot t$

$$P \cdot t = \frac{m v^2}{2} + m g h$$

$$v = \frac{dh}{dt}$$

$$P \cdot t = \frac{m}{2} \cdot \left(\frac{dh}{dt}\right)^2 + m g h \rightarrow h(t)$$



lahko rešimo numerično.

približno: (ker je razdalja dovolj velika)

$$P \cdot t = m g x$$

$$t = \frac{m g x}{P} = \frac{100 kg \cdot 10 \frac{m}{s^2} \cdot 20 m}{1000 W} = \boxed{20 s}$$

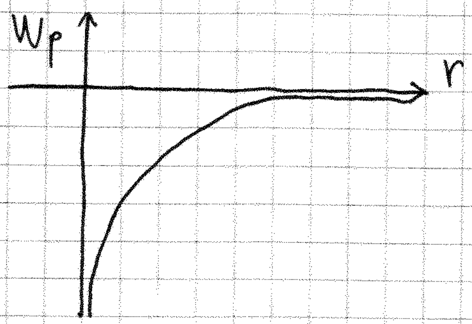
$$\frac{x}{t} = \boxed{1 m/s}$$

Gravitacijska potencialna energija

m_1 r m_2

$$V_g = -\frac{G m_1}{r}$$

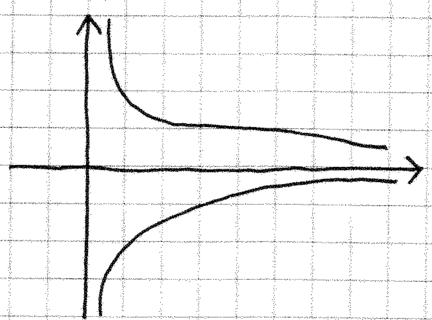
$$W_p = m_2 \cdot V_g = -\frac{G m_1 m_2}{r}$$



q_1 q_2

$$V = q_1 / 4\pi\varepsilon_0 r$$

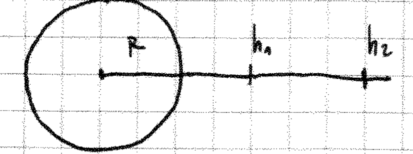
$$W_p = q_1 q_2 / 4\pi\varepsilon_0 r = V \cdot q_2$$



(pot. oz. neg.)

Koliko dela morajo najmanj opraviti raketni motorji, da dvignejo satelit z $m = 100\,kg$ iz $2000\,km$ na $5000\,km$. g_0, $R = 6400\,km$

$A = ?$



① po def. dela

$$A = \int_{h_1}^{h_2} F(h)\, dh$$

$$g = g_0 \left(\frac{R}{R+h}\right)^2$$

$u = R + h$
$du/dh = 1 \quad du = dh$

$$A = m g_0 R^2 \int_{h_1}^{h_2} \frac{dh}{(R+h)^2} = m g_0 R^2 \left(-\frac{1}{u}\right)\Bigg|_{R+h_1}^{R+h_2} =$$

$$A = mg_0 R^2\left(\frac{1}{R+h_1} - \frac{1}{R+h_2}\right) = mg_0 R^2 \cdot \left(\frac{\cancel{R}+h_2-\cancel{R}-h_1}{(R+h_1)(R+h_2)}\right) =$$

$$= \boxed{\frac{(h_2-h_1)\, mg_0 R^2}{(R+h_1)(R+h_2)}}$$

(2) Razlika potencialnih energij

$$A = W_p(h_2) - W_p(h_1)$$

$$= -\frac{G m_z m}{R+h^2} - \left(-\frac{G m_z m}{R+h_1}\right) =$$

$$\left(\cancel{m} g_0 = \frac{G m_z \cancel{m}}{R^2}\right)$$

$$= \boxed{m g_0 R^2 \left(\frac{h_2 - h_1}{(R+h_1)(R+h_2)}\right)}$$

Nekje v vesolju sta dva planeta.

Prvi: $r_1 = 8000$ km $g_1 = 12\ m/s^2$

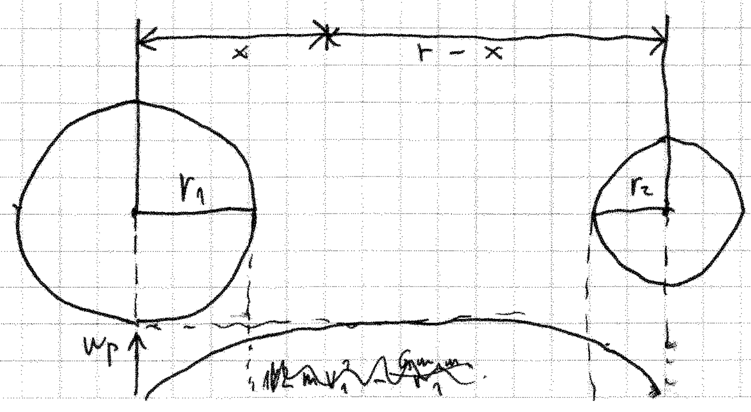
Drugi: $r_2 = 6000$ km

$m_1/m_2 = 16$

Razdalja je 900 000 km.

S kolikšno v moramo izstreliti izstrelek iz večjega, da bo dosegel manjšega.

S kolikšno v bo zadel manjši planet.



$$W_p = -\frac{G m_1 m}{x} - \frac{G m_2 \cdot m}{r-x} = -Gm\left[m_1 x^{-1} + m_2 (r-x)^{-1}\right]$$

$$\frac{dW_p}{dx} = 0 = -m_1 x^{-2} + \cancel{(-1)} m_2 (r-x)^{-2} \cdot \cancel{(-1)} = 0$$

$$= \frac{m_2 \cdot x^2 - m_1 (r-x)^2}{(r-x)^2 \cdot x^2}$$

$$m_1 (r-x)^2 = m_2 x^2$$

$$\sqrt{m_1}(r-x) = \sqrt{m_2}\, x$$

$$\sqrt{m_1}\, r - \sqrt{m_1}\, x = \sqrt{m_2}\, x$$

$$x = \frac{\sqrt{m_1}\, r}{\sqrt{m_1} + \sqrt{m_2}}$$

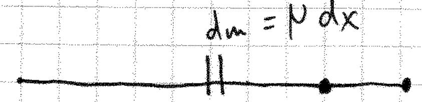
$$x = \frac{r \sqrt{m_1 m_2 / m_2}}{\sqrt{m_1 m_2 / m_2} + \sqrt{m_2^2}}$$

$$\boxed{x = 4/5}$$

$$\frac{1}{2} m v_1^2 - \frac{G m_1 m}{r_1} - \frac{G m_2 m}{(r - r_1)} = -\frac{G m_1 m}{\frac{4}{5} r} - \frac{G m_2 m}{\frac{r}{5}}$$

$$v_1 = \sqrt{\frac{2 G m_1}{r_1} + \frac{2 G m_2}{(r - r_1)} - \frac{10 G m_1}{4 r} - \frac{10 G m_2}{r}}$$

Homogena ravna palica je dolga 2m in ima m na dolžinsko enoto $\mu = 0.03\,kg/cm$. Majhna utež miruje v točki $a = 20\,cm$ od krajišča.



