FIZIKA II
zapiski z avditornih vaj

Šolsko leto 2007 / 2008
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UREJANJE DOKUMENTA

VERZIJA 01 **REVIZIJA** 01
DATUM 1. 3. 2009

ZADNJI POPRAVLJAL /
PREGLEDAL Blaž Potočnik, Aljoša Praznik

OPOMBE

POPRAVKI

www.stromar.si
zbirka študijske literature na spletu

v dokumentu lahko obstajajo napake

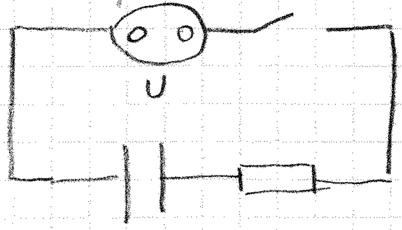
FIZ 2 - GYERGYEK

07/08 VAJE

U^2

POLNENJE IN PRAZNENJE KOND.

Kondenzator s kapaciteto 0,2 μF in 3 MΩ upor vežemo zaporedno in priključimo na baterijo. Po kolikšnem času po priključitvi je napetost na kondenzatorju 3× večja od padca napetosti na uporu.



$$U_C = U\cdot(1-e^{-\frac{t}{RC}})$$

$$U_U = U\cdot e^{-\frac{t}{RC}}$$

$$U = 3\,U_U$$

$$U_C$$

$$U(1-e^{-\frac{t}{RC}}) = 3\,U\,e^{-\frac{t}{RC}}$$

$$1 = 4\,e^{-\frac{t}{RC}}$$

$$t = -RC\,\ln\frac{1}{4}$$

izpeljava tega iz Kirchovega zakona

$$U + U_R + U_C = 0$$

$$U - I_{(t)}R - \frac{e_{(t)}}{C} = 0 \qquad I = \frac{de}{dt}$$

$$I = \frac{de}{dt}$$

$$U - R\frac{de}{dt} - \frac{e}{C} = 0 \qquad (\cdot C)(t)$$

$$U - \frac{e}{C} = R\cdot\frac{de}{dt}$$

$$dt = \frac{R\cdot de}{U-\frac{e}{C}} = \frac{R\cdot C\cdot de}{U\cdot C - e}$$

$$\int_0^t dt = RC\int_0^e \frac{de}{UC-e} \qquad UC - e = x,\quad dx = -de$$

$$= -RC\int_{UC}^{UC-e}\frac{dx}{x}$$

$$t = -RC\,\ln\frac{UC-e}{UC} \qquad e = Q$$

$$e^{-\frac{t}{RC}} = \frac{UC-Q}{UC} \qquad Q = UC(1-e^{-\frac{t}{RC}})$$

$$e^{-\frac{t}{RC}} = 1 - \frac{Q}{UC} \qquad U_C = \frac{e}{C} = U(1-e^{-\frac{t}{RC}})$$

$$I = \frac{de}{dt} = U C \frac{1}{RC} e^{-\frac{t}{RC}}$$

$$= \frac{U}{R} e^{-\frac{t}{RC}}$$

$$U_R = I \cdot R = U \cdot e^{-\frac{t}{RC}}$$

2. način

$$U - I(t) \cdot R - \frac{e(t)}{C} = 0 \quad / \frac{d}{dt}$$

$$-\frac{dI}{dt} R - \frac{I}{C} = 0$$

$$\frac{dI}{dt} R = -\frac{I}{C}$$

$$\int_0^t dt = -RC \int_{\frac{U}{R}}^{I} \frac{dI}{I}$$

$$t = -RC \cdot \ln \frac{IR}{U}$$

$$I = \frac{U}{R} e^{-\frac{t}{RC}}$$

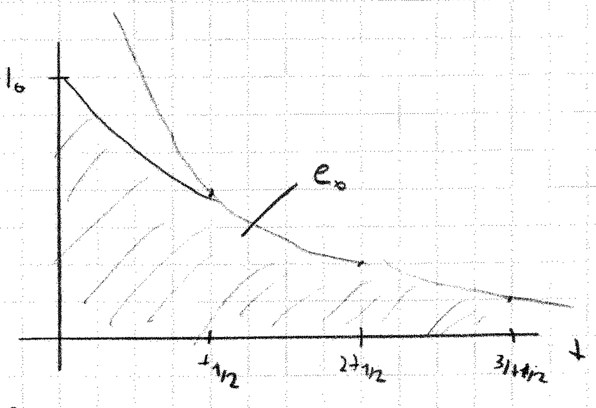
Fožo $R = 8\,\Omega$

$e_0 = 30\,As$

$t_{1/2} = 24\,s$

$Q = A_{el} = ?$

$$I(t) = I_0 e^{\frac{t}{\tau}}$$



$$Q = A_{el} = \int_0^\infty P\,dt = \int_0^\infty R \cdot I^2(t) \cdot dt$$

$$= \int_0^\infty R \cdot I_0^2 \cdot e^{-\frac{2t}{\tau}} dt$$

$$= R \cdot I_0^2 \int_0^\infty e^{-\frac{2t}{\tau}} dt$$

$$= R \cdot I_0^2 \left. \frac{e^{-\frac{2t}{\tau}}}{-\frac{2}{\tau}} \right|_0^\infty$$

$$= -\frac{\tau R \cdot I_0^2}{2} \cdot [\overbrace{e^{-\infty}}^{0} - \overbrace{e^{-0}}^{1}]$$

$$= \frac{\tau R \cdot I_0^2}{2} = \dots$$

$t = t_{1/2}$: $\frac{I_0}{2} = I_0 \cdot e^{-\frac{t_{1/2}}{\tau}}$

$$e^{\frac{t_{1/2}}{\tau}} = 2 \quad \ln$$

$$\frac{t_{1/2}}{\tau} = \ln 2$$

$$\tau = \frac{24\,s}{\ln 2}$$

$$\Rightarrow (e^{\ln 2})^{-\frac{t}{t_{1/2}}}$$

$$2^{-\frac{t}{t_{1/2}}}$$

$$e_0 = \int_0^\infty I \cdot dt$$

$$= \int I_0 e^{-\frac{t}{\tau}} = I_0 \left. \frac{e^{-\frac{t}{\tau}}}{-\frac{1}{\tau}} \right|_0^\infty = -I_0 \tau [0 - 1]$$

$$e_0 = I_0 \cdot \tau \quad \Rightarrow \quad I_0 = \frac{e_0}{\tau}$$

ELEKTRIČNO POLJE DIPOLA

FIZ II

električni dipol je sestavljen iz 2 točkastih nalojev e_1 in e_2, ki sta med seboj razmaknjena 1mm. Kolikšna je jakost E v točki, ki je oddaljena 10 cm od dipola, zveznica pa oklepa kot 20° (med središčem in premico)

F_1, F_2

$\sin\vartheta(\cos\vartheta - \ldots)$

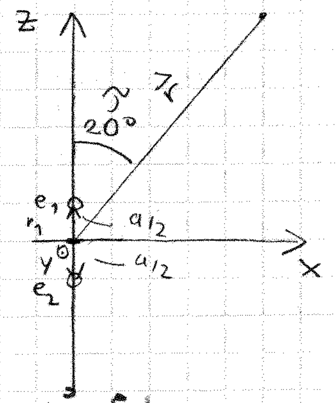
$e_1 = 2\cdot 10^{-8}$ As

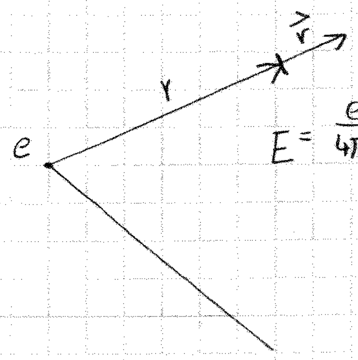
$\underline{e_2 = 2\cdot 10^{-8} \text{ As}}$

$r = 10$ cm

$a = 1$ mm

20°

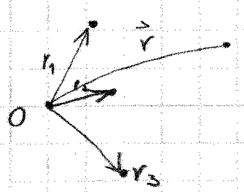




$$E = \frac{e}{4\pi\varepsilon_0 r^2}\frac{\vec r}{r}$$

nasprotno znak

$$E = \frac{e_1}{4\pi\varepsilon_0}\frac{\vec r - \vec r_1}{|r-r_1|^3} + \frac{e_2}{4\pi\varepsilon_0}\frac{|r-\vec r_2|}{|r-r_2|}$$



$$E = \frac{e_1}{4\pi\varepsilon_0}\frac{\vec r - \vec r_1}{|r-r_1|^3} + \frac{e_2}{4\pi\varepsilon_0}\frac{\vec r - \vec r_2}{|\vec r - \vec r_2|^3} + \ldots$$

$$E = \frac{e}{4\pi\varepsilon_0}\left(\frac{\vec r - \vec r_1}{|r-r_1|^3} - \frac{\vec r - \vec r_2}{|r-r_2|^3}\right)$$

$\Rightarrow$ na koncu mora priti

$$E = \sqrt{E_x + E_y + E_z}$$

$r_1 = (0, 0, \frac{a}{2})$

$r_2 = (0, 0, -a/2)$

$r = (r\cdot\sin\vartheta, 0, r\cdot\cos\vartheta)$

$$r^3\left(1 - \frac{3a}{2r\cos\vartheta}\right) = |r - r_1|^3 = \left((r_x - r_{1x})^2 + (r_y - r_y)^2 + (r_z - r_{1z})^2\right)^{3/2}$$

$$= \left(r^2\sin^2\vartheta + (r\cos\vartheta - a/2)^2\right)^{3/2}$$

$$= \left(\underbrace{r^2\sin^2\vartheta + r^2\cos^2\vartheta}_{r^2} - ar\cos\vartheta + a^2/4\right)^{3/2}$$

$$= r^3\left(1 - \frac{a}{r}\cos\vartheta + \cancel{\frac{1}{4}\frac{a^2}{r^2}}\right)^{3/2}$$

temu rečem linearizacija

$$\approx r^3\left(1 - \frac{a}{r}\cos\vartheta\right)^{3/2}$$

$$\approx r^3\left(1 - \frac{3a}{2r}\cos\vartheta\right)$$

Taylorjeva vrsta

$$|r - r_2|^3 = r^3\left(1 + \frac{3a}{2r}\cos\vartheta\right)$$

ostane se samo predznak pri $-a/2$

$$\frac{a}{r} = 10^{-2}$$

$$\frac{a^2}{r^2} = 10^{-4}$$

$$E = \frac{e}{4\pi\varepsilon_0 r^3}\left(\frac{\vec r - \vec r_1}{1 - 3a/2r\cdot\cos\vartheta} - \frac{r - r_2}{1 + 3a/2r\cdot\cos\vartheta}\right)$$

$$E = \frac{e}{4\pi\varepsilon_0 r^3}\frac{(\vec r - \vec r_1)(1 + 3a/2r\cdot\cos\vartheta) - (\vec r - \vec r_2)(1 - 3a/2r\cos\vartheta)}{1 - \cancel{9a^2/4r^2\cos\vartheta}}$$

$$E = \frac{e}{4\pi\varepsilon_0 r^3}\left(\cancel{\vec r} - \vec r_1 + \frac{3a}{2r}\cos\vartheta\frac{r}{r} - \frac{3a}{2r}\cos\vartheta\frac{\vec r_2}{r} - \cancel{\vec r} + \vec r_2 + \frac{3a}{2r}\cos\vartheta\frac{r}{r} - \frac{3a}{2r}\cos\vartheta\, r_2\right)$$

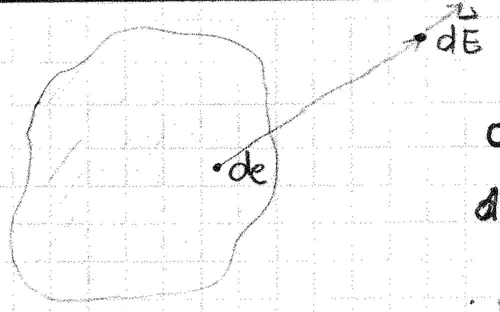
$$E = \frac{e}{4\pi\varepsilon_0 r^3}\left(3a\cos\vartheta\frac{\vec r}{r} + \vec r_2 - \vec r_1\right)\left(\frac{3a}{2r}\cos\vartheta(\vec r_1 + \vec r_2)\right)$$

$$E_x = \frac{3ea\sin\vartheta\cdot\cos\vartheta}{4\pi\varepsilon_0 r^3}$$

$$E_y = 0$$

$$E_z = \frac{e}{4\pi\varepsilon_0 r^3}(3a\cos^2\vartheta - a)$$

$$= \frac{ea(3\cos^2\vartheta - 1)}{4\pi\varepsilon_0 r^3}$$

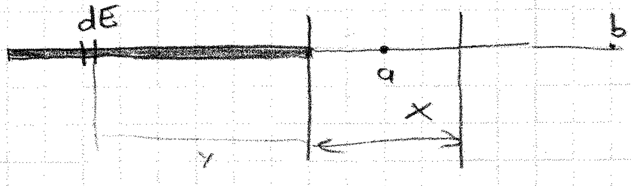


$$d\vec{E} = \frac{de}{4\pi\varepsilon_0 r^2}\frac{\vec{r}}{r}$$

$$d\vec{E} = \int \frac{\rho(r)\cdot dV}{4\pi\varepsilon_0 r^2}\frac{\vec{r}}{r}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r)\cdot dV\cdot\vec{r}}{r^3}$$

Po 50 cm dolgem ravnem tankem vodniku je enakomerno porazdeljen naboj $+10^{-7}$ As/cm vodnika. Točkasti naboj $-3\cdot 10^{-7}$ As se nahaja na isti premici kot vodnik in je 10 cm oddaljen od krajišča vodnika. Koliko dela moramo opraviti, če ga želimo spraviti na 20 cm od vodnika. (krajišča)



$$dE = \frac{de}{4\pi\varepsilon_0 (x+y)^2} = \frac{\mu\cdot dy}{4\pi\varepsilon_0 (x+y)^2}$$

$$E = \frac{\mu}{4\pi\varepsilon_0}\int_0^l \frac{dy}{(x+y)^2} = \frac{\mu}{4\pi\varepsilon_0}\int_{x}^{x+l}\frac{du}{u^2}$$

$x+y = u$

$dy = du$

$$= \frac{\mu}{4\pi\varepsilon_0}\left(-\frac{1}{u}\right)\Big|_0^{x+l}$$

$$E = \frac{\mu}{4\pi\varepsilon_0}\left(-\frac{1}{x+l}+\frac{1}{x}\right)$$

$$E = \frac{\mu}{4\pi\varepsilon_0}\,\frac{-x+x+l}{x\cdot(x+l)}$$

$$E = \frac{\mu\cdot l}{4\pi\varepsilon_0}\,\frac{l}{x(x+l)}$$

$$F_x = \frac{\mu\cdot e\cdot l}{4\pi\varepsilon_0 x\cdot(x+l)}$$

$$A = -\int_a^b F(x)\,dx = -\frac{\mu\cdot e\cdot l}{4\pi\varepsilon_0}\int_a^b \frac{dx}{x(x+l)}$$

$$= -\frac{\mu\cdot e\cdot l}{4\pi\varepsilon_0}\left(\frac{1}{l}\int_a^b \frac{dx}{x\cdot l} - \frac{1}{l}\int \frac{dx}{x+l}\right)$$

$$= -\frac{\mu\cdot e\cdot \cancel{l}}{4\pi\varepsilon_0}\left(\frac{1}{l}\ln\frac{b}{a} - \ln\frac{b+l}{a+l}\right)$$

$$= \frac{\mu\cdot e}{4\pi\varepsilon_0}\ln\frac{b\cdot(a+l)}{a(b+l)}\Big]$$

$$\frac{A}{x}+\frac{B}{x+l}$$

$$\frac{Ax+Al+Bx}{x\,(x+l)}$$

$(A+B)=0$

$Al = 1$

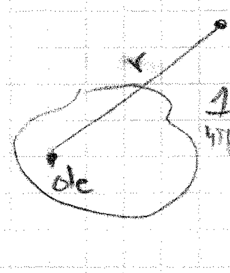
$A = \frac{1}{l} \quad B = -\frac{1}{l}$

lahko pa s potencialom

$$V = \frac{e}{4\pi\varepsilon_0 r}$$

$$W_p = e_2\cdot V = \frac{e\,e_2}{4\pi\varepsilon_0 r}$$

$$dV = \frac{de}{4\pi\varepsilon_0 r}$$



$$\frac{1}{4\pi\varepsilon_0}\int\frac{\rho(r)\,dV}{r}$$

4

$$dU = \frac{de}{4\pi\varepsilon_0 (x+y)} = \frac{\mu\, dy}{4\pi\varepsilon_0 (x+y)}$$

$$= \int_0^l \frac{\mu\, dy}{4\pi\varepsilon_0 (x+y)} = \frac{\mu}{4\pi\varepsilon_0} \int_0^l \frac{dy}{x+y} = \frac{\mu}{4\pi\varepsilon_0} \ln\left|\frac{x+l}{x}\right|$$

$$A = W_p(b) - W_p(a) = \frac{\mu \cdot e}{4\pi\varepsilon_0} \ln\left(\frac{b+l}{b}\right) - \frac{\mu e}{4\pi\varepsilon_0} \ln\left(\frac{a+l}{a}\right)$$

$$= \frac{\mu_0 e}{4\pi\varepsilon_0} \ln \frac{ab+al}{ab+bl}$$

Po zelo dolgem ravnem tankem vodniku je enakomerno porazdeljen naboj $+10^{-7}$ As na centimeter. Negativni točkasti naboj $5 \cdot 10^{-9}$ As miruje v točki, ki je 10 cm oddaljena od vodnika in enako oddaljena od krajišč. Koliko dela moralo opraviti, da premaknemo za 20 cm.

$$E(y) = \frac{\mu}{2\pi\varepsilon_0 y} \qquad F(y) = \frac{\mu e}{2\pi\varepsilon_0 y} \qquad A = \int_a^b F(y)\, dy = -\frac{\mu e}{2\pi\varepsilon_0} \ln \frac{b}{a}$$

FIZ II
22. 2. 2008

KINETIČNA TEORIJA PLINOV

p, V, T ↓ → ↑↘

$$pV = \frac{m}{M} RT$$

M – masa kilomola

R – splošna plinska konstanta.

N – št. gradnikov

$N_A = 6{,}02 \cdot 10^{26}$ kmola
$k_B = 1{,}38 \cdot 10^{-23}$ J/K

$$R = N_A \cdot k_B$$

k_B – BOLTZMANOVA

$$pV = \frac{N \cdot m_1}{N_A \cdot m_1} N_A \cdot k_B \cdot T$$

$$pV = N \cdot k_B T$$

$$\langle \vec{v} \rangle = 0 \qquad \frac{1}{2} m_1 \langle v_x^2 \rangle = \frac{1}{2} k_B T$$

$$\langle v^2 \rangle \neq 0$$

$$\langle W_{k\,trans} \rangle = \frac{1}{2} m_1 \langle v^2 \rangle = \frac{1}{2} m_1 \langle v_x^2 \rangle + \frac{1}{2} m_1 \langle v_y^2 \rangle + \frac{1}{2} m_1 \langle v_z^2 \rangle$$
$$= \frac{3}{2} kT$$

če je 2-atomna molekula – rotacija

$$\langle W_{k\,rot.} \rangle = \frac{2}{2} k_B T$$

Imamo kisik (O_2). Kolikšno celotno povprečno kinetično energijo imajo molekule pri 20°C celzija. Kolikšna je povprečna usrezna hitrost.

$T = 20°C$
$M_{O_2} = 32$ kg/kmol

$\langle W_k \rangle = ?$
$\langle v \rangle = ?$

$$\langle W_k \rangle = \frac{5}{2} k_B T$$
$$= \frac{5}{2} \cdot 1{,}38 \cdot 10^{-23}\,\text{J/K} \cdot 293\,\text{K}$$
$$\approx 10^{-20}\,\text{J}$$

$$\frac{1}{2} m_1 \langle v^2 \rangle = \frac{3}{2} k_B T$$

$$\langle v^2 \rangle = \frac{3 k_B \cdot T}{m_1} \qquad m_1 = \frac{M_{O_2}}{N_A}$$

$$\langle v \rangle = \sqrt{\frac{3 k_B \cdot T \cdot N_A}{M_{O_2}}}$$

Kolikšen je volumen posode, v kateri je $N = 2 \cdot 10^{20}$ molekul kisika, s povprečnim kvadratom hitrosti $3 \cdot 10^4$ m²/s². V posodi je tlak $1{,}20 \cdot 10^3$ Pa

$p, N, \langle v^2 \rangle$

$$pV = N \cdot k_B T \qquad \langle v^2 \rangle = \frac{3 k_B \cdot T}{m_1}$$

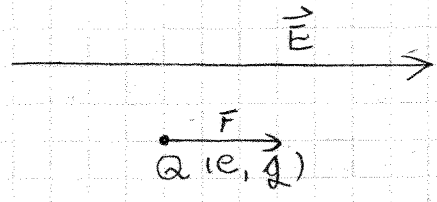
$$V = \frac{2 \cdot 10^{20} \cdot \cancel{1{,}38 \cdot 10^{-23}} \langle v^2 \rangle \cdot M_{O_2}}{1{,}28 \cdot 10^{3} \cdot 3 \cancel{k_B} \cdot N_A} \qquad = 3RT / M_{O_2}$$

$$V = 9{,}96 \cdot 10^{-4} \cong 1\,\text{dm}^3 \qquad T = \frac{\langle v^2 \rangle \cdot M_{O_2}}{3R = 3 \cdot k_B \cdot N_A}$$

6

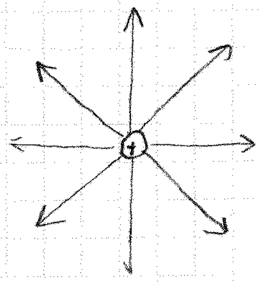
EL POLJE

$F = \vec{E} \cdot \vec{e}$

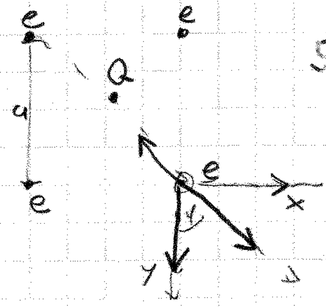


POLJE TOČKASTEGA NABOJA:

$E = \frac{e}{4\pi\varepsilon_0 r^2} \frac{\vec{r}}{r}$



4. Identični točkasti naboji se nahajajo v ogliščih kvadrata $e_1 = e_2 = e_3 = e_4 = 1\,C$. Kam moramo postaviti še 5 naboj da bo sistem v ravnovesju



$e = ?$ - bo v središču.

v ravnovesju: $\varphi = 45°$

$\sum_i F_i = 0$

$x: F_1 + F_2 \cdot \cos 45° - F_Q \cdot \cos 45° = 0$

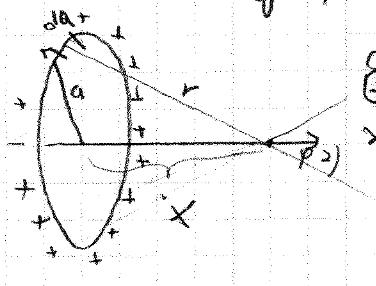
$\frac{e^2}{4\pi\varepsilon_0 a^2} + \frac{e^2}{4\pi\varepsilon_0 (a\sqrt{2})^2} \cdot \frac{\sqrt{2}}{2} - \frac{e \cdot Q \sqrt{2}}{4\pi\varepsilon_0 \frac{a\sqrt{2}}{2} \cdot 2} = 0$

$\frac{e^{\cancel{2}}}{\cancel{4\pi\varepsilon_0}} \left(\frac{1}{a^2} + \frac{\sqrt{2}}{4a^2}\right) = \frac{\cancel{e} \cdot Q \frac{\sqrt{2}}{2}}{\cancel{4\pi\varepsilon_0} a^2}$

$Q = \frac{e\left(\frac{1}{a^2} + \frac{\sqrt{2}}{4a^2}\right) \cdot a}{\frac{\sqrt{2}}{a^2}}$

$Q = \frac{e + \frac{\sqrt{2}}{4}}{\sqrt{2}} \qquad Q = \frac{e}{\sqrt{2}} + \frac{e}{4} = 0{,}957$

Izračunaj polje na osi enakomerno naelektrene zanke:



$a = 1\,m$
$Q = 10^{-7}\,As$
$x = 3\,m$

$E = \int dE \cdot \cos\varphi$

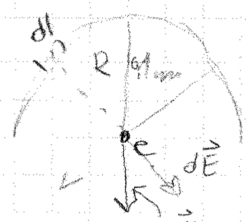
$\cos\varphi = \frac{x}{r} \qquad r = x^2 + a^2$

$dE = \int \frac{dQ}{4\pi\varepsilon_0 r^2} \cdot \frac{x}{\sqrt{x^2 + a^2}}$

$dE = \int \frac{dQ}{4\pi\varepsilon_0} \frac{x}{(x^2+a^2)\sqrt{x^2+a^2}}$

$dE = \frac{Q}{4\pi\varepsilon_0} \frac{x}{(x^2+a^2)^{3/2}} = 85 \frac{V}{m}$

Žico zvijemo v polkrožno zanko s polmerom 0,1 m. Naboj je $q(\lambda) = 10^{-4}$ C/m.
$e = 10^{-9}$ C



Določite smer in velikost trenutne sile

$dE \sin\varphi$ $\quad E = \int dE \cdot \overset{\sin}{\cancel{\cos}}\varphi$ $\quad dl = r \cdot d\varphi$

$$E = \int \frac{q \cdot dl}{4\pi\varepsilon_0} \cdot \sin\varphi$$

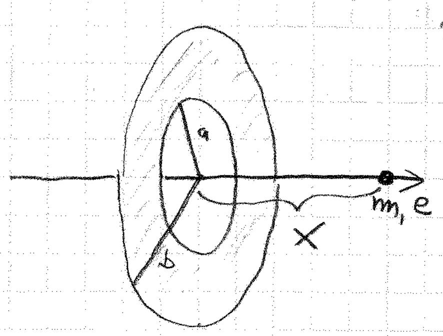
$$E = \int_0^{\pi} \frac{q \cdot r \cdot d\varphi}{4\pi\varepsilon_0 r^2} \cdot \sin\varphi$$

$$E = \frac{q \cdot r}{4\pi\varepsilon_0}(-\cos)\Big|_0^{\pi}$$

$$E = \frac{q \cdot r}{4\pi\varepsilon_0}(-(-1) + 1)$$

$$E = \frac{q \cdot \cancel{r}}{2\pi\varepsilon_0 r^{\cancel{2}}} \qquad F = e \cdot E$$

Imamo kroglico z nabojem -10^{-5} As in maso 0,1 kg. Se lahko giblje vzdolž geometrijske osi tankega nabitega kolobarja s polmeroma a in b. in površinsko gostoto $\sigma = 10^{-5}$ As/m². Kolikšen je nihajni čas, če jo malo smaknemo iz ravnovesja



$x \ll a$ (majhni odmiki)
$t_0 = ?$

$e = 0,1$ g
$m = -10^{-5}$ As
$a = 5$ cm
$b = 10$ cm
$\sigma = 10^{-5}$ As/m²

$F = m \cdot a$
$eE = m \cdot a \qquad a = \ddot{x}$

$\sigma\, dS = Q$

$$e \cdot \frac{\sigma \cdot x}{2\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = m \cdot \ddot{x}$$

$$\ddot{x} = \frac{e \cdot \sigma \cdot \cancel{x}}{m \cdot 2\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) x$$

$\omega^2 = \ldots$

$\omega = \frac{2\pi}{t_0} \qquad \omega = \frac{2\pi}{t_0}$

$$t_0 = \sqrt{\frac{4\pi^2 \cdot m \cdot 2\varepsilon_0}{e \cdot \sigma \left(\frac{1}{a} - \frac{1}{b}\right)}}$$

$$t_0 = \sqrt{\frac{4\pi^2 \cdot a \cdot b \cdot m \cdot 2 \cdot \varepsilon_0}{e \cdot \sigma\,(b - a)}} = 8{,}36 \cdot 10^{-3}$$

$$dE = \frac{\sigma \cdot 2\pi r \cdot dr}{4\pi\varepsilon_0} \frac{x}{(x^2 + r^2)^{3/2}}$$

$$E = \int \frac{\sigma\, 2\pi r \cdot dr \cdot x}{4\pi\varepsilon_0\, (x^2 + r^2)^{3/2}}$$

$$E = \frac{\sigma\, 2\pi\, x}{4\pi\varepsilon_0} \int_0^a \frac{r \cdot dr}{(x^2 + r^2)^{3/2}}$$

$$E = \frac{\sigma \cdot x}{2\varepsilon_0} \int_0 \frac{r \cdot dr}{(x^2 + r^2)^{3/2}} \qquad x^2 + r^2 = u \quad 2r\,dr = du \quad \frac{du}{2r}$$

$$E = \frac{\sigma \cdot x}{2\varepsilon_0\, 2} \int_0^b \frac{du}{u^{3/2}}$$

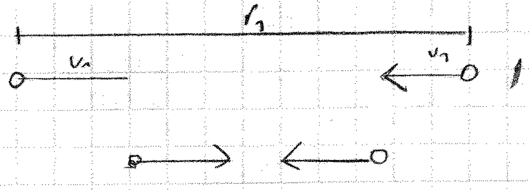
$$E = \frac{\sigma \cdot x}{2\varepsilon_0 \cancel{2}} \cdot \cancel{2} \left(-\cancel{\tfrac{1}{2}}\right) u^{-\frac{1}{2}}\Big|_a^b$$

$$E = \frac{\sigma x}{2\varepsilon_0}\left(-(x^2 + r^2)^{-\frac{1}{2}}\right)\Big|_0$$

$$E = \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{\sqrt{x^2 + a^2}} - \frac{1}{\sqrt{x^2 + b^2}}\right) \approx \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Majhna kroglica z maso 2 g je naelektrena z nabojem $3\cdot10^{-8}$ As, druga pa z maso 3 g pa z nabojem $-2\cdot10^{-8}$ As. V trenutku, ko sta kroglici 80 cm narazen ima prva kroglica hitrost 4 m/s, druga pa 7 m/s. Kroglici pa se gibljeta druga proti drugi. Kolikšni sta hitrosti obeh kroglic, ko je razdalja 50 cm?

$m_1 = 2g$
$e_1 = +3\cdot10^{-8}$ As
$m_2 = 3g$
$e_2 = -2\cdot10^{-8}$ As
$r_1 = 80$ cm
$v_1 = 4$ m/s
$v_2 = 7$ m/s
$r_2 = 50$ cm
$v_{3,4} = ?$



$$m_1v_1 - m_2v_2 = m_1v_3 - m_2v_4$$

$$W_{p_1} + W_{k_1} = W_{p_2} + W_{k_2}$$

$$\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} + \frac{e_1\cdot e_2}{4\pi\varepsilon_0 r_1} = \frac{m_1 v_3^2}{2} + \frac{m_2 v_4^2}{2} + \frac{e\cdot e}{4\pi\varepsilon_0 r_2} \Big/ 4\pi\varepsilon_0 r_1 r_2$$

v_3

$$2\pi\varepsilon_0 r_1 r_2 m_1 v_1^2 + 2\pi\varepsilon_0 r_1 r_2 v_2^2\cdot m_2 + e_1\cdot e_2\cdot r_2 = 2\pi\varepsilon_0 r_1 r_2 m_1 v_3^2 + 2\pi\varepsilon_0 r_1 r_2 v_4^2\cdot m_2 + e_1\cdot e_2 r_1$$

$$v_3 = \frac{m_1v_1 - m_2v_2 + m_2v_4}{m_1} = v_1 - \frac{m_2}{m_1}(v_2 - v_4)$$

$$\left(v_1^2 - 2\frac{m_2}{m_1}v_1(v_2-v_4) + \frac{m_2^2}{m_1^2}(v_2^2 + v_4^2 - 2v_2v_4)\right)$$

$$2\pi\varepsilon_0 r_1 r_2 m_1 v_1^2 + 2\pi\varepsilon_0 r_1 r_2 m_2 v_2^2 + e_1\cdot e_2\cdot r_2 = 2\pi\varepsilon_0 r_1 r_2 m_1 \left(v_1 - \frac{m_2}{m_1}(v_2-v_4)\right)^2 + 2\pi\varepsilon_0 r_1 r_2 m_2 v_4^2$$

$$\cancel{2\pi\varepsilon_0 r_1 r_2 m_1 v_1^2} + 2\pi\varepsilon_0 r_1 r_2 m_2 v_2^2 + e_1\cdot e_2\cdot r_2 = \cancel{2\pi\varepsilon_0 r_1 r_2 m_1 v_1^2} - 4\pi\varepsilon_0 r_1 r_2\cdot m_2 v_1 (v_2 - v_4) + 2\pi\varepsilon_0 r_1 r_2 \frac{m_2^2}{m_1}(v_2^2 + v_4^2 - 2v_2\cdot v_4) + e_1\cdot e_2 r_1$$

$$2\pi\varepsilon_0 r_1 r_2 m_2 v_2^2 + e_1 e_2 r_2 = -4\pi\varepsilon_0 r_1 r_2 m_2 v_1 v_2 + 4\pi\varepsilon_0 r_1 r_2 m_2 v_1 v_4 + 2\pi\varepsilon_0 r_1 r_2 \frac{m_2^2}{m_1} v^2 - 4\pi\varepsilon_0 r_1 r_2 \frac{m_2^2}{m_1} v_2\cdot v_4 + 2\pi\varepsilon_0 r_1 r_2 \frac{m_2^2}{m_1} v_4^2 + e_1\cdot e_2\cdot r_1 + 2\pi\varepsilon_0 r_1 r_2 m_2 v_4^2$$

$$v_4^2\, 2\pi\varepsilon_0 r_1 r_2 m_2 \left(1 + \frac{m_2}{m_1}\right) + v_4\, 4\pi\varepsilon_0 r_1 r_2 m_2 \left(v_1 - \frac{m_2}{m_1}v_2\right) + 2\pi\varepsilon_0 r_1 r_2 m_2 v_2^2 \left(\frac{m_2}{m_1} - 1\right) + e_1\cdot e_2 (r_1 - r_2) - 4\pi\varepsilon_0 r_1 r_2 m_2 v_{12} = 0$$

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te na začetku mirujejo:

$$m_1 v_1 - m_2 v_2 = 0 \qquad v_1 = \frac{m_2}{m_1} \cdot v_2$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{e_1 e_2}{4\pi\varepsilon_0 r_2} = \frac{e_1 e_2}{4\pi\varepsilon_0 r_1}$$

$$\frac{1}{2} \frac{m_2^2 v_2^2}{m_1} + \frac{1}{2} m_2 v_2^2 = \frac{e_1 \cdot e_2}{4\pi\varepsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\frac{1}{2} m_2 v^2 \left(m_2/m_1 + 1\right) = \frac{e_1 \cdot e_2 (r_2 - r_1)}{4\pi\varepsilon_0 r_1 r_2}$$

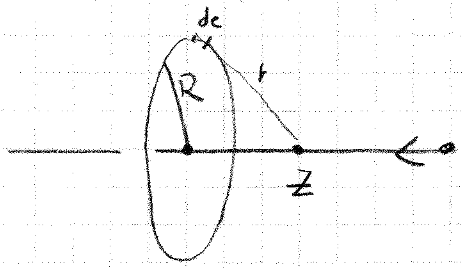
$$\frac{1}{2} m_2 v^2 \left(\frac{m_2 + m_1}{m_1}\right)$$

$$\frac{m_1}{m_2}$$

$$v_2 = \pm\sqrt{\frac{e_1 \cdot e_2 (r_2 - r_1)\, m_1}{2\pi\varepsilon_0 r_1 \cdot r_2 (m_2 + m_1)}}$$

$$v_1 = \sqrt{\frac{e_1 \cdot e_2 (r_2 - r_1)\, m_2}{2\pi\varepsilon_0 r_1 \cdot r_2 \cdot m_1 (m_1 + m_2)}}$$

Tanka žica je zvita v krožno zanko s polmerom 20 cm, po žici pa je enakomerno nanešen naboj $2 \cdot 10^{-8}$ As/cm. Majhna kroglica z maso 2 g se giblje po geometrijski osi zanke proti središču zanke. V zelo veliki razdalji ima hitrost 5 m/s. Do kolikšne razdalje se približa zanki.



