# Kazalo

# 1 Množice

1. Dokaži enakosti:

   (a) $\mathcal{A} \cup \mathcal{A} = \mathcal{A} \cap \mathcal{A} = \mathcal{A}$

   (b) $\mathcal{A} \cap \mathcal{B} = \mathcal{B} \cap \mathcal{A}$

   (c) $\mathcal{A} \cup \mathcal{B} = \mathcal{B} \cup \mathcal{A}$

   (d) $\mathcal{A} \cap (\mathcal{B} \cap \mathcal{C}) = (\mathcal{A} \cap \mathcal{B}) \cap \mathcal{C}$

   (e) $\mathcal{A} \cup (\mathcal{B} \cup \mathcal{C}) = (\mathcal{A} \cup \mathcal{B}) \cup \mathcal{C}$

   (f) $\mathcal{A} \cap (\mathcal{B} \cup \mathcal{C}) = (\mathcal{A} \cap \mathcal{B}) \cup (\mathcal{A} \cap \mathcal{C})$

   (g) $\mathcal{A} \cup (\mathcal{B} \cap \mathcal{C}) = (\mathcal{A} \cup \mathcal{B}) \cap (\mathcal{A} \cup \mathcal{C})$

   (h) $(\mathcal{A} \cup \mathcal{B}) \cap \mathcal{A} = (\mathcal{A} \cap \mathcal{B}) \cup \mathcal{A} = \mathcal{A}$

   (i) $(\mathcal{A} \cap \mathcal{B}) \cup (\mathcal{C} \cap \mathcal{D}) = (\mathcal{A} \cup \mathcal{C}) \cap (\mathcal{B} \cup \mathcal{C}) \cap (\mathcal{A} \cup \mathcal{D}) \cap (\mathcal{B} \cup \mathcal{D})$

2. Dokaži enakosti:

   (a) $\overline{\overline{\mathcal{A}}} = \mathcal{A}$

   (b) $\mathcal{A} \cup \overline{\mathcal{A}} = \mathcal{U}$

   (c) $\mathcal{A} \cap \overline{\mathcal{A}} = \emptyset$

   (d) $\overline{\mathcal{A} \cap \mathcal{B}} = \overline{\mathcal{A}} \cup \overline{\mathcal{B}}$

   (e) $\overline{\mathcal{A} \cup \mathcal{B}} = \overline{\mathcal{A}} \cap \overline{\mathcal{B}}$

   (f) $(\mathcal{A} \cap \mathcal{B}) \cup (\mathcal{A} \cap \overline{\mathcal{B}}) = \mathcal{A}$

   (g) $(\mathcal{A} \cup \mathcal{B}) \cap (\mathcal{A} \cup \overline{\mathcal{B}}) = \mathcal{A}$

   (h) $(\overline{\mathcal{A}} \cup \mathcal{B}) \cap \mathcal{A} = \mathcal{A} \cap \mathcal{B}$

3. Dokaži enakosti:

   (a) $\mathcal{A} - (\mathcal{B} \cup \mathcal{C}) = (\mathcal{A} - \mathcal{B}) \cap (\mathcal{A} - \mathcal{C})$

   (b) $\mathcal{A} - (\mathcal{B} \cap \mathcal{C}) = (\mathcal{A} - \mathcal{B}) \cup (\mathcal{A} - \mathcal{C})$

   (c) $\mathcal{A} - (\mathcal{A} - \mathcal{B}) = \mathcal{A} \cap \mathcal{B}$

   (d) $\mathcal{A} - \mathcal{B} = \mathcal{A} - (\mathcal{A} \cap \mathcal{B})$

   (e) $(\mathcal{A} - \mathcal{B}) - \mathcal{C} = (\mathcal{A} - \mathcal{C}) - (\mathcal{B} - \mathcal{C})$

   (f) $\mathcal{A} \cup \mathcal{B} = \mathcal{A} \cup (\mathcal{B} - \mathcal{A})$

   (g) $\mathcal{A} \cap (\mathcal{B} - \mathcal{A}) = \emptyset$

   (h) $(\mathcal{A} \cup \mathcal{B}) - \mathcal{C} = (\mathcal{A} - \mathcal{C}) \cup (\mathcal{B} - \mathcal{C})$

   (i) $\mathcal{A} - (\mathcal{B} - \mathcal{C}) = (\mathcal{A} - \mathcal{B}) \cup (\mathcal{A} \cap \mathcal{C})$

   (j) $\mathcal{A} - (\mathcal{B} \cup \mathcal{C}) = (\mathcal{A} - \mathcal{B}) - \mathcal{C}$

   (k) $\mathcal{A} \cap (\mathcal{B} - \mathcal{C}) = (\mathcal{A} \cap \mathcal{B}) - (\mathcal{A} \cap \mathcal{C}) = (\mathcal{A} \cap \mathcal{B}) - \mathcal{C}$

4. Naj bo $\mathcal{A} \triangle \mathcal{B} = (\mathcal{A} \cup \mathcal{B}) - (\mathcal{A} \cap \mathcal{B})$. Dokaži enakosti:

   (a) $\mathcal{A} \triangle \mathcal{B} = \mathcal{B} \triangle \mathcal{A}$

   (b) $\mathcal{A} \triangle (\mathcal{B} \triangle \mathcal{C}) = (\mathcal{A} \triangle \mathcal{B}) \triangle \mathcal{C}$

   (c) $\mathcal{A} \cap (\mathcal{B} \triangle \mathcal{C}) = (\mathcal{A} \cap \mathcal{B}) \triangle (\mathcal{A} \cap \mathcal{C})$

   (d) $\mathcal{A} \triangle (\mathcal{A} \triangle \mathcal{B}) = \mathcal{B}$

   (e) $\mathcal{A} \cup \mathcal{B} = \mathcal{A} \triangle \mathcal{B} \triangle (\mathcal{A} \cap \mathcal{B})$

   (f) $\mathcal{A} - \mathcal{B} = \mathcal{A} \triangle (\mathcal{A} \cap \mathcal{B})$

   (g) $\mathcal{A} \triangle \emptyset = \mathcal{A}$

   (h) $\mathcal{A} \triangle \mathcal{A} = \emptyset$

(i) $\mathcal{A} \triangle \mathcal{U} = \overline{\mathcal{A}}$

(j) $\mathcal{A} \cup \mathcal{B} = (\mathcal{A} \triangle \mathcal{B}) \cup (\mathcal{A} \cap \mathcal{B})$

5. Dokaži ekvivalence:

(a) $\mathcal{A} \cup \mathcal{B} \subseteq \mathcal{C} \iff \mathcal{A} \subseteq \mathcal{C} \wedge \mathcal{B} \subseteq \mathcal{C}$

(b) $\mathcal{A} \subseteq \mathcal{B} \cap \mathcal{C} \iff \mathcal{A} \subseteq \mathcal{B} \wedge \mathcal{A} \subseteq \mathcal{C}$

(c) $\mathcal{A} \cap \mathcal{B} \subseteq \mathcal{C} \iff \mathcal{A} \subseteq \overline{\mathcal{B}} \cup \mathcal{C}$

(d) $\mathcal{A} \subseteq \mathcal{B} \cup \mathcal{C} \iff \mathcal{A} \cap \overline{\mathcal{B}} \subseteq \mathcal{C}$

(e) $(\mathcal{A} - \mathcal{B}) \cup \mathcal{B} = \mathcal{A} \iff \mathcal{B} \subseteq \mathcal{A}$

(f) $(\mathcal{A} \cap \mathcal{B}) \cup \mathcal{C} = \mathcal{A} \cap (\mathcal{B} \cup \mathcal{C}) \iff \mathcal{C} \subseteq \mathcal{A}$

(g) $\mathcal{A} \subseteq \mathcal{B} \implies \mathcal{A} \cup \mathcal{C} \subseteq \mathcal{B} \cup \mathcal{C}$

(h) $\mathcal{A} \subseteq \mathcal{B} \implies \mathcal{A} \cap \mathcal{C} \subseteq \mathcal{B} \cap \mathcal{C}$

(i) $\mathcal{A} \subseteq \mathcal{B} \implies (\mathcal{A} - \mathcal{C}) \subseteq (\mathcal{B} - \mathcal{C})$

(j) $\mathcal{A} \subseteq \mathcal{B} \implies (\mathcal{C} - \mathcal{B}) \subseteq (\mathcal{C} - \mathcal{A})$

(k) $\mathcal{A} \subseteq \mathcal{B} \implies \overline{\mathcal{B}} \subseteq \overline{\mathcal{A}}$

(l) $\mathcal{A} \cup \mathcal{B} = \mathcal{A} \cap \mathcal{B} \implies \mathcal{A} = \mathcal{B}$

(m) $\mathcal{A} = \overline{\mathcal{B}} \iff \mathcal{A} \cap \mathcal{B} = \emptyset \wedge \mathcal{A} \cup \mathcal{B} = \mathcal{U}$

6. Dokaži:

(a) $\mathcal{A} \subseteq \mathcal{A}$ *refleksivnost*

(b) $\mathcal{A} \subseteq \mathcal{B} \wedge \mathcal{B} \subseteq \mathcal{C} \Rightarrow \mathcal{A} \subseteq C$ *tranzitivnost*

(c) $\mathcal{A} \cap \mathcal{B} \subseteq \mathcal{A} \subseteq \mathcal{A} \cup \mathcal{B}$

(d) $\mathcal{A} \cap \mathcal{B} \subseteq \mathcal{B} \subseteq \mathcal{A} \cup \mathcal{B}$

(e) $\mathcal{A} - \mathcal{B} \subseteq \mathcal{A}$

7. Dokaži:

(a) $\mathcal{A} \triangle \mathcal{B} = \emptyset \iff \mathcal{A} = \mathcal{B}$

(b) $\mathcal{A} \cap \mathcal{B} = \emptyset \implies \mathcal{A} \cup \mathcal{B} = \mathcal{A} \triangle \mathcal{B}$

(c) $\mathcal{A} \triangle \mathcal{B} = \mathcal{C} \iff \mathcal{B} \triangle \mathcal{C} = \mathcal{A} \iff