Koliko A, da spravimo utež v točko b, ki je 50 cm od krajišča

(1) $A = W_{pb} - W_{pa}$

$$dW_p = -\frac{G \cdot m \cdot \mu\, dx}{x}$$

$$W_{pa} = -Gm\mu \int_a^{a+l} \frac{dx}{x} \qquad W_{pb} = -Gm\mu \int_b^{b+l} \frac{dx}{x}$$

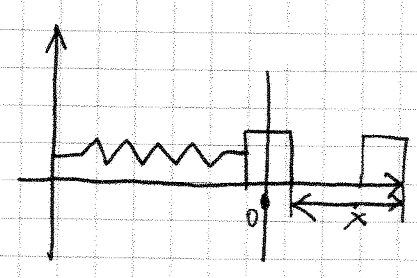
$$= -Gm\mu \ln x \Big|_a^{a+l} \qquad = -Gm\mu \ln x \Big|_b^{b+l}$$

$$= -Gm\mu \ln \frac{a+l}{a} \qquad = -Gm\mu \ln \frac{b+l}{b}$$

$$\boxed{A = -Gm\mu \cdot \ln\left(\frac{(a+l)\,b}{a \cdot (b+l)}\right)}$$

11.12.07

Nihalo na vijačno vzmet je sestavljeno iz uteži $m=0{,}2\,kg$ in vzmeti s $k=16\,N/m$. Utež odmaknemo za 12 cm od ravnovesne lege in pustimo, da zaniha. Kolikšni so, ko gre utež 0,1s po tem, ko gre utež prvič skozi ravnovesno lego hitrost, odmik in pospešek.



$$m\ddot{x} = -k\cdot x$$

$$\ddot{x} + \frac{k}{m}\cdot x = 0 \rightarrow x(t)$$

$\rightarrow x$

$$\ddot{x} + \omega_0^2 x = 0$$

$$\boxed{\omega_0 = \sqrt{\frac{k}{m}}\ [s^{-1}]}$$

$$x(t) = a\cdot\sin\omega t + b\cdot\cos\omega_0 t$$

$$x(t=0)=0 \longrightarrow b=0$$

$$\boxed{x = x_0 \sin\omega_0 t}$$

$$x(t) = A\sin(\omega_0 t + \delta) = A\sin\omega_0 t\cos\delta + A\cos\omega_0 t\sin\delta$$

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{16\,N}{m\,0{,}2\,kg}} = 8{,}9\ s^{-1}$$

$$x = x_0\sin\omega_0 t = 0{,}12\,m\ \sin(\underbrace{0{,}89}_{rad}) = 0{,}093\,m$$

$$v = \dot{x} = x_0\omega_0\cos\omega_0 t = 12\,cm\cdot 8{,}9\,s^{-1}\cos(0{,}89) = 67\,cm/s$$

$$a = \ddot{x} = -x_0\omega_0^2\sin\omega_0 t = -12\,cm\cdot 8{,}9^2 s^{-2}\sin(0{,}89 = -790\,cm/s^2$$

Akta

Nihalo na vijačno vzmet,
vijačna vzmet $k = 16\ N/m$
masa uteži $m = 0{,}2\ kg$.

Vzmet odmaknemo tako, da se v vzmeti nakopiči prožnostna energija 4 J. Kolikšna je amplituda nihanja in s kolikšno hitrostjo gre utež skozi ravnovesno lego, če ni izgub.

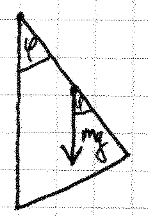
$$\frac{kx_0^2}{2} = W_0 \rightarrow x_0 = \sqrt{\frac{8\ J\ m}{16\ N}} = 0{,}7\ m$$

$$\frac{mv^2}{2} = W_0 \rightarrow v = \sqrt{\frac{2W_0}{m}} = \sqrt{\frac{8\ J}{0{,}2\ kg}} = 6{,}3\ m/s$$

$$v = x_0 \omega_0 =$$

$$= \sqrt{\frac{2W_k}{\cancel{k}}}\sqrt{\frac{\cancel{k}}{m}} = \sqrt{\frac{2W_k}{m}}$$

Drog je vrtljiv okoli osi, ki gre skozi njegovo zgornje krajišče, os pa je pravokotna na drog. Drog je dolg en meter. S kolikšnim t_0 zaniha, ko ga MALO odmaknemo iz ravnovesne lege.



$$M = J \cdot \alpha$$

$$-m \cdot g \cdot l/2 \cdot \sin\varphi = \frac{ml^2}{3}\ddot{\varphi}$$

$$\ddot{\varphi} + \frac{3g}{2l} \cdot \sin\varphi = 0 \longrightarrow \varphi(t)$$

$$\varphi(t) \ll 1$$

$$\varphi \approx \sin\varphi$$

$$\ddot{\varphi} + \frac{3g}{2l}\varphi = 0$$

$$\ddot{\text{odmik}} + k \cdot \text{odmik} = 0$$

$$\downarrow$$

$$\omega_0^2 = \left(\frac{2\pi}{t_0}\right)^2$$

$$\frac{3g}{2l} = \left(\frac{2\pi}{t_0}\right)^2 \rightarrow t_0 = 2\pi\sqrt{\frac{2l}{3g}}$$

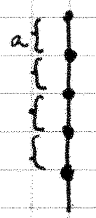
$$t_0 = 2\pi\sqrt{2\ \cancel{m}\ s^2 / 3 \cdot 9{,}81\ \cancel{m}}$$

$$\boxed{t_0 = 2\pi\sqrt{\frac{J}{mgr_t}}}$$

$J = ml^2/3$ $\quad r_t = l/2 \quad$ r težišča

$$\boxed{t_0 = 2\pi\sqrt{\frac{2l}{3g}}}$$

Homogen drog brez mase je vrtljiv okoli osi skozi zgornje krajišče. Na drog so v enakomernih razmikih pritrjene 4 uteži (razmik $a = 20\,cm$)



$t_0 = ?$

$$J = m_0 a^2 + m_0(2a)^2 + m_0(3a)^2 + m_0(4a)^2$$

$$t_0 = 2\pi\sqrt{\frac{J}{mgr_t}} \qquad r_t = \frac{\sum_i \vec{r}_i m_i}{\sum_i m_i}$$

Nehomogen drog $l = 1m$, vrtljiv je okoli krajišča. Gostota od osi vrtenja narašča po enačbi

$$\rho(x) = \rho_0(1 + x/l)$$

S kolikšnim t_0 zaniha drog.