$v = 5$ m/s
$m = 2$ g
$e = +3 \cdot 10^{-8}$ As
$R = 20$ cm
$\mu = 2 \cdot 10^{-8}$ As/cm

$$dV = \frac{de}{4\pi\varepsilon_0 r}$$

$$V = \frac{1}{4\pi\varepsilon_0 r} \int de = \frac{\mu 2\pi R}{4\pi\varepsilon_0 r} = \frac{\mu R}{2\varepsilon_0 r}$$

$$= \frac{\mu R}{2\pi\varepsilon_0 (R^2 + z^2)^{1/2}}$$

$$\frac{m_1 \cdot v_1^2}{\cancel{2}} = \frac{e \cdot \mu \cdot R}{\cancel{2} \varepsilon_0 \sqrt{R^2 + z^2}}$$

$$m_1 v_1^2 \varepsilon_0 \sqrt{R^2 + z^2} = e \cdot \mu \cdot R$$

$$\sqrt{R^2 + z^2} = \frac{e \mu \cdot R}{m_1 \cdot v_1^2 \varepsilon_0}$$

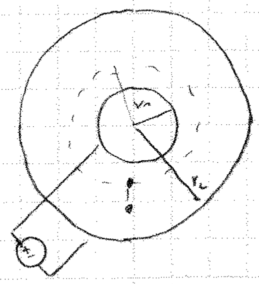
$$z = \sqrt{\left(\frac{e \mu R}{m_1 v_1^2 \varepsilon_0}\right)^2 - R^2}$$

$$z = R \sqrt{\left(\frac{e\mu}{m_1 v_1^2 \varepsilon_0}\right)^2 - 1}$$

GAUSSOV IZREK

$$\oint_S D \cdot dS = \int_V \rho_{(r)} dV$$

Krogelni kondenzator ima krogli s polmerom $r_1 = 2$ in r_2 5 cm. Med elektrodama pa je vakum. Priključen je na napetost 6000 V, tako da je notranja elektroda poz. zunanja pa negativna. Proton z maso $1{,}672 \cdot 10^{-27}$ kg in svojim osnovnim nabojem ima v trenutku, ko je 3 cm oddaljen od kondenzatorja zanemarljivo majhno hitrost. Kakšno ima hitrost, ko je 4 cm oddaljen od središča.



$$W_K = \Delta W_P$$

$$\frac{mv^2}{2} = e_0 (V_4 - V_3)$$

$$\oint D \cdot dS = e$$

$$\oint \varepsilon_0 E \cdot dS = e$$

$$\varepsilon_0 E \, 4\pi r^2 = e$$

$$E = \frac{e}{4\pi\varepsilon_0 r^2}$$

$$V_4 - V_3 = \int_{r_3}^{r_4} E \cdot dr = \frac{e}{4\pi\varepsilon_0} \int_{r_3}^{r_4} \frac{dr}{r^2} = \frac{e}{4\pi\varepsilon_0} \left(\frac{r_4 - r_3}{r_3 \cdot r_4} \right)$$

$$e = CU$$

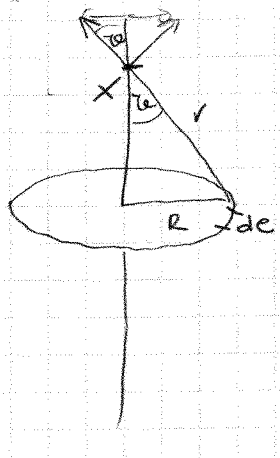
$$e = \frac{4\pi\varepsilon_0 r_1 \cdot r_2}{r_2 - r_1} U$$

$$V_4 - V_3 = \frac{\frac{4\pi\varepsilon_0 r_1 \cdot r_2}{r_2 - r_1} \cdot U}{4\pi\varepsilon_0} \left(\frac{r_4 - r_3}{r_3 \cdot r_4} \right)$$

$$V_4 - V_3 = U \cdot \frac{r_1 \cdot r_2}{r_2 - r_1} \left(\frac{r_4 - r_3}{r_3 \cdot r_4} \right)$$

$$v^2 = \sqrt{\frac{2 \cdot e_0}{m} \frac{\cancel{4\pi\varepsilon_0} r_1 \cdot r_2 \cdot U (r_4 - r_3)}{(r_2 - r_1)(r_3 \cdot r_4)}}$$

Tanka žica je zvita v krožno zanko s polmerom 30 cm. Po njej je enakomerno porazdeljen naboj $+2 \cdot 10^{-8}$ As/cm dolžine žice. Majhna kroglica ima maso $-3 \cdot 10^{-8}$ As in je prosto gibljiva po geometrijski osi zanke. S kolikšnim nihajnim časom niha ta kroglica, amplituda nihanja pa je mnogo manjša od polmera zanke.



$$F = m \cdot a$$

$$-k \cdot x = m \cdot \ddot{x}$$

$$\ddot{x} + \frac{k}{m} x = 0$$

$$\left(\frac{2\pi}{t_0} \right)^2 = \omega^2$$

$$r = \sqrt{x^2 + R^2}$$

$$\cos \vartheta = \frac{x}{(x^2 + r^2)^{\frac{1}{2}}}$$

$$q \cdot 2\pi R$$

$$dE = \frac{de}{4\pi\varepsilon_0 r^2} \cdot \cos \vartheta$$

$$E = \frac{\cos \vartheta}{4\pi\varepsilon_0 r^2} \int de$$

$$E = \frac{\cos \vartheta \cdot 2\pi R}{4\pi\varepsilon_0 r^2} = \frac{q \cdot R \cdot \cos \vartheta}{2\pi\varepsilon_0 r^2}$$

$$E = \frac{q \cdot R \cdot x}{2\varepsilon_0 (R^2 + x^2)^{3/2}}$$

$$m\ddot{x} + \frac{e_0 \cdot e \cdot R \cdot x}{2\varepsilon_0 (x^2 + r^2)^{3/2}}$$

$$\ddot{x} + \frac{e_0 \cdot R \cdot e}{2\varepsilon_0 (x^2 + r^2)^{3/2} \cdot m} x = 0$$

$$t_0 = 2\pi \sqrt{\frac{2m \cdot \varepsilon_0 \cdot R^{3}}{e_0 \cdot e \cdot R}}$$

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v kateri točki na osi zanke pa je električno polje največje in kolikšno je.

$$E = \frac{\mu R}{2\varepsilon_0} \cdot x (x^2+R^2)^{-3/2}$$

$$\frac{dE}{dx} = 1 \cdot (x^2+R^2)^{-3/2} + x\left(-\frac{3}{2}\cdot(x^2+R^2)^{-5/2}\cdot 2x\right) = 0$$

$$= \frac{1}{(x^2+R^2)^{-3/2}} - \frac{3x^2}{(x^2+R^2)^{5/2}} = 0 \quad (x^2+R^2)^{5/2}$$

$$= 1 - \frac{3x^2}{x^2+R^2} = 0$$

$$= x^2+R^2) - 3x^2 = 0$$

$$-2x^2+R^2 = 0$$

$$2x^2 = R^2$$

$$x = \pm\sqrt{\frac{R^2}{2}}$$

maksimalno E:

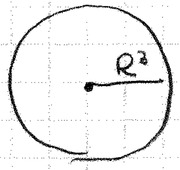
$$E = \frac{\mu R}{2\varepsilon_0} \frac{\frac{R}{\sqrt{2}}}{\left(\frac{R^2}{2}+R^2\right)^{3/2}} = \frac{\frac{q}{\pi R} R}{2\varepsilon_0} \frac{R}{\sqrt{2}\left(\frac{3R^2}{2}\right)^{3/2}}$$

$$= \frac{q R^2 \cdot 2\sqrt{2}}{2\pi\sqrt{2}\,\varepsilon_0\, 3\sqrt{3} R^3} = \frac{q}{\varepsilon_0 R\, 8\sqrt{3}}$$

Izračunaj energijo električnega polja krogle, ki ga ustvarja krogla s polmerom 3 cm, ki je po prostornini enakomerno nabita z nabojem $\rho = 10^7$ As/cm³.

$$w_e = \frac{1}{2}\vec{E}\cdot\vec{D} \rightarrow \frac{1}{2} E \cdot D$$

$$= 1/2\, E^2_{\text{?}} \varepsilon \cdot \varepsilon_0 \varepsilon$$



$$W = \int w_e \cdot dV$$

$$e = \rho_e \cdot \frac{4}{3}\pi R^3$$

e

znotraj $\oint D \cdot d\vec{S} = e$

$$\varepsilon_0 E \cdot \cancel{4\pi} r^2 = \rho\, \frac{\cancel{4\pi} r^3}{3}$$

$$E = \frac{\rho r}{3\varepsilon_0} = \frac{e r \cancel{3}}{4\pi r^3 \cancel{3}\varepsilon_0}$$

$$E = \frac{e \cdot r}{4\pi\varepsilon_0 R^3}$$

$$w = \frac{1}{2} E^2 \cdot \varepsilon_0 = \frac{1}{2}\varepsilon_0 \cdot \frac{e^2 \cdot r^2}{(4\pi\varepsilon_0)^2 R^6} = \frac{e^2 r^2}{32\pi^2 \varepsilon_0 R^6}$$

$$dV = r^2 dr \cdot \sin\vartheta\, d\vartheta\, d\varphi$$

$$W_e = \int w_e\, dV = \frac{e}{\cancel{32}\pi^2\varepsilon_0 R^6}\int_0^R r^4 dr \int_0^\pi \sin\vartheta\, d\vartheta \int_0^{2\pi} d\varphi$$

$$= \frac{e}{16\pi\varepsilon_0 R^6}\int_0^R r^4 dr \int_0^\pi \sin\vartheta\, d\vartheta$$

$$= \frac{e^2}{40\cancel{8}\pi\varepsilon_0 \cancel{R^6}} = \frac{e^2}{40\pi\varepsilon_0 R}$$

$$\int_0^\pi \sin\vartheta\, d\vartheta = -\cos\Big|_0^\pi$$

$$= (-(-1)+1) = 2$$

$$\int r^4 dr \qquad \frac{R^5}{5}$$

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Zunaj krogle:

$$E = \frac{e}{4\pi\varepsilon_0 r^2} \qquad w_e = \frac{1}{2}\frac{e^2}{16\pi^2\varepsilon_0 R^4} = \frac{e^2}{32\pi^2\cdot\varepsilon_0 R^4}$$

$$W = \frac{e^2}{32\pi^2\varepsilon_0}\int_R^{\infty}\frac{dr}{r^2}\underbrace{\int_0^{\pi}\sin\vartheta\, d\vartheta\int_0^{2\pi}d\varphi}_{4\pi}$$

$$= \frac{e^2}{8\pi\varepsilon_0}\int_R^{\infty}\frac{dr}{R^2}$$

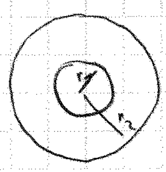
$2\pi r\, dr$

$$= \frac{e^2}{8\pi\varepsilon_0 R}$$

$$W = W_{zun} + W_{not} = \frac{e^2}{\pi\varepsilon_0 R}\left(\frac{1}{40} + \frac{1}{8}\right) = \frac{6e^2}{40\varepsilon_0\pi R} = \frac{3e^2}{20\pi\varepsilon_0 R}$$

Koaksialni vodnik ima žilo s polmerom $r_1 = 1mm$ in notranji polmer plašča $r_2 = 4mm$. Znotraj koaksialnega kabla pa ima dielektrik poljsko jakost $= 8 kV/cm$. Kakšno največjo napetost lahko priključimo na ta vodnik.

$$U_p = E_p \cdot r_1 \cdot \ln\frac{r_2}{r_1}$$

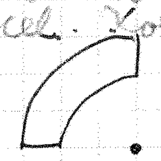


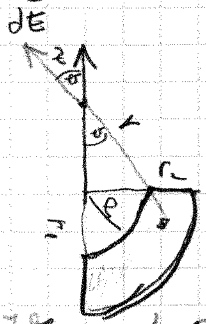
$U_p = C \cdot U_p$ $\qquad E = \ldots$

-||- ||- ||-

$$\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Ravna plošča ima obliko 1/4 kolobarja s polmeroma 3 in 7 cm. Po plošči je enakomerno porazdeljen naboj $3\cdot 10^{-8}\ As/cm^2$. Kolikšna je jakost $\vec{E}$ v točki ki je 5 cm oddaljena od središča kolobarja, če bi bil cel in leži na geometrijski osi, če bi bil cel. Kolikšen kot oklepa vektor $\vec{E}$ z geometrijsko osjo

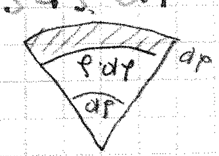




$$dE_\rho = \frac{de\cdot\sin\vartheta}{4\pi\varepsilon_0 r^2}$$

$$dE_z = \frac{de\cos\vartheta}{4\pi\varepsilon_0 r^2}$$

$$de = \sigma\cdot dS = \qquad dS = \rho\, d\rho\cdot d\varphi$$



$$dE_\rho = \frac{\sigma\cdot d\rho\cdot d\varphi\sin\vartheta}{4\pi\varepsilon_0 r^2} = \frac{\sigma\cdot d\varphi\cdot\rho^2\cdot d\rho}{4\pi\varepsilon_0(\rho^2+z^2)^{3/2}}$$

$$dE_z = \frac{\sigma\cdot d\varphi\cdot d\rho\cdot\rho\cdot\cos\vartheta}{4\pi\varepsilon_0 r^2}\cdot\cos\varphi$$

$$= \frac{\sigma\cdot z\cdot d\varphi\cdot\rho\cdot d\rho}{4\pi\varepsilon_0(\rho^2+z^2)^{3/2}}$$

$$r = \sqrt{\rho^2+z^2}$$

$$\cos\vartheta = \frac{z}{\sqrt{\rho^2+z^2}}$$

$$\sin\vartheta = \frac{\rho}{\sqrt{\rho^2+z^2}}$$

$$E_z = \frac{\sigma \cdot z}{4\pi\varepsilon_0}\int_0^{\pi/2} d\varphi \int_{r_1}^{r_2} \frac{\rho \cdot d\rho}{(\rho^2+z^2)^{3/2}} = \frac{\sigma \cdot z}{8\varepsilon_0}\left(-\frac{1}{(\rho^2+z^2)^{1/2}}\right)\Bigg|_{r_1}^{r_2} \qquad \rho^2+z^2=u$$

$$= \frac{\sigma \cdot z}{8\varepsilon_0}\left(\frac{1}{(r_1^2+z^2)^{1/2}} - \frac{1}{(r_2^2+z^2)^{1/2}}\right) \qquad du = 2\rho\, d\rho$$

$$E_\rho = \frac{\sigma}{4\pi\varepsilon_0}\int_0^{\pi/2} d\varphi \int_{r_1}^{r_2} \frac{\rho^2 \cdot d\rho}{(\rho^2+z^2)^{3/2}} = \frac{\sigma}{8\varepsilon_0}\int \frac{\rho^2}{(\rho^2+z^2)^{3/2}} = \frac{\sigma}{8\varepsilon_0}\left(-\frac{\rho}{(\rho^2+z^2)^{1/2}} + \ln\left(\rho + \sqrt{\rho^2+z^2}\right)\right)\Bigg|_{r_1}^{r_2}$$

$M = J\alpha$

$(F_e - F_g)\, l \sin\varphi = ml^2 \cdot \ddot{\varphi}$

$\ddot{\varphi}$

$t_0 = 2\pi\sqrt{\frac{ml}{mg - qE}}$

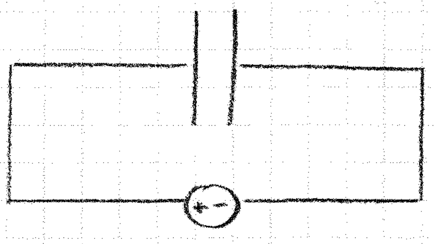
FIZ II-V
6. 3. 2008

ELEKTRIČNI TOK

$$I = \frac{de}{dt}$$

Ploščni kondenzator ima elektrodi s ploščino 300 cm², v začetku pa sta razmaknjeni 2 cm. Kondenzator je priključen na vir 900 V. V nekem času se začne oddaljevati s hitrostjo 1,7 mm/s. Kakšen električni tok teče skozi vir napetosti po 0,3 s.

KOLOKVIJ !!!



$$e = CU$$

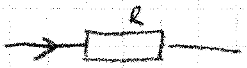
$$e(t) = C(t)\, U(t) \quad /\frac{d}{dt}$$

$$I = \frac{dC}{dt} U + C \underbrace{\frac{dU}{dt}}_{konst}$$

$$C = \frac{\varepsilon_0 S}{(x_0 + vt)} = \varepsilon_0 S\, (x_0 + v_0 t)^{-1}$$

$$\frac{dC}{dt} = \varepsilon_0 S\, (-1)\, (x_0 + v_0 t)^{-2} \cdot v = -\frac{\varepsilon_0 S v}{(x_0 + vt)^2}$$

$$I = -\frac{U \varepsilon_0 S \cdot v}{(x_0 + v_0 t)^2}$$



$U = IR$

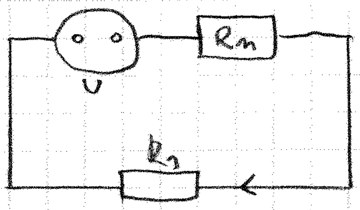
$R = \frac{U}{I} \quad \left[\frac{V}{A} = \text{OHM}\right]$

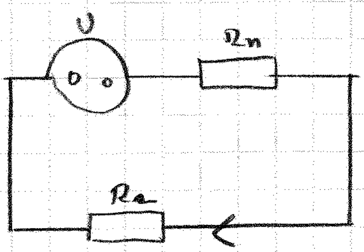
$P = UI \quad [VA = W]$

$P = I^2 R = \frac{U^2}{R}$ PAZI, TO JE PADEC NAP.

BOLJ UPORABNO

Ko na neko baterijo priključimo upor 25 Ω se troši moč 25 W, če pa priključimo 18 Ω pa se troši moč 30,6 W. Kolikšni sta notranji upornost in gonilna nap. baterije





$$P_1 = R_1 I_1^2 = \frac{U^2 \cdot R_1}{(R_1+R_n)^2} = P_1$$

$$P_2 = R_2 I_2^2 = \frac{U^2 \cdot R_2}{(R_2+R_n)^2} = P_2$$

delimo

$$\frac{P_1}{P_2} = \frac{R_1 (R_2+R_n)^2}{(R_1+R_n)^2 R_2}$$

$$R_1 (R_2+R_n)^2 \cdot P_2 = P_1 (R_1+R_n)^2 \cdot R_2$$

$$\sqrt{R_1 \cdot P_2} \cdot R_2 + \sqrt{R_1 P_2} \cdot R_n = \sqrt{P_1 R_2}\, R_1 + \sqrt{P_1 \cdot R_2} \cdot R_n$$

$$R_n (\sqrt{R_1 P_2} - \sqrt{P_1 R_2}) = \sqrt{P_1 R_2} \cdot R_1 - \sqrt{R_1 P_2}\, R_2$$

$$R_n = \frac{\sqrt{P_1 R_2} \cdot R_1 + \sqrt{P_2 R_1}\, R_2}{\sqrt{R_1 P_2} - \sqrt{P_1 R_2}}$$

$$R_n = \frac{\sqrt{25 \cdot 18} \cdot 25 - \sqrt{25 \cdot 30{,}6} \cdot 18}{\sqrt{25 \cdot 30{,}6} - \sqrt{25 \cdot 18}}$$

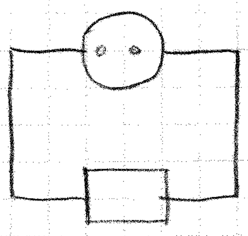
$$R_n = 5 \cdot 3\sqrt{2} \cdot 25 - \sqrt{\quad}\ 5$$

$$R_n = 5{,}04\,\Omega$$

$$U = \sqrt{\frac{P_1}{R_1}}\,(R_1 + R_n) \simeq 25\,V$$

Še kvadratno doma:

Električni grelec z močjo 300 W je priključen na napetost 220 V. Kolikšen električen tok teče čez ta grelec in koliko časa potrebujemo, da segrejemo 0,5 kg vode za 20 °C. c_p vode = 4200 J/kgK.



$$P = U \cdot I$$

$$I = \frac{P}{U}$$

$$\Delta Q = m c_p \cdot \Delta T \qquad P = \frac{dA}{dt} \qquad P = \frac{dQ}{dt}$$

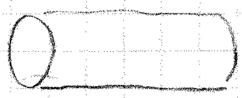
$$dQ = m c_p \, dT \qquad m c_p \frac{dT}{dt} = P$$

$$\int_0^t dt = \frac{m c_p}{P} \int_0^{\Delta T} dT$$

$$t = \frac{m c_p}{P} \Delta T$$

$$t = \frac{0{,}5 \cdot 4200}{300} \cdot 20$$

$$t = 140\,s$$



$$R = \overset{\text{pri OE}}{\zeta(\rho)} \cdot \frac{l}{S} \qquad [\Omega m, \frac{Vm}{A}]$$

$$\Omega \frac{mm^2}{m} = \Omega \frac{10^{-6}\,m^2}{m} = 10^{-6}\,\Omega m$$

Iz 20 g koša bakra, ki ima gostoto 8,9 g/cm³ in specifično upornost 0,017 $\Omega \frac{mm^2}{m}$ bi radi naredili žico ~~dolžine~~ upornosti $R = 0{,}1\,\Omega$. Kakšen bo presek in dolžina žice.

$$R = \zeta \frac{l}{S} \Leftarrow \qquad m = \rho \cdot V = \rho \cdot l \cdot S = m$$

$$l = \frac{R \cdot S}{\zeta}$$

$$m = \rho \cdot \frac{R \cdot S}{\zeta} \cdot S$$

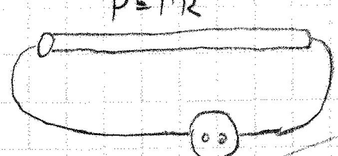
$$S = \sqrt{\frac{m \cdot \zeta}{R \cdot \rho}}$$

$$l = \frac{R \cdot \sqrt{\frac{m \cdot \zeta}{\rho \cdot R}}}{\zeta}$$

Žica je dolga 20 m in ima presek 0,5 mm². Priključimo na baterijo 1,5 V in zanemarljivo notranjo upornostjo. Žica je popolnoma toplotno izolirana. Za koliko mA se vsako sekundo zmanjša tok po žici zaradi segrevanja žice. $\zeta = 0{,}017\,\Omega \frac{mm^2}{m}$, $\alpha = 4 \cdot 10^{-3}\,K^{-1}$ gostota bakra 8,9 g/cm³, specifična toplota pa 380 J/kgK

$$\Delta \zeta = \zeta \alpha \Delta T$$

$P = I^2 R$



$$\frac{dI}{dt} = \frac{dI}{dR}\cdot\frac{dR}{dT}\cdot\frac{dT}{dt}$$

$$I = \frac{U}{R}$$

$$\frac{dI}{dR} = -\frac{U}{R^2}$$

$$d\rho = \rho\,\alpha\, dT \;/\; \frac{l}{S}$$

$$\frac{d\rho\, l}{S} = \frac{\rho l}{S}\,\alpha\, dT$$

$$dR = R\cdot\alpha\, dT$$

$$\frac{dR}{dT} = \alpha\cdot R$$

$$P = \frac{dQ}{dt} = m c_p\, dT/dt = \frac{U^2}{R}$$

$$\frac{dT}{dt} = \frac{U^2}{R\, m c_p}$$

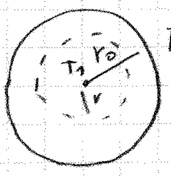
$$\frac{dI}{dt} = -\frac{U}{R^2}\cdot\alpha\cdot R\cdot\frac{U^2}{R\, m c_p} = -\frac{U^3\cdot\alpha}{R^2 m c_p} = -\frac{U^3 S^2}{\rho^2 l^2 \rho\cdot l\cdot S c_p}$$

$$= -\frac{U^3\cdot\alpha\cdot S}{\rho^2 l^3 \rho\, c_p}$$

$m = \rho\cdot l\cdot S$

$R = \rho\,\frac{l}{S}$

Kolikšen tok sme največ teči po valjasti bakreni žici s polmerom 2 mm, če temperatura na osi žice ne sme preseči 80°C, na površini pa ves čas vzdržujemo $T = 10°C$. $\rho = 0{,}017\ \Omega\,\mathrm{mm^2/m}$, $\lambda = 380\ \mathrm{W/mK}$

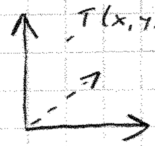


T_2

$P = -\lambda\nabla T \qquad P = \int j\cdot dS =$

$j = -\lambda\cdot\nabla T$

$T(x,y,z) \quad \left(\frac{\partial T}{\partial t}\right)$



$$P_c = -\int j\cdot d\vec{S} = -\lambda\int \nabla T\cdot d\vec{S}$$

$\int dS = 2\pi r l$

$$P_c = -\lambda\cdot\frac{\partial T}{\partial r}\cdot 2\pi r l \qquad P_c = I^2\cdot R$$

$$= \left(\frac{I_0}{\pi r_0^2}\cdot\pi r^2\right)^2\cdot\rho\,\frac{l}{\pi r^2} = \frac{I_0^2 r^2 \rho l}{r_0^4\cdot\pi}$$

$$\frac{I_0^2 r^2\cdot\rho l}{r_0^4\cdot\pi} = -\lambda\cdot 2\pi l r\,\frac{dT}{dr}$$

$$\frac{I_0^2\,\rho}{2\pi^2\cdot r_0^4\lambda}\cdot r = -\frac{dT}{dr}$$

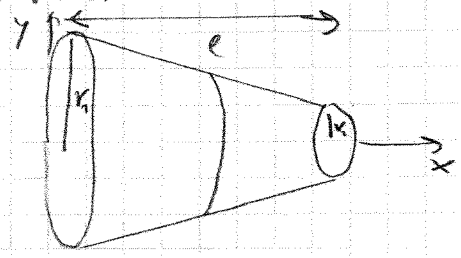
$$\int_{T_1}^{T_2} dT = -\frac{I_0^2\rho}{2\pi^2\cdot r_0^4\lambda}\int r\cdot dr$$

$$T_2 - T_1 = -\frac{I_0^2\rho}{4\pi^2 r_0^4\cancel{2}}\, r_0^2$$

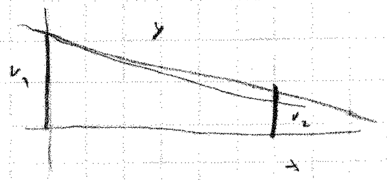
$$I_0 = \sqrt{\frac{4\pi^2 r_0^2 (T_1 - T_2)}{\rho}}$$

Specifično upornost iz katerega naredimo upor je 300 Ωm, upor pa naredimo v obliki prisekanega stožca, ki ima osnovni ploskvi 15 in 7 mm v medsebojni oddaljenosti 5 cm. Kolikšna je upornost tega upora.

$\rho = 300\ \Omega m$
$r_1 = 15\ mm$
$r_2 = 7\ mm$
$l = 5\ cm$



$$dR = \varrho \frac{dx}{\pi \cdot y^2(x)}$$



$$dR = \frac{\varrho}{\pi} \frac{dx}{\left(\frac{r_2-r_1}{l}x + r_1\right)^2}$$

$$y = \frac{r_2-r_1}{l}x + r_1$$

$$R = \frac{\varrho}{\pi}\int_0^l \frac{dx}{\left(\frac{r_2-r_1}{l}\cdot x + r_1\right)^2}$$

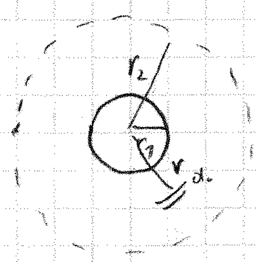
$$dy = \frac{r_2-r_1}{l}dx \qquad dx = \frac{l\,dy}{r_2-r_1}$$

$$R = \frac{\varrho l}{\pi(r_2-r_1)}\int_{r_1}^{r_2}\frac{dy}{y^2} = \frac{\varrho l}{\pi(r_2-r_1)}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{\varrho l \cancel{(r_2-r_1)}}{\pi\cancel{(r_2-r_1)}(r_1 \cdot r_2)} = R$$

Krogelni kondenzator ima koncentrični elektrodi s polmeroma 1 in 4 cm. Med elektrodama pa je izolator s specifično upornostjo $10^8\ \Omega m$. Kond. je priključen na napetost 2000 V. Kolikšen elekt. tok teče med elektrodama zaradi puščanja izolacije. Kolikšna je temp. notranje elektrode, če zunanjo ohranjamo pri 5°C, toplotna prevodnost pa je .

