

# Rešitve MATEMATIKA I

28. januar 2008

## 1. naloga

Poiščite vse rešitve enačbe

$$|x + 3| - |2x - 1| = x - 4$$

rešitev:

$$\begin{aligned} x < -3 \quad \text{in} \quad & -(x + 3) + (2x - 1) = x - 4 \\ & -x - 3 + 2x - 1 = x - 4 \\ & x - 4 = x - 4 \end{aligned}$$

za vsak  $x$

$$\begin{aligned} -3 \leq x < \frac{1}{2} \quad & x + 3 + 2x - 1 = x - 4 \\ & 3x + 2 = x - 4 \\ & 2x = -6 \\ & x = -3 \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \leq x \quad & x + 3 - 2x + 1 = x - 4 \\ & -x + 4 = x - 4 \\ & -2x = -8 \\ & x = 4 \end{aligned}$$

$$\underline{(-\infty, -3] \cup \{4\}}$$

## 2. naloga

Izračunajte limito

$$\lim_{n \rightarrow \infty} \frac{3n^2 + n}{2n^2 - 4}$$

rešitev:

$$\lim_{n \rightarrow \infty} \frac{3n^2 + n}{2n^2 - 4} = \lim_{n \rightarrow \infty} \frac{3 + \frac{1}{n}}{2 - \frac{4}{n^2}} = \frac{3}{2}$$

### 3. naloga

Poiščite ekstreme funkcije

$$f(x) = \frac{\ln x}{\sqrt{x}}$$

**rešitev:**

$$f'(x) = \frac{\frac{1}{x}\sqrt{x} - \ln x \frac{1}{2\sqrt{x}}}{x} = \frac{1 - \frac{1}{2} \ln x}{x\sqrt{x}}$$

$$f'(x) = 0 \text{ pri } \ln x = 2 \text{ oz. } x = e^2.$$

$$x < e^2 \rightarrow f' > 0 \rightarrow f \text{ narašča}$$

$$x > e^2 \rightarrow f' < 0 \rightarrow f \text{ pada in točka } M(e^2, 2/e) \text{ je maksimum.}$$

### 4. naloga

Izračunajte integral

$$\int_0^{\infty} e^{-2x}(x+1) dx$$

**rešitev:**

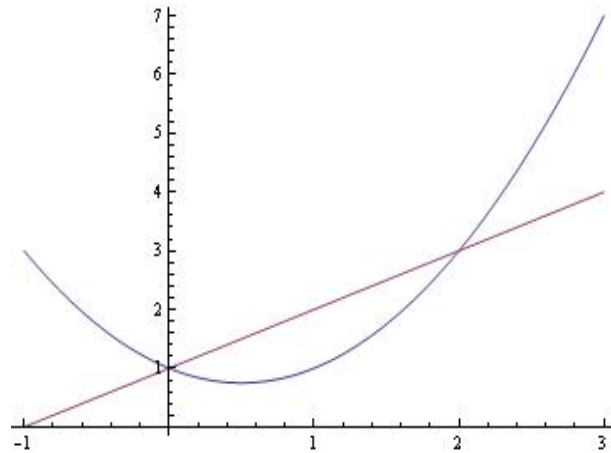
$$\int_0^{\infty} e^{-2x}(x+1) dx = (x+1) \frac{e^{-2x}}{-2} \Big|_0^{\infty} - \int_0^{\infty} \frac{e^{-2x}}{-2} dx = \frac{1}{2} - \frac{e^{-2x}}{4} \Big|_0^{\infty} =$$

$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

### 5. naloga

Izračunajte ploščino lika med parabolo  $y = x^2 - x + 1$  in premico  $y = x + 1$ !

**rešitev:**



$$x^2 - x + 1 = x + 1$$

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

$$x_1 = 0, x_2 = 2$$

$$P = \int_0^2 [x+1 - (x^2 - x + 1)] dx = \int_0^2 dx (2x - x^2) = x^2 - \frac{x^3}{3} \Big|_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$$