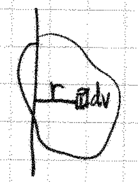
$$t_0 = 2\pi\sqrt{\frac{J}{mgr_t}}$$

$$m = \int_0^l \rho(x)\,dx = \rho_0\int_0^l \left(1 + \frac{x}{l}\right)dx = \rho_0\left(x + \frac{x^2}{2l}\right)\Big|_0^l =$$

$$\boxed{m = \rho_0\,(3l/2)} \qquad \vec{r}_t = \frac{\int \vec{r}\cdot\rho(\vec{r})\,dV}{\int \rho(\vec{r})\,dV}$$

$$r_t = \frac{\int_0^l x\rho(x)\,dx}{\int_0^l \rho(x)\,dx} = \frac{\rho_0\int_0^l x\left(1 + \frac{x}{l}\right)dx}{\frac{3}{2}l\rho_0} = \frac{2\left(\frac{x^2}{2} - \frac{x^3}{3l}\right)\Big|_0^l}{3l} =$$

$$= \frac{2\left(\frac{l^2}{2}+\frac{l^2}{3}\right)}{3l} = \frac{\cancel{2}\cdot\frac{5l^{\cancel{2}}}{\cancel{6}\,3}}{3\cancel{l}} = \boxed{\frac{5l}{9}} = r_t$$



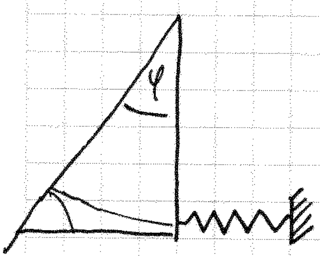
$$J = \int r^2 \rho(\vec r)\, dV$$

$$J = \rho_0\int_0^l x^2\left(1+\frac{x}{l}\right)dx = \rho_0\left(\frac{x^3}{3}+\frac{x^4}{4l}\right)\Big|_0^l = \boxed{\rho_0\,\frac{7l^3}{12}}$$

$$t_0 = 2\pi\sqrt{\frac{7l^3\cancel{\rho_0}\cdot 2\cdot 9}{12\cdot\cancel{\rho_0}\,3l\cdot 5l\cdot 9{,}81}} = \boxed{2\pi\sqrt{\frac{7l}{10g}}}$$

Homogen drog 1m, m=0,8kg. Vrtljiv je okoli vodoravne osi skozi krajišče. Na spodnje krajišče je pritrjena vzmet k=10N/m, da je v ravnovesni legi vodoravna.

S kolikšnim nihajnim časom zaniha ta drog, ko ga malo odmaknemo



↑ majhen odmik

$$M = J\cdot\alpha$$

$$-\left(mg\frac{l}{2}\sin\varphi + k\cdot x\, l\cos\varphi\right) = \frac{ml^2}{3}\cdot\ddot\varphi$$

$\varphi \ll 1 \rightarrow$ φ $\quad l\cdot\varphi \quad 1$

$$-\left(mg\frac{l^2}{2}\varphi + kl^2\varphi\right) = \frac{ml^2}{3}\ddot\varphi$$

$$-\varphi\cdot\cancel{l}\left(\frac{mg}{2}+kl\right) = \frac{ml^{\cancel{2}}}{3}\ddot\varphi$$

$$\ddot\varphi + k\varphi = 0 \qquad k \to (2\pi/t_0)^2$$

$$\ddot\varphi + \frac{3}{ml}\left(\frac{mg}{2}+kl\right)\varphi = 0$$

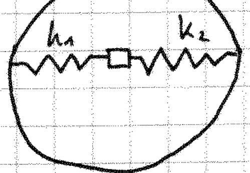
$$\frac{2\pi^2}{t_0} = \frac{3}{ml} \quad \frac{mg}{2} + 2kl = \frac{3mg+6kl}{2ml}$$

$$\boxed{t_0 = 2\pi\sqrt{\frac{2ml}{3mg+6kl}}}$$

Nihalo je sestavljeno iz uteži $m=0{,}3\,kg$ in dveh vijačnih vzmeti s koeficientoma $k_1 = 12\,N/m$, $k_2 = 15\,N/m$ (slika). Nahaja se na okrogli plošči, da v ravnovesju utež miruje v središču plošče. Plošča je vrtljiva okoli geometrijske osi. Na začetku utež in plošča mirujeta.

Plošča se začne vrteti s $\omega = 4\,s^{-1}$. Utež izmaknemo

$\boxed{t_0 = ?}$



$$m\ddot{x} = -k_1x - k_2x + m\omega^2 x$$

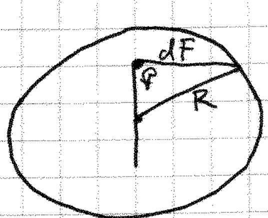
$$\ddot{x} + kx = 0$$

$$\ddot{x} + x\underbrace{\boxed{\frac{k_1+k_2-m\omega^2}{m}}}_{\omega_0^2} = 0$$

$$\left(\frac{2\pi}{t_0}\right)^2 = \frac{k_1+k_2-m\omega^2}{m}$$

$$t_0 = 2\pi\sqrt{\frac{m}{k_1+k_2-m\omega^2}}$$

Vesoljska postaja ima obliko tankega homogenega obroča z $m = 50\,000\,t$ in velikim polmerom



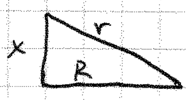
Neko telo, ki ima mnogo manjšo maso je gravitacijsko vjeto na osi postaje in niha pravokotno na ravnino postaje.

Koliko je t_0, če je amplituda $\ll R$!

$t_0 = ?$

$m\ddot{x}$

$$dF = \frac{G \cdot m\, dM}{r^2} \cdot \cos\vartheta$$



$$F = \frac{G m M \cos\vartheta}{r^2}$$

$$r = \sqrt{x^2+R^2}$$

$$\cos\vartheta = \frac{x}{\sqrt{x^2+R^2}}$$

$$= \frac{G M m x}{(x^2+R^2)^{3/2}}$$

$$m\ddot{x} = -\frac{G M m x}{(R^2+x^2)^{3/2}}$$

$$\ddot{x} + \frac{G M x}{(R^2+x^2)^{3/2}} = 0$$

$$\ddot{x} + Kx = 0$$

$x \ll R$

$$\ddot{x} + \frac{GM}{R^3}x = 0$$

$$\boxed{t_0 = 2\pi\sqrt{\frac{R^3}{GM}}}$$

Postaja ima radij 800 m,

Nihalo je sestavljeno iz majhne uteži z $m = 0{,}2\ \mathrm{kg}$ in $k = 16\ \mathrm{N/m}$
Odmaknemo $x_0 = 15\ \mathrm{cm}$ in spustimo, da zaniha, 3 s za tem ko je šlo prvič skozi ravnovesno lego, je amplituda nihanja samo še 4 cm.
Kolikšna sta β in t_0?

$$\boxed{x_1 = x_0 \cdot e^{-\beta t_1}}$$

$$\boxed{\beta = -\frac{\ln\frac{x_1}{x_0}}{t_1}} = \frac{-\ln\frac{4}{15}}{3\,\mathrm{s}} = 0{,}44\ \mathrm{s}^{-1}$$

$$\boxed{\omega = \sqrt{\omega_0^2 - \beta^2}} = \sqrt{\frac{k}{m} - \beta^2} = \sqrt{\frac{16}{0{,}2} - 0{,}44^2}$$