$T_2 = 5°C$
$r_1 = 1\ cm$
$r_2 = 4\ cm$
$\varrho = 10^8\ \Omega m$
$U = 2000\ V$
$\lambda = 30\ W/mK$



$$dR = \frac{\varrho\, dr}{4\pi r^2}$$

$$R = \int_{r_1}^{r_2}\frac{\varrho\, dr}{4\pi r^2} = \frac{\varrho}{4\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad I = \frac{U}{R} = \frac{U 4\pi r_1 \cdot r_2}{\cancel{4\pi (r_2-r_1)}\,\varrho\,(r_2-r_1)}$$

$$P = \int j \cdot dS = -\int \lambda \cdot \nabla T \cdot dS$$

$$I^2 \cdot R = -\lambda \frac{dT}{dr}\cdot 4\pi r^2$$

$$\frac{U^2 \cdot 16\pi^2 r_1^2 \cdot r_2^2}{\varrho^2 \cdot (r_2-r_1)^2}\cdot\frac{\varrho(r-r_1)}{\cancel{4\pi r \cdot r}} = -\lambda\frac{dT}{dr}4\pi r^2$$

$$\frac{U^2 \cdot r_1 \cdot r_2^2}{\varrho(r_2-r_1)^2}\cdot\frac{r-r_1}{r} = -\lambda\frac{dT}{dr}r^2$$

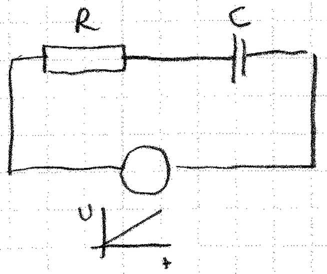
$$\int_{T_1}^{T_2} dT = -\frac{U^2 r_1 \cdot r_2^2}{\lambda\varrho(r_2-r_1)^2}\int_{r_1}^{r_2}\frac{(r-r_1)}{r^3}\cdot dr \qquad = \int_{r_1}^{r_2}\frac{dr}{r^2} - r_1\int_{r_1}^{r_2}\frac{dr}{r^3} =$$

$$= \left(-\frac{1}{r}\right)\Big|_{r_1}^{r_2} + r_1\frac{r^{-2}}{2}\Big|_{r_1}^{r_2}$$

$$= \frac{1}{r_1} - \frac{1}{r_2} + \frac{r_1}{2}\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)$$

$$= \frac{(r_2-r_1)}{r_1 \cdot r_2} + \frac{1}{2}\frac{(r_1^2 - r_2^2)}{r_1 \cdot r_2^2}$$

Kondenzator s kapaciteto 0,2 μF in 3 MΩ upor zvežemo zaporedno in priključimo na vir žagaste napetosti na katerem je napetost na začetku enaka 0, v trenutku pa začne naraščati linearno $U=At$. Kolikšna je napetost po 1,2 s na vsakemu od elementov $A=2\,V/s$



$$U + U_R + U_C = 0$$

$$At + I(t)R + \frac{Q(t)}{C} = 0 \quad , \quad \frac{d}{dt}\ I = \frac{dQ}{dt}$$

$$I = \frac{dQ}{dt}$$

$$I(t) \quad = A - R\frac{dI}{dt} - \frac{I}{C} = 0$$

$$R\frac{dI}{dt} = A - \frac{I}{C}$$

$$R\cdot dI = \frac{AC-I}{C}\cdot dt$$

$$\int_0^t dt = \int_0^I RC\frac{dI}{AC-I}$$

$AC - I = x$
$-dx = dI$

$$t = -RC\int_{AC}^{AC-I}\frac{dx}{x} =$$

$$t = RC\ln\frac{AC}{AC-I}$$

$$\ln\frac{AC-I}{AC} = -\frac{t}{RC}$$

$$AC - I = ACe^{-\frac{t}{RC}}$$

$$I(t) = AC(1-e^{-\frac{t}{RC}})$$

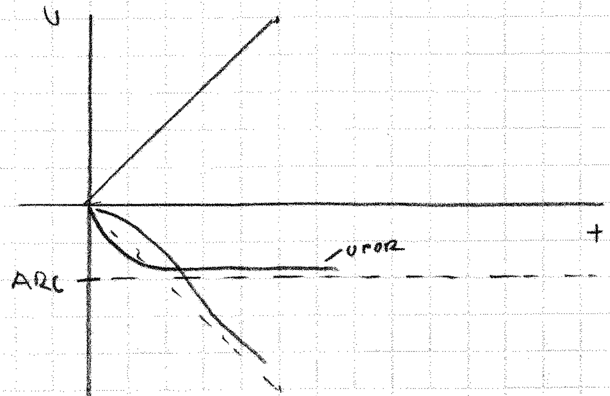
tok na upor

$$U = I\cdot R$$

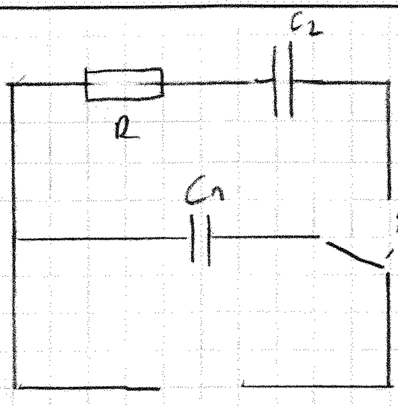
$$= ARC(1-e^{-\frac{t}{RC}})$$

$$U_C = At - U_R$$

$$U_C = A[t - RC(1-e^{-\frac{t}{RC}})]$$



Kondenzator s kapaciteto nabijemo na 3000 V $C = 2\,\mu F$ nato pa ga praznimo prebo 2 MΩ upora in drugega kondenzatorja s kapaciteto 3 μF. Kolikšne so vse napetosti po 0,8 s



$U_1 + U_2 + U_3 = 0$

$$\boxed{\frac{Q_1(t)}{C_1} - \frac{Q_2(t)}{C_2} - I(t)R = 0 \quad \Big| \frac{d}{dt}}$$

$$I = -\frac{dQ_1}{dt}$$

$$I = +\frac{dQ_2}{dt}$$

$$-\frac{I}{C_1} - \frac{I}{C_2} - R\frac{dI}{dt} = 0$$

$$\frac{R\cdot dI}{dt} = -I\,\frac{C_2+C_1}{C_1C_2}$$

$$\int_0^t dt = -\frac{RC_1C_2}{C_1+C_2}\int_{I=U_0/R}^{I}\frac{dI}{I}$$

$$t = -\frac{RC_1C_2}{C_1+C_2}\ln\frac{I\cdot R}{U_0}$$

$$-\frac{t(C_1+C_2)}{RC_1C_2} = \ln\frac{I\cdot R}{U_0}$$

$$\frac{IR}{U_0} = e^{-\frac{t(C_1+C_2)}{RC_1C_2}}$$

$$I(t) = \underbrace{\frac{U_0}{R}}_{I_0}\, e^{-\frac{t(C_1+C_2)}{RC_1C_2}}$$

$$U_R = I\cdot R = U_0\, e^{-\frac{t(C_1+C_2)}{RC_1C_2}}$$

$$dQ_2 = I\cdot dt$$

$$\int_0^{Q_2} dQ_2 = \frac{U_0}{R}\int_0^t e^{-\frac{t(C_1+C_2)}{RC_1C_2}}\,dt$$

$$Q_2 = -\frac{U_0C_1C_2}{C_1+C_2}\, e^{-\frac{t(C_1+C_2)}{RC_1C_2}}\Big|_0^t$$

$$Q_2 = \frac{U_0C_1C_2}{C_1+C_2}\left(1 - e^{-\frac{t(C_1+C_2)}{RC_1C_2}}\right)$$

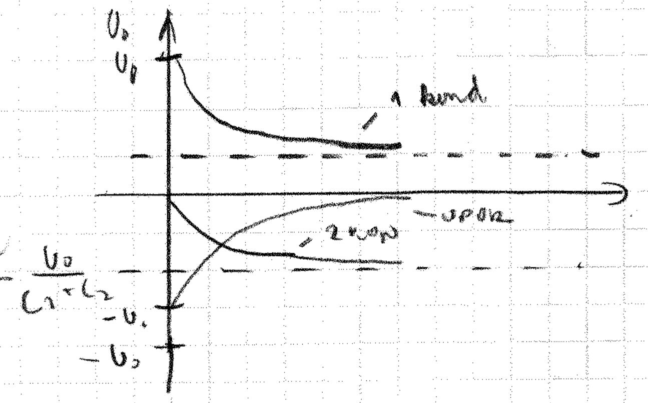
$$U_2 = \frac{Q_2}{C_2} = \frac{U_0C_1}{C_1+C_2}\left(1 - e^{-\frac{t(C_1+C_2)}{RC_1C_2}}\right)$$

$$U_1 = U_2 + U_3$$

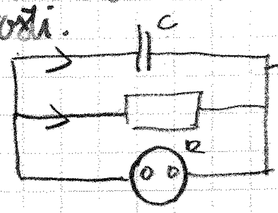
$$U_1 = U_0\left[e^{-\frac{t(C_1+C_2)}{RC_1C_2}} + \frac{C_1}{C_1+C_2} - \frac{C_1}{C_1+C_2}e^{-\frac{t(C_1+C_2)}{RC_1C_2}}\right]$$

$$= U_0\left[\frac{C_1}{C_1+C_2} + e^{-\frac{t(C_1+C_2)}{RC_1C_2}}\left(\frac{C_1+C_2}{C_1+C_2} - \frac{C_1}{C_1+C_2}\right)\right]$$

$$= U_0\,\frac{1}{C_1+C_2}\left[C_1 + C_2e^{-\frac{t(C_1+C_2)}{RC_1C_2}}\right]$$



$R = 60\,\Omega$ upor in kondenzator z $C = 0{,}2\,\mu F$ vežemo vzporedno in priključimo na generator sinusne napetosti $1{,}5\,V$ $= U_0$ in frekvenco $50\,000\ s^{-1}$. Kolikšna je amplituda toka, ki teče skozi vir napetosti. Kolikšna povprečna moč se porablja na uporu in kolikšno povprečno moč mora zagotavljati generator napetosti.

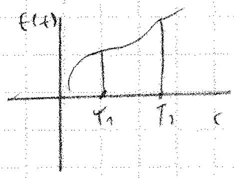


$$I_0 = U_0 \underbrace{\sqrt{\frac{1}{R^2} + \omega^2 C^2}}_{\text{impedanca}} \ !!!$$

$$I_0 = \frac{U_0}{|Z|} \qquad I = I_0 \sin(\omega t + \delta) \qquad \tan\delta = R\omega C$$

$$P(t) = \frac{U^2}{R} = \frac{U_0^2}{R} \sin^2 \omega t \qquad U_{ef} = \frac{U_0}{\sqrt{2}} = \frac{U_{ef}^2}{R} = \frac{U_0}{\sqrt{2}} \frac{U_0}{\sqrt{2}} \frac{1}{R}$$

$$\langle P \rangle = \frac{U_0^2}{2R} = \frac{U_{ef}^2}{R}$$

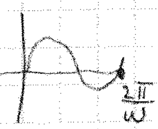


$$\langle f \rangle = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f(t)\,dt$$

$$\langle f^2 \rangle = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f^2(t)\,dt$$

$$f_{ef} = \sqrt{\langle f^2 \rangle}$$

$$\langle \sin^2(\omega t) \rangle = \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} \sin^2(\omega t)\,dt$$



$$= \frac{\omega}{2\pi} \frac{1}{2} \int_0^{\frac{2\pi}{\omega}} dt - \int_0^{\frac{2\pi}{\omega}} \cos(2\omega t)\,dt$$

$$= \frac{\omega}{2\pi} \frac{1}{2} \left[\frac{2\pi}{\omega} + \frac{1}{2\omega} \sin(2\omega t) \Big|_0^{\frac{2\pi}{\omega}} \right.$$

$$= \frac{1}{2}$$

povprečna moč vira

$$P = U \cdot I = U_0 \sin \omega t \cdot I_0 \cdot \sin(\omega t + \delta)$$

$$= U_0^2 \sqrt{\frac{1}{R^2} + \omega^2 C^2}\ \underline{\sin \omega t \cdot \sin(\omega t + \delta)}$$

$$\frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} \sin(\omega t) \sin(\omega t + \delta)\,dt$$

$$\frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} \sin^2 \omega t \cdot \cos\delta\,dt + \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} \sin \omega t \cos \omega t \cdot \sin\delta\,dt$$

$$x = \sin \omega t \qquad \frac{\sin\delta}{\omega} \int_0^0 x\,dx = 0$$

$$= \frac{\cos\delta}{2}$$

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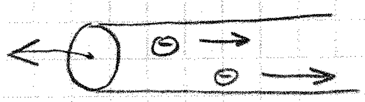
$$\langle P \rangle = U_0^2 \sqrt{\frac{1}{R^2+\omega^2C^2}} \cdot \cos\vartheta / 2 = \frac{U_{ef}^2}{\cdot} \sqrt{\frac{1}{R^2+\omega^2C^2}}$$

$\mathrm{tg}\,\vartheta = R\omega C$

$$\cos\vartheta = \frac{1}{\sqrt{1+\mathrm{tg}^2\vartheta}} = \frac{1}{\sqrt{1+R^2\omega^2C^2}} = \frac{1}{\sqrt{\frac{R^2}{R^2}+R^2\omega^2C^2}} = \frac{1}{R\sqrt{\frac{1}{R^2}+\omega^2C^2}}$$

$$= \frac{U_{ef}^2}{R}$$

Bakrena žica ima presek 3 mm², po njej pa teče električni tok 1 A. Gostota prostih elektronov v bakru je $8 \cdot 10^{28}$ e/m³. Kolikšna je povprečna hitrost v žici



$J = \rho \langle \vec{v} \rangle$

$J = e_0 \cdot n \cdot \langle v \rangle$

$I = \int \vec{j} \cdot d\vec{S} \qquad J = \frac{I}{S}$

$\frac{I}{S} = e_0 \cdot n \cdot \langle v \rangle$

$$\langle v \rangle = \frac{I}{S e_0 n} = \frac{1 \cdot 1{,}6 \cdot 10^{+19} \cdot 8 \cdot 10^{-28}}{3 \cdot 10^{-6}\ \mathrm{mm}^2}$$

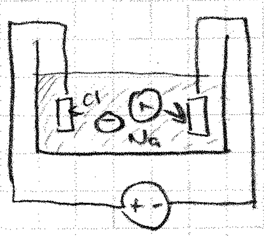
$$= 1 \frac{1}{4{,}8 \cdot 8}\ \mathrm{mm/s}$$

Pri elektrolizi raztopini NaCl se na neg. elektrodi v 5h nloži 0,2 g natrija. Kolikšen električni tok teče po raztopini.

$z = 1$ ← vsak ima 1 prost el.

$M = 23$ kg/kmol

$I = ?$



$$I = \frac{dQ}{dt} = \frac{Q}{t}$$

$$Q = z \cdot e_0 \cdot N$$

$$= z \cdot e_0 \cdot \frac{m}{m_{1.\,\mathrm{jon}}}$$

$$m_1 = \frac{M}{N_A}$$

$$N = \frac{m}{m_1}$$

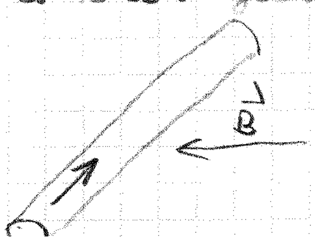
$$= z \cdot e_0 \cdot \frac{m}{\frac{M}{N_A}} = z \cdot e_0 \frac{m \cdot N_A}{M}$$

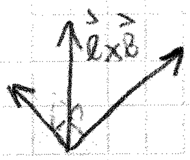
$$I = \frac{z \cdot e_0 \cdot N_A \cdot m}{M \cdot t} = 0{,}05\,A$$

MAGNETNE SILE

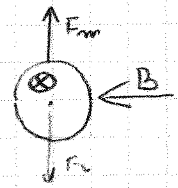
Po vodoravni bakreni žici s presekom 3 mm² teče električni tok. Žica se nahaja v homog. polju 0,2 T, ki je pravokoten na žico. Kolikšen elekt. tok mora teči po žici, da uravnovesi silo teže žice. Gostota bakra je 8,9 g/cm³.

$$\vec{F} = I \cdot d\vec{l} \times \vec{B}$$





$$F = \int I \cdot d\vec{l} \times \vec{B}\,(T)$$

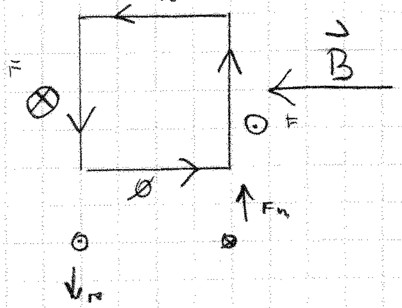


$$I \cdot l \cdot B = m \cdot g$$

$$I = \frac{m \cdot g}{l \cdot B} = \frac{\rho \cdot S \cdot l \cdot g}{l \cdot B} = \frac{\rho S g}{B} = \frac{8900\,kg/m^3 \cdot 3 \cdot 10^{-6}\,m^2 \cdot 9{,}81\,m/s^2}{0{,}2\,Vs/m^2}$$

$$I = 1{,}3\,A$$

Po kvadratni zanki s stranico 30 cm teče električni tok 8 A. Zanka se nahaja v homogenem magnetnem polju z gostoto 0,3 T, ki je pravokotno na 2 stranici zanke in vzporedno z drugima 2. Kolikšen magnetni navor deluje na zanko.



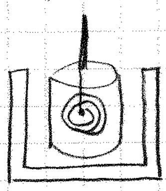
$$M = F \cdot a = I \cdot l \cdot B \cdot a = I \cdot a^2 \cdot B = p_m B$$

$$M = \vec{p}_m \times \vec{B}$$

$$p_m = N\, I\, S$$

če je več ovojev

Galvanometer ima vrtljivo tuljavico s 60 ovoji in presekom 2 cm². Nahaja se v homogenem magnetnem polju z jakostjo 0,6 T. Kazalec predstavlja podaljšek geometrijske osi tuljavice, tuljavica pa je vrtljiva okoli osi, ki je ⊥ tako na geom. os kot tudi na mag. polje. Na os je pripeta polžasta vzmet s koef. D = 5·10⁻⁴ Nm, ki tuljavico vrača v navp. lego. Vztr. moment tuljavice in vzmeti je 2·10⁻⁴ kgm². Za kolikšen tok se odkloni kazalec tuljavice, če skozi tuljavo teče I = 0,5 A. J kolikšno amplitudo zaniha, če skozi ovoje spustimo tokovni sunek 8 μAs



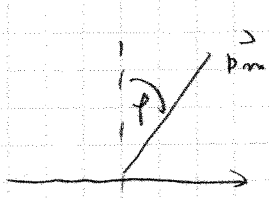
$$D\varphi = |\vec{p}_m \times \vec{B}|$$

$$D\varphi = |I \cdot S \cdot N \cdot B \cdot \cos\varphi|$$

$$\varphi = \frac{N \cdot I \cdot S \cdot B}{D} \cdot \cos\varphi = \frac{60 \cdot 0{,}1\,A \cdot 2 \cdot \cancel{10^{-4}}\,m^2 \cdot 0{,}6\,T}{5 \cdot \cancel{10^{-4}}}$$

1,10
0,53

1,00
0,77



$$1{,}44 \cdot \cos\varphi$$

φ	$1{,}44 \cdot \cos\varphi$
0,2	1,41
1,41	0,23
0,23	1,40
1,40	0,24
0,24	
1,39	

$$M \cdot dt = \Delta\Gamma$$

$$\underbrace{N \cdot I \Delta t S \cdot B}_{\text{sunek navor}} = J\omega$$

$$t_0 = 2\pi\sqrt{\frac{J}{D}}$$

$$N \cdot I \cdot \Delta t \cdot S \cdot B = J \cdot \frac{2\pi}{t_0} \cdot \varphi \Rightarrow N \cdot I\Delta t \cdot S \cdot B = J \cdot \sqrt{\frac{D}{J}} \cdot \varphi_0$$

$$\varphi_0 = \frac{N \cdot I\Delta t \cdot S \cdot B \cdot \sqrt{J}}{\ldots} = 1{,}81 \cdot 10^{-4}$$

Magnetna igla se nahaja v homogenem mag. polju, ki je vzp. z iglo, ki je vrtljiva okoli osi. V začetku je $B = 0{,}02\,T$ in ko iglo malo odmaknemo zaniha z $t_0 = 1{,}27\,s$. Nato mag. polje spremenimo in nato zaniha z $0{,}92\,s$. $B = ?$

$$M = J \cdot \alpha$$

$$-p_m \cdot B \cdot \sin\varphi = J \cdot \ddot{\varphi}$$

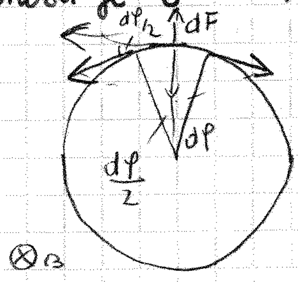
$$\ddot{\varphi} + \frac{p_m B}{J}\varphi = 0$$

$$\left(\frac{2\pi}{t_1}\right)^2 = \frac{p_m B_1}{J_1}$$

$$\left(\frac{2\pi}{t_2}\right)^2 = \frac{p_m B_2}{J}$$

$$\frac{t_2^2}{t_1^2} = \frac{B_1}{B_2} = \qquad B_2 = B_1 \frac{t_1^2}{t_2^2}$$

Imamo žico 1 mm² je sklenjena v krožno zanko s polmerom 10 cm in leži v ravnini s homogenim poljem 1 T. Kolikšen največji električni tok sme teči, da se zanka ne pretrga. Meja natezne trdnosti je $\sigma = 30\ N/mm^2$



$$dF = I\,dl \times B \qquad dF = r\,d\varphi \cdot I \cdot B$$

$$= 2N \cdot \sin\frac{d\varphi}{2}$$

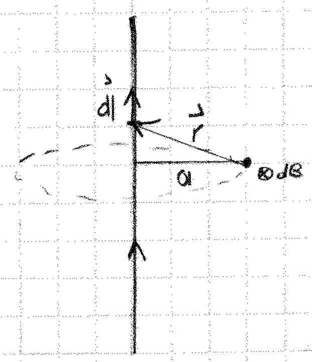
$$IBr\,d\varphi = N\,d\varphi$$

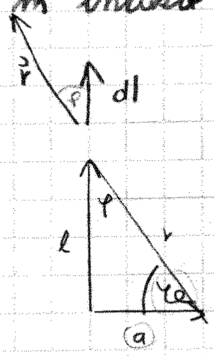
$$\sigma = \frac{N}{S} \qquad I = \frac{N}{Br}$$

$$N = \sigma \cdot S \qquad I = \frac{\sigma S}{B r}$$

BIOT-SAVAROV ZAKON

Po zelo dolgi ravni tanki žici teče tok 27 A. Kolikšna je gostota magnetnega polja, ki je 12 cm od žice in enako od obeh krajišč.





$$\int d\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{r \times dl}{r^3}$$

$$dB = \frac{\mu_0 I}{4\pi}\,\frac{dl \cdot \sin\varphi}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi} \cdot \int_{\pi/2} \frac{\sin\varphi\, dl}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-\pi/2} \frac{\cos\vartheta\, a\cdot d\vartheta \cdot \cos^2\vartheta}{\cos^2\vartheta\, a^2}$$

$$\sin\varphi = \cos\vartheta \qquad l = a\,\mathrm{tg}\,\vartheta$$

$$r = \frac{a}{\cos\vartheta} \qquad dl = \frac{a\,d\vartheta}{\cos^2\vartheta}$$

$dl =$

$$B = \frac{\mu_0 I}{4\pi a}\int_{-\pi/2}^{\pi/2} \cos\vartheta\, d\vartheta$$

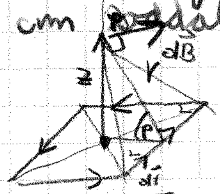
$$B = \frac{\mu_0 I}{4\pi a}\,\sin\vartheta\Big|_{-\pi/2}^{\pi/2}$$

$$B = \frac{\mu_0 I}{2\pi a}$$

če pa imamo končno dolg vodnik

$$\frac{\mu_0 I}{4\pi a}(\sin\alpha + \sin\beta)$$

Po kvadratni zanki s stranico 20 cm teče tok 80 A. Kolikšna je gostota mag. polja, ki se nahaja na geom. osi zanke in je 8 cm oddaljena od središča zanke.



vodoravne komp. se odštejejo

$$B = \frac{\mu_0 I}{4\pi\sqrt{a^2/4 + z^2}} \cdot \frac{a}{\sqrt{z^2 + \frac{a^2}{2}}} \cdot 4 \qquad 2\cdot\sin\alpha = \frac{\frac{a}{2}}{\sqrt{z^2 + \left(\frac{a\sqrt{2}}{2}\right)^2}}$$

$$r = \sqrt{\frac{a^2}{4} + z^2} \qquad B = \frac{\mu_0 I a}{\sqrt{z^2 + \frac{a^2}{4}}\sqrt{z^2 + \frac{a^2}{2}}\ \pi} \cdot \frac{a}{2\sqrt{z^2 + \frac{a^2}{4}}}$$

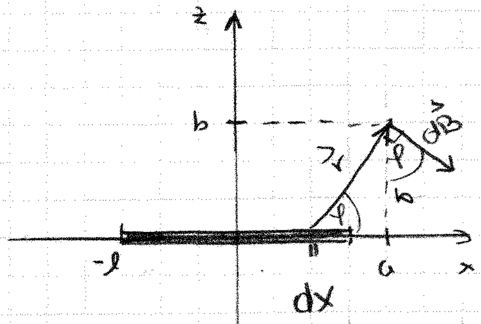
$$= \frac{2a}{2\sqrt{z^2 + \frac{a^2}{2}}}$$

$$B = \frac{\mu_0 I a^2}{2\pi\left(z^2 + \frac{a^2}{4}\right)\sqrt{z^2 + \frac{a^2}{2}}}$$

$$\cos\varphi = \frac{\frac{a}{2}}{\sqrt{z^2 + \frac{a^2}{4}}} = \frac{a}{2\sqrt{z^2 + \frac{a^2}{4}}}$$

$$B(z=0) = \frac{2\mu_0 I\sqrt{2}}{\pi a}$$

Zelo dolg tanek kovinski trak, ki je širok 50 cm leži v ravnini xy, tako da os y predstavlja vzdolžno simetralo traku. Po traku teče električni tok 300 A in je razporejen po celotnem prerezu traku. Kolikšna je magnetna gostota B v točki (x,z), kjer $x = 32$ cm, $z = 23$ cm



$$dB_x = \frac{\mu_0 dI \cdot \sin\varphi}{2\pi r}$$

Ni treba znat za kol.

$$dB_z = \frac{\mu_0 dI \cdot \cos\varphi}{2\pi r}$$

$$dI = \frac{I}{2l}\,dx$$

$$dB_x = \frac{\mu_0 dI \cdot \sin\varphi}{2\pi r} = \frac{\mu_0 I \sin\varphi \cdot dx}{4\pi l r}$$

$$dB_z = \frac{\mu_0 I \cos\varphi\, dx}{4\pi l r}$$

$$r = \sqrt{b^2+(a-x)^2}$$

$$\sin\varphi = \frac{b}{\sqrt{b^2+(a-x)^2}}$$

$$\cos\varphi = \frac{a-x}{\sqrt{b^2+(a-x)^2}}$$

$$dB_x = \frac{\mu_0 I\, b\, dx}{4\pi l\,(b^2+(a-x)^2)}$$

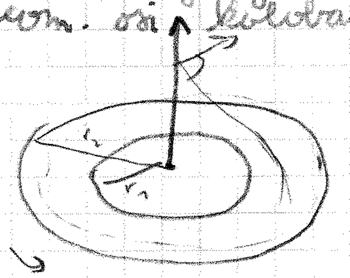
$$dB_z = \frac{\mu_0 I\,(a-x)\,dx}{4\pi l\,(b^2+(a-x)^2)}$$

$$B_x = \frac{\mu_0 I b}{4\pi l}\int_{-l}^{l}\frac{dx}{b^2+(a-x)^2} = \frac{\mu_0 I}{4\pi l}\left(\mathrm{arctg}\left(\frac{a+l}{b}\right) - \mathrm{arctg}\left(\frac{a-l}{b}\right)\right).$$

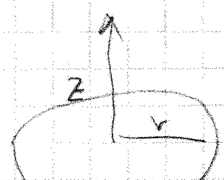
$$\int \frac{dx}{b^2+(a-x)^2} = -\frac{\mathrm{arctg}\left(\frac{a-x}{b}\right)}{b}$$

$$B_z = \frac{\mu_0 I (a-x)}{4\pi l}\int_{-l}^{l}\frac{(a-x)}{b^2+(a-x)^2} = \frac{\mu_0 I}{8\pi l}\ln\frac{b^2+(a+l)^2}{b^2+(a-l)^2}$$

Ravna tanka plošča ima obliko kolobarja s polmeroma 2 in 5 cm. Po plošči je enakomerno porazdeljen naboj $\sigma = 10^{-4}$ As/cm^2. Plošča se vrti okoli svoje geometrijske osi s hitrostjo 200 s^{-1}. Kolikšna je gostota mag. polja v točki, ki se nahaja na geom. osi kolobarja in je 6 cm oddaljena od središča kolobarja.



$$dB = \frac{\mu_0\, dI\, \rho^2}{2(z^2+\rho^2)^{3/2}} = \frac{\mu_0 \sigma \omega\, \rho^3 d\rho}{2(z^2+\rho^2)^{3/2}}$$



$$\frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}$$

$$dQ = \sigma\, dS = \sigma\,\rho\, d\rho\, d\varphi$$

$$B = \frac{\mu_0 \sigma \omega}{2}\int \frac{\rho^3 d\rho}{(z^2+\rho^2)^{3/2}} = \frac{\mu_0 \sigma \omega}{4}\int \frac{\overbrace{2\rho\, d\rho}^{du}\ \overbrace{\rho^2}^{u-z^2}}{u^{3/2}}$$

$$\frac{dQ}{dt} = \sigma\rho\, d\rho\,\frac{d\varphi}{dt} = \sigma\rho\, d\rho\,\omega = dI$$

$$z^2+\rho^2 = u$$

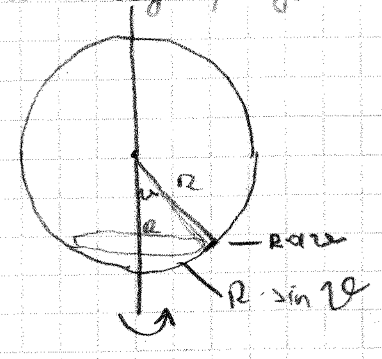
$$2\rho\, d\rho = du$$

$$B = \frac{\sigma \mu_0 \omega}{4} \int_{z^2+r_1^2}^{z_2+r_2^2} \frac{du\,(u-z^2)}{u^{3/2}}$$

$$= \frac{\mu_0 \sigma \omega}{4}\left[\int_{r_1^2+z^2}^{z^2+r_2^2} \frac{du}{u^{1/2}} - z^2 \int_{z_2^2+r_1^2}^{z^2+r_2^2} \frac{du}{u^{3/2}}\right] =$$

$$= \frac{\mu_0 \sigma \omega}{4}\left[2\sqrt{u}\Big|_{z_2+r_1^2}^{z^2+r_2^2} + z^2 \frac{2}{\sqrt{u}}\Big|_{z^2+r_1^2}^{z^2+r_2^2}\right] = \frac{\mu_0 \sigma \omega}{2}\left[\sqrt{r_2^2+z^2} - \sqrt{r_1^2+z^2} + \frac{z^2}{\sqrt{r_2^2+z^2}} - \frac{z^2}{\sqrt{r_1^2+z^2}}\right]$$

Po notranjosti tanke krogelne lupine s polmerom 8 cm je enakomerno porazdeljen naboj 10^{-7} As/cm^2. Krogla se vrti okoli enega od svojih polmerov s hitrostjo 400 s^{-1}. Kolikšna je gostota mag. polja v središču krogle.



$$dQ = \sigma\, dS = \sigma R \cdot \cancel{\sin\vartheta}\, d\vartheta \cdot R \cdot \sin\vartheta \cdot d\varphi$$

$$= \sigma R^2 \sin\vartheta\, d\vartheta\, d\varphi$$

$$\frac{dQ}{dt} = \sigma R^2 \sin\vartheta\, d\vartheta \cdot \omega = dI$$

$$dB = \frac{\mu_0\, dI\, R^2 \sin^2\vartheta}{2(R^2\sin^2\vartheta + R^2\cos^2\vartheta)^{3/2}}$$

$$dB = \frac{\mu_0\, dI\, R^2 \sin^2\vartheta}{2R^3} = \frac{\mu_0 \sigma \omega R \sin^3\vartheta\, d\varphi}{2}$$

$$B = \frac{\mu_0 \sigma \omega R}{2}\int_0^{\pi} \sin^3\vartheta\, d\vartheta = \frac{\mu_0 \sigma \omega R}{2}\left(-\cos\vartheta + \frac{1}{3}\cos^3\vartheta\right)\Big|_0^{\pi}$$

$$= \frac{\mu_0 \sigma \omega R}{2}\left(+1 - \frac{1}{3} + 1 - \frac{1}{3}\right) = \frac{2}{3}\mu_0 \sigma \omega R$$

$$p_m = I\, S$$

$$S = \pi (R \cdot \sin\vartheta)^2$$

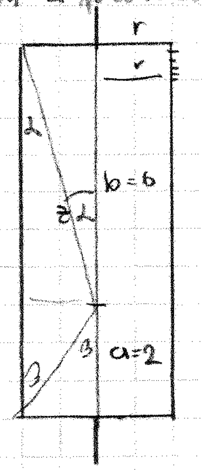
$$dp_m = dI \cdot S = \sigma R^2 \omega \sin\vartheta\, d\vartheta\, \pi\, (R^2 \sin^2\vartheta)$$

$$= \sigma R^4 \pi\, \omega \sin^3\vartheta\, d\vartheta$$

$$p_m = \sigma \pi R^4 \omega \underbrace{\int_0^{\pi} \sin^3\vartheta\, d\vartheta}_{4/3} =$$

$$= \sigma \pi R^4 \omega \,\frac{4}{3} = \frac{4\pi R^4 \omega \sigma}{3}$$

8 cm dolga tuljava s polmerom 1 cm ima gosto navite ovoje, tako da pride 5 ovojev na mm dolžine tuljave. Po ovojih teče tok 3 A. Kolikšna je gostota magnetnega polja, ki se nahaja v geom. osi tuljave in je oddaljena od 1.6 cm in 2 ploskve 2 cm.