Z Newtonovim zakonom:

$$m\ddot{x} = -kx - A\cdot\dot{x}$$

$$\ddot{x} + \frac{k}{m}x + \frac{A}{m}\dot{x} = 0$$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0 \rightarrow x(t)$$

$$x(t) = e^{-\beta t} y(t)$$

$$\dot{x} = -\beta \cdot e^{-\beta t}\cdot y + e^{-\beta t}\cdot\dot{y} = e^{-\beta t}(\dot{y} - \beta y)$$

$$\ddot{x} = (-\beta)\cdot e^{-\beta t}(\dot{y} - \beta y) + e^{-\beta t}(\ddot{y} - \beta\dot{y}) =$$

$$= e^{-\beta t}(\ddot{y} - 2\beta\dot{y} + \beta^2 y)$$

$$/ \cdot e^{-\beta t}$$

$$\ddot{y} - \cancel{2\beta\dot{y}} + \beta^2 y \;\; \cancel{+2\beta\dot{y}} - 2\beta^2 y + y\omega_0^2 = 0$$

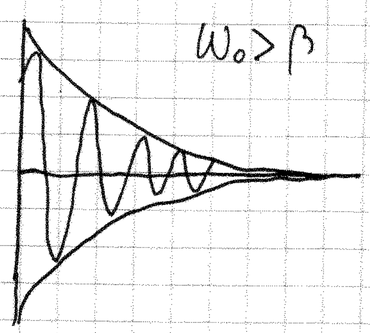
$$\ddot{y} + \underbrace{(\omega_0^2 - \beta^2)}_{} y = 0$$

$$\left(\frac{2\pi}{t_0}\right)^2 = \omega^2 = \sqrt{\omega_0^2 - \beta^2}$$

$$y = A\sin(\omega t + \delta)$$

$$x = A e^{-\beta t}\sin(\omega t + \delta)$$

$\omega_0 > \beta$



Neko nihalo odmaknemo za 20 cm od ravnovesne lege in pustimo, da zaniha. Čez 5 s je amplituda 7 cm. Po kolikšnem času se amplituda zniža na 2 cm?

$t_2 = ?$

$$x_1 = x_0 e^{-\beta t_1} \rightarrow \beta = -\frac{\ln \frac{x_1}{x_0}}{t_1} \rightarrow x_2 = x_0 e^{\frac{t_2}{t_1} \ln \frac{x_1}{x_0}}$$

$$x_2 = x_0 e^{-\beta t_2}$$

$$\ln \frac{x_2}{x_0} = \frac{t}{t_1} \ln \frac{x_1}{x_0}$$

$$t_2 = t_1 \cdot \frac{\ln \frac{x_2}{x_0}}{\ln \frac{x_1}{x_0}} = 5\,s \cdot \frac{\ln \frac{1}{10}}{\ln \frac{7}{20}} \approx 11\,s$$

! Nekemu nihalu, ki ima $\omega = 300$ rad/s in koef. dušenja 100 rad/s. Vsiljujemo nihanje s krožno frekvenco z $\omega_1 = 320\ s^{-1}$. Amplituda $A_1 = 6$ cm.

Kolikšna je amplituda, če frekvenco vsiljevanja zmanjšamo na 200/s. S kolikšno freq bi morali vsiljevati, da bi bila amplituda največja in kolikšna bi bila.

$$\boxed{A = \frac{\frac{F}{m}}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}}}$$

$$\frac{dA}{d\omega} = 0 \qquad A = \frac{F}{m}\left((\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2\right)^{-1/2}$$

$$\hookrightarrow = \left(-\frac{1}{2}\right)\left((\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2\right)^{-3/2}\left(2(\omega_0^2 - \omega^2)(-2\omega) + 8\beta^2\omega\right)$$

$$-4\omega_0^2\omega + 4\omega^3 + 8\beta^2\omega = 0$$

$$-\omega_0^2 + \omega^2 + 2\beta^2 = 0$$

$$\boxed{\omega = \sqrt{\omega_0^2 - 2\beta^2}}$$

18.12.07

vaje 11.1.08

$$A_{max} = \frac{\frac{F}{m}}{\sqrt{(\omega_0^2-\omega_0^2+2\beta^2)+4\beta^2(\omega_0^2-2\beta^2)}} = \frac{\frac{F}{m}}{\sqrt{4\beta^2(\omega_0^2-\beta^2)}}$$

$$F/m = A_1 \cdot \sqrt{(\omega_0^2-\omega_1^2)^2+4\beta^2\omega_1^2}$$

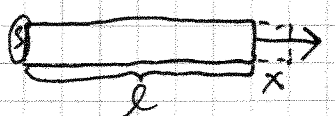
$$\boxed{A_{max} = \frac{A_1\sqrt{(\omega_0^2-\omega_1^2)^2+4\beta^2\omega_1^2}}{\sqrt{4\beta^2(\omega_0^2-\beta^2)}}}$$

$$A_1 = \frac{\frac{F}{m}}{\sqrt{(\omega_0^2-\omega_1^2)^2+4\beta^2\omega_1^2}}$$

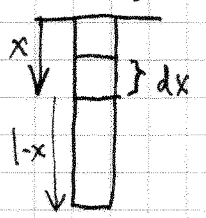
$$A_2 = \frac{F/m}{\sqrt{(\omega_0^2-\omega_2^2)^2+4\beta^2\omega_2^2}} = \boxed{\frac{A_1\sqrt{(\omega_0^2-\omega_1^2)^2+4\beta^2\omega_1^2}}{\sqrt{(\omega_0^2-\omega_2^2)^2+4\beta^2\omega_2^2}}}$$

Deformacije teles

1 m dolga svinčena palica ima gostoto $\rho = 11{,}3\ g/cm^3$ in prožnostni modul $E = 2\cdot 10^6\ N/cm^2$. Palico obesimo za eno krajišče na strop. Za koliko se podaljša zaradi lastne teže.



$$\frac{F}{S} = E\,\frac{x}{l}$$



$$\frac{F(x)}{S} = E\cdot\frac{du}{dx}$$

$$F(x) = \rho\cdot S\,(l-x)\cdot g$$

$$\rho g\,(l-x) = E\,\frac{du}{dx}$$

$$\int_0^u du = \frac{\rho g}{E}\int_0^l (l-x)\,dx$$

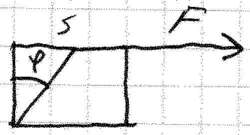
$$u = \frac{\rho g}{E}\,\frac{l^2}{2} = \boxed{2{,}77\cdot 10^{-6}\ m}$$

Pri kolikšni dolžini bi se pretrgala, če je meja natezne trdnosti 2000 N/cm²

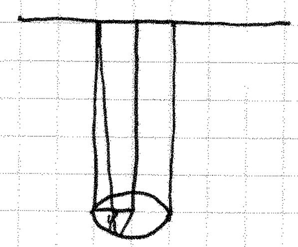
$\sigma = 2000\, N/cm^2$

$$\frac{F}{S} = \rho g l = \sigma \qquad \boxed{l = \frac{\sigma}{\rho g}}$$

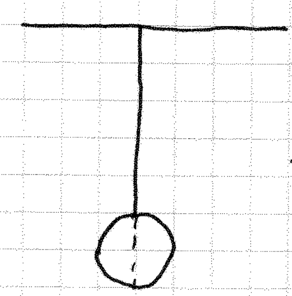
2 m dolgo žico s presekom $S = 10\, mm^2$. Strižni modul te žice je $G = 8 \cdot 10^4\, N/mm^2$. Na spodnje krajišče pritrdimo kroglo z $r = 8cm$ in $m = 0{,}7\, kg$. Zavrtimo in pustimo. Kolikšen je t_0 lastnega torzijskega nihanja.



$$\boxed{\frac{F}{S} = G \cdot \varphi}$$



$$M = D\varphi = \frac{G S^2}{2\pi l}\varphi = \frac{G \pi R^4}{2l}\varphi$$



newtonov zakon za vrtenje

$$M = J\alpha$$

$$-D\varphi = J \cdot \ddot{\varphi}$$

$$\ddot{\varphi} + \frac{D}{J}\varphi = 0$$

$$\left(\frac{2\pi}{t_0}\right)^2 = D/J$$

$$t_0 = \sqrt{\frac{J}{D}} \cdot 2\pi = \boxed{2\pi \sqrt{\frac{2mr^2 \, 2\pi l}{5 G S^2}}}$$

Stisljivost

$$\frac{\Delta V}{V} = -\chi \Delta p$$

$$\frac{dV}{V} = -\chi \, dp$$

$$\chi \, [m^2/N]$$

$N/m^2 = Pa$

$1 \, bar = 10^5 \, Pa$

$1 \, bar = 750 \, torr$

$1 \, mbar = 100 \, Pa$

V posodi je $30 \, dm^3$ etanola. Ko tlak v posodi zvišamo za 500 bar, se volumen zmanjša na $V_1 = 28,35 \, dm^3$. Kolikšna je stisljivost etanola?

$V_0 = 30 \, dm^3$
$\Delta p = 5 \cdot 10^7 \, N/m^2$
$V_1 = 28,35 \, dm^3$

$\chi = ?$

$$\frac{V_1 - V_0}{V_0} = -\chi \Delta p \rightarrow \boxed{\chi = \frac{V_0 - V_1}{V_0 \Delta p}} \quad 1,1 \cdot 10^{-9} \, m^2/N$$

$$\boxed{A = -\int_{V_{zač}}^{V_{konč}} p \, dV}$$

Ocenite koliko dela opravimo, ko 2 l vode s $\chi = 5 \cdot 10^{-10} \, m^2/N$ stisnemo iz začetnega $p_1 = 2 \cdot 10^5 \, N/m^2$ na $p_2 = 8 \cdot 10^5 \, N/m^2$.

Predpostavimo, da se volumen ni spremenil.

$$\frac{\Delta V}{V} = -\chi \Delta p$$

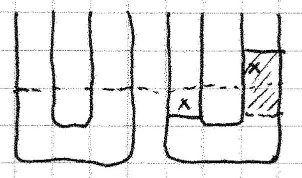
$$\Delta V = -\chi V \Delta p = -5 \cdot 10^{-10} \, m^2/N \cdot 2 \cdot 10^{-3} \, m^3 \cdot 6 \cdot 10^5 \, N/m^2 = 6 \cdot 10^{-7} \, m^3$$

$$A = p \, \Delta V = \frac{p_1 + p_2}{2} \Delta V = 5 \cdot 10^5 \, N/m^2 \; 6 \, 10^{-6} \, m^3 = \underline{0,3 \, J}$$

ali

$$A = -\int_{V_1}^{V_2} p \, dV = -\int_{p_1}^{p_2} p \frac{dV}{dp} dp = \chi V \int_{p_1}^{p_2} p \, dp = \frac{1}{2} \chi V (p_2^2 - p_1^2) = \underline{0,3 \, J}$$

V cevko oblike U, ki ima presek $1 cm^2$ nalijemo 80g vode, ki ima $\rho = 1 g/cm^2$



S kolikšnim ~~ta~~ to zaniha ~~ta~~ stolpec, ko ga spravimo iz ravnovesja?

$S = 1 cm^2$
$m = 80 g$
$\rho = 1 g/cm^3$

$t_0 = ?$

$F = m \cdot a = m \cdot \ddot{x}$

$-\rho g\, 2xS = m\ddot{x}$

$\ddot{x} + \left(\frac{2\rho g S}{m}\right) x = 0$

$\left(\frac{2\pi}{t_0}\right)^2 = \frac{2\rho g S}{m} \rightarrow \boxed{t_0 = 2\pi\sqrt{\frac{m}{2\rho g S}}}$

Oceni kolikšna sta gostota vode in hidrostatični tlak v oceanu na globini 8000 m.

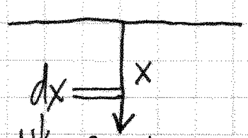
$x = 8000 m$
$\rho_0 = 1025 kg/m^3$ na gladini.

Stisljivost vode $\chi = 5 \cdot 10^{-10} m^2/N$.

Predpostavi, da χ ni odvisna od tlaka in globine

$p_0 = 10^5 N/m^2$

$p(x),\ \rho(x) = ?$



$dp = \rho(x) g\, dx$

$\frac{dV}{V} = -\chi\, dp$

$\frac{dV}{V} = -\chi g \cdot \rho(x) dx$

$m = \rho \cdot V$
$V = m/\rho$

$\frac{dV}{d\rho} = -\frac{m}{\rho^2} = -\frac{m}{\rho} \cdot \frac{1}{\rho} = -\frac{dV}{d\rho} \rightarrow \frac{dV}{V} = -\frac{d\rho}{\rho}$

$$\frac{d\rho}{\rho} = \chi g \rho dx$$

$$\int_0^x dx = \frac{1}{\chi g}\int_{\rho_0}^{\rho}\frac{d\rho}{\rho^2}$$

$$x = -\frac{1}{\chi g}\,\frac{1}{\rho}\Big|_{\rho_0}^{\rho} \qquad x = -\frac{1}{\chi g}\left(\frac{1}{\rho}-\frac{1}{\rho_0}\right)$$

$$-\chi g x \rho\rho_0 = \rho_0 - \rho$$

$$\rho(1-\chi g \rho_0 x) = \rho_0$$

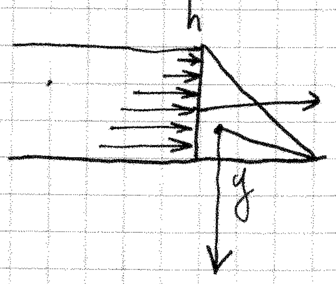
$$\boxed{\rho(x) = \frac{\rho_0}{1-\chi g \rho_0 x}}$$

$$\int_{p_0}^{p} dp = \rho_0 g \int_0^x \frac{dx}{1-\chi g \rho_0 x} \qquad \begin{aligned} 1-\chi g \rho_0 x &= y \\ -\chi g \rho_0 dx &= dy \\ dx &= -\frac{dy}{\chi g \rho_0} \end{aligned}$$

$$p - p_0 = -\frac{1}{\chi}\int_1^{1-\chi\rho_0 g x}\frac{dy}{y} = -\frac{1}{\chi}\log(1-\chi\rho_0 g x)$$