$5\,mm^{-1}$ $n = \frac{N}{l}$

$dz \rightarrow n \cdot dz = N$

$dI = n \cdot I \cdot dz = \frac{N}{l} I\, dz$

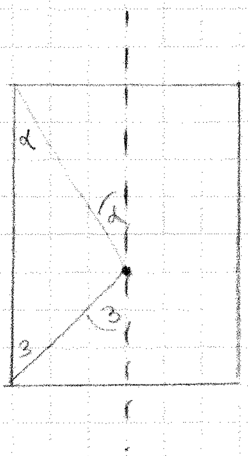
$$dB = \frac{\mu_0\, dI \cdot R^2}{2(z^2+R^2)^{3/2}} = \frac{\mu_0 I \cdot N R^2 dz}{2l\,(z^2+R^2)^{3/2}}$$

$$B = \frac{\mu_0 N \cdot I R^2}{2l} \int_{-a}^{l-a} \frac{dz}{(z^2+R^2)^{3/2}}$$

$$= \frac{\mu_0 N \cdot I \cancel{R^2}}{2l} \left. \frac{z}{\cancel{R^2}\sqrt{z^2+R^2}} \right|_{-a}^{l-a}$$

$$= \frac{\mu_0 N I}{2l} \left[\frac{l-a}{\sqrt{(l-a)^2+R^2}} + \frac{a}{\sqrt{a^2+R^2}} \right]$$

$$= \frac{\mu_0 N \cdot I}{2l} (\cos\alpha + \cos\beta)$$



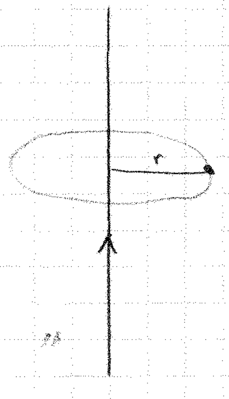
$$B(z = l/2) = \frac{\mu_0 N \cdot I}{2l} \left[2 \frac{\frac{l}{2}}{\sqrt{\frac{l^2}{4}+R^2}} \right] = \frac{\mu_0 N \cdot I}{\sqrt{4R^2+l^2}}$$

$$l >> R \Rightarrow B = \frac{\mu_0 N \cdot I}{l}$$

ZAKON O MAGNETNI NAPETOSTI

$$\oint \vec{H} \cdot d\vec{s} = \int_S \left(j + \frac{\partial D}{\partial t}\right) \cdot d\vec{S}$$

gostota el. toka — premikalni tok, posl. mr. polja

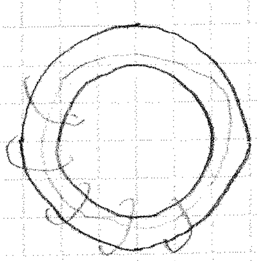


$$H \cdot 2\pi r = I$$

$$H = \frac{I}{2\pi r}$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Zgled: Na železno jedro toroida je enakomerno navitih 500 ovojev. Srednji obseg toroida pa je 50 cm. Permeabilnost železa je 4000. Po ovojih najprej teče tok 2 A, nato v železno jedro naredimo 1 mm široko režo. Kolikšen tok mora sedaj teči po ovojih, da se ohrani magnetni pretok skozi jedro.



$$\Phi_m = \int \vec{B}\, d\vec{S}$$

$$\Phi_m = B \cdot S$$

$$\oint H \cdot d\vec{s} = N \cdot I_1$$

$$H_0\, l = N \cdot I_1$$

$$H_0 = \frac{N \cdot I_1}{l} \Rightarrow B = \mu \mu_0 \frac{N \cdot I_1}{l}$$

$$\oint H \cdot d\vec{s} = N \cdot I_2$$

$$H_1 \cdot (l - d) + H_2 \cdot d = N \cdot I_2$$

$$\mu \cancel{\mu_0} H_1 = \cancel{\mu_0} H_2$$

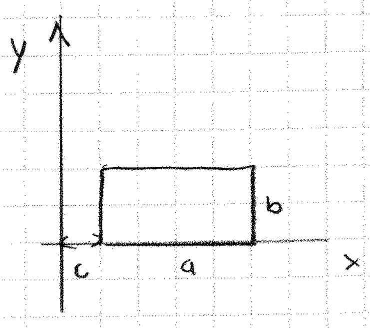
$$H_1 (l - d) + \mu H_1 \cdot d = N \cdot I_2$$

$$H_1 = \frac{N \cdot I_2}{l - d + \mu d} = \frac{N \cdot I_2}{l + d(\mu - 1)}$$

$$\frac{N \cdot I_2}{l + d(\mu - 1)} = \frac{N \cdot I_1}{l}$$

$$I_2 = I_1 \cdot \frac{l + d(\mu - 1)}{l}$$

Po zelo dolgi ravni tanki žici teče električni tok 72 A. Pravokotna zanka s stranicama 15 in 8 cm leži tako, da krajša leži vzporedno z vodnikom. Bližja je oddaljena 4 cm. Kolikšen je mag. pretok skozi zanko.

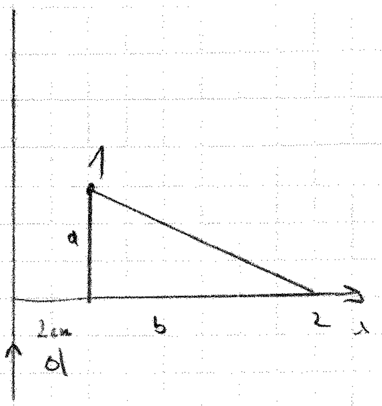


$$\Phi_m = \int \vec{B} \cdot d\vec{S}$$

$$= \frac{\mu_0 I}{2\pi} \int_c^{a+c} \frac{dx}{x} \int_0^b dy$$

$$= \frac{\mu_0 I \cdot b}{2\pi} \ln \frac{a+c}{c}$$

Po zelo dolgi ravni tanki žici teče tok 72 A. Zanka, ki ima obliko pravokotnega Δ, leži v ravnini tako, da je krajša kateta vzporedna z vod. ~~za~~ Zanka je obrnjena tako kot kaže slika. Krajša kateta je 2 cm oddaljena od vodnika.



$$\phi_m = \frac{\mu_0 I}{2\pi} \int^{d+b} \frac{dx}{x} \int^{y(x)} dy$$

$$= \frac{\mu_0 I}{2\pi} \int_d^{d+b} \frac{y(x)\,dx}{x}$$

$$= \frac{\mu_0 I}{2\pi} \frac{a}{b} \int_d^{d+b} \frac{(x+b-d)\,dx}{x}$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - a = \frac{a}{b} \cdot (x - d)$$

$$y = \frac{ax}{b} - \frac{ad}{b} + a = \frac{ax - ad + ab}{b} = \frac{a}{b}(x - d + b)$$

$$= \frac{\mu_0 I}{2\pi} \frac{a}{b} \int_d^{d+b} \overset{x}{dx} + b \int_d^{d+b} \frac{dx}{x}^{\ln x} + d \int_d^{d+b} \frac{dx}{x}^{\ln x}$$

$$= \frac{\mu_0 I}{2\pi} \frac{a}{b} * (d+b-d) + b \ln \frac{d+b}{d} - d \ln \frac{d+b}{d}$$

$$= \frac{\mu_0 I}{2\pi} \frac{a}{b} \left[b + \ln \frac{d+b}{d} (b-d) \right]$$

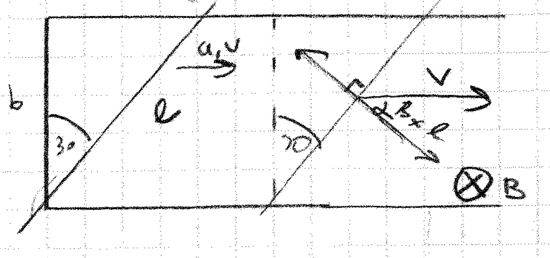
INDUCIRANA NAPETOST

FIZ II-V
27.3.2008

$$U_i = \int \vec{v}\cdot(\vec{B}\times d\vec{l})$$
$$= \int dl\,(\vec{v}\times\vec{B})$$
$$= \int B\,(d\vec{l}\times\vec{v})$$

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int B\cdot d\vec{S}$$
$$U_i = -\frac{d\Phi_m}{dt} \qquad U_i\,\Delta t = -\Delta\Phi_m$$

Dva zelo dolga vzporedna ravna vodnika sta med seboj oddaljena 5 cm. Levi krajišči sta povezana s prečko, ki ima enak premer in spec. upornost. Homogeno magnetno polje je pravokotno na ravnino, druga kovinska prečka z enakim presekom in specifično upornostjo minije, da s 1. prečko oklepa 30° kot kaže slika. V nekem trenutku se ta prečka začne gibati s konst. pospeškom vzdolž vodnikov. Kolikšna je razdalja med prečkama v trenutku, ko je induciran tok največji.



$$I = \frac{U_i}{R} = \frac{v\,B\,b\,S}{\zeta\,(b+\frac{b}{\cos\alpha}+x+x+b\,\mathrm{tg}\,\alpha}$$

obseg zanke

l

$$U_i = \vec{v}\cdot(\vec{B}\times\vec{l})$$
$$U_i = v\cdot B\cdot l\cdot\cos\alpha = v\cdot B\,\frac{b}{\cos\alpha}\cdot\cos\alpha = v\cdot B\cdot b$$

$v = at$
$x = \frac{at^2}{2}$

$$I_i = \frac{at\,B\cdot b\,S}{\zeta\,(at^2+b(1+\frac{1}{\cos\alpha}+\mathrm{tg}\,\alpha))}$$

$$I_i = \frac{B\cdot b\cdot S\,a\cos\alpha\cdot t}{\zeta\,(\frac{at^2}{\cos\alpha}+b\,(1+\cos\alpha+\sin\alpha))}$$

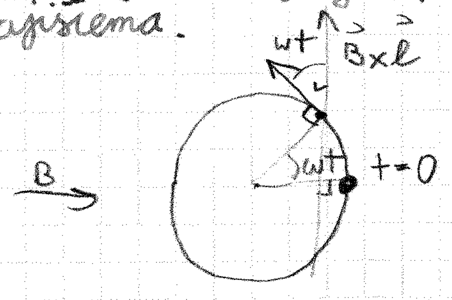
$$I_i = \frac{B\cdot b\cdot S\cdot a\cdot\cos\alpha}{\zeta}\;t\,\Big(\underbrace{\frac{at^2}{\cos\alpha}+b\,(1+\cos\alpha+\sin\alpha)}_{Y}\Big)^{-1}$$

$$\frac{dI}{dt} = 0 = \Big(\frac{at^2}{\cos\alpha}+b\,(1+\cos\alpha+\sin\alpha)\Big)^{-1} + t(-1)\,\big(at^2+b\,(1+\cos\alpha+\sin\alpha)\big)^{-2}\cdot 2at\cos\alpha \quad /\cdot Y^2$$

$$at^2\cos\alpha + b\,(1+\cos\alpha+\sin\alpha) - 2at^2\cos\alpha = 0$$

$$t^2 = \frac{b\,(1+\cos\alpha+\sin\alpha)}{a\cdot\cos\alpha} \Rightarrow x = \frac{at^2}{2} = \frac{b\,(1+\cos\alpha+\sin\alpha)}{2\cos\alpha}$$

0,5 m ravna tanka prečka kroži po plašču valja s polmerom 12 cm s konstantno hitrostjo 70 s^{-1} Tako da je vseskozi vzporedna z geometrijsko osjo. Mag. polje je ima jakost 0,6 T. Kolikšna je ef. vrednost napetosti, ki se inducira med krajiščema.



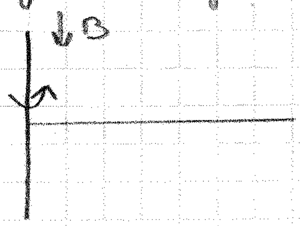
$$U_i = \vec v \cdot (\vec B \times \vec l)$$

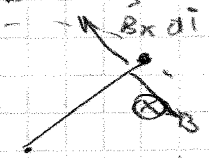
$$U_i = B l \cdot \vec v \cdot \cos \omega t$$

$$U_i = B l \, r \cdot \omega \cos \omega t$$

$$U_{ief} = \frac{B l r \omega}{\sqrt{2}}$$

50 cm dolga ravna kovinska prečka se vrti okoli navpične osi, ki je pravokotna na prečko s konst. hitrostjo 60 s^{-1} v krajišču. Polje je homogeno z jakostjo B = 0,2 T. Kakšna U_i se inducira?

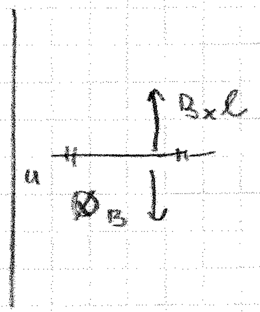




$$U_i = \int \vec v \cdot (\vec B \times d\vec l) \qquad dl \to dr, \quad v = \omega r$$

$$U_i = B \cdot \omega \int_0^l \vec r \cdot d\vec r = B \cdot \omega \frac{l^2}{2}$$

Po zelo dolgem ravnem tankem vodniku teče tok 8 A. 20 cm dolga ravna tanka kovinska prečka se giblje vzdolž vodnika s konst. hitrostjo 8 cm/s. Tako da je ves čas ⊥ vodnik, bližje krajišče pa je ves čas a=5 cm oddaljeno od vodnika. U_i = ?



$$U_i = \int v \cdot (\vec B \times d\vec l)$$

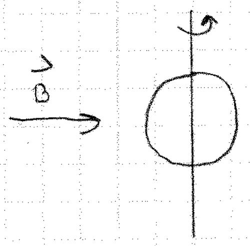
$$B = \frac{\mu_0 I}{2\pi x}$$

$$dl = dx$$

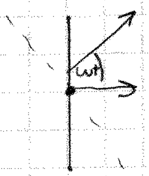
$$U_i = \frac{\mu_0 I}{2\pi} \cdot v \int_a^{a+l} dx/x$$

$$U_i = \frac{\mu_0 I v}{2\pi} \ln \frac{a+l}{a}$$

Tanko žico zvijemo v krožno zanko z $r = 7$ cm. Upornost zanke je $R = 0{,}1\,\Omega$. Zanko vrtimo okoli polmera z $\omega = 30\,s^{-1}$. Polje $B = 0{,}5\,T$ je $\perp$ na os vrtenja. Kolikšen izmeničen magnetni navor deluje na to zanko.



$$U_i = -\frac{d}{dt}\int \vec{B}\cdot d\vec{S}$$
$$= -\frac{d}{dt}(B\cdot S)$$
$$= -\frac{d}{dt}(B\cdot S\cdot \cos\omega t)$$
$$= B\cdot S\omega \sin\omega t$$
$$= B\pi r^2 \omega \sin\omega t$$



KOLOKVIJ !!!

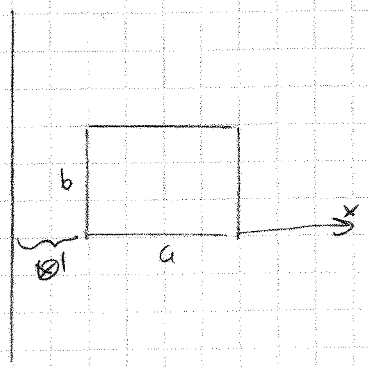
$$I_i = \frac{U_i}{R} = \frac{B\pi R^2\omega}{R}\cdot \sin\omega t$$

$$M = p_m \times \vec{B}$$
$$M = I_i S \times \vec{B}$$
sin ωt

$$M = I_i\cdot S\cdot B\cdot \sin\omega t$$
$$M = \frac{B\cdot\pi\cdot r^2\omega}{R}\cdot \sin\omega t\cdot \pi\cdot r^2\cdot B\cdot \sin\omega t$$
$$M = \frac{B^2\pi^2\cdot r^4\cdot\omega}{R}\sin^2\omega t$$
$$\langle M\rangle = \frac{B^2\pi^2 r^4\omega}{2R}$$

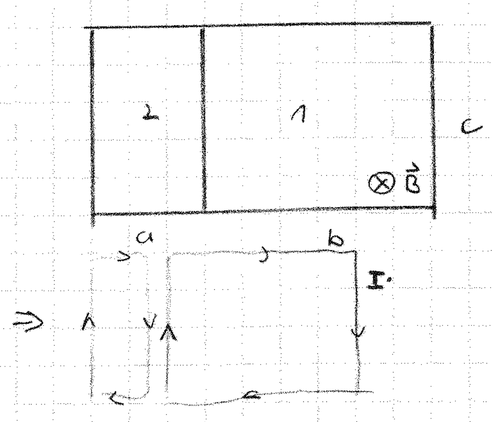
Po zelo dolgem ravnem tankem vodniku teče tok 100 A. Pravokotna zanka s stranicama 12 in 7 cm leži v ravnini vodnika tako, da sta krajši stranici vzporedni z vodnikom. Bližja stranica je oddaljena 5 cm od vodnika. Tok začne eksp. pojemati z enačbo $I = I_0 e^{-t/\tau}$, $\tau = 7$ s. Kolikšna napetost se inducira v zanki 2 s po ugasnitvi toka.

KOLOKVIJ !!!



$$U_i = -\frac{d\Phi_m}{dt}$$
$$\Phi_m = \int B\cdot dS = \frac{\mu_0 I}{2\pi}\int_0^b dy \int_d^{a+d}\frac{dx}{x}$$
$$B = \frac{\mu_0 I}{2\pi x} \qquad = \frac{\mu_0 I\cdot b}{2\pi}\ln\frac{a+d}{d}$$
$$dS = dx\cdot dy$$
$$\Phi_m = \frac{\mu_0 I_0 b}{2\pi}\ln\frac{a+d}{d}\, e^{-\frac{t}{\tau}}$$
$$U_i = -\frac{d\Phi_m}{dt} = \frac{\mu_0 I_0\cdot b}{2\pi}\ln\frac{a+d}{d}\cdot\frac{1}{\tau}\, e^{-\frac{t}{\tau}}$$

* Iz bakrene žice, ki ima presek $1\,mm^2$ in $\rho = 0{,}017\,\Omega \frac{mm^2}{m}$ naredimo pravokotno zanko, ki jo kaže slika. Zanka leži v ravnini, ki je pravokotna na mag. polje z gostoto 0,3 T. V nekem trenutku začne polje ugašati z enačbo $B = B_0(1-\frac{t}{\tau})^2$, $\tau = 1\,s$, $t = 2\,s$. Kolikšen tok teče po srednji prečki 2 s po ugašanju polja.



$$I = \frac{U_1 - U_2}{R} = \frac{(U_{i1} - U_{i2})S}{\rho\, c}$$

$$I_1 = \frac{U_1}{R_1} \qquad U_1 = -\frac{d}{dt}(B \cdot b\, c) = B_0\, b \cdot c \frac{d}{dt}\left(1-\frac{t}{\tau}\right)^2 = -B_0 b c\, 2\left(1-\frac{t}{\tau}\right)\left(-\frac{1}{\tau}\right) = \frac{2B_0 b c}{\tau}\left(1-\frac{t}{\tau}\right)$$

$$I_1 = \frac{\cancel{2} B_0 b c \cdot S}{\rho \cdot \tau\, \cancel{2}(b+c)}\left(1-\frac{t}{\tau}\right)$$

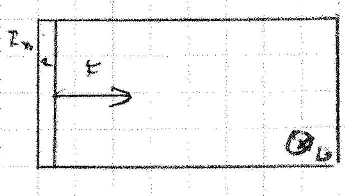
presek žice

samo a namesto b

$$I_2 = \frac{B_0 a c \cdot S}{\rho \cdot \tau\,(a+c)}\left(1-\frac{t}{\tau}\right)$$

$$I_1 - I_2 = \frac{B_0 c S}{\rho \tau}\left(1-\frac{t}{\tau}\right)\left[\frac{b}{b+c} - \frac{a}{a+c}\right]$$

2 zelo dolga vzporedna ravna vodnika sta med seboj razmaknjena 30 cm. Levi krajišči obeh vodnikov sta povezani s prečko. Upornost je pri vodnikih in prečki zanemarljivo majhna. 2 kovinska prečka z maso 70 g miruje ob 1 prečki. R prečke je 0,2 Ω. Homogeno mag polje pa je pravokotno na ravnino $B = 0{,}6\,T$. V nekem trenutku začnemo prečko vleči s konst. silo 0,02 N. Kakšna je v po 3 s hitrost prečke.



$$F - F_m = m \cdot a$$

$$F - I \cdot l \cdot B = m \cdot a$$

$$F - lB\frac{U_i}{R} = m \cdot a$$

$$F - lB\frac{v \cdot B \cdot l}{R} = m\frac{dv}{dt}$$

$$F - \frac{B^2 l^2}{R} \cdot v = m\frac{dv}{dt}$$

$$\int_0^t dt = \int_0^v \frac{m\, dv}{F - \frac{B^2 l^2}{R} \cdot v}$$

$$t = -\frac{mR}{B^2\ell^2}\int_{F}^{F-\frac{B^2\ell^2}{R}v} \frac{dx}{x} = -\frac{mR}{B^2\ell^2}\ln \frac{F-\frac{B^2\ell^2}{R}v}{F}$$

$$F-\frac{B^2\ell^2}{R}v = x$$

$$-\frac{B^2\ell^2}{R}\,dv = x$$

$$dv = \frac{-dx\,R}{B^2\ell^2}$$

$$F-\frac{B^2\ell^2}{R}v = F\cdot e^{-\frac{B^2\ell^2 t}{mR}}$$

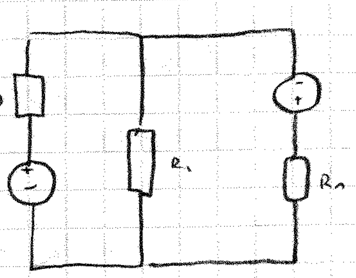
$$v = \frac{FR}{B^2\ell^2}\left(1-e^{-\frac{B^2\ell^2 t}{mr}}\right)$$

* Še enkrat

$$R_0 = \frac{\varsigma c}{S}$$

$$R_2 = \frac{\varsigma(2a+c)}{S}$$

$$R_1 = \frac{\varsigma(2b+c)}{S}$$



$$U_1 = -\frac{d\Phi_1}{dt} = \frac{2B_0bc}{2}\left(1-\frac{t}{\tau}\right)$$

$$B = B_0\left(1-\frac{t}{\tau}\right)^2$$

$$\frac{dB}{dt} = -2B_0\left(1-\frac{t}{\tau}\right)\left(-\frac{1}{\tau}\right)$$

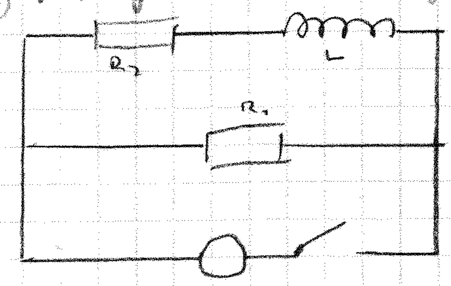
$$U_2 = \frac{2B_0ac}{\tau}\left(1-\frac{t}{\tau}\right)$$

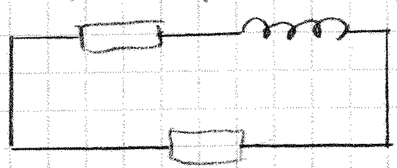
INDUKTIVNOST

$$\Phi_m = L I \qquad L = \left[\frac{Vs}{A} = H\right]$$

$$L = \frac{\mu_0 N^2 S}{\ell}$$

12 Ω in 15 Ω upor in tuljava so priključeni na vir napetosti 1,5 V tako kot kaže slika. L = 1 mH. V nekem trenutku prekinemo stikalo na sliki. Kolikšen tok teče skozi krog, ki je ostal sklenjen t = 0,1 ms po odklopu.





$$U_i = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}(LI)$$

$$U_i = -L\frac{dI}{dt}$$

$$U_L + U_{R_1} + U_{R_2} = 0$$

$$L\frac{dI}{dt} + IR_1 + IR_2 = 0$$

$$L\frac{dI}{d} = -I(R_1+R_2)$$

$$dt = -\frac{L\,dI}{(R_1+R_2)\,I}$$

$$\int_0^t dt = -\frac{L}{R_1+R_2}\int_{I_2}^{I}\frac{dI}{I}$$

$$t = -\frac{L}{R_1+R_2}\ln\frac{I}{I_2}$$

$$I = I_2\, e^{\frac{-(R_1+R_2)t}{L}}$$

$$I_1+I_2=\frac{U}{\frac{R_1R_2}{R_1+R_2}} \qquad I_1\cdot R_1=I_2\cdot R_2$$

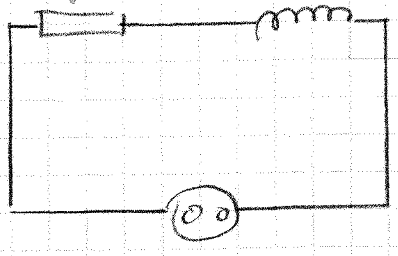
$$I_2\frac{R_1}{R_2}+I_2=\frac{U(R_1+R_2)}{R_1\cdot R_2}$$

$$I_2R_2^2+I_2R_1R_2=U(R_1+R_2)$$

$$I_2R_2(R_1+R_2)=(R_1+R_2)U$$

$$I=\frac{U}{R_2}e^{-\frac{(R_1+R_2)t}{L}}$$

20 ohmski upor in tuljavo z induktivnostjo 1 mH vežemo zaporedno in priključimo na generator izmenične napetosti $U=U_0\sin\omega t$ z amplitudo 3V in $\omega=30000\,s^{-1}$ in zanemarljivo notranjo upornostjo. Kolikšna povprečna moč se porablja na uporu.



$$U+U_R+U_L=0$$

$$U_0\sin\omega t-IR-L\frac{dI}{dt}=0$$

$$\frac{dI}{dt}+\frac{R}{L}I=\frac{U_0}{L}\sin\omega t$$

$$I=I_0\sin(\omega t+\delta)$$

$$\frac{dI}{dt}=I_0\omega\cos\omega t\cos\delta-I_0\omega\sin\omega t\sin\delta$$

$$I_0\omega\cos\omega t\cos\delta-I_0\omega\sin\omega t\sin\delta+\frac{RI_0}{L}\sin\omega t\cos\delta+\frac{RI}{L}\sin\omega t\cos\delta$$
$$=\frac{U}{L}\sin\omega t$$

$$I_0\omega\cos\omega t+\frac{RI_0}{L}\sin\delta=0 \qquad \Rightarrow \operatorname{tg}\delta=-\frac{\omega L}{R}$$

$$-I_0\omega\sin\omega t+\frac{RI_0}{L}\cos\delta=\frac{U_0}{L}$$

$$I_0^2\omega^2+\frac{R^2I_0^2}{L^2}=\frac{U_0^2}{L^2}$$

$$I_0^2(\omega^2L^2+R^2)=U_0^2$$

$$I_0=\frac{U_0}{\sqrt{R^2+\omega^2L^2}}$$

$$P=I^2\cdot R=$$

$$=I_0^2\cdot R\sin^2(\omega t+\delta)$$

$\langle P\rangle$

$$\langle\sin^2(\omega t+\delta)\rangle \qquad \omega t+\delta=x \quad dx=\omega dt \quad dt=\frac{dx}{\omega}$$

$$=\frac{\omega}{\pi}\int_0^{\pi/\omega}\sin^2(\omega t+\delta)\,dt=$$

$$=\frac{1}{\pi}\int_\delta^{\pi+\delta}\sin^2x\,dx$$

$$=\frac{1}{\pi}\left[\frac{1}{2}x-\frac{1}{4}\sin 2x\right]\Big|_\delta^{\pi+\delta}$$

$$=\frac{1}{\pi}\left[\frac{\pi+\delta}{2}-\frac{1}{4}\sin(2\pi+2\delta)-\frac{\delta}{2}+\frac{1}{4}\sin2\delta\right]$$

$$= \frac{1}{2} \cdot 1 \quad \frac{1}{4}\sin 2\delta - \frac{1}{4}\sin(2\pi + 2\delta)$$

$$= \frac{1}{4}\left(\sin 2\delta - \cancel{\sin 2\pi \cos 2\delta + \cos 2\pi \sin 2\delta}\right)$$

$$= \frac{1}{2} = <$$

$$<P> = \frac{1}{2} I_0^2 \cdot R = \frac{U_0^2 R}{(R^2 + \omega^2 L^2) 2}$$

VAJE 15

$P = \frac{U_0}{}$

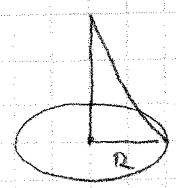
$7{,}50W = \frac{}{(1{,}5)^2 \cdot 40}$

$2r = 0{,}3\,m$

$Q = -10^{-10}\,As$

$m = 3{,}4 \cdot 10^{-27}\,kg$

$$0 = \frac{1}{2}mv^2 + e_0 \frac{q}{4\pi\varepsilon_0 R}$$



$$v = \sqrt{\frac{-e_0 q}{2\pi\varepsilon_0 R m}}$$

$$= \sqrt{\frac{1{,}6 \cdot 10^{-19} \cdot 10^{-10}\,As^2}{2 \cdot \pi \cdot 8{,}86 \cdot 10^{-12} \frac{As}{Vm} \cdot 0{,}15\,m \cdot 3{,}4 \cdot 10^{-27}\,kg}}$$

$$=$$

16. $R = 3\,cm$

$e_2 \; 0{,}1 \cdot 10^{-6}\,As$

$v = 30\,km/h$

$$\frac{1}{2}mv^2 = \frac{Q \cdot e_2}{4\pi\varepsilon_0 \sqrt{R^2+x^2}}$$

$$2\pi\varepsilon_0 m v^2 \sqrt{R^2+x^2} = Q \cdot e_2$$

$$\sqrt{R^2+x^2} = \frac{Q \cdot e_2}{2\pi\varepsilon_0 m v^2}$$

$$x = \sqrt{\left(\frac{Q \cdot e_2}{2\pi\varepsilon_0 m v^2}\right)^2 - R^2}$$

Krogelni kondenzator ima koncentrični elektrodi s polmeroma 2 in 5 cm. Med elektrodama pa je vakum. Priključen je na 6000V. Proton z maso $1{,}672 \cdot 10^{-27}$ kg in je nabit z $1{,}6 \cdot 10^{-19}$ As. Če je $r_3 = 3\,cm$ od središča je $v = 0$. Kakšna je hitrost po $r_4 = 4\,cm$.

$$e_0 U_3 = \frac{1}{2}mv^2 + e_0 V_4$$

$$v = \sqrt{\frac{2 e_0 (U_3 - U_4)}{m}}$$

$$\oint D \cdot d\vec{S} = e$$

$$4\pi\varepsilon_0 r^2 \cdot E = CU$$

$$E = \frac{CU}{4\pi\varepsilon_0 r^2}$$

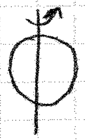
$$\int_{r_3}^{r_4} E \cdot dr = \frac{CU\,dr}{4\pi\varepsilon_0 r^2}$$

$$= \frac{CU}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)\Big|_{r_3}^{r_4} = \frac{CU}{4\pi\varepsilon_0}\left(\frac{1}{r_3} - \frac{1}{r_4}\right)$$

$$= \frac{\frac{4\pi\varepsilon_0 r_3 r_4}{r_4 - r_3} U (r_4 - r_3)}{4\pi\varepsilon_0 r_3 \cdot r_4}$$

$$=$$

Tanko žico zvijemo v krožno zanko z $r = 6cm$. Vrti se okoli enega od svojih premerov z $\omega = 70 s^{-1}$, $B = 0,7T$ je $\perp$ na os vrtenja. R zanke je $0,1\Omega$. Kolikšen električen ef I teče po zanki in kolikšen $\langle M \rangle$ deluje na zanko.



$$U_i = \frac{-d\phi}{dt} \qquad \phi_m = B \cdot S \cdot \cos\varphi$$

$$U_i = -\frac{d\phi}{dt} \qquad \phi_m = B \cdot S \cdot \cos\omega t$$

$$I = \frac{U_i}{R} = -\frac{B \cdot S \cdot \cos\omega t}{dt}$$

$$I = \frac{B \cdot S \cdot \omega \cdot \sin\omega t}{R}$$

$$I_{ef} = \frac{B \cdot S \cdot \omega \cdot \sin\omega t}{\sqrt{2} R}$$

$$M = p_m \times \vec{B}$$

$$M = I \cdot S \cdot B \cdot \cancel{\cos\omega t} \sin\omega t$$

$$M = \frac{B \cdot S \cdot \omega}{R} \cdot S \cdot B \cdot \cancel{\cos\omega t} \cdot \sin\omega t \cdot \sin\omega t$$

$$M = \frac{B^2 \cdot S^2 \cdot \omega}{R} \cdot \sin^2\omega t$$

$$\langle M \rangle = \frac{B^2 \cdot S^2 \cdot \omega}{2R}$$

NIHAJNI KROGI

FIZ II
3. 4. 2008

Iz tuljave $L = 1H$ in kond. $C = 1\mu F$ sestavimo idealni nihajni krog. Najprej kond. nabijemo na 3000 V potem pa sklenemo krog. Po kolikšnem času po sklenitvi se energiji kond. in tuljave med seboj prvič izenačita. Kolikšen je I_0 el. toka, ki niha po krogu.

$$\frac{CU_0^2}{2} = \frac{Q^2(t)}{2C} + \frac{LI^2(t)}{2} \quad / \cdot \frac{d}{dt} \cdot 2$$

$$I = \frac{dQ}{dt}$$

$$0 = \frac{1}{C} 2Q \cdot \frac{dQ}{dt} + L 2 I \frac{dI}{dt} \quad /:2$$

$$0 = \frac{-Q}{C} I + LI \frac{dI}{dt} \quad /:I$$

$$0 = -\frac{Q}{C} + L\frac{dI}{dt} \quad / \frac{d}{dt}$$

$$0 = \frac{I}{C} + L\frac{d^2I}{dt^2}$$

$$\frac{d^2I}{dt^2} + \frac{1}{LC} I = 0 \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = -\int I \, dt = -a \int \sin\omega_0 t \, dt$$

$$Q = \frac{a}{\omega_0} \cos\omega_0 t$$

$$\frac{d^2I}{dt^2} = \omega_0^2 I = 0$$

$$I = a \cdot \sin\omega_0 t + \overset{0}{b} \cos\overset{1}{\omega_0 t} \Rightarrow I = a \sin\omega t$$

$Q(t=0) = CU_0 = \frac{a}{\omega_0}$

$a = CU_0\omega_0 = U_0\sqrt{\frac{C}{L}}$

$I = U_0\sqrt{\frac{C}{L}}\sin\omega t \qquad Q_{(t)} = CU_0 \cdot \cos\omega_0 t$

$\Rightarrow \frac{Q^2}{2C} = \frac{LI^2}{2}$

$\frac{\cancel{C^2} U_0^2 \cos^2\omega_0 t}{2\cancel{C}} = LU_0^2 \frac{C}{L}\sin^2\omega t$

$\cos^2\omega_0 t = \sin^2\omega t$

$\omega_0 t = \frac{\pi}{4} + \frac{n\pi}{2}$

$t = \frac{\pi}{4\omega_0} = \frac{\pi}{4}\sqrt{LC}$

Iz tuljave in kondenzatorja sestavimo idealni nihajni krog. Za koliko odstotkov se spremeni lastna krožna frekvenca nihajnega kroga, če razmik med ploščama kond. povečamo za 20%.