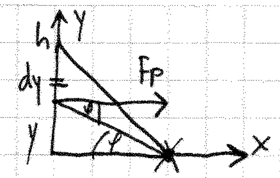
$$\boxed{p = p_0 - \frac{1}{\chi}\log(1-\chi g \rho_0 x)}$$

Betonski jez ima obliko ležeče tristrane prizme, ki ima za osnovno ploskev pravokotni trikotnik s katetama g in h.



Jez je do vrha poln vode, ki ima gostoto $1000\,kg/m^3$, beton pa ima $2700\,kg/m^3$.

Kolikšno mora biti razmerje med b in h, da bo navor teže jezu 3x večji od navora tlaka vode.



$$dF = p(y)\cdot dS = p(y)\cdot a \cdot dy$$

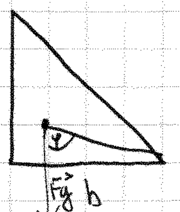
$$dF = \rho_0 g a (h-y)\, dy$$

$$dM_p = \rho_0 g a (h-y) dy \cancel{\sqrt{b^2+y^2}} \cdot \cancel{\sin \sqrt{b^2+y^2}} =$$

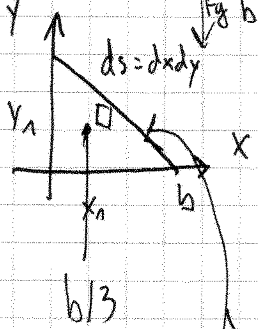
$$dM_p = \rho_0 g a\, y (h-y)\, dy$$

$$M_p = \rho_0 g a \int_0 (h-y) y\, dy = \rho_0 g a \left[\frac{h^3}{2} - \frac{h^3}{3}\right] =$$

$$= \boxed{\frac{\rho_0 g a h^3}{6}}$$ havor tlaha



$$M_g = \rho_1 \cdot \frac{b\cdot h}{2} a \cdot g \cdot r \cdot \sin\varphi$$



$$r_t = \frac{\int \vec{r} \rho(\vec{r}) dV}{\int \rho(\vec{r}) dV} = \frac{\frac{hb^2}{6}}{\frac{hb}{2}} = \boxed{\frac{b}{3}}$$

$$\vec{x}_t = \frac{\cancel{\rho}\iint_S x\, dS}{\cancel{\rho_1}\int dS} =$$

$$\int_0^{y(x)} dy \int_0^b x\, dx =$$

$$y(x) = -\frac{h}{b}x + h = h\left(1 - \frac{x}{b}\right)$$

$$\int_0^{h(1-x/b)} dy \int_0^b x\, dx = h\int_0^b \left(1 - \frac{x}{b}\right) x\, dx =$$

$$= h\left[\frac{b^2}{2} - \frac{b^2}{3}\right] = \boxed{\frac{hb^2}{6}}$$

$$y_T = \frac{\int y\,ds}{\int ds} = \frac{\frac{h^2 b}{6}}{\frac{hb}{2}} = h/3$$

$$Mg = \rho_1 \frac{bh}{2} a g r \sin\varphi = \rho_1 \frac{bh}{2} a g \cancel{\sqrt{\left(\frac{2b}{3}\right)^2 + \left(\frac{h}{3}\right)^2}} \cdot \frac{2b}{3\cancel{\sqrt{\left(\frac{2b}{3}\right)^2 + \left(\frac{h}{3}\right)^2}}}$$

$$\boxed{Mg = \frac{\rho_1 b^2 h a g}{3}}$$

$$Mg = 3 Mp$$

$$\frac{\rho_1 b^2 h a g}{3} = 3 \frac{\rho_0 g a h^3}{6}$$

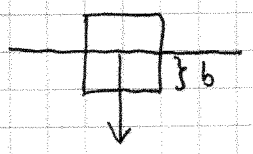
$$\frac{b^2}{h^2} = \frac{9 \rho_0}{6 \rho_1} = \frac{3 \rho_0}{2 \rho_1}$$

$$\frac{b}{h} = \sqrt{\frac{3 \rho_0}{2 \rho_1}}$$

Vzgon

$a = 10\,cm$
$\rho_1 = 0{,}7\,g/cm^3$
$\rho_0 = 1\,g/cm^3$

kako globoko je kocka potopljena v ravnovesni legi



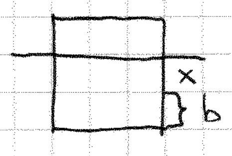
$$F_g = F_{vzg}$$

$$\rho_1 a^3 g = \rho_0 a^2 b \cdot g$$

$$\boxed{b = a^2 \frac{\rho_1}{\rho_0}}$$

Plavajočo kocko še malo potopimo. Kako zaniha?

$t_0 = ?$, če zanemorimo dušenje zaradi viskoznosti.



$$F = m \cdot a$$

$$\rho_1 \cdot a^3 \cdot \ddot{x} = -\rho_0 \cdot a^2 \cdot x \cdot g$$

$$\ddot{x} + \frac{\rho_0 g}{\rho_1 a} x = 0$$

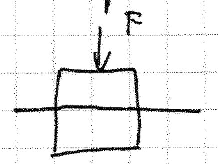
$$\left(\frac{2\pi}{t_0}\right)^2 = \frac{\rho_0 g}{\rho_1 a}$$

$$\boxed{t_0 = 2\pi \sqrt{\frac{\rho_1 a}{\rho_0 g}}}$$

Kocko potopimo tako, da je cela potopljena, nato pa spustimo.

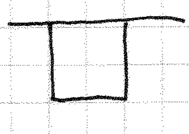
Do kolikšne h se kocka dvigne?

$A = W_p$



$$A = \int_0^{a-b} F(x)\,dx = \rho_0 a^2 g \int_0^{a(1-\frac{\rho_1}{\rho_0})} x\,dx = \frac{1}{2}\rho_0 a^4 g \left(1-\frac{\rho_1}{\rho_0}\right)^2$$

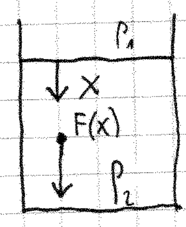
$$\frac{1}{2}\rho_0 a^4 g \left(1-\frac{\rho_1}{\rho_0}\right)^2 = \rho_1 a^3 g h$$



$$\boxed{h = \frac{\rho_0}{2\rho_1}\left(1-\frac{\rho_1}{\rho_0}\right)^2 a}$$

Gostota v 2m globoki posodi narašča linearno z globino in je na gladini enaka $1 g/cm^3$ in na dnu $1{,}2 g/cm^3$.

Koliko dela moramo opraviti, če želimo majhno kroglico z $V = 0{,}7 cm^3$ in $\rho_0 = 0{,}8 g/cm^3$ spraviti na dno te posode.



$$F(x) = F_{vzg}(x) - F_g \qquad A = \int_0^h F(x)\,dx$$

$$F(x) = \rho(x) \cdot V \cdot g - \rho_0 V \cdot g =$$

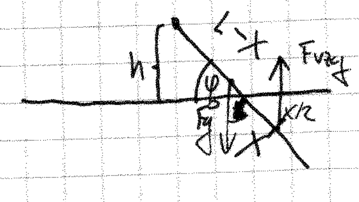
$$= V \cdot g\,(\rho(x) - \rho_0)$$

$$\rho(x) = \frac{\rho_2 - \rho_1}{h} \cdot x + \rho_1$$

$$= V \cdot g \left[\frac{\rho_2 - \rho_1}{h} x + \rho_1 - \rho_0\right]$$

$$A = \int_0^h \left(\frac{\rho_2 - \rho_1}{h} x + \rho_1 - \rho_0\right) dx = V g h \left[\frac{\rho_2 - \rho_1}{2} + \rho_1 - \rho_0\right] = \boxed{V g h \frac{\rho_2 + \rho_1 - 2\rho_0}{2}}$$

Homogena ravna tanka lesena palica je dolga 1m, njena gostota pa je $0{,}7 g/cm^3$. Palica je vrtljiva okoli vodoravne osi, ki je pravokotna na palico in gre skozi njeno krajišče, os pa se nahaja 30 cm nad gladino, voda pa ima $\rho = 1 g/cm^3$. Kolikšen kot oklepa palica z gladino v stabilni ravnovesni legi.



$$M_g = M_{vzg}$$

$$\rho_1 \cdot S \cdot l g \cdot \frac{l}{2} \sin\alpha = \rho_0 \cdot S x g \left(l - \frac{x}{2}\right) \sin\alpha$$

$$\rho_1 \frac{l^2}{2} = \rho_0 x (l - x/2)$$

$$\rho_1 l^2 = 2\rho_0 x l - \rho_0 x^2$$

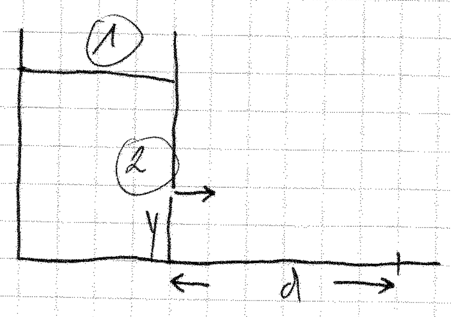
$$\rho_0 x^2 - 2\rho_0 l x + \rho_1 l^2 = 0$$

$$x_{1,2} = \frac{2\rho_0 l \pm \sqrt{4\rho_0^2 l^2 - 4\rho_0 \rho_1 l^2}}{2\rho_0}$$

$$x_{1,2} = l\left(1 \pm \sqrt{1 - \rho_1/\rho_0}\right)$$

8.1.08

Pokončna posoda je do višine 2m napolnjena z vodo. V stransko steno posode izvrtamo luknjico z majhnim prečnim presekom.
Na kolikšni višini moramo izvrtati luknjico, da bo domet iztekajočega curka največji.



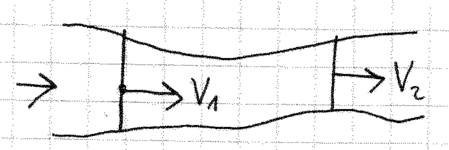
$D = v \cdot t$

$y = gt^2/2 \rightarrow t = \sqrt{2y/g}$

$D = v\sqrt{\frac{2y}{g}}$

bernoullijeva enačba

$$\frac{1}{2}\rho v_1^2 + \rho g h_1 + p_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2 + p_2$$



$$\rho g H = \frac{1}{2}\rho v^2 + \rho g y$$

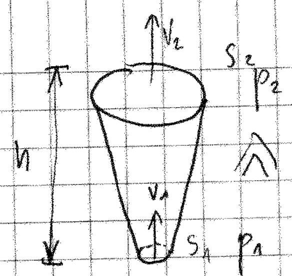
$$v = \sqrt{2g(H-y)} \longrightarrow v\sqrt{\frac{2y}{g}} \rightarrow = \sqrt{4y(H-y)}$$

$$\frac{dD}{dy} = 0 = \frac{1}{2}\left(4y(H-y)\right)^{-1/2}\underbrace{\left(4(H-y)+4y(-1)\right)}_{0} = 0$$

$H - 2y = 0$

$\boxed{y = H/2}$ $\quad D_{max} = \sqrt{4\frac{H}{2}\frac{H}{2}} = \boxed{H}$

Navpična cev je visoka 2m. Na spodnjem koncu ima presek $50\ cm^2$, na zgornjem $90\ cm^2$. Vodo z $\rho = 1000\ kg/m^3$ poganja navzgor tlačna razlika $\Delta p = 95000\ N/m^2$



$$\frac{1}{2}\rho v_1^2 + 0 + p_1 = \frac{1}{2}\rho v_2^2 + \rho g h + p_2$$

$$\frac{1}{2}\rho v_1^2 + \underbrace{p_1 - p_2}_{\Delta p} = \frac{1}{2}\rho v_2^2 + \rho g h$$

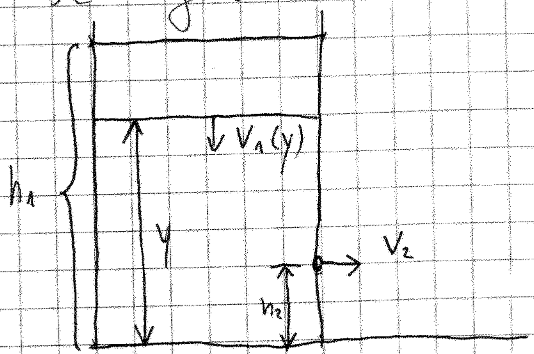
$$\rho v_1^2 + 2\Delta p = \rho v_2^2 + 2\rho g h \qquad \text{z hitrosti izrazimo}$$

pretok $\quad S_1 v_1 = S_2 v_2 \longrightarrow \quad v_1 = v_2 \frac{S_2}{S_1}$

$$\rho v_2^2 \frac{S_2^2}{S_1^2} + 2\Delta p + \rho v_2^2 + 2\rho g h \cdot$$

$$v_2^2 = \frac{2(\rho g h - \Delta p)\, S_1^2}{\rho(S_2^2 - S_1^2)}$$

Pokončna posoda $S = 800\,cm^2$ je do $h = 2m$ napolnjena z vodo. Na $h = 40\,cm$ od dna izvrtamo luknjico $S = 1\,cm^2$. V kolikšnem času se gladina vode zniža na polovico.