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L\frac{\varepsilon_0 S}{x}}} = \sqrt{\frac{x}{L\varepsilon_0 S}}$

$\omega_0 + \Delta\omega_0 = \sqrt{\frac{x+\Delta x}{L\varepsilon_0 S}} \quad : \omega_0$

$1 + \frac{\Delta\omega_0}{\omega_0} = \frac{1}{\omega_0}\sqrt{\frac{x+\Delta x}{L\varepsilon_0 S}}$

$1 + \frac{\Delta\omega_0}{\omega_0} = \sqrt{\frac{\cancel{L\varepsilon_0 S}}{x}} \cdot \sqrt{\frac{x+\Delta x}{\cancel{L\varepsilon_0 S}}}$

$1 + \frac{\Delta\omega_0}{\omega_0} = \sqrt{1+\frac{\Delta x}{x}}$

$\frac{\Delta\omega_0}{\omega_0} = \sqrt{1+\frac{\Delta\omega_0}{\omega_0}} - 1 = \sqrt{1{,}2} - 1$

Tuljavo z induktivnostjo 1 mH in $C = 1\,\mu F$ in upor z upornostjo $0{,}1\,\Omega$ vežemo v nihajni krog. Kolikšen je nihajni čas nihanja v tem krogu in po kolikšnem času se začetna amplituda zmanjša na ½ začetne vrednosti.

$U_C + U_R + U_L = 0$

$\frac{Q(t)}{C} + I \cdot R - L\frac{dI}{dt} = 0 \qquad I = -\frac{dQ}{dt}$

$-\frac{I}{C} - R\frac{dI}{dt} - L\frac{d^2I}{dt^2} = 0$

$\frac{d^2I}{dt} + \underbrace{R/L}_{2\beta}\frac{dI}{dt} + \underbrace{\frac{1}{LC}}_{\omega_0^2} I = 0$

$\beta = \frac{R}{2L}$ - koef. dušenja

$$I_0 = I_0 e^{-\beta t} \cdot \cos \omega t$$

$$\tilde{\omega} = \sqrt{\omega_0^2 - \beta^2}$$

$$t_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}} = \frac{2\pi}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}$$

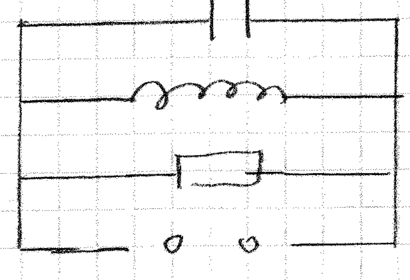
$$\frac{I_0}{4} = I_0 e^{-\beta t}$$

$$t = -\frac{1}{\beta} \ln \frac{1}{4} = -\frac{2L}{R} \ln \frac{1}{4}$$

20 Ω upor, tuljavo z induktivnostjo $L = 1\,mH$ vežemo zaporedno in priključimo na generator izmenične napetosti z amplitudo 1,5 V in krožno frekvenco $40000\ s^{-1}$. Kolikšen ef tok teče skozi vir napetosti in kolikšna bi morala biti ω na generatorju, da bi bil I_{ef} največji, če bi se U_0 ohranila.

$$I_0 = U_0 \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$\omega = \frac{1}{\sqrt{LC}}$$



$$I_{ef} = \frac{I_0}{\sqrt{2}}$$

1. $U_0 \cdot \sin \omega t - I_R \cdot R = 0$

$$I_R = \frac{U_0}{R} \cdot \sin \omega t$$

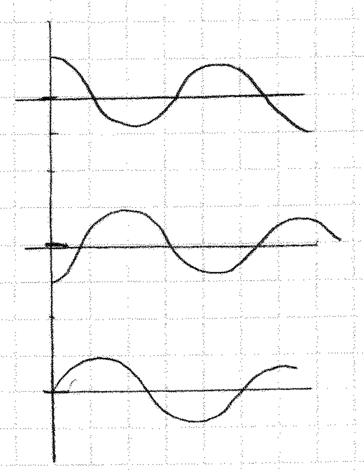
2. $U_0 \sin \omega t - L \frac{dI}{dt} = 0$

$$I_L = \frac{U_0}{L} \int \sin \omega t\, dt = -\frac{U_0}{L\omega} \cos \omega t$$

3. $U_0 \sin \omega t - \frac{Q}{C} = 0$

$$I_C = \frac{dQ}{dt}$$

$$I_C = U_0 \omega C \cos \omega t$$



$$I = I_R + I_L + I_C$$

$$= \frac{U_0}{R} \sin \omega t - \frac{U_0}{\omega L} \cos \omega t + \frac{U_0}{U_0} \omega C \cos \omega t$$

$$I = \frac{U_0}{R} \sin \omega t + U_0 \cos \omega t \left(\omega C - \frac{1}{\omega L}\right)$$

$$I = I_0 \sin(\omega t + \delta) = I_0 \sin \omega t \cos \delta + I_0 \cos \omega t \sin \delta$$

$$I_0 \sin \delta = U_0 \left(\omega C - \frac{1}{\omega L}\right)$$

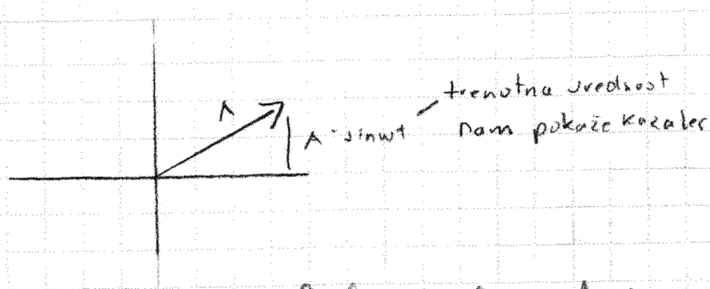
$$I_0 \cos \delta = \frac{U_0}{R}$$

$$\operatorname{tg} \delta = R \cdot \left(\omega C - \frac{1}{\omega L}\right)$$

$$I_0^2 = U_0^2 \left(\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right)$$

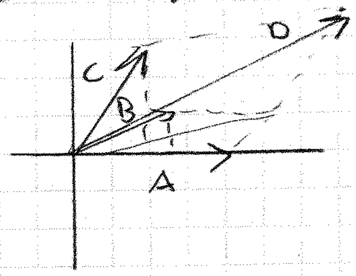
$$I_0 = U_0 \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

KAZALCI

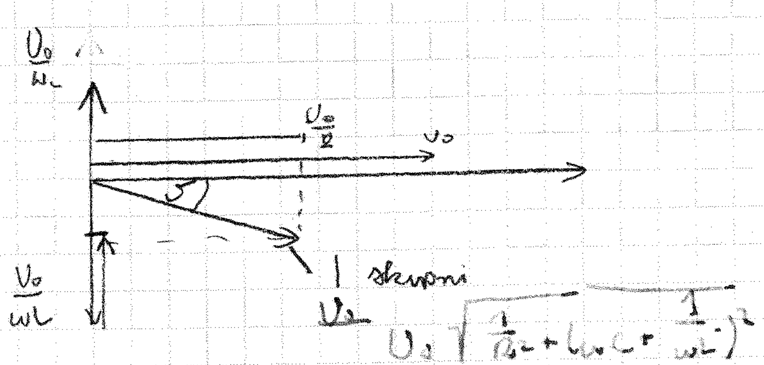


Če imamo več količin, ki nihajo

$D = A \cdot \sin\omega t + B \sin(\omega t + \delta_1) - C \cdot \sin(\omega t + \delta_2)$



v našem primeru bi rekli napetost:



ZAPOREDNA VEZAVA

$20\,\Omega$ upor in tuljavo $L = 1\,mH$ in kondenzator s kapaciteto $C = 1\,\mu F$ vežemo skupaj zaporedno in priključimo na $U = U_0 \sin\omega t$, $U_0 = 1{,}5\,V$ $\omega_0 = 10000\,s^{-1}$

$I = I_0 \sin(\omega t - \delta)$ $\qquad I_0 = \frac{U_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$ $\qquad \tan\delta = \frac{\omega L - \frac{1}{\omega C}}{R}$

$$P = I^2 R = I_0^2 R \sin^2(\omega t - \delta)$$

$$\langle P \rangle = \frac{I_0^2 R}{2} = \frac{U_0^2 R}{2\left(R^2 + (\omega L - \frac{1}{\omega C})^2\right)}$$

$$= \underbrace{\frac{U_0}{\sqrt{2}}}_{U_{ef}} \cdot \underbrace{\frac{U_0}{\sqrt{2}\,\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}}_{I_{ef}} \cdot \underbrace{\frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}}_{\cos\delta}$$

Upor, tuljavo in kondenzator vežemo zaporedno na vir napetosti $U = U_0 \sin\omega t$ z amplitudo 2V. in krožno frekvenco $\omega = 50000\ s^{-1}$. Kolikšna moč se porablja. $R = 50\ \Omega$, $L = 1mH$, $C = 1\mu F$.

$$I = I_0 \sin\omega t \qquad Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega L})^2}$$

$$tg\,\delta = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$P = I^2 R$$

$$\langle P \rangle = \frac{I_0^2 R}{2} = \frac{U_0^2 R}{(R^2 + (\omega L - \frac{1}{\omega C})^2)2}$$

IZPELJAVA

$$U + U_L + U_R + U_C = 0$$

1. $U_0 \sin\omega t - L\, dI/dt - IR - Q/C = 0 \quad / \frac{d}{dt}$

2. $I = \frac{dQ}{dt}$

$$U_0 \omega \cos\omega t - L\frac{d^2I}{dt^2} - R\frac{dI}{dt} - \frac{I}{C} = 0 \quad /:L$$

$$\frac{d^2I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{1}{LC}\cdot I = \frac{U_0\omega}{L}\cdot\cos\omega t \quad \Rightarrow a\sin\omega t + b\cdot\cos\omega t$$

$\Rightarrow$

$$I = I_0 \sin(\omega t - \delta)$$

$$= I_0 \sin\omega t\cos\delta - I_0\cos\omega t\cdot\sin\delta$$

$$\frac{dI}{dt} = I_0\omega\cos\omega t\cos\delta + I_0\omega\sin\omega t\sin\delta$$

$$\frac{d^2I}{dt^2} = -I_0\omega^2\sin\omega t\cos\delta + I_0\omega^2\cos\omega t\sin\delta$$

$$\Rightarrow -I_0\omega^2\sin\omega t\cos\delta + I_0\omega^2\cos\omega t\cdot\sin\delta + \frac{R}{L}\cdot I_0\omega\cos\omega t\cos\delta + \frac{R}{L}\omega\cdot I_0\sin\omega t\sin\delta$$
$$+ \frac{1}{LC}\cdot I_0\sin\omega t\cos\delta - \frac{1}{LC}I_0\cos\omega t\sin\delta = \frac{U_0\omega}{L}\cdot\cos\omega t$$

$$\Rightarrow (-I_0\omega^2\cos\delta + \frac{R}{L}\cdot\omega\cdot I_0\sin\delta + I_0\frac{1}{LC}\cdot\cos\delta) = 0$$

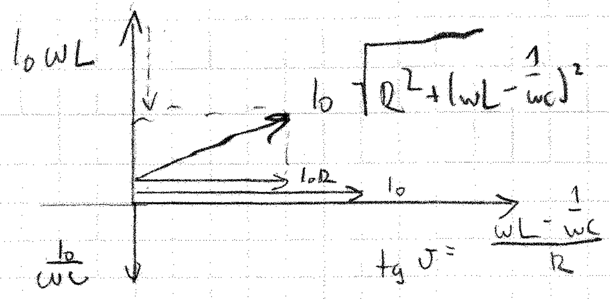
$$\Rightarrow (I_0\omega^2\sin\delta + \frac{R}{L}I_0\omega\cos\delta - \frac{I_0}{LC}\sin\delta) = \frac{U_0\omega}{L}$$

$$I_0\cos\delta(\frac{1}{LC} - \omega^2) + \frac{R}{L}I_0\omega\sin\delta = 0$$

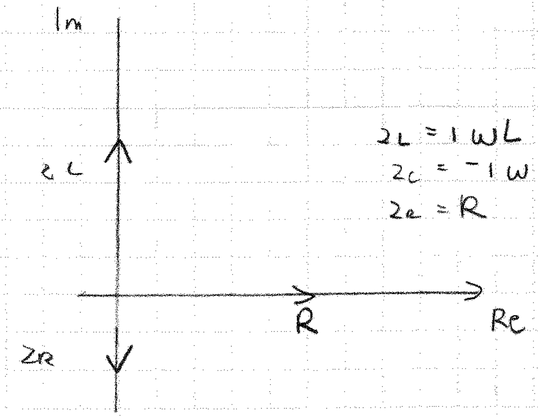
$$-I_0\sin(\frac{1}{LC} - \omega^2) + \frac{R}{L}I_0\omega\cos\delta = \frac{U_0\omega}{L}$$

$$tg\,\delta = \frac{\frac{1}{LC} - \omega^2}{\frac{R\omega}{L}} = \frac{\frac{1\omega}{\omega C} - \omega^2\cdot L}{R\omega} = \frac{\frac{1}{\omega C} - \omega L}{R} \quad // \quad tg\,\delta = \frac{\omega L - \frac{1}{\omega C}}{R}$$

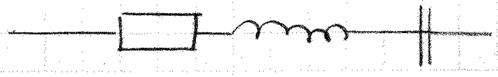
$$I_0 = \frac{U_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$



$$\operatorname{tg} \vartheta = \frac{\omega L - \frac{1}{\omega C}}{R}$$



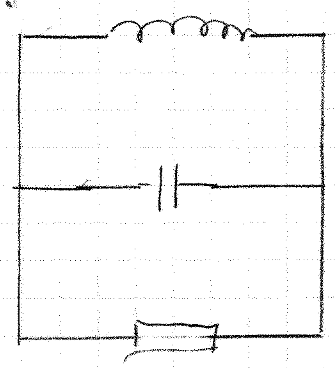
$$z_L = i\omega L$$
$$z_C = -i\omega C = \frac{1}{i\omega C}$$
$$z_R = R$$



$$z = R + i\omega L + \left(-\frac{1}{\omega C}\right)$$
$$z = R + i\left(\omega L - \frac{1}{\omega C}\right)$$
$$|z| = \sqrt{z \cdot z^*} = \sqrt{\left(R^2 + i\left(\omega L - \frac{1}{\omega C}\right)\right)\left(R - i\left(\omega L - \frac{1}{\omega C}\right)\right)}$$

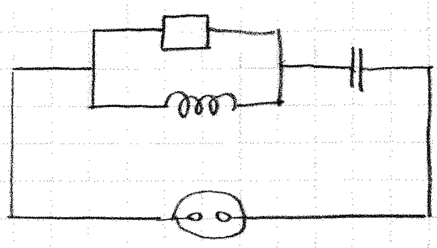
$$\operatorname{tg} \vartheta = \frac{\operatorname{Im} z}{\operatorname{Re} z} = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$|z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)}$$



$$\frac{1}{z} = \frac{1}{R} + \frac{1}{i\omega L} + \frac{1}{\left(-\frac{i}{\omega C}\right)^2}$$
$$= \frac{1}{R} + \frac{1}{\omega L} - \frac{\omega C}{i}$$
$$= \frac{1}{R} - \frac{1}{\omega L} + i\omega C$$
$$= \frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right)$$

$$\frac{1}{|z|} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$



$I_0 = ?$

$$I_0 = \frac{U_0}{|z|}$$

$$z = \left(\frac{1}{R} + \frac{1}{i\omega L}\right)^{-1} + \left(-\frac{i}{\omega C}\right)$$
$$= \left(\frac{i\omega L + R}{iR\omega L}\right)^{-1} + \frac{i}{\omega C} =$$
$$= \frac{i\omega R L}{R - i\omega L} - \frac{i}{\omega C} =$$
$$= \frac{i\omega R L \omega C - iR + i\omega L}{\omega C (R + i\omega L)} = \frac{\omega L + iR(\omega^2 LC - 1)(R - i\omega L)}{\omega C (R + i\omega L)(R - i\omega L)}$$

$\omega C R + i\omega C \omega L$

$$= \frac{\omega L R + iR^2(\omega^2 LC - 1) - i\omega^2 L^2 + R\omega L(\omega^2 LC - 1)}{\omega C (R^2 + \omega^2 L^2)}$$

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$$= \frac{\omega L R(\omega^2 LC - 1 + 1) + i[\omega^2 R^2 LC - R^2 - \omega^2 L^2]}{\omega C(R^2 + \omega^2 L^2)}$$

$$= \frac{\omega^3 L^2 R C + i[\omega^2 R^2 LC - R^2 - \omega^2 L^2]}{\omega C(R^2 + \omega^2 L^2)} = Z$$

$$|Z| = \sqrt{\frac{(\omega^3 R L^2 C + i[\omega^2 R^2 LC - R^2 - \omega^2 L^2])(\omega^3 R L^2 C - i[\omega^2 R^2 LC - R^2 - \omega^2 L^2])}{\omega^2 C^2 (R^2 + \omega^2 L^2)^2}}$$

$$|Z| = \sqrt{\frac{\omega^6 R^2 L^4 C^2 + (\omega^2 R^2 LC - R^2 - \omega^2 L^2)^2}{\omega^2 C^2 (R^2 + \omega^2 L^2)^2}}$$

50 Ω upor, tuljavo in kondenzator $C = 1\,mF$ priključimo na generator krožne sinusne napetosti s frekvenco $5000\,s^{-1}$. Skozi vezje teče tok 0,9 A. Nato upor zamenjamo s 45 Ω uporom in skozi vezje sedaj teče tok 0,98 A. Kolikšna je induktivnost tuljave

$$U = I_1 \sqrt{R_1^2 + (\omega L - \tfrac{1}{\omega C})^2}$$

$$U = I_2 \sqrt{R_2^2 + (\omega L - \tfrac{1}{\omega C})^2}$$

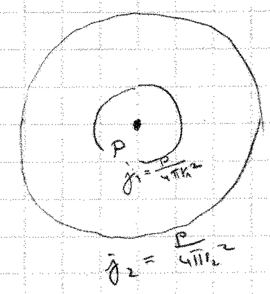
$$I_1^2 R_1^2 + I_1^2 (\omega L - \tfrac{1}{\omega C})^2 = I_2^2 R_2^2 + I_2^2 (\omega L - \tfrac{1}{\omega C})^2$$

$$(\omega L - \tfrac{1}{\omega C})^2 (I_1^2 - I_2^2) = (I_2^2 R_2^2 - I_1^2 R_1^2)$$

$$\omega L = \underbrace{\frac{1}{\omega C} \pm \sqrt{\frac{I_2^2 R_2^2 - I_1^2 R_1^2}{I_1^2 - I_2^2}}}_{\text{mora biti + in šele nato izberem +/− pred } \sqrt{}}$$

ELEKTROMAG. VALOVANJE

Radijska postaja oddaja valove enakomerno v vse smeri. 3 km od postaje je povprečna gostota energije enaka $10^{-6}\,W/m^2$. V kolikšni največji razdalji še lahko poslušamo radio, ki potrebuje amplitudo jakosti električnega polja $4 \cdot 10^{-8}\,V/m$.



$$j_2 = \frac{1}{2} \varepsilon \varepsilon_0 E_0^2 \cdot c = \frac{P}{4\pi r_2^2}$$

$$j_1 = \frac{P}{4\pi r_1^2}$$

$$P = j\, 4\pi r_1^2$$

$$\frac{1}{2} \varepsilon \varepsilon_0 E_0^2 \cdot c = \frac{j_1 4\pi r_1^2}{4\pi r_2^2}$$

$$\cancel{E_0}\; r_2 = r_1 \sqrt{\frac{2 j_1}{\varepsilon \varepsilon_0 E_0^2 c}}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$j = \frac{1}{2} w \cdot c = \left(\frac{1}{2} \varepsilon \varepsilon_0 E_0^2 \sin^2 \omega t + \frac{1}{2} \frac{B^2}{\mu_0} \sin^2 \omega t\right)$$

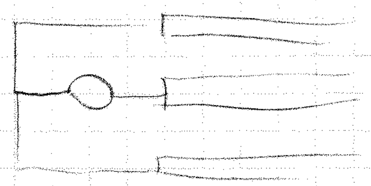
$$E = B \times c$$

$$= \left(\frac{1}{2} \varepsilon_0 E_0^2 \sin^2 \omega t + \frac{1}{2} \frac{E_0^2 \varepsilon_0 \mu_0}{\mu_0} \sin^2 \omega t\right)$$

$$E = \frac{B}{\sqrt{\varepsilon_0 \mu_0}} \Rightarrow B = E \sqrt{\varepsilon_0 \mu_0}$$

$$j = \frac{1}{2}\varepsilon_0 E_0'^2 c$$

Koaksialni vodnik ima žilo s polmerom 1mm in zunanjega plašča =3 mm. Med žilo in plaščem je izolator z dielektričnostjo 4. Na vodnik priključimo generator sinusne izmenične napetosti z amplitudo 2 V. Kolikšen $\langle j \rangle$ se širi po vodniku.



$$j = \frac{1}{2}\varepsilon\varepsilon_0 E_0^2 \cdot \frac{1}{\sqrt{\varepsilon\varepsilon_0\mu_0}} = \frac{1}{2}\sqrt{\frac{\varepsilon\varepsilon_0}{\mu_0}}\, E_0^2(r)$$

$$\vec{D}\cdot d\vec{S} = e$$

$$\varepsilon\varepsilon_0 E \cdot 2\pi r l = U_0 \frac{2\pi\varepsilon\varepsilon_0 l}{\ln r_2/r_1} \qquad = \frac{1}{2}\sqrt{\frac{\varepsilon\varepsilon_0}{\mu_0}}\frac{U_0^2}{r^2\ln^2(r_2/r_1)}$$

$$E = \frac{U_0}{r\ln r_2/r_1} \quad !!!$$

$$P = \int j\cdot d\vec{S} = \frac{1}{2}\sqrt{\frac{\varepsilon\varepsilon_0}{\mu_0}}\frac{U_0^2\cdot 2\pi}{\ln^2(r_2/r_1)}\int_{r_1}^{r_2}\frac{\cancel{r}\,dr}{r^2}$$

$$dS = r\,dr\,d\varphi = 2\pi r\,dr$$

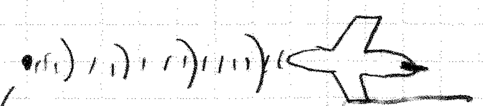
$$= \frac{1}{1}\sqrt{\frac{\varepsilon\varepsilon_0}{\mu_0}}\frac{\pi U_0^2}{\ln(r_2/r_1)}$$

Dopplerjev efekt velja tudi za elektromag. valovanje

$\nu = \nu_0\sqrt{\frac{c-v}{c+v}}$ - oddalj.

$\nu = \nu_0\sqrt{\frac{c+v}{c-v}}$ - približ. oddaj.

$\nu_0 = 1{,}2\cdot 10^{10}$ Hz oddaja valove, ki se odbijajo od bližajočega se letala. Te valove sprejmejo v radarski postaji in ugotovijo, da je njihova frekvenca za $4\cdot 10^4$ Hz višja od oddanih valov. S kolikšno hitrostjo se bliža letalo



$$\nu_0+\Delta\nu = \nu_0\sqrt{\frac{c+v}{c-v}}\sqrt{\frac{c+v}{c-v}}$$

$$\nu_0+\Delta\nu = \nu_0\cdot\frac{c+v}{c-v}$$

$$(\nu_0+\Delta\nu)(c-v) = \nu_0 c + \nu_0 v$$

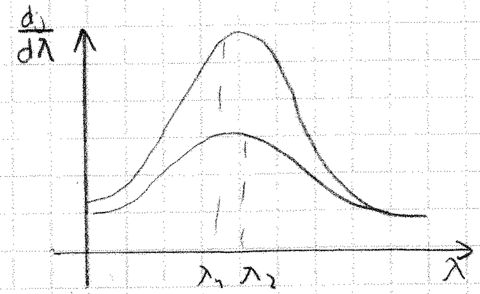
$$\cancel{\nu_0 c} + \Delta\nu c - \nu_0 v - \Delta\nu v = \cancel{\nu_0 c} + \nu_0 v$$

$$\Delta\nu c = 2\nu_0 v + \cancel{\Delta\nu v}$$

$$v = \frac{\Delta\nu c}{2\nu_0} = \frac{4\cdot 10^4\cdot 3\cdot 10^8}{2\cdot 1{,}2\cdot 10^{10}} = 500\ \mathrm{m/s}$$

Iz Planckovega zakona izpelji Stefanov zakon

$$\frac{dj}{d\lambda} = \frac{2\pi h c^2}{\lambda^5 \left(e^{\frac{hc}{\lambda kT}} - 1\right)}$$



$j = \sigma T^4$

$\sigma = 5{,}67\ 10^{-8}\ \frac{W}{m^2K^4}$

$\lambda T = k_w$ Vinov zakon

$k_w = 2{,}898\ 10^{-3}\ mK$

$k = 1{,}38\ 10^{-23}\ J/K$

$h = 6{,}62\ 10^{-34}\ Js$

$$\int_0^\infty \frac{2\pi h c^2\, d\lambda}{\lambda^5 \left(e^{\frac{hc}{\lambda kT}} - 1\right)} \qquad \frac{hc}{\lambda kT} = x \qquad \lambda = \frac{hc}{kTx} \qquad d\lambda = -\frac{hc}{kTx^2}$$

$$= -\frac{2\pi h^2 c^3 k^5 T^5}{kT c^5 h^5} \int_\infty^0 \frac{x^5\, dx}{x^2 (e^x - 1)}$$

$$= \frac{2\pi k^4 T^4}{h^3 c^2} \underbrace{\int_0^\infty \frac{x^3\, dx}{e^x - 1}}_{\frac{\pi^4}{15}} = \underbrace{\frac{2\pi^5 k^4}{15 h^3 c^2}}_{\text{Stef. konst.}} T^4$$

Vinov zakon

$$\frac{2\pi h c^2 k^5 T^5 x^5}{h^5 c^5 (e^x - 1)}$$

$$\left(x^5 (e^x - 1)^{-1} = 0\right)'$$

$$5x^4 (e^x - 1)^{-1} + x^5 (-1)(e^x - 1)^{-2} e^x = 0 \quad /: x^4 \cdot (e^x - 1)^2$$

$$5e^x - 5 - xe^x = 0$$

$$x = 5(1 - e^{-x})$$

x	$5(1-e^{-x})$
5	4,9663
4,9663	4,9652
4,96523	4,9651
4,9651	4,9651

$$\Rightarrow x = \frac{hc}{\lambda kT} = 4{,}9651$$

$$\lambda T = \frac{hc}{4{,}9651 k} = k_w$$

Tanka žica ima premer 0,1mm in specifično upornost 0,05 $\frac{\Omega mm^2}{m}$. Tok je 3A. Kolikšna je T žice v ravnovesju, če žička vso električno moč odda, sevajoč kot črno telo

$$P_{el} = P_{izs}$$
$$I^2 R = \sigma T^4 \cdot 2\pi r l \qquad = j \cdot S$$
$$I^2 \cdot \zeta \frac{l}{\pi r^2} = \sigma T^4 \cdot 2\pi r l$$
$$T = \left(\frac{I^2 \zeta}{2\pi^2 r^3 \sigma}\right)^{1/4}$$

Majhno črno telo ima površino $S = 50cm^2$ in kapaciteto 0,01K. V začetku ima temperaturo 2400K. Po kolikšnem času se temp telesa zniža na 1190K če seva kot črno telo.

$$Q = C_p \Delta T$$
$$\downarrow$$
$$m c_p$$

$$-\frac{dQ}{dt} = \sigma T^4 \cdot S$$
$$-C_p \frac{dT}{dt} = \sigma T^4 \cdot S$$
$$\int_0^t dt = \int_{T_1}^{T_2} -\frac{C_p}{\sigma S}\frac{dT}{T^4}$$
$$t = +\frac{C_p}{\sigma S 3}\frac{1}{T^3}\Big|_{T_1}^{T_2}$$
$$t = \frac{C_p}{3\sigma S}\left(\frac{1}{T_2^3} - \frac{1}{T_1^3}\right)$$
$$t = \frac{C_p}{3\sigma S}\frac{T_1^3 - T_2^3}{(T_2 T_1)^3}$$

če ne seva kot črno telo

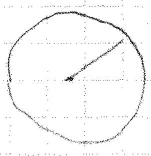
$$j = c\,\sigma T^4 \qquad c \leq 1$$

lantanoheksaborid

Okrogla tanka ploščica LaB_6 ima polmer 2cm. Na zadnji strani jo grejemo s spiralo tako da ima v središču 2300K, potem pa T linearno pada do roba $T_2 = 1200K$. Kolikšen energijski tok seva iz neogrevane polovice. Če predpostavljamo, da seva kot črno telo.

$$P = \int j\, dS = \int \sigma T^4\, dS = 2\pi\sigma \int_0^{r_0} \left(T_1 - \frac{T_1 - T_2}{r_0}\rho\right)^4 \rho\, d\rho$$

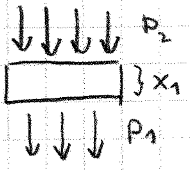
$$dS = \rho\, d\varphi\, d\rho$$
$$dS = 2\pi \rho\, d\rho$$



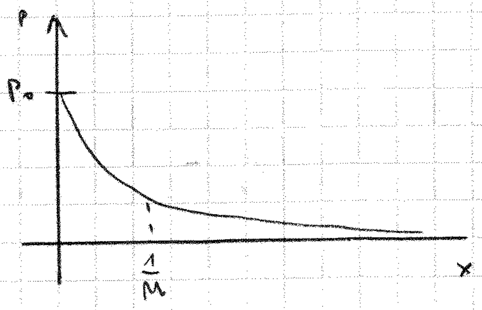
$$T = T_1 + \frac{T_2 - T_1}{r_0}\rho$$
$$dT = \frac{T_2 - T_1}{r_0} d\rho$$
$$d\rho = \frac{r_0}{T_2 - T_1} dT$$
$$\rho = \frac{r_0}{T_2 - T_1}(T - T_1)$$

$$= \frac{2\pi\sigma r_0^2}{(T_2 - T_1)^2}\int_{T_1}^{T_2} T^4 (T - T_1)\, dT$$
$$= \frac{2\pi\sigma r_0^2}{(T_2 - T_1)^2}\left[\int_{T_1}^{T_2} T^5 dT - T_1 \int_{T_1}^{T_2} T^4 dT\right]$$
$$= \frac{2\pi\sigma r_0^2}{(T_2 - T_1)^2}\left[\frac{1}{6}(T_2^6 - T_1^6) - \frac{T_1}{5}(T_2^5 - T_1^5)\right]$$

2 cm debela siva ploščica prepusti 50% vpadnega svetlobnega toka. Koliko odstotkov prepusti 3 cm debela ploščica iz istega materiala.



$P_1 = p_1 P_0$



$$P(x) = P_0 \cdot e^{-\mu x}$$

$$P_1 = p_1 P_0 = P_0 e^{-\mu x_1}$$

$$P_2 = p_2 P_0 = P_0 e^{-\mu x_2}$$

$$\Rightarrow p_1 = e^{-\mu x_1} \quad \Rightarrow \mu = -\frac{\ln p_1}{x_1}$$

$$p_2 = e^{-\mu x_2} \quad \mu = -\frac{\ln p_2}{x_2}$$

$$p_2 = e^{\ln p_1 \frac{x_2}{x_1}}$$

Kolikšna je razpolovna debelina:

$$p_1 = 2^{-\frac{x}{x_{1/2}}}$$

$$\log_2 p_1 = -\frac{x_1}{x_{1/2}} \qquad x_{1/2} = \frac{x_1}{\log_2 p_1} = -\frac{x \ln 2}{\ln p_1} = \frac{\ln 2}{\mu}$$

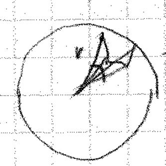
FOTOMETRIJA

P [W] - osnovna enota [lm] - obstajajo tudi lumni

$$I = \frac{dP}{d\Omega} \; [W/st] = [lm/st, cd]$$

steradian

prostorski kot



krogla

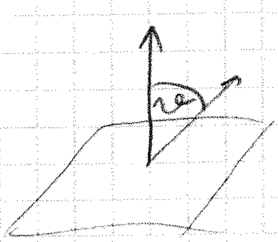
$$\Omega = \frac{S}{r^2} \quad \text{poln} \quad \frac{4\pi r^2}{r^2} = 4\pi$$

$$dV = r^2 dr \underbrace{\sin\vartheta \, d\vartheta \, d\varphi}_{d\Omega}$$

svetlost

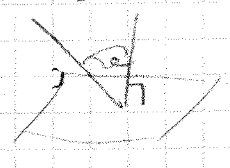
$$B = \frac{dI}{dS} = \frac{d^2 P}{dS \, d\Omega} \quad [W/m^2 st]$$

$$[lm/m^2 ; stilb]$$



$I = B \cdot S \cos\vartheta$ če sveti po Lambertovem zakonu

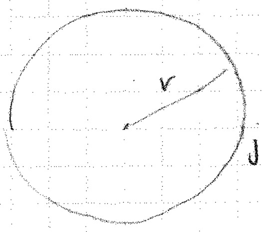
svetilnost:



$$E = j \cdot \cos\vartheta \; \left[\frac{W}{m^2}\right]$$

$$\left[\frac{lm}{m^2}, lux\right]$$

Majhna žarnica sveti enakomerno na vse strani. V razdalji 2 m pa $j = 0,1\ W/m^2$. Kolikšna je svetilnost te žarnice.



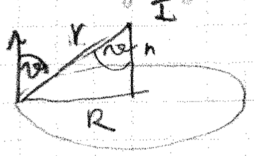
$$j = \frac{P}{S} = \frac{P}{4\pi r^2}$$

$$I = \frac{dP}{d\Omega} \qquad I = \frac{P}{4\pi}$$

$$I = \frac{j\,4\pi r^2}{4\pi} = j\,r^2 = 0,4\,W$$

Majhna žarnica ki seva na vse strani je 60 cm nad okroglo $R = 0,7$ ravno mizo. Kolikšna je osvetljenost roba mize, če je $I = 0,5\,W$. Koliko nad središčem bi žarnica morala biti obešena, da bi bila osvetljenost največja - roba in kolikšna bi bila.

$E?$ $\frac{I}{r^2}\cos\alpha$



$$E = j \cdot \cos\vartheta$$

$E = j \cos\alpha$

$$= \frac{P}{S}\cos\vartheta$$

$$= \frac{P}{4\pi r^2}\cos\vartheta = \frac{I}{r^2}\cos\vartheta = E$$

$$= \frac{I\,h}{(h^2+r^2)^{3/2}}$$

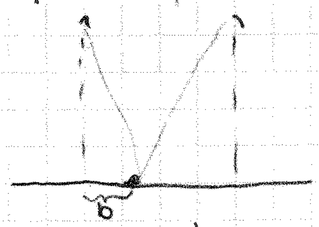
$$= (Ih(h^2+R^2)^{-3/2})' = 0$$

$$= I(h^2+R^2)^{-3/2} + h(-3/2)(h^2+R^2)^{-5/2}\,2h \;/(h^2+R^2)^{5/2}$$

$$h^2 + R^2 - 3h^2 = 0$$

E

Dve majhni žarnici ki obe sevata enakomerno na vse strani sta obešeni obe v enaki višini $1\,m$ nad vodoravnimi tlemi. Razdalja med žarnicama je $3,0\,W/m^2$. Kolikšna je svetilnost 2 žarnice, če je najbolj osvetljena točka na tleh ista in se nahaja na zveznici med mednima točkama 1 m od prve



$$E = \frac{I_1\cos\vartheta_1}{r^2} + \frac{I_2\cos\vartheta_2}{r^2}$$

$$= \frac{I_1\,h}{(h^2+b^2)^{3/2}} + \frac{I_2\,h}{(h^2+(a-b)^2)^{3/2}}$$

$$\cos\vartheta_1 = \frac{h}{\sqrt{h^2+b^2}}$$

$$= I_1\,h\,(h^2+b^2)^{-3/2} + I_2\,h\,(h^2+(a-b)^2)^{-3/2}$$

$$\cos\vartheta_2 = \frac{h}{\sqrt{h^2+(a-b)^2}}$$

$$\frac{\partial E}{\partial b} = \ldots\ldots$$

$$0 = \frac{I_1 b}{(h^2+b^2)^{5/2}} - \frac{I_2(a-b)}{(h^2+(a-b)^2)^{5/2}}$$

$$I_2 = I_1\,\frac{a\,(h^2+(a-b)^2)^{5/2}}{(h^2+b^2)^{5/2}\,(a-b)}$$

Ploščata luč s ploščino $40\,cm^2$ sveti po Lambertovem zakonu z $B = 0{,}08\ W/mm^2$. Kolikšen svetlobni tok seva svetilo

$I = B \cdot S \cos\vartheta$

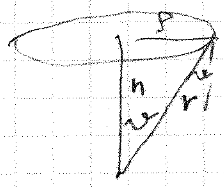
$I = \frac{dP}{d\Omega} \qquad P = \int I \cdot d\Omega = BS \int_0^{\pi/2} \sin\vartheta \, d\vartheta \int_0^{2\pi} d\varphi =$

$= 2\pi BS \int_0^{\pi/2} \sin\vartheta \cdot \cos\vartheta \, d\vartheta$

$\sin\vartheta = x$
$\cos\vartheta \, d\vartheta = x$
$= 2\pi BS \int_0^1 x \, dx = \pi BS$

Na 2 m visokem stropu je vrtljiva luč, ki ima krogelno obliko s polmerom $B = 0{,}1\ W/m^2$. Kolikšna je osvetljenost na v sredini

$dE = \frac{dI \cos\vartheta}{r^2} \qquad dI = B\,dS \cdot \cos\vartheta$



$= \frac{B \cdot dS \cdot \cos^2\vartheta}{r^2} \qquad dS = 2\pi \rho \, d\rho$

$dS = \rho \, d\rho \, d\varphi$

$= \frac{2\pi B \rho \, d\rho \cos^2\vartheta}{r^2}$

$r = \sqrt{\rho^2 + h^2}$

$\cos\vartheta = \frac{h}{\sqrt{\rho^2+h^2}}$

$dE = \frac{2\pi B h^2 \rho \, d\rho}{(\rho^2 + h^2)^2}$

$E = 2\pi B h^2 \int_0^R \frac{\rho \, d\rho}{(\rho^2+h^2)^2} \qquad \rho^2 + h^2 = x, \quad 2\rho \, d\rho = dx$

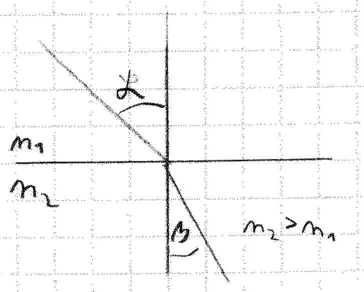
$= \pi B h^2 \int_{h^2}^{h^2+R^2} \frac{dx}{x^2}$

$= \pi B h^2 \frac{1}{x}\Big|_{h^2}^{h^2+R^2} = \frac{\pi B \cancel{h^2} R^2}{(R^2+h^2)\cancel{h^2}}$

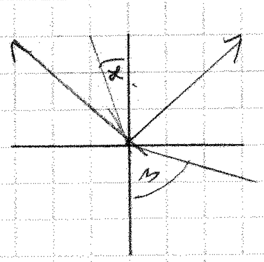
LOMNI KOLIČNIK

FIZ II-V
24. 4. 2008

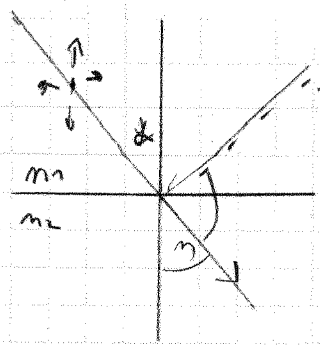
$n = \frac{c_0}{c} > 1$



$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}$

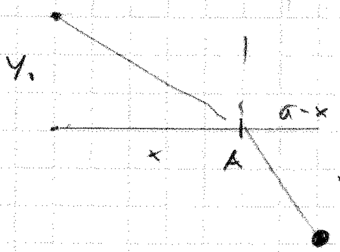


$\sin\alpha_{mejni} = \frac{n_2}{n_1}$



$$\frac{\sin\alpha}{\cos\alpha}=\mathrm{tg}\,\alpha=\frac{n_2}{n_1}$$

Tekmovalec na triatlonu mora čim hitreje priti od štarta do bazena z bojo. Štart je oddaljen od bazena $y_1=20\,m$, boja pa je od roba bazena 18 m. Bolja je vodoravno in pravokotno glede na roba bazena 50 m. Hitrost tekmovalca na kopnem $v_1=6\,m/s$ v vodi pa $2\,m/s$. Določi točko A da bo najhitreje pri boji



$$s_1=\sqrt{y_1^2+x^2}$$

$$s_2=\sqrt{y_2^2+(a-x)^2}$$

$$t=\frac{s_1}{v_1}+\frac{s_2}{v_2}=\frac{\sqrt{y_1^2+x^2}}{v_1}+\frac{\sqrt{y_2^2+(a-x)^2}}{v_2}$$

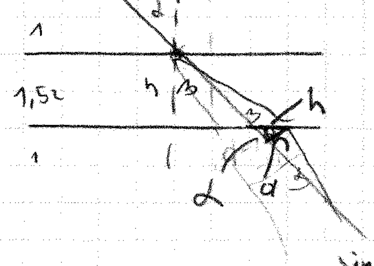
$$\frac{\partial t}{\partial x}=0=\frac{1}{v_1}\cdot\frac{1}{2}\frac{2x}{\sqrt{y_1^2+x^2}}+\frac{1\cdot 2(a-x)\cdot(-1)}{v_2\cdot 2\sqrt{y_2^2+(a-x)^2}}$$

$$=\frac{x}{v_1\sqrt{y_1^2+x^2}}-\frac{(a-x)}{v_2\sqrt{y_2^2+(a-x)^2}}=0$$

$$\frac{v_1}{v_2}=\frac{\overset{\sin\alpha}{x}\sqrt{y_2^2+(a-x)^2}}{\sqrt{y_1^2+x^2}\,(a-x)}=\frac{\sin\alpha}{\sin\beta}$$

$$\frac{\sin\alpha}{\sin\beta}=\frac{\frac{c_0}{n_1}}{\frac{c_0}{n_2}}=\frac{n_2}{n_1}$$

Plast debeline $h=3\,m$ je narejena iz snovi z lomnim količnikom $n=1,52$. Za koliko se spremakne žarek, če je vpadni kot žarka $\alpha=50°$.



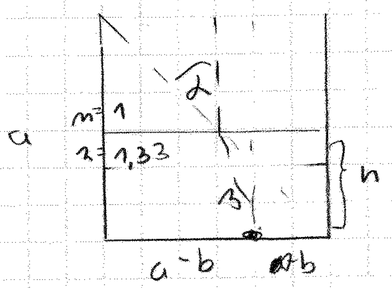
$$\frac{\sin\alpha}{\sin\beta}=n\rightarrow\sin\beta=\frac{\sin\alpha}{n}$$

$$d=h(\mathrm{tg}\,\alpha-\mathrm{tg}\,\beta)\cdot\cos\alpha$$

$$\mathrm{tg}\,\beta=\frac{\sin\beta}{\cos\beta}=\frac{\sin\alpha}{n\sqrt{1-\frac{\sin^2\alpha}{n^2}}}=\frac{\sin\alpha}{\sqrt{n^2-\sin^2\alpha}}$$

$$d=h\cdot\sin\alpha\cdot\cos\alpha\left(\frac{1}{\cos\alpha}-\frac{1}{\sqrt{n^2-\sin^2\alpha}}\right)$$

Posoda ima obliko kocke z obliko 30 cm. Na dno postavimo kovanec 7 cm od roba kocke. Z lomim očesom škilimo in vidimo ravno stičišče dna in roba. Do kolikšne višine moramo natočiti vode, da bomo kovanec spet videli.



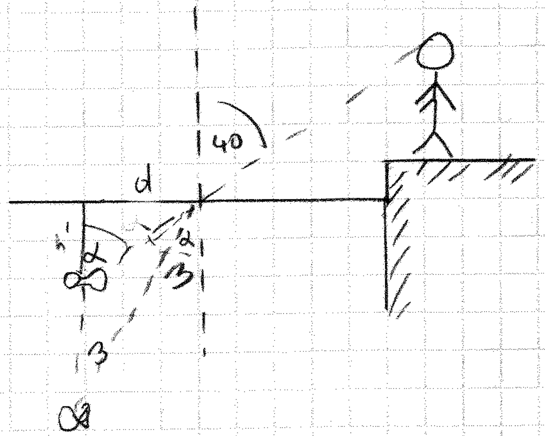
$$\frac{\sin\alpha}{\sin\beta} = n \qquad b = h(\mathrm{tg}\,\alpha - \mathrm{tg}\,\beta)$$

$$h = \frac{b}{\mathrm{tg}\,\alpha - \mathrm{tg}\,\beta}$$

$$h = \frac{b}{\frac{\sin\alpha}{\cos\alpha} - \frac{\sin\alpha}{\sqrt{n^2-\sin^2\alpha}}}$$

$$= \frac{b \cdot \cos\alpha\,\sqrt{n^2-\sin^2\alpha}}{\sin\alpha\,(\sqrt{n^2-\sin^2\alpha} - \cos\alpha)}$$

Ribič opazuje ribo, ki plava v vodi z lomnim količnikom 1,33. Ribič gleda pod kotom 40°, vidi, da je riba 1,2 m pod gladino. Kako globoko je riba v resnici 'h'



$$\frac{\sin\alpha}{\sin\beta} = n$$

$$\frac{d}{h'} = \mathrm{tg}\,\alpha$$

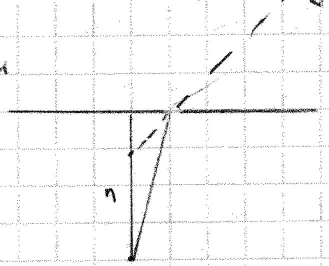
$$\frac{d}{h} = \mathrm{tg}\,\beta$$

$$\frac{h}{h'} = \frac{\mathrm{tg}\,\alpha}{\mathrm{tg}\,\beta}$$

$$h = h'\frac{\mathrm{tg}\,\alpha}{\mathrm{tg}\,\beta} = \frac{h'\cancel{\sin\alpha}\,\sqrt{n^2-\sin^2\alpha}}{\cos\alpha \cdot \cancel{\sin\alpha}}$$

$$h = h'\frac{\sqrt{n^2-\sin^2\alpha}}{\cos\alpha}$$

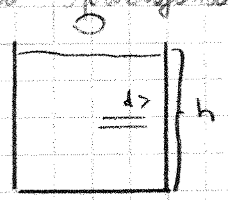
če gledamo pravokotno



$$h = h' \cdot n$$

$$h' = h \cdot n$$

Pokončna posoda je do višine 1 m napolnjena s prozorno tekočino ki ima na gladini $n = 1,52$, na dnu pa 1,53 vmes pa lomni količnik linearno narašča. Kako globoka se nam zdi posoda, če dno opazujemo v navpični smeri.



$$dh = \frac{dx}{n(x)}$$

$$\int dh = \int \frac{dx}{n(x)} = \int_0^h \frac{dx}{n_1 + \frac{n_2-n_1}{h}x} = \ldots\ldots$$

$$= \frac{h}{n_2-n_1}\int_{n_1}^{n_2}\frac{dz}{z}$$

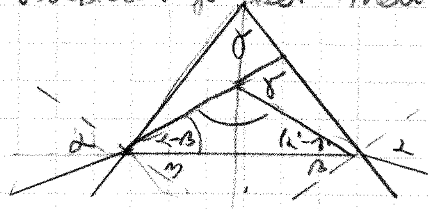
$$h' = \frac{h}{n_2-n_1}\ln\frac{n_2}{n_1} = 1\,m\;10{,}07\;\ln\frac{1{,}53}{1{,}52} = 64\,cm$$

Prizma je narejena iz stekla z lomnim količnikom 1,52, kot ob vrhu prizme pa je 20°. Žarek pade na stransko ploskev prizme pod kot 12°, se lomi, pride do druge ploskve in spet lomi in izstopi. Kolikšen je kot med vstopnim in izstopnim žarkom.

n

$\alpha = 12°$

$\gamma = 20°$



$\frac{\sin\alpha}{\sin\beta} = n$

$\frac{\sin\alpha'}{\sin\beta'} = n$

$$\delta = \alpha - \beta + \alpha' - \beta'$$

$$\beta + \beta' = \gamma$$

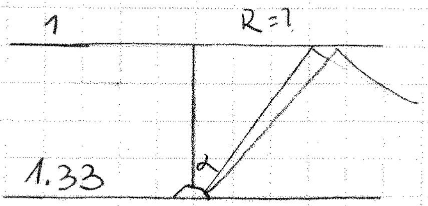
$$\beta = \arcsin\left(\frac{\sin 12}{1{,}52}\right) = 7{,}9°$$

$$\beta' = 12{,}1°$$

$$\alpha' = \arcsin(1{,}52 \cdot \sin 12{,}1) = 18{,}6°$$

$$\delta = 12 - 7{,}9 + 18{,}6 - 12{,}1 = 10{,}6°$$

Zelo velik bazen je do višine 10 cm napolnjen z vodo, $n = 1{,}33$. Na dnu bazena je majhna lučka, ki sveti v vse smeri.



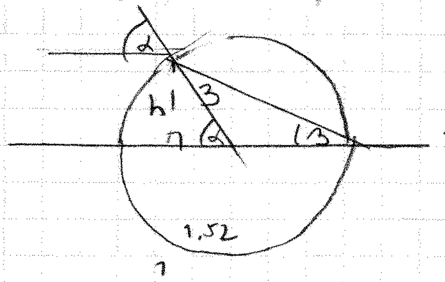
$$\sin\alpha = \frac{1}{n}$$

$$\frac{1}{n} = \frac{R}{\sqrt{h^2+R^2}}$$

$$h^2 + R^2 = n^2 R^2$$

$$R^2(n^2-1) = h^2 \qquad R = \frac{h}{\sqrt{n^2-1}}$$

Vedeževalkina krogla je narejena iz stekla $n = 1{,}52$ in polmera 10 cm. Žarek pade na kroglo in se na površini zlomi tako, da izstopi iz krogle točno na presečišču med optično osjo in površino krogle. Kolikšna je bila razdalja med žarkom in optično osjo na začetku



$$\frac{\sin\alpha}{\sin\beta} = n \quad \Rightarrow \sin\alpha = n \cdot \sin\frac{\alpha}{2}$$

$$= n \cdot \sqrt{\frac{1-\cos\alpha}{2}}$$

$$\sin\alpha = \frac{h}{R}$$

$$\alpha = 2\beta$$

$$\frac{h}{R} = n\sqrt{\tfrac{1}{2}\left(1 - \sqrt{1 - \frac{h^2}{R^2}}\right)}$$

$$\frac{2h^2}{n^2R^2} = 1 - \sqrt{1 - \frac{h^2}{R^2}}$$

$$\sqrt{1 - \frac{h^2}{R^2}} = 1 - \frac{2h^2}{n^2R^2}$$

$$\cancel{1} - \frac{h^2}{R^2} = \cancel{1} - \frac{4h^2}{n^2R^2} + \frac{4h^4}{n^4R^4} \quad / \frac{R^2}{h^2}$$

$$-1 = -\frac{4}{n^2} + \frac{4h^2}{n^4R^2} \quad / n^4R^2$$

$$-n^4R^2 = -4n^2R^2 + 4h^2$$

$$4h^2 = n^2R^2(4-n^2)$$

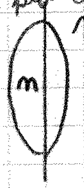
$$h = \frac{nR}{2}\sqrt{(4-n^2)}$$

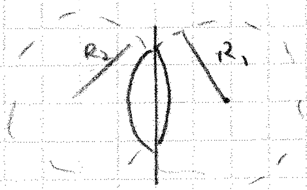
LEČE

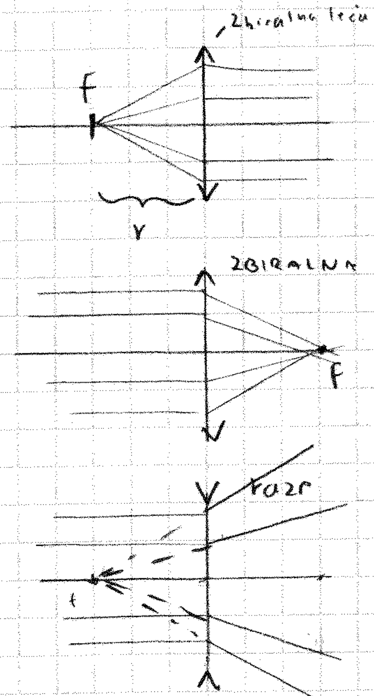
enačba za izdelavo leče

$$\frac{1}{f} = \left(\frac{n}{n_0} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

krivinski polmer ploskve







Konveksno-konkavna leča je narejena iz stekla 1'52. Krivinski polmer izbočene ploskve je 20 cm, drugi pa 12 cm. Kolikšna je goriščna razdalja leče če jo potopimo v glicerinu z $n_0 = 1,47$.

$$\frac{1}{f} = \frac{n-n_0}{n_0} \cdot \frac{R_1+R_2}{R_1R_2}$$

$$f = \frac{n_0 R_1 R_2}{(R_1+R_2)(n-n_0)} = \frac{1,47 \cdot 20 \cdot (-12)\ \mathrm{cm}^2}{0,05 \cdot (20+(-12))\ \mathrm{cm}} = 8,82\ \mathrm{cm}$$

lahko pa

$$\frac{1}{f} = \frac{n-n_0}{n_0} \cdot \frac{R_2-R_1}{R_1 R_2} = \ldots \quad \text{vse pride neg.}$$

Bikonveksna leča ima na zraku goriščno razdaljo 10 cm. Če jo potopimo v neko tekočino pa se poveča za 29 cm. Lomni količnik stekla $n = 1'52$. Kolikšen je n_0 tekočine

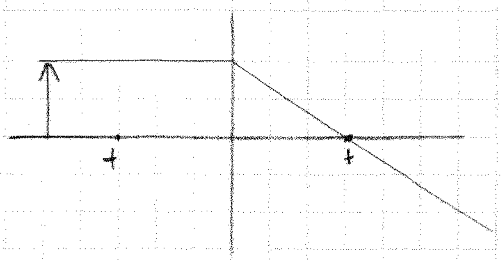
$$\frac{1}{f_1} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{f_2} = \left(\frac{n}{n_0} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{f_2}{f_1} = \frac{n-1}{\frac{n}{n_0} - 1} = \frac{n_0(n-1)}{n-n_0} \quad / f_1(n-n_0)$$

f_2 ⋮

$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$$

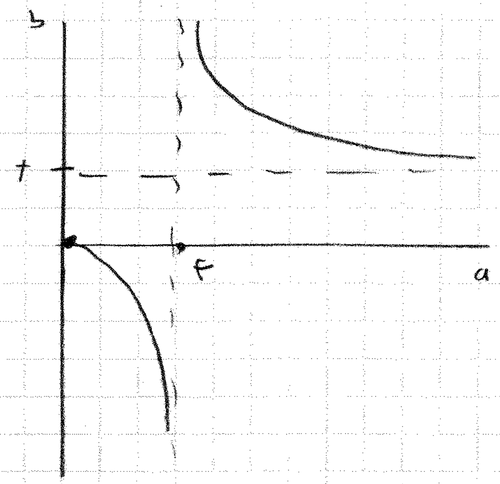


$a = f + x$
$b = f + y$

$$\frac{1}{f} = \frac{1}{f+x} + \frac{1}{f+y}$$

$(f+x)(f+y) = f(f+y) + f(f+x)$
$\cancel{f^2} + fx + fy + xy = \cancel{f^2} + \cancel{fy} + f^2 + \cancel{fx}$
$f^2 = xy$

Tanka zbiralna leča ima goriščno razdaljo 20 cm. Kako daleč od leče moramo postaviti predmet, da je razdalja med predmetom in realno sliko najmanjša možna in ~~kolik~~? Skiciraj graf funkcije razdalje med predmetom in to sliko.



$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$$

$$b = \frac{af}{a-f}$$

$$\lim_{a \to \infty} \frac{af}{a-f} = \lim a\left(1 - \frac{f}{a}\right) = f$$

$$d = a + b = a + \frac{af}{a-f} = a\left(1 + \frac{f}{a-f}\right)$$

$$d' = 0 = \left(1 + \frac{f}{a-f}\right) + a\,\frac{f}{(a-f)^2}\,(a-f)^2$$

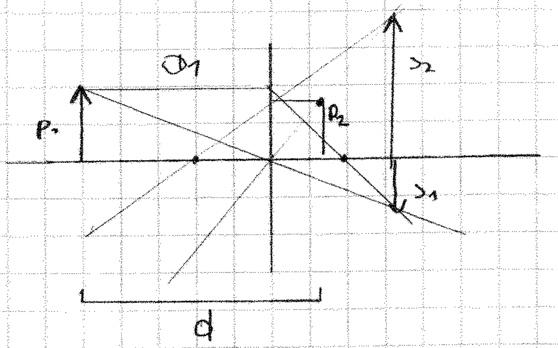
$$= (a-f)^2(1 + f(a-f) + af = 0$$

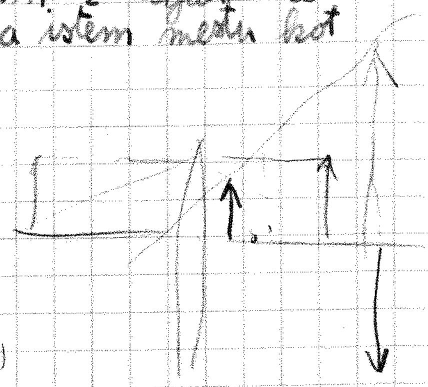
$$= a^2 - 2af + \cancel{f^2} - fa - \cancel{f^2} - \cancel{a f} = 0$$

$a - 2af = 0$
$a(1 - 2f) = 0$
$a = 0$
$a = 2f$

Dva predmeta postavimo na skupno geometrično os. Med njima pa premikamo zbiralno lečo z $f = 10$ cm. Pri 2 legah leče se zgodi, da navidezna slika 1. pred. nastane na istem mestu kot slika 2 predmeta.

$d = 30$ cm
$f = 10$ cm





$$\left.\begin{aligned} 1/f &= 1/a_1 + 1/b \\ 1/f &= 1/a_2 - 1/b \end{aligned}\right\} \text{ poiščemo } a_1 \text{ in } a_2$$

$d = a_1 + a_2$

$$\frac{2}{f} = \frac{1}{a_1} + \frac{1}{a_2} \quad / a_1 a_2 f$$

$$2a_1a_2 = a_2 f + a_1 f$$

$$a_1(2a_2 - f) = a_2 f$$

$$a_1 = \frac{a_2 f}{2a_2 - f}$$

$$d = a_2 + \frac{a_2 f}{2a_2 - f}$$

$$d = a_2\left(1 + \frac{f}{2a_2 - f}\right)$$

$$d = a_2 \frac{2a_2 - f + f}{2a_2 - f}$$

$$\frac{2a^2}{2a_2 - f} = d \quad \Rightarrow \quad 2a_2^2 - 2a_2 d + fd = 0$$

$$a_2 = \frac{2d \pm \sqrt{4d^2 - 8fd}}{4}$$

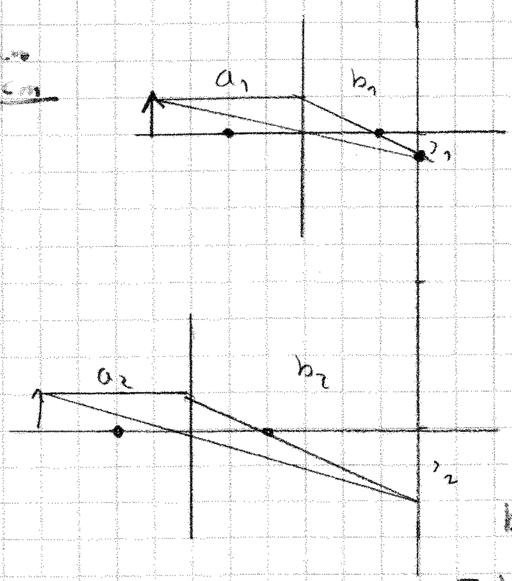
$$a_2 = \frac{d}{2}\left(1 \pm \sqrt{1 - \frac{2f}{d}}\right)$$

$$a_1 = d - a_2 = d - \left[1 - \frac{1}{2}\left(1 \pm \sqrt{1 - \frac{2f}{d}}\right)\right]$$

$$= d\left[\frac{1}{2} \mp \frac{1}{2}\sqrt{1 - \frac{2f}{d}}\right]$$

Predmet stoji pred pokončnim zaslonom. Med predmetom in zaslonom pa premikamo tanko zbiralno lečo. Ostro sliko predmeta na zaslonu dobimo pri 2 legah leče in sicer $b_1 = 30$ cm in $b_2 = 60$ cm od zaslona (oddaljenost leče). Kolikšna je f in d, ter kolikšno je razmerje med višinama slik

$b_1 = 30$ cm
$b_2 = 60$ cm
$f = ?$
$d = ?$
$s_1/s_2 = ?$



$$1/f = 1/a_1 + 1/b_1 \qquad a_1 = \frac{b_1 f}{b_1 - f}$$

$$1/f = 1/a_2 + 1/b_2 \qquad a_2 = \frac{b_2 f}{b_2 - f}$$

$$a_1 + b_1 = a_2 + b_2$$

$$\frac{b_1 f}{b_1 - f} + b_1 = \frac{b_2 f}{b_2 - f} + b_2$$

$$b_1 f(b_2 - f) + b_1(b_1 - f)(b_2 - f) = b_2 f(b_1 - f) + b_2(b_1 - f)(b_2 - f)$$

$$\cancel{b_1 b_2 f} - b_1 f^2 + b_1(b_1 b_2 - b_1 f - b_2 f + f^2)$$

$$= \cancel{b_1 b_2 f} - b_2 f^2 + b_2(b_1 b_2 - b_1 f - b_2 f + f^2)$$

$$-\cancel{b_1 f^2} + b_1^2 b_2 - \cancel{b_1 b_2 f} - b_1^2 f + \cancel{b_1 f^2} = -\cancel{b_2 f^2} + b_2^2 b_1 - \cancel{b_1 b_2 f} - b_2^2 f + \cancel{b_2 f^2}$$

$$b_1^2 b_2 - b_1^2 f = b_2^2 b_1 - b_2^2 f$$

$$b_1^2 b_2 - b_2^2 b_1 = f(b_1^2 - b_2^2)$$

$$f = \frac{b_1 b_2 \cancel{(b_1 - b_2)}}{\cancel{(b_1 - b_2)}(b_1+b_2)}$$

$$f = \frac{b_1 b_2}{b_1+b_2}$$

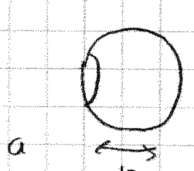
$$a_1 = \frac{b_1 \frac{b_1 b_2}{b_1+b_2}}{b_1 - \frac{b_1 b_2}{b_1+b_2}} = \frac{b_1 b_2}{b_1+b_2-b_2} = b_2 \qquad a_1 = b_2, \quad a_2 = b_1$$

$\frac{S_1}{S_2}$: iz podobnih:

$$\frac{P}{a_2} = \frac{S_2}{b_2} \quad ; \quad \frac{P}{a_1} = \frac{S_1}{b_1}$$

$$\Rightarrow \frac{a_2}{a_1} = \frac{S_2 b_1}{S_1 b_2} \Rightarrow \frac{S_1}{S_2} = \frac{b_1^2}{b_2^2}$$

oko



$$\frac1f = \frac1a + \frac1b \qquad \frac1f = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad \frac1f$$

normalna zorna razdalja: $a = 25$ cm.