$$\frac{1}{2}\rho \cancel{v_1}^2 + \rho g y = \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$S_1 v_1 = S_2 v_2$$

$$v_1 = -\frac{dy}{dt}$$

$$v_2 = v_1 \frac{S_1}{S_2}$$

$$\cancel{2} v_1^2 + 2gy = v_1^2 \frac{S_1^2}{S_2^2} + 2gh_2$$

$$v_1^2\left(1 - \frac{S_1^2}{S_2^2}\right) = 2g(h_2 - y)$$

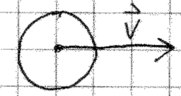
$$v_1 = \sqrt{\frac{2g S_2^2 (h_2 - y)}{S_2^2 - S_1^2}}$$

$$\sqrt{\frac{2g S_2^2 (y - h_2)}{S_1^2 - S_2^2}} = -\frac{dy}{dt} \quad \longrightarrow \quad y(t)$$

$$dt = -\sqrt{\frac{S_1^2 - S_2}{2g S_2^2}} \frac{dy}{\sqrt{y - h^2}} \qquad \int_0^t dt = -\sqrt{\frac{S_1^2 - S_2^2}{2g S_2^2}} \int_{h_1}^{h_3} \frac{dy}{\sqrt{y - h_2}}$$

$$y - h_2 = x$$
$$dy = dx$$

$$t = \sqrt{\frac{S_1^2 - S_2^2}{2 g S_2^2}} \int_{h_1-h_2}^{h_3-h_2} \frac{dx}{\sqrt{x}} = -\sqrt{\frac{2(S_1^2-S_2^2)}{g S_2^2}}\, \sqrt{x}\,\Big|_{h_1-h_2}^{h_3-h_2}$$

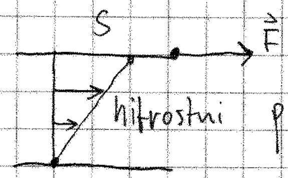


zastojni tlak

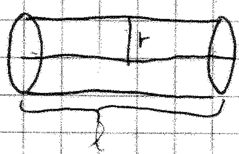
$p = \frac{1}{2}\rho v^2$

$F_u = \frac{1}{2}\rho v^2 \cdot S \cdot c$ (faktor upora - empirični)

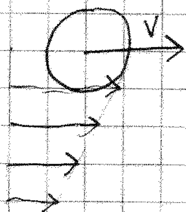
Viskoznost



$$\frac{F}{S} = \eta \frac{dv}{dz} \quad \left[\frac{kg}{m\,s}\right]$$



$$\boxed{\Phi_v = \frac{\pi r^4 \Delta p}{8\eta \cdot l}}$$



kroglica $\boxed{F_u = 6\pi r \eta v}$ linearni zakon upora

reynoldsovo število

$$\boxed{Re = \frac{D\rho v}{\eta}}$$

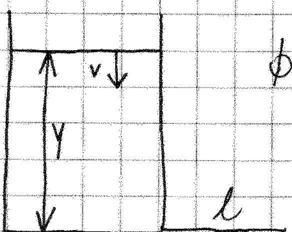
$Re < 1$ linearni zakon

$Re > 1000$ kvadratni zakon

Posoda s $S = 600\,cm^2$ je do $h = 2\,m$ napolnjena z vodo $\rho = 1000\,kg/m^3$ in $\eta = 10^{-3}\,kg/ms$

Tik ob dnu skozi steno vstavimo 15 cm dolgo kapilaro s polmerom $r = 0{,}7\,mm$.

Po kolikšnem času se gladina zniža za polovico. Predpostavi, da za volumski pretok skozi kapilaro velja Poiseuilleov zakon.



$\phi_v = S \cdot v$

$$\phi_v = \frac{\pi r^4 \Delta p}{8\eta l} = \frac{\pi r^4}{8\eta l}\overbrace{\rho g y}^{\Delta p}$$ tlačna razlika

$$v = -\frac{dy}{dt}$$

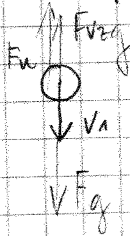
$$-S\cdot\frac{dy}{dt} = \frac{\pi r^4 \rho g}{8\eta l}\, y$$

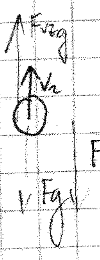
$$\int_0^t dt = -\frac{8S\eta l}{\pi r^4 \rho g}\int_{h_1}^{h_2}\frac{dy}{y} \qquad t = \frac{8S\eta l}{\pi r^4 \rho g}\ln\frac{h_1}{h_2}$$

Majhna kroglica z $r = 0{,}4\,mm$ in $m = 5{,}2\,mg$ v tekočini pada z $v = 6{,}2\,mm/s$

Druga kroglica z enakim polmerom in $m = 0{,}17\,mg$ pa se v tej tekočini dviga z $v = 0{,}04\,mm/s$

Kolikšni sta ρ in η te tekočine. Velja linearni zakon upora





① $m_1 g - \frac{4}{3}\pi r^3 \rho g - 6\pi r\eta v_1 = 0$

② $m_2 g - \frac{4}{3}\pi r^3 \rho g + 6\pi r\eta v_2 = 0$

$$g(m_1 - m_2) - 6\pi r\eta(v_1 + v_2) = 0$$

$$\boxed{\eta = \frac{g(m_1 - m_2)}{6\pi r(v_1 + v_2)}}$$ enako za ρ.

$r = 0{,}6\,mm$
$m = 0{,}4\,mg$ spustimo v tekočini z $\rho = 0{,}9\,g/cm^3$ in viskoznostjo $\eta = 4\,kg/ms$

Po kolikšnem času doseže 99% svoje končne hitrosti? Velja linearni zakon upora.

$$mg - \frac{4}{3}\pi r^3 \rho g - 6\pi r\eta v = m\frac{dv}{dt}$$

$$\int_0^t dt = \int_0^v \frac{m\,dv}{mg - \frac{4}{3}\pi r^3 \rho g - 6\pi r \eta v}$$

$$mg - \frac{4}{3}\pi r^3 \rho g - 6\pi r\eta v = x$$

$$-6\pi r\eta\, dv = dx$$
$$dv = -\frac{dx}{6\pi r\eta}$$

$$t = -\frac{m}{6\pi r\eta}\int_{mg-4/3\pi r^3\rho g}^{mg-\frac{4}{3}\pi r^3\rho g-6\pi r\eta v}\frac{dx}{x}$$

$$t = -\frac{m}{6\pi r\eta}\ln\frac{mg-\frac{4}{3}\pi r^3\rho g-6\pi r\eta v}{mg-4/3\pi r^3\rho g}$$

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veliko sreče

pri učenja

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