Daljnovidno oko ne more jasno videti predmetov, ki so bližji od 1 metra. Kolikšna mora biti dioptrija (f) od očal da bo to oko videlo normalno.

$$\frac{1}{a_{min}} + \frac1b = \frac{1}{f_{oko}} \qquad \frac{1}{f_1}+\frac{1}{f_2}-\frac{s(f_1,f_2)}{f_1 f_2}=\frac{1}{f_{skupni}}$$

$$\frac{1}{a_0}+\frac1b = \frac{1}{f_{oko}} + \frac{1}{f_{oč}}$$

$$\frac{1}{a_0} + \cancel{\frac1b} = \frac{1}{a_{min}} + \cancel{\frac1b} + \frac{1}{f_{oč}}$$

$$\frac{1}{f_{oč}} = \frac{1}{a_0} - \frac{1}{a_{min}} = \frac{a_{min}-a_0}{a_0 a}$$

$$f = \frac{a_0 a}{a - a_0} = \frac{1\cdot 0{,}25}{0{,}75} = 0{,}33$$

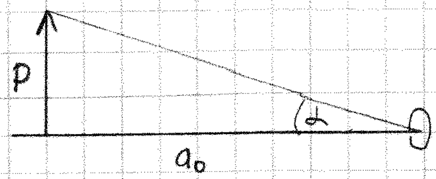
Kratkovidno oko ne more bit jasno videti predmetov, ki so bolj oddaljeni kot 2 m. Kolikšna je f od očal.

$$\frac{1}{a_{max}} + \frac1b = \frac{1}{f_{oko}}$$

$$\frac{1}{\infty}+\frac1b = \frac{1}{f_{oko}} + \frac{1}{f_{oč}}$$

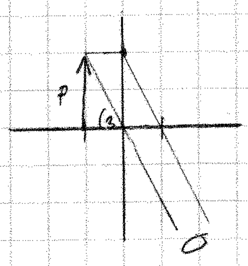
$$\frac{1}{f_{oč}} = -\frac{1}{a_{max}}$$

$$f_{oč} = f = 2\text{ m}$$

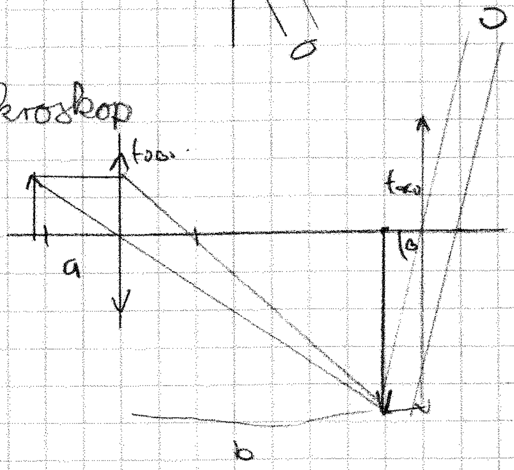


$$M = \frac{\operatorname{tg}\beta}{\operatorname{tg}\alpha}$$

$$M = \frac{\operatorname{tg}\beta}{\operatorname{tg}\alpha} = \frac{a_0}{f} \qquad \operatorname{tg}\alpha = \frac{p}{a_0}$$



mikroskop



$$M = \frac{\operatorname{tg}\beta}{\operatorname{tg}\alpha} = \frac{s\,a_0}{f_{ok}\,p}\cdot\frac{p}{a} = \frac{b\,a_0}{f_{ok}\,a} = \frac{a_0(b-f_{ob})}{f_{ok}\,f_{ob}} = \frac{a_0\,e}{f_{ob}\,f_{ok}} = M$$

$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$$

$$a = \frac{b f}{b - f}$$

Nek mikroskop ima povečavo 300, $f_{ok} = 4\,mm$, objekt pa je 7mm oddaljen od objektiva.

$M = 300$
$f_{ok} = 4\,mm$
$a = 7\,mm$

$$M = \frac{a_0\,e}{f_{ob}\,f_{ok}} \qquad e = \frac{M\,f_{ob}\,f_{ok}}{a_0}$$

$$\frac{1}{f_{ob}} = \frac{1}{a} + \frac{1}{b} \quad /\cdot f_{ob}\,a\,b \qquad b = e + f_{ob}$$

$$a\cdot(e+f_{ob}) = f_{ob}(e+f_{ob}) + a f_{ob}$$

$$a\cdot e + \cancel{a f_{ob}} = e\cdot f_{ob} + f_{ob}^2 + \cancel{a f_{ob}}$$

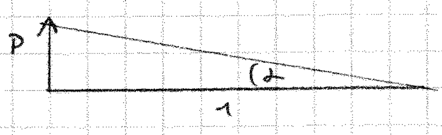
$$\frac{M f_{ob} f_{ok}\, a}{a_0} = \frac{M\, f_{ob}^2\, f_{ok}}{a_0} + f_{ob}^2 \quad /: f_{ob}\cdot a_0$$

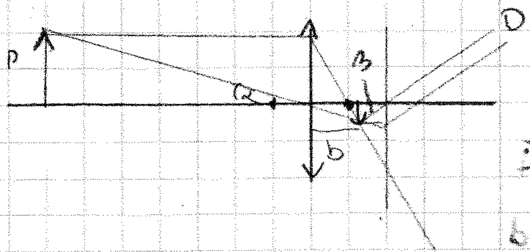
$$M\cdot f_{ok}\cdot a = M\cdot f_{ob}\cdot f_{ok} + f_{ob}\, a_0$$

$$f_{ob}\,(M\cdot f_{ok} + a_0) = M\cdot f_{ok}\cdot a$$

$$f_{ob} = \frac{M\cdot f_{ok}\cdot a}{M\cdot f_{ok} + a_0}$$

Daljnogled ima $f_{ob} = 50\,cm$ in $f_{ok} = 5\,cm$. Z tem daljnogledom posnemamo 100 m oddaljen predmet. Kakšna je povečava?





$$M = \frac{\operatorname{tg}\beta}{\operatorname{tg}\alpha} = \frac{\frac{s}{f_{ok}}}{\frac{s}{b}} = \frac{b}{f_{ok}} = \frac{a\cdot f_{ob}}{f_{ok}(a - f_{ob})} = \frac{100\cdot 0{,}5}{0{,}05\cdot 99{,}5} = 10{,}05$$

$$\frac{1}{f_{ob}} = \frac{1}{a} + \frac{1}{b}$$

$$b = \frac{a\,f_{ob}}{a - f_{ob}}$$

$$\lim_{a\to\infty} \frac{a\,f_{ob}}{f_{ok}(a - f_{ob})} = \frac{f_{ob}}{f_{ok}} = M$$

povečava astronomskega daljnogleda

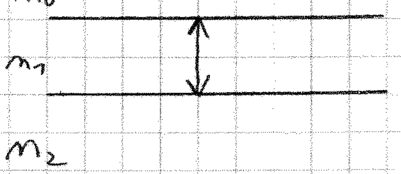
Astronomski daljnogled je naravnan na neskončnost, njegova povečava je 50, razdalja med objektivom in okularjem pa $d=60$. Kolikšni sta goriščni razdalji

$$M = \frac{f_{ob}}{f_{ok}} \qquad d = M f_{ok} + f_{ok}$$

$$d = f_{ob} + f_{ok} \qquad f_{ok} = \frac{d}{M+1}$$

$$f_{ob} = \frac{Md}{M+1}$$

Izpelji izraz za razliko optičnih poti med lomnim in odbitim žarkom na tanki plasti. To razliko izrazi z debelino, α, n_0, n_1, n_2



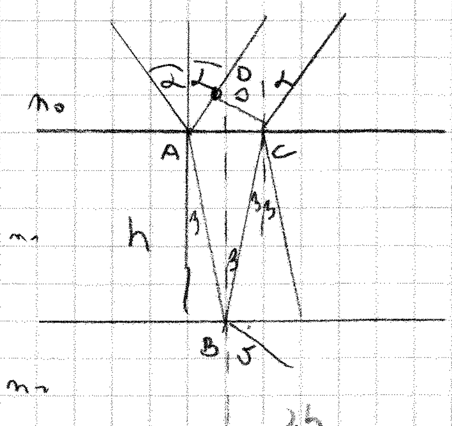
optična pot je produkt geometrijske od in lomnega količnika

$$c_0 = \lambda_0 \nu$$

$$\frac{c_0}{n_1} = \lambda_1 \nu$$

$$n_1 = \lambda_0 / \lambda_1$$

$$\lambda_0 = n_1 \cdot \lambda_1$$



$$\cos\beta = \sqrt{1 - \frac{n_0^2}{n_1^2}\sin^2\alpha}$$

$$\Delta = n_1 \overline{ABC} - n_0 \overline{AD}$$

$$\frac{\sin\alpha}{\sin\beta} = \frac{n_1}{n_0} \qquad \sin\beta = \frac{n_0}{n_1}\sin\alpha$$

$$\Delta = n_1 \cdot 2h/\cos\beta - n_0 2h \cdot tg\,\beta \cdot \sin\alpha$$

$$\Delta = \frac{2h}{\cos\beta}\left[n_1 - n_0 \sin\alpha \cdot \sin\beta\right]$$

$$\Delta = \frac{2h}{\sqrt{1-\frac{n_0^2}{n_1^2}\sin^2\alpha}}\left[n_1 - \frac{n_0^2 n_1}{n_1^2}\sin^2\alpha\right]$$

$$= \frac{2h n_1}{\sqrt{1-\frac{n_0^2}{n_1^2}\sin^2\alpha}}\left(1 - \frac{n_0^2}{n_1^2}\sin^2\alpha\right)$$

$$\Delta = 2h\sqrt{n_1^2 - n_0^2\sin^2\alpha}$$

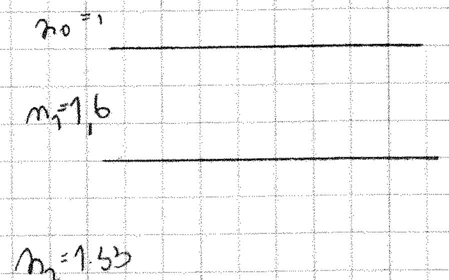
Če je

$$\Delta = \begin{cases} N\cdot\lambda & \text{ojačitev} \\ \frac{2N+1}{2}\lambda & \text{oslabitev} \end{cases} \qquad n_0 < n_1 < n_2$$

če se odbije na optično gostejši snovi se faza obrne.

$$\Delta = \begin{cases} N\cdot\lambda & \text{oslabitev} \qquad n_0 < n_1 > n_2 \\ \frac{2N+1}{2} & \text{ojačitev} \qquad n_0 > n_1 < n_2 \end{cases}$$

Na vodi z $n_2 = 1.33$ plava 5 μm debela plast olja z lomnim količnikom 1.6. Jo osvetljujemo pod $\alpha = 30°$ z belo svetlobo. Poišči valovno dolžino, ki se odbije in je najbližja 550 nm.



$$2h \sqrt{n_1^2 - \sin^2\alpha} = \frac{2N+1}{2} \lambda$$

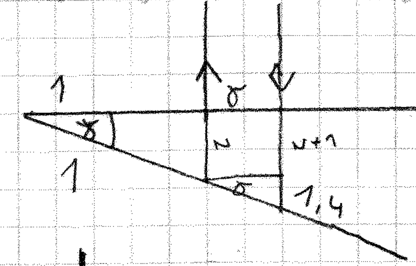
$$2N+1 = \frac{4h}{\lambda} \sqrt{n_1^2 - \sin^2\alpha}$$

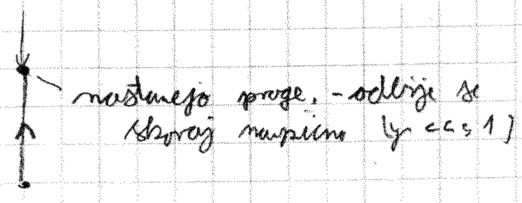
$$N = \frac{2h}{\lambda} \sqrt{n_1^2 - \sin^2\alpha} - \frac{1}{2} =$$

$$= \frac{10 \cdot 10^{-6}\,m}{550 \cdot 10^{-9}\,m} \sqrt{1{,}6^2 - 0{,}25} - 0{,}5 = 27{,}13$$

$$\lambda_m = \frac{4h}{2N+1} \sqrt{n_1^2 - \sin^2\alpha} = 552{,}67 \cdot 10^{-9}\,m$$

Milnata plast ima $n_1 = 1{,}4$. Kot ob robu klina pa je enak $\delta = 10^{-4}$ rad. Plast osvetljujemo v navpični smeri s svetlobo določene valovne dolžine. Na osvetljeni ploskvi nastanejo svetle in temne proge enake debeline. Razdalja med 2 svetlima svetlima progama je $\delta = 2.5$ mm. Kolikšna je svetlobna dolžina s katero osvetljujemo plast.





$$s = 2h \sqrt{n_1^2 - n_0 \sin^2\alpha}$$

$$2h_N n_1 = \frac{2N+1}{2} \lambda \Rightarrow h_N = \frac{(2N+1) \cdot \lambda}{4n_1}$$

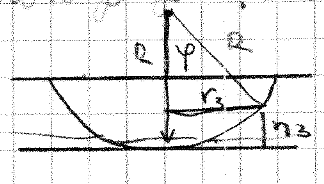
$$2h_{N+1} \cdot n_1 = \frac{2N+3}{2} \cdot \lambda \qquad h_{N+1} = \frac{(2N+3) \cdot \lambda}{4n_1}$$

$$\operatorname{tg} \delta = \frac{h_{N+1} - h_N}{\delta}$$

$$\operatorname{tg} \delta = \frac{(2N+3-2N-1)\lambda}{4n_1 \delta} = \frac{\lambda}{2n_1\delta} = \operatorname{tg}\delta$$

$$\lambda = 2n_1 \delta \operatorname{tg} \delta$$

Plankonveksno lečo položimo z izbočeno stranjo na debelo vodoravno debelo plast iz enakega stekla kot je leča. Ravno stran leče navpično osvetljujemo s svetlobo valovne dolžine 5?? nm. Na ravni ploskvi se pokažejo Newtonovi kolobarji. Polmer 3 svetlega kolobarja je 1,2 mm. Kolikšen je krivinski radij leče.



$$n_0 > n_1 < n$$

$$2h_3 = \frac{5 \cdot \lambda}{2}$$

$$\cos\varphi = \frac{R - h}{R}$$

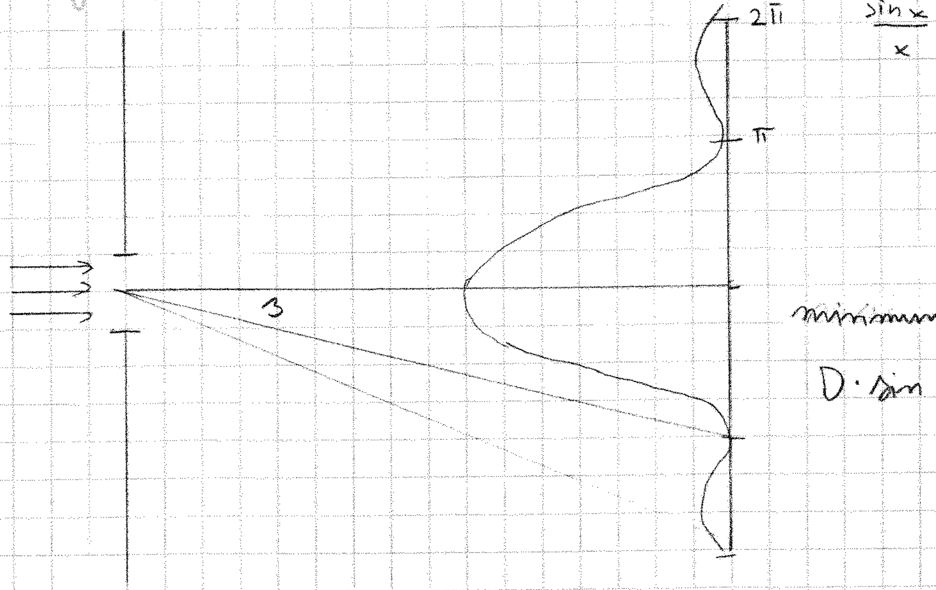
$$\sin\varphi = \frac{r_3}{R}$$

$$\sin^2\varphi + \cos^2\varphi = 1$$

$$\frac{r_3^2}{R^2} + \frac{R^2 - 2Rh + h^2}{R^2} = 1$$

$$\cancel{R^2} = r_3^2 + \cancel{R^2} - 2Rh_3 + h^2 \qquad R = \frac{r_3^2 + h_3^2}{2h_3} \doteq \frac{r_3^2 + \frac{25\lambda^2}{16}}{5\lambda}$$

Svetlobo $\lambda = 550$ nm pravokotno osvetljujemo režo s širino $D = 0,1$ mm. Uklonsko sliko opazujemo na zaslonu in na tem zaslonu vidimo, da je 2 temna stranska lisa 3 cm oddaljena od svetle pege. Kako daleč od reže je zaslon.



minimum za eno režo

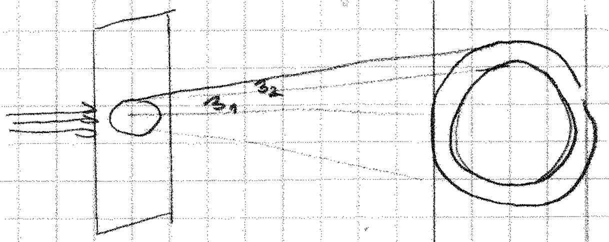
$$D \cdot \sin \beta_N = N \cdot \lambda$$

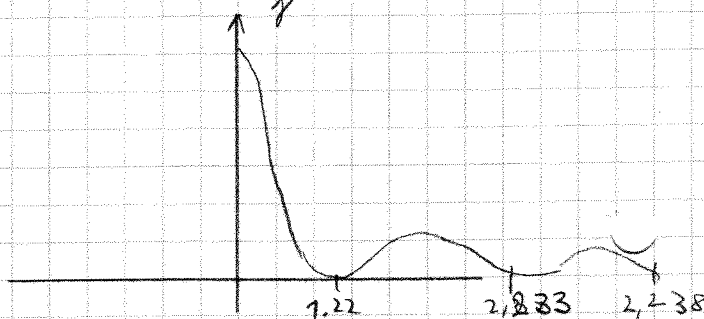
$$D \cdot \sin \beta_N = N \cdot \lambda$$

$$D \sin \beta_N = 2\lambda \qquad \sin \beta_N = \frac{2\lambda}{D} \sim \beta_2$$

$$x_2 = a \cdot tg\, \beta_2 \qquad a = \frac{x_2}{tg\, \beta_2} \sim \frac{x_2}{\beta_2} = \frac{x_2 \cdot D}{2\lambda}$$

S svetlobo $\lambda = 650$ v pravokotni smeri osvetljujemo okroglo odprtino s premerom 0,01 mm. Sliko opazujemo na 1 m oddaljenem zaslonu. Kolikšen je na zaslonu polmer osrednje pege.



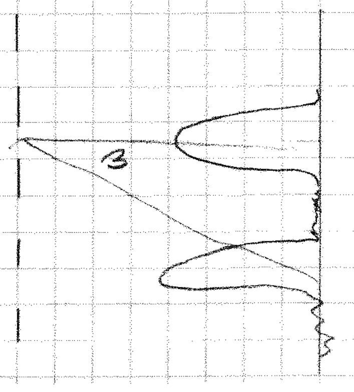


enačba za 1 minimum:

$$D \sin \beta_1 = 1,22\,\lambda \qquad r = a \cdot tg\, \beta \sim a \beta_1 = a \frac{1,22\lambda}{D}$$

$$\beta_1 \sim \frac{1,22\lambda}{D}$$

Uklonska mrežica ima ozke reže, ki so med seboj v povprečju med seboj razmaknjene 2 μm. Rešico osvetljujemo pravokotno s svetlobo $\lambda = 600$ nm. Kolikšno je največje število jačitev, ki jih še vidimo.



maksimum

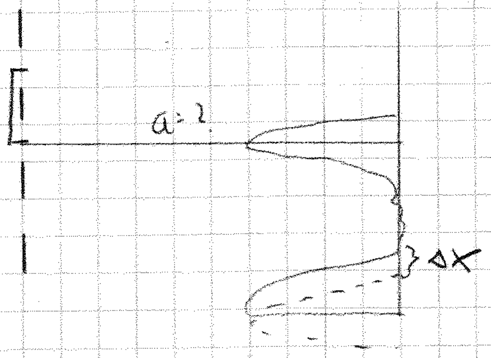
$$d \cdot \sin \alpha_n = N \cdot \lambda \quad (\text{do } 1)$$

$$N = \frac{2 \cdot 10^{-6}\,m}{6 \cdot 10^{-7}\,m}$$

$$N = \frac{20}{6} = 3$$

Uklonska rešica ima število rež 10000 na cm. Uklonsko rešico v pravokotni smeri osvetljujemo s svetlobo, ki ima $\lambda_1 = 590$ nm. Če jo povečamo na 591 nm. Je sprememba $\Delta x = 1$ cm. Kako daleč je zaslon?

$k = 10000$ cm $\qquad k = \frac{1}{d}$



$$\frac{1}{k}\sin\alpha_{11} = 1\lambda_1 \;\Rightarrow\; \sin\alpha_{11} = k\lambda_1 \;\rightarrow\; \operatorname{tg}\alpha_{11} = \frac{k\lambda_1}{\sqrt{1-k^2\lambda_1^2}}$$

$$\frac{1}{k}\sin\alpha_{12} = 1\cdot\lambda_2$$

$$\Delta x = x_2 - x_1 = a(\operatorname{tg}\alpha_{12} - \operatorname{tg}\alpha_{11})$$

$$= a\left(\frac{k\lambda_2}{\sqrt{1-k^2\lambda_2^2}} - \frac{k\lambda_1}{\sqrt{1-k^2\lambda_1^2}}\right) = \Delta x$$

$$a = \frac{\Delta x}{k\left[\frac{\lambda_2}{\sqrt{1-k^2\lambda_2^2}} - \frac{\lambda_1}{\sqrt{1-k^2\lambda_1^2}}\right]}$$

2. način

$$\frac{\Delta x}{\Delta\lambda} \rightarrow \frac{dx}{d\lambda}$$

$$x = a\cdot\operatorname{tg}\alpha$$

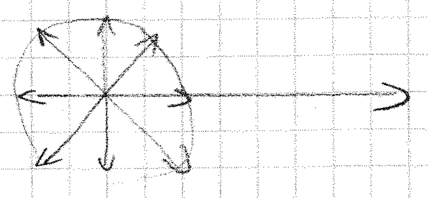
$$x = a\frac{k\lambda}{\sqrt{1-k^2\lambda^2}} = ak\lambda(1-k^2\lambda^2)^{-1/2}$$

$$\frac{dx}{d\lambda} = ak\left[(1-k^2\lambda^2)^{-1/2} + \lambda\left(-\tfrac{1}{2}\right)(1-k^2\lambda^2)^{-3/2}(-2k^2\lambda)\right]$$

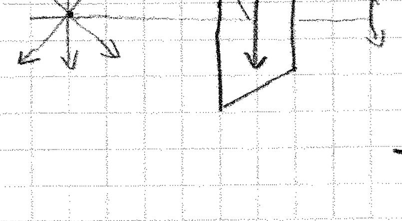
$$= ak\left[\frac{1}{\sqrt{1-k^2\lambda^2}} + \frac{k^2\lambda^2}{(1-k^2\lambda^2)^{3/2}}\right] = \frac{ak}{(1-k^2\lambda^2)^{3/2}} = \frac{dx}{d\lambda}$$

$$a = \frac{\Delta x}{\Delta\lambda}\,\frac{(1-k^2\lambda^2)^{3/2}}{k}$$

POLARIZACIJA



Nepolarizirana svetloba z $j_0 = 0{,}1\ W/m^2$ pada na polarizator v pravokotni smeri. Kakšna je gostota prepuščenega svetlobnega toka.

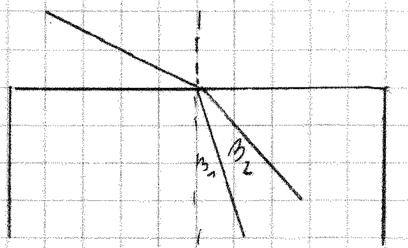


$$j = \frac{1}{2}\varepsilon E^2 c$$

$$j = \frac{1}{2}\varepsilon_0 E^2 \cos^2\varphi \cdot c \qquad \frac{1}{2\pi}\int_0^{2\pi}\cos^2\varphi\, d\varphi = \frac{1}{2}$$

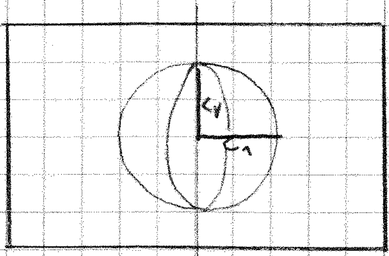
$$\frac{j}{j_0} = \frac{1}{2}$$

dvolomni kristal - 2 lomna količnika



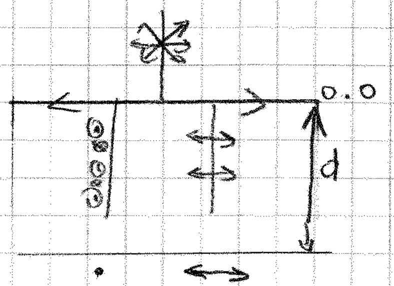
$\frac{c_0}{n_r}$ — redni, ši se v vse smeri enako hitro

$\frac{c_0}{n_i}$ — v odvisna od smeri



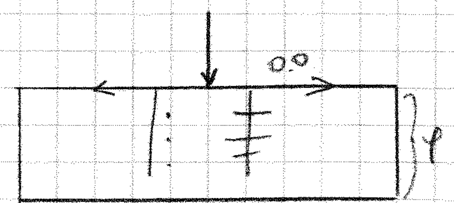
optična os - tu potujejo enako hitro

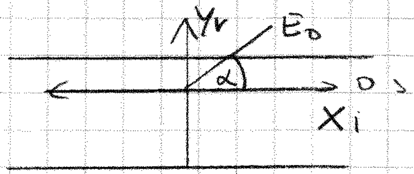
oba žarka sta linearno polarizirana in med sabo pravokotna.



$$\varphi = \frac{2\pi d}{\lambda}(n_r - n_i)$$

Linearno polarizirana svetloba z $\lambda = 550$ nm pada v pravokotni smeri na ploščico dvolomnega kristala, ki je rezana tako, da optična os leži na plošči, na katero pada svetloba $n_r = 2{,}211$, $n_i = 2{,}204$. Določi debelino ~~ploščine~~ tega kristala tako, da bo izstopajoče valovanje krožno polarizirano, debelina pa bo največja 60 μm. Pred vstop v kristal, vpadna svetloba kot s stranjo 45°.





$$E_i = E_x = E_0 \cos\alpha \sin(\omega t + \varphi)$$

$$E_r = E_y = E_0 \sin\alpha \cdot \sin(\omega t)$$

$$E_0 \cos\alpha \sin\omega t \cos\varphi + E_0 \cos\alpha \cos\omega t \sin\varphi$$

$$(\sin\omega t) = \frac{E_y}{E_0 \sin\alpha} \qquad \cos\omega t = \sqrt{1 - \frac{E_y^2}{E_0^2 \sin^2\alpha}}$$

$$E_x = \cancel{E_0} \cos\alpha \frac{E_y}{\cancel{E_0} \sin\alpha} \cos\varphi + E_0 \cos\alpha \sqrt{1 - \frac{E_y^2}{E_0^2 \sin^2\alpha}} \sin\varphi$$

$$E_x - E_y \operatorname{ctg}\alpha \cos\varphi = E_0 \cos\alpha \sqrt{\frac{E_0^2 \sin^2\alpha - E_y^2}{E_0^2 \sin^2\alpha}} \sin\varphi$$

$$E_x^2 - 2E_xE_y \operatorname{ctg}\alpha \cos\varphi + E_y^2 \operatorname{ctg}^2\alpha \cos^2\varphi = \cancel{E_0^2} \cos^2\alpha \frac{E_0^2 \sin^2\alpha - E_y^2}{\cancel{E_0^2} \sin^2\alpha} \sin^2\varphi \,/\, \sin^2\alpha$$

$$E_x^2 \sin^2\alpha - 2E_xE_y \cos\alpha \sin\alpha \cos\varphi + E_y^2 \cos^2\alpha \cos^2\varphi = E_0^2 \cos^2\alpha \sin^2\alpha \sin^2\varphi - E_y^2 \cos^2\alpha \sin^2\varphi$$

$$E_x^2 \sin^2\alpha - 2E_xE_y \cos\alpha \sin\alpha \cos\varphi + E_y^2 \cos^2\alpha = E_0^2 \cos^2\alpha \sin^2\alpha \sin^2\varphi$$

1. $\varphi = 2\pi, 4\pi, 6\pi$
$\sin\varphi = 0 \quad \cos\varphi = 1$

$\Rightarrow E_x^2 \sin^2\alpha - 2E_x E_y \cos\alpha \sin\alpha + E_y^2 \cos^2\alpha = 0$
$(E_x \sin\alpha - E_y \cos\alpha)^2 = 0$

$E_y = E_x \operatorname{tg}\alpha$ $\Rightarrow$ kot da kristala ni

2. $\varphi = \pi, 3\pi, 5\pi$
$\sin\varphi = 0 \quad \cos\varphi = -1$

$\Rightarrow (E_x \sin\alpha + E_y \cos\alpha)^2 = 0$

$E_y = -E_x \operatorname{tg}\alpha$ $\rightarrow$

3. $\varphi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$
$\cos\varphi = 0 \quad \sin\varphi = \pm 1$

$= E_x^2 \sin^2\alpha + E_y^2 \cos^2\alpha = E_0^2 \cos^2\alpha \sin^2\alpha$

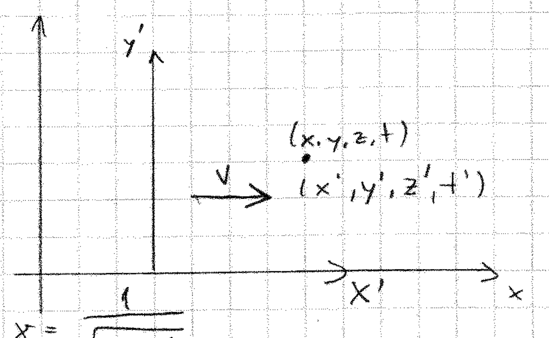
$\frac{E_x^2}{E_0^2 \cos^2\alpha} + \frac{E_y^2}{E_0^2 \sin^2\alpha} = 1$ enačba elipse $\Rightarrow$ valovanje je eliptično polarizirano

$\alpha = 45$ $\Rightarrow$ krožno polarizirano

Določi debelino ploščice:

$$\frac{2\pi d}{\lambda}(n_r - n_i) = \frac{\pi}{2}(2N+1)$$
$$2N+1 = \frac{4d}{\lambda}(n_r - n_i)$$
$$N = \frac{2d}{\lambda}(n_r - n_i) - \frac{1}{2}$$
$$N = \frac{120 \cdot 10^{-6}}{558 \cdot 10^{-9}} \cdot 0{,}01 - 0{,}5 = 1{,}7$$
$$\varphi = \frac{5\pi}{2} = \frac{2\pi d}{\lambda}(n_r - n_i)$$
$$d = \frac{5\lambda}{4(n_r - n_i)} \sim 67\ \mu m.$$

TEORIJA RELATIVNOSTI



GALILEJEVA
$x = x' + vt'$
$y = y'$
$z = z'$
$t = t'$

LORENTZOVE
$x = \gamma(x' + vt')$
$y = y'$
$z = z'$
$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$

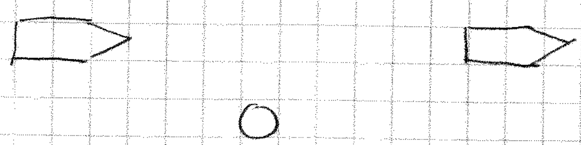
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$x' = \gamma(x - vt)$
$y = y'$
$z = z'$
$t' = \gamma\left(t - \frac{vx}{c^2}\right)$

Raketa leti mimo zemlje s $v = 0{,}995\,c$ glede na zemljo. Vesoljec v raketi se spravlja spat in prižge luč v trenutku, 20 min bere knjigo in nato luč ugasne. Koliko časa je trajalo branje knjige za opazovalca na zemlji.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0{,}995^2}} = 10$$

$$t = \gamma\,\tau = 200\ \text{min}$$



$t_1' = 0$
$t_2' = \tau$
$x_1' = 0$
$x_2' = 0$

$$t_1 = \gamma\left(t_1' + \frac{v x_1'}{c^2}\right) = \gamma\left(0 + \frac{v \cdot 0}{c^2}\right) = 0$$

$$t_2 = \gamma\left(t_2' + \frac{v x_2'}{c^2}\right) = \gamma\left(\tau + \frac{v \cdot 0}{c^2}\right) = \gamma \cdot \tau$$

Kolikšno pot naredi raketa, ko luč gori

$x_1' = 0$ $\qquad x_1 = 0$
$x_2' = 0$ $\qquad x_2 = \gamma \cdot (x_2' + v \cdot t') = \gamma(0 + v \cdot t') = v\,t$

Palica je v lastnem sistemu dolga 2 m. Kolikšno dolžino palice izmeri opazovalec, ki se giblje v smeri palice s hitrostjo $v = 0{,}85\,c$ glede na palico.

$$l' = \frac{l}{\gamma} = \frac{l}{\frac{1}{\sqrt{1-0{,}85^2}}} = 2\,\text{m} \cdot \sqrt{1 - 0{,}85^2} = 1{,}06\,\text{m}$$

$x_1 = 0$
$x_2 = l$
$t_1' = 0$
$t_2' = 0$

$$x_1' = \gamma(x_1 - v t_1) = \gamma\left(x_1 - v \cdot \gamma \cdot \left(t_1' + \frac{v x_1'}{c^2}\right)\right)$$

$$x_1' = \gamma x_1 - v\gamma^2 t_1' - \frac{v^2\gamma^2}{c^2} x_1'$$

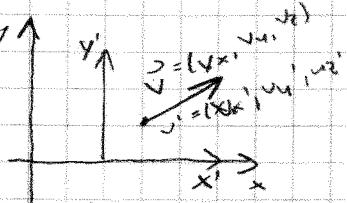
$$x_1'\left(\frac{1}{\gamma^2} + \frac{v^2\gamma^2}{c^2}\right) = \gamma(x_1 - v\gamma t_1')$$

$$x_1' = \frac{x_1 - v\gamma t_1'}{\gamma} \qquad l' = x_2' - x_1' = \frac{l}{\gamma}$$

$$x_2' = \frac{x_2 - v\gamma t_2'}{\gamma}$$

$$1 + \frac{v^2}{c^2}\,\frac{1}{1 - v^2/c^2} = 1 + \frac{v^2}{c^2 - v^2} = \frac{c^2 - v^2 + v^2}{c^2 - v^2} = \gamma^2$$

Iz diferencialne Lorentzove transformacije sledi Lorentzove transformacije za hitrosti



$$dx = \gamma(dx' + v\,dt') \qquad dx' = \gamma(dx - v\,dt)$$
$$dy = dy' \qquad dy' = dy$$
$$dz = dz' \qquad dz' = dz$$
$$dt = \gamma\left(dt' + \frac{v}{c^2}dx'\right) \qquad dt' = \gamma\left(dt - \frac{v}{c^2}dx\right)$$

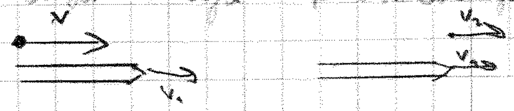
$$v_x = \frac{dx}{dt} = \frac{\cancel{\gamma}(dx' + v\,dt')}{\cancel{\gamma}(dt' + v/c^2\,dx')} = \frac{dt'\left(\frac{dx'}{dt'} + v\right)}{dt'\left(1 + v/c^2\,\frac{dx'}{dt'}\right)} = \frac{v_x' + v}{1 + \frac{v v_x'}{c^2}}$$

$$v_y = \frac{dy}{dt} = \frac{dy'}{\gamma(dt' + \frac{v}{c^2}dx')} = \frac{dt'\,\frac{dy'}{dt'}}{\gamma\,dt'\left(1 + \frac{v}{c^2}\frac{dx'}{dt'}\right)} = \frac{v_y'}{\gamma\left(1 + \frac{v v_x'}{c^2}\right)}$$

$$v_z = \frac{v_z'}{\gamma\left(1 + \frac{v v_x'}{c^2}\right)}$$

$$V_x' = \frac{V_x - V}{1 - \frac{V V_x}{c^2}} \qquad V_y' = \frac{V_y}{\gamma\left(1 + \frac{V U_x}{c^2}\right)} \qquad V_z' = \frac{V_z}{\gamma\left(1 + \frac{V V_z}{c^2}\right)}$$

Vesoljska ladja Galaktika leti mimo Zemlje s hitrostjo 0,95 c v lastnem sistemu pa je dolga 2500 m. Vesoljski lovec Raptor leti s hitrostjo 0,998 c glede na opazovalca na Zemlji. Koliko časa traja prehitevanje za opazovalca na Galaktiki?



$$V_2' = \frac{V_2 - V_1}{1 - \frac{V_1 V_2}{c^2}} = \frac{(0{,}998 - 0{,}995)c}{1 - \frac{0{,}998 \cdot 0{,}995\, c^2}{c^2}} = 0{,}925\,c$$

$$t' = \frac{l}{V_2'} = \frac{2500\,m}{0{,}925 \cdot 3 \cdot 10^8\,m} = 9 \cdot 10^{-6}\,s$$

Koliko bi pa to bilo za opazovalca na Zemlji

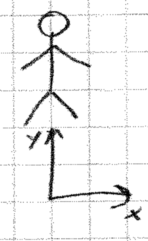
$$t'' = \gamma$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{V_1}{c^2}}}$$

Koliko časa pa bi trajal glede na lovca

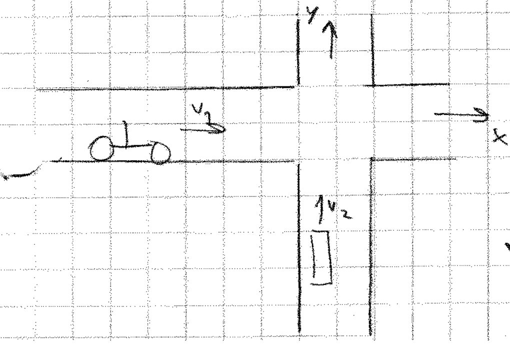
nasprotno galaktiko $v = 0{,}925 \Rightarrow \gamma \Rightarrow l' \Rightarrow t$

V pospeševalniku se elektron in pozitron gibljeta drug proti drugem po premici. Hitrost prvega = 0,98 c, drugega pa −0,99 c. S kolikšno v se giblje



$$V_2' = \frac{V_2 - V_1}{1 - \frac{V_1 V_2}{c^2}} = \frac{-0{,}99c - 0{,}98c}{1 - \frac{0{,}98c \cdot (-0{,}99c)}{c^2}} = \frac{-1{,}97}{1 + 0{,}98 \cdot 0{,}99} = -0{,}9998\,c$$

Policist stoji v križišču 2 pravokotnih cest. Iz 1 smeri prihaja motorist z 0,98 c, po drugi pa auto 0,98 c. Kolikšna je hitrost automobila glede na motor in pod kolikšnim kotom motorist vidi, da se mu auto približuje

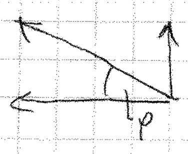


$$\vec{V}_1 = (V_{1x}, 0) = (0{,}98c, 0)$$
$$\vec{V}_2 = (0, V_{2y}) = (0, 0{,}98c)$$

$$V_{2x}' = \frac{V_{2x} - V_{1x}}{1 - \frac{V_{1x} V_{2x}}{c^2}} = \frac{0 - 0{,}98c}{1 - \frac{0 \cdot 0{,}98c}{c^2}} = -0{,}98c$$

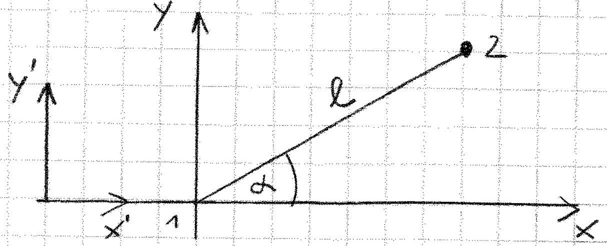
$$V_{2y}' = \frac{V_{2y}}{\gamma\left(1 - \frac{V_{1x} V_{2x}}{c^2}\right)} = \frac{0{,}98c}{5(1-0)} = 0{,}19c$$

$V_2' = \sqrt{V'^2_{2x} + V'^2_{2y}} = 0'998\,c$



$tg\,\varphi \; \frac{0'19}{0'98} = 10'1°$

Palica je v lastnem sistemu dolga $l = 2m$ in leži v ravnini xy, da z osjo x oklepa $30°$. Kolikšno dolžino palice izmeri opazovalec, ki se giblje z $0'958c$ v smeri x. Kolikšen kot med palico in osjo x izmeri opazovalec.



$x_1 = 0$
$x_2 = l \cdot \cos\alpha$
$y_1 = 0$
$y_2 = l \cdot \sin\alpha$

$t_1' = t_2' = 0$

$$x_1' = \gamma \cdot (x_1 - v \cdot t_1) = \gamma\left(x_1 - v\gamma\left(t_1' + \frac{v}{c^2}x_1'\right)\right)$$

$$\Rightarrow x_1' = \gamma x_1 - v\gamma^2 t_1' - \frac{v^2}{c^2}\gamma^2 x_1'$$

$$x_1' \underbrace{\left(1 + \frac{v^2}{c^2}\right)}_{\gamma^2} = \gamma\,(x_1 - v\gamma t_1')$$

$$x_1' = \frac{x_1 - v\gamma t_1'}{\gamma} \qquad x_2' = \frac{x_2 - v\gamma t_2'}{\gamma}$$

$$l_x' = x_2' - x_1' = \frac{x_2 - v\gamma t_2'}{\gamma} - \frac{x_1 - v\gamma t_1'}{\gamma} = \frac{x_2 - x_1}{\gamma} = \frac{2\cos\alpha}{15'8}$$

$y_1' = y_1 = 0$
$y_2' = y_2 = l \cdot \sin\alpha$

$$l_y' = y_2' - y_1' = l \cdot \sin\alpha$$

$$l' = \sqrt{l'^2_x + l'^2_y} = l\sqrt{\frac{\cos^2\alpha}{\gamma^2} + \sin^2\alpha}$$

$$tg\,\alpha' = \frac{l_y'}{l_x'} = \frac{l \cdot \sin\alpha \cdot \gamma}{l \cdot \cos\alpha} = \gamma \cdot tg\,\alpha$$

GIBALNA KOLIČINA IN KINETIČNA ENERGIJA

$E = mc^2$ mirovna energija $m_0 c^2 + W_k \to v \to \gamma$

$$m_0 c^2 + W_k = \underbrace{m_0 \gamma}_{m_r} \cdot c^2$$

relativistična masa

$$W_k = m_0 c^2 (\gamma - 1) = m_0 c^2 \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right) \qquad (1-x)^{-1/2} = 1 + \frac{1}{2} x -$$

$$= m_0 c^2 \left(\cancel{1} + \frac{1}{2} \frac{v^2}{c^2} - \ldots - \cancel{1} \right)$$

$$= m_0 \frac{v^2}{2} \text{ za majhne hitrosti}$$

$$p = m_0 \gamma \cdot v = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}$$

Kolikšni sta kinetična energija in gibalna količina elektrona, ki ima $m_0 = 0.51\ \mathrm{MeV}/c^2$ in hitrost $0.9c$

$$p = m_0 \gamma v = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{0.51\ \mathrm{MeV} \cdot 0.9 c}{c^2 \sqrt{1-0.9^2}}$$

$$= 1.05 \frac{\mathrm{MeV}}{c}$$

$1\ \mathrm{eV} = 1.6 \cdot 10^{-19}\ \mathrm{J}$

$1\ \mathrm{MeV} = 1.6 \cdot 10^{-13}\ \mathrm{J}$

$$\frac{1.6 \cdot 10^{-13}\ \mathrm{J\,s^2}}{9 \cdot 10^{16}\ \mathrm{m^2}}$$

$$W_k = m_0 c^2 (\gamma - 1) = 0.51 \frac{\mathrm{MeV}}{c^2} \cdot \cancel{c^2} \left(\frac{1}{\sqrt{1-0.9^2}} - 1 \right) = 0.66\ \mathrm{MeV}$$

Kolikšni sta mirovna masa in $\vec{v}$ delca z $W_k = 628.64\ \mathrm{MeV}$ in gibalno količino $1254.95\ \mathrm{MeV}/c$

1 način

$$\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} \; : \; W_k = m_0 c^2 \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1 \right)$$

$$\Rightarrow \frac{p}{W_k} = \frac{v}{\sqrt{1-\frac{v^2}{c^2}}\; c^2 \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right)}$$

$$= \frac{v}{1 - \sqrt{1-\frac{v^2}{c^2}}} \Rightarrow p c^2 \left(1 - \sqrt{1-\frac{v^2}{c^2}} \right) = W_k \cdot v$$

$$pc^2 - pc^2 \sqrt{1 - v^2/c^2} = W_k v$$

$$pc^2 - W_k v = pc^2 \sqrt{1-\frac{v^2}{c^2}} \quad /^2$$

$$\cancel{p^2 c^4} - 2pc^2 W_k \cancel{v} + v^{\cancel{2}} W_k^2 = \cancel{p^2 c^4} - p^2 v^{\cancel{2}} c^2 \quad / : v$$

$$+ 2pc^2 W_k = W_k^2 v + p^2 c^2 v$$

$$v = \frac{2pc^2 W_k}{p^2 c^2 + W_k^2}$$

$$v = \frac{2 \cdot 1254{,}95 \frac{MeV}{c} \cdot c^2\, 628{,}64\, MeV}{1254{,}95^2 \frac{MeV^2}{c^2} + 628{,}64^2\, MeV^2}$$

$$= \ldots c$$

2 način

$$p \qquad m_0 = \frac{p}{v}\sqrt{1-\frac{v^2}{c^2}}$$

$$= \frac{p^2c^2 - Wk^2}{2Wk\,c^2} = m_0$$

2 način formula

$$(m_0 c^2 + Wk)^2 = m_0^2 c^4 + p^2 c^2$$

Elektrodo kondenzatorja obsevamo s svetlobo, tako da iz elektrode izbijamo elektrone, ki imajo zanemarljivo mejhno hitrost $m_0 = 0{,}51\ MeV/c^2$; $e_0 = 1{,}6 \cdot 10^{-19}$. V nekem trenutku na to elektrodo - priključimo kond. na $E = 2000\ V/m$. Kolikšna je v po 1 ms v sistemu elektrona po priključitvi

$$\vec{F} = e_0 E = m_0 \cdot a$$

$$v = a \cdot t = \frac{e_0 E}{m_0} \cdot t = \frac{2000\, eV \cdot c^2 \cdot 10^{-3}\, s}{m \cdot 0{,}51 \cdot 10^6\, eV} = v > c \text{ ne moremo klasično}$$

moramo relativistično

$$F \cdot t = \Delta p \qquad e_0 \cdot E \cdot t = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$e_0^2 E^2 t^2 = \frac{m_0^2 v^2 c^2}{c^2 - v^2}$$

$$e_0^2 E^2 t^2 c^2 - E^2 e_0^2 t^2 v^2 = m_0^2 v^2 c^2$$

$$v^2 \cdot m_0^2 c^2 + e_0^2 t^2 v^2 E^2 = e_0^2 E^2 t^2 c^2$$

$$v = \frac{e_0 E t c}{\sqrt{m_0^2 c^2 + e_0^2 t^2 E^2}}$$

Vesoljska ladja v začetku miruje v brezteženem prostoru - galaktičnem prostoru. V nekem trenutku pa starta s pospeškom $0{,}2\ m/s^2$ merjeno v sistemu ladje. Kolikšna je v ladje, ko v ladji mineta 2 leti.

$$dp = F \cdot dt$$

$$F = \frac{dp}{dt}$$

$$F = \frac{d}{dt}\left(\frac{m_0 v}{\sqrt{1-\frac{v(t)^2}{c^2}}}\right)$$

$$a = \frac{F}{m_0}$$

$$a = \frac{d}{dt}\left(v\left(1-\frac{v^2}{c^2}\right)^{-1/2}\right)$$

$$\int_0^t a\cdot dt = \int_0^v d\left[v\left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$at = \frac{v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$a = \frac{dv}{dt}\left(1-\frac{v^2}{c^2}\right)^{-1/2} + v\left(-\tfrac{1}{2}\right)\left(1-\tfrac{1}{2}\right)^{-3/2}\left(-\frac{2v}{c^2}\frac{dv}{dt}\right)$$

$$a = \frac{dv}{dt}\left[\frac{1}{(1-v^2/c^2)^{1/2}} + \frac{v^2/c^2}{(1-v^2/c^2)^{3/2}}\right] = \frac{dv}{dt}\cdot\frac{1}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

$$\int_0^v \frac{dv}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} = \int_0^t a\cdot dt$$

$$at = c\int_0^{v/c}\frac{dx}{(1-x^2)^{3/2}} \qquad \frac{v}{c} = x \qquad dv = c\cdot dx$$

$$= c\cdot\frac{x}{\sqrt{1-x^2}} = \frac{v}{\sqrt{1-v^2/c^2}}$$

FOTOEFEKT

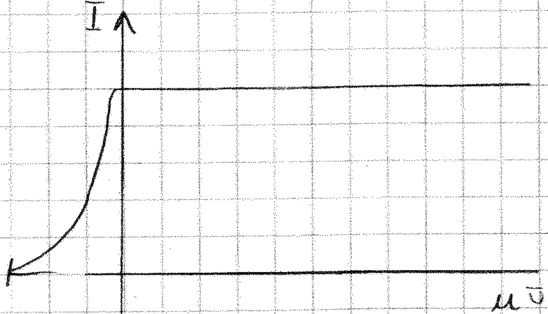
$E = h\nu$ $\qquad c = \lambda\nu$

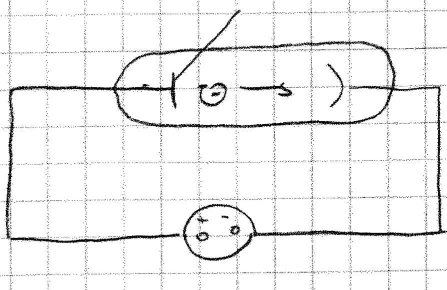
$h = 6{,}62\cdot 10^{-34}\ \mathrm{Js}$

$$E = \frac{hc}{\lambda} \qquad G_p = \frac{h}{\lambda} \qquad F = \frac{dG}{dt} = \frac{d}{dt}\left(\frac{E}{c}\right) = \frac{1}{c}P$$

$$hc = 1240\ \mathrm{eVnm} \qquad p = \frac{F}{S} = \frac{1}{c}\frac{P}{S} = \frac{j}{c}$$

Katodo fotocelice osvetljujemo s svetlobo 500 nm in spreminjamo napetost na fotocelici ~~[illegible]~~ Izkaže se, da zaporna napetost 1.5V. Kolikšno je izstopno delo.





$$\frac{hc}{\lambda} = e_0U + A_i$$

$$A_i = \frac{hc}{\lambda} - e_0U = \frac{1240\ \cancel{eVnm}}{500\ \cancel{nm}} - 1{,}5\ \mathrm{eV} = 0{,}98\ \mathrm{eV}$$

Katodo fotocelice osvetljujemo s svetlobo valovne dolžine 560 nm. Svetlobni tok pa je enak 0'5 W. Nasičeni električni tok skozi fotocelico je $I = 10^{-4}$. Kolikšen je kvantni izkoristek te foto katode.

$$\eta = \frac{N_e}{N_f}$$

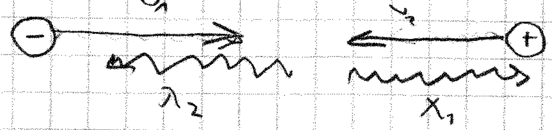
$$I = \frac{dQ}{dt} = \frac{Q}{t} = \frac{N_e \cdot e_0}{t} \Rightarrow N_e = \frac{It}{e_0}$$

$$P = \frac{dE}{dt} = \frac{E}{t} = \frac{N_f \cdot \frac{hc}{\lambda}}{t} \qquad N_f = \frac{Pt\lambda}{hc}$$

$$\eta = \frac{I \cdot t \cdot hc}{e_0 P t \lambda}$$

$$\eta = \frac{10^{-4} \cdot 1240 \, \cancel{eV\,nm}}{560 \, \cancel{nm} \cdot 0{,}5 \, \cancel{W}} = \frac{1240 \cdot 10^{-4}}{0{,}5 \cdot 560} = 4{,}4 \cdot 10^{-4}$$

Elektron in pozitron letita drug proti drugemu s hitrostima $v_1 = 0'95$, $v_2 = 0'98\,c$ (glede na opazovalca). Delca trčita in se anihilirata v dva fotona od katerih odleti eden v smer e^- in pozitron. Kolikšni sta λ_1 in λ_2 nastalih fotonov.



$$p_1 = \frac{m_0 v_1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad p_2 = \frac{m_0 v_2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$W_1 = m_0 c^2 \left(\frac{1}{\sqrt{1-\frac{v_1^2}{c^2}}} - 1\right)$$

$$- p_1 - p_2 = \frac{h}{\lambda_1} - \frac{h}{\lambda_2}$$

$$- 2 m_0 c^2 + W_{k_1} + W_{k_2} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$2 m_0 c^2 + W_1 + W_2 + p_1 c - p_2 c = \frac{2hc}{\lambda_1}$$

$$\lambda_1 = \frac{2hc}{2m_0c^2 + W_1 + W_2 + c(p_1 - p_2)}$$

$$\lambda_2 = \frac{2hc}{2m_0c^2 + W_1 + W_2 + c(p_2 - p_1)}$$

Dolžnost

$$p = \frac{h}{\lambda} \qquad \lambda_B = \frac{h}{p} \qquad \lambda_c = \frac{h}{m_0 c}$$

Kolikšna je λ_B za elektron, če ga je pospešila napetost 2 MV

$$\lambda_B = \frac{h}{p}$$

$$(e_0 U + m_0 c^2)^2 = m_0^2 c^4 + c^2 p^2$$

$$e_0^2 U^2 + 2 e_0 U m_0 c^2 = c^2 p^2$$

$$p = \frac{\sqrt{e_0 U (e_0 U + 2 m_0 c^2)}}{c}$$

$$\lambda_B = \frac{hc}{\sqrt{e_0 U (e_0 U + 2 m_0 c^2)}}$$

$$= 5 \cdot 10^{-4} \text{ nm.}$$

Ciklotron

$$e r B = p$$

FL

Izračuni hitrost protona z [illegible] $m_0c^2 = 938$ MeV ki v $B=2T$ kroži z $r=$ [illegible] ...

$$e_0 r B = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$e_0^2 r^2 B^2 = \frac{m_0^2 c^2 v^2}{c^2 - v^2}$$

$$e_0^2 r^2 B^2 c^2 - e_0^2 r^2 B^2 v^2 = m_0^2 v^2 c^2$$

$$v^2 (m_0^2 c^2 + e_0^2 r^2 B^2) = e_0^2 r^2 B^2 c^2$$

$$v = \frac{e_0 r B c}{\sqrt{m_0^2 c^2 + e_0^2 r^2 B^2}} = \ldots =$$

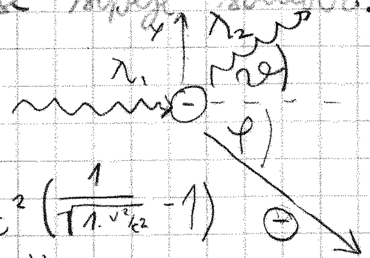
$$\lambda_B = \frac{h}{p}$$

$$\lambda_c = \frac{h}{m_0 c} \quad (\text{el } 0{,}0024\ \text{nm})$$

compt.

$$p = e r B$$

Svetloba z valovno dolžino 0,003 nm se comptonsko sipje na mirujočih e. Po sipanju ima svetloba valovno dolžino 0,005 nm. V ~~kolikšno~~ v in glede na začetno ~~stara~~ smer, pod kolikšnim kotom se sipje svetloba. In pod kolikšnim kotom se odbijajo e?



$$\frac{hc}{\lambda_1} = \frac{hc}{\lambda_2} + W_k$$

$$W_k = m_0 c^2 \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$$

$$p = \frac{m_0 v}{\sqrt{1-v^2/c^2}}$$

$$v\ x: \quad \frac{h}{\lambda_1} = \frac{h}{\lambda_2}\cos\vartheta + p\cos\varphi$$

$$y: \quad 0 = \frac{h}{\lambda_2}\sin\vartheta - p\sin\varphi$$

prišli bi: $\lambda_2 - \lambda_1 = \lambda_c (1 - \cos\vartheta)$

mirovna masa e

$$m_0 = 0{,}51 \frac{\text{MeV}}{c^2}$$

$$W_k = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = \frac{hc(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}$$

$$= m_0 c^2 \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$$

$$\frac{1}{\sqrt{1-v^2/c^2}} = 1 + \frac{hc(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2} \Big/ ^2$$

$$\frac{c^2}{c^2 - v^2} = 1 + \frac{h^2 c^2 (\lambda_2 - \lambda_1)^2}{\lambda_1^2 \lambda_2^2} + 2\frac{hc(\lambda_2-\lambda_1)}{\lambda_1 \lambda_2}$$

$$c^2 = \left(\frac{h(\lambda_2 - \lambda_1)}{m_0 c \lambda_1 \lambda_2}\right) c^2 + \left(\frac{h(\lambda_2-\lambda_1)}{m_0 c \lambda_1 \lambda_2} + 1\right)^2 v^2$$

$$c^2 \left(1 - \left(\frac{h(\lambda_2-\lambda_1)^2}{m_0 c \lambda_1 \lambda_2} + 1\right)^2\right) = \left(\frac{h(\lambda_2 - \lambda_1)}{m_0 c \lambda_1 \lambda_2} + 1\right)^2 v^2$$

$$v = \frac{c}{\frac{h(\lambda_2-\lambda_1)}{m_0 c \lambda_1 \lambda_2} + 1} \sqrt{1 - \left(\frac{h(\lambda_2-\lambda_1)}{m_0 c \lambda_1 \lambda_2} + 1\right)^2}$$

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pravilno gre nekako takole

$$c^2 (\ )^2 - v^2 (\ )^2 = c^2$$

$$c^2 \left[(\ )^2 - 1 \right] = v^2 (\ )^2$$

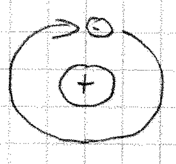
$$v = \frac{c}{\frac{h c (\lambda_2 - \lambda_1)}{m_0 c^2 \lambda_1 \lambda_2} + 1} \cdot \sqrt{\left(\frac{h c (\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2 m_0 c^2} + 1\right)^2 - 1}$$

$$\cos \vartheta = 1 - \frac{\lambda_2 - \lambda_1}{\lambda_1}$$

iz prejšnjih 2 enačb za P dobimo

$$\operatorname{tg} \varphi = \frac{h/\lambda_2 \sin \vartheta}{h/\lambda_1 - h/\lambda_2 \cos \vartheta}$$

BOROV MODEL H



$$\Gamma = n \hbar \qquad \hbar = \frac{h}{2\pi}$$

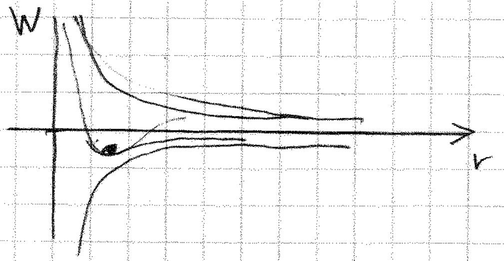
v, l r, l

$$\hbar c = 197\,eV\,nm$$

Izpelji izraz za Borov radij elektronovega tira v osnovnem stanju z vrtilno količino po Borovem modelu.

$$\frac{p^2}{2 m_e} - \frac{e_0^2}{4\pi\varepsilon_0 r^2} = W \qquad \Gamma = r \cdot p \qquad p = \frac{\Gamma}{r} = \frac{n\hbar}{r}$$

$$\frac{n^2 \hbar^2}{2 m_e r^2} - \frac{e_0^2}{4\pi\varepsilon_0 r} = W$$



$$W = \frac{n^2 \hbar^2}{2 m_e} r^{-2} - \frac{e_0^2}{4\pi\varepsilon_0} r^{-1}$$

$$\frac{dW}{dr} = 0 = (-2) \frac{n^2 \hbar^2}{2 m_e} r^{-3} - (-1) \frac{e_0^2}{4\pi\varepsilon_0} r^{-2} \ / r^3$$

$$= -\frac{n^2 \hbar^2}{m_e} + \frac{e_0^2 r}{4\pi\varepsilon_0} = 0$$

$$r = \frac{4\pi\varepsilon_0 n^2 \hbar^2}{m_e e_0^2}$$

$$r_B = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e_0^2} = 53 \ 10^{-12}\,m$$

$$r = r_B \cdot n^2$$

$$W = \frac{n^2 \hbar^2}{2 m_e n^4 r_B^2} - \frac{e_0^2}{4\pi\varepsilon_0 n^2 r_B}$$

$$W = \frac{1}{n^2 r_B} \left[\frac{\hbar^2}{2 m_e r_B} - \frac{e_0^2}{4\pi\varepsilon_0} \right]$$

$$W = \frac{1}{n^2 r_B} \left[\frac{\cancel{\hbar^2}\ \cancel{m_e} e_0^2}{2 \cancel{m_e} 4\pi\varepsilon_0 \cancel{\hbar^2}} - \frac{e_0^2}{4\pi\varepsilon_0} \right]$$

$$W = -\frac{e_0^2}{8\pi\varepsilon_0 r_B} \frac{1}{n^2} = 13{,}6\,eV$$

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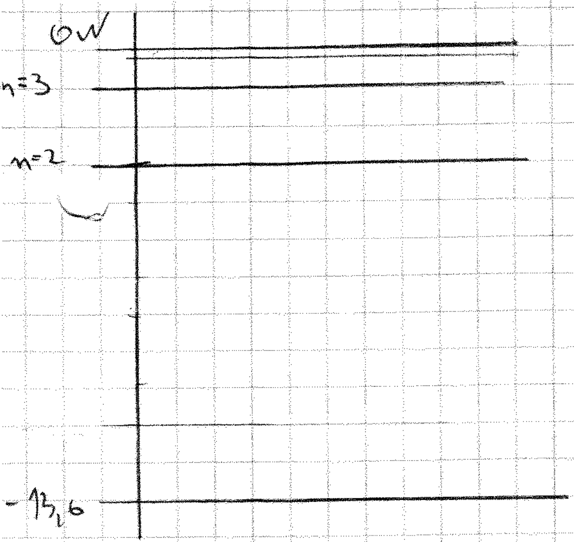


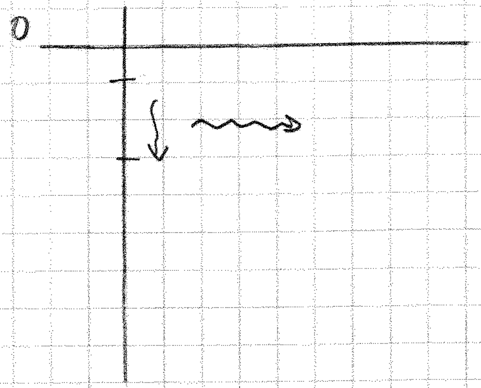
Oceni dodno hitrost e^- v vodikovem atomu v 1. vzbujenem stanju s pomočjo Borovega modelu.

$n=2$

$$\Gamma = m_0 r \cdot v = 2\hbar$$

$$v = \frac{2\hbar}{m_0 r} = \frac{2\hbar}{m_0 4 r_0} = \frac{\hbar}{2 m_0 r_0} = 3,7 \cdot 10^{-3}\ c$$

Elektron v H atomu preide iz 2 v 1 vzbujeno stanje. Kolikšna je λ, ki se pri tem izseva.



$$\frac{hc}{\lambda} = W_2 - W_1$$

$$\frac{hc}{\lambda} = W_0 \left(\frac{1}{4} - \frac{1}{9}\right)$$

$$= W_0 \frac{5}{36} = \frac{hc}{\lambda}$$

$$\lambda = \frac{36\, hc}{5 W_0} = \frac{36 \cdot 1240\ eVnm \cdot c}{5 \cdot 13,6\ eV}$$

$$\lambda = 656\ nm.$$

RADIOAKTIVNI RAZPAD

Polonij 210 razpada, pri razpadu oddaja α delce. Razpolovno doba je 138,4 dni. V vzorcu polonija je bilo na začetku 10^{20} atomov polonija. Koliko delcev α izseva ta ~~dolo~~ delec v 1. 124 dneh.

$$N(t) = N_0 e^{-\lambda t}$$

$$N_\alpha = N_0 - N_{(124\ DNI)} = N_0 (1 - e^{-\lambda t})$$

$$= N_0 (1 - e^{-\frac{t \ln 2}{t_{1/2}}})$$

$$= N_0 10^{20} (1 - e^{-\frac{124 \cdot \ln 2}{138,4}}) = 4,6 \cdot 10^{19}$$

$$N(t) = N_0 e^{-\lambda t}$$

$$N(t) = N_0 2^{-\frac{t}{t_{1/2}}}$$

$$t = \frac{-\ln N/N_0}{\lambda}$$

$$t = -t_{1/2} \log_2 \frac{N}{N_0}$$

$$= -t_{1/2} \frac{\ln N/N_0}{\ln 2}$$

$$-\ln \frac{N}{N_0} / \lambda = -t_{1/2} \frac{\ln N/N_0}{\ln 2}$$

$$\lambda \cdot t_{1/2} = \ln 2$$

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Predpostavimo, da so vsi delci α tvorili He. Količina cilinda V pri 0°C in tlaku 10^5 Pa. $M = 4$ kg/kmol

$$pV = \frac{m}{M} RT$$

$$V = \frac{mRT}{pM} = \frac{N(t)\, m_1 \cdot RT}{pM} = \frac{N \cdot \frac{M}{N_A} \cdot RT}{pM} = \frac{N\, RT}{N_A p}$$

$$= \frac{4{,}6 \cdot 10^{12} \cdot 273 \cdot 8314}{10^5 \cdot 6 \cdot 10^{26}}$$

$$=$$

V začetku imamo tolikšno količino polonija z molekulsko maso 210 kg/kmol, da na sekundo razpade $1{,}86 \cdot 10^7$ njegovih jeder. Koliko g Po nam je ostalo v roku še 100 dni, razpolovna doba polonija pa 138,4 dni.

$$N(t) = N_0\, e^{-\lambda t} = N_0\, e^{-\frac{t \ln 2}{t_{1/2}}} \qquad A = -\frac{dN(t)}{dt}$$

$$= N_0 \left(+\frac{\ln 2}{t_{1/2}}\right) e^{-\frac{t \ln 2}{t_{1/2}}}$$

$$A(t=0) = \frac{N_0 \ln 2}{t_{1/2}}$$

$$N_0 = t_{1/2} \cdot A(t=0) / \ln 2$$

$$N_{100} = \frac{t_{1/2} \cdot A(t=0)}{\ln 2}\, e^{-\frac{100 \ln 2}{138{,}4}} = 1{,}93 \cdot 10^{15}$$

$$m = N(t=100) \cdot m_1 = N(t=100) \cdot \frac{M}{N_A} = 6{,}74 \cdot 10^{-10}\ \text{kg}$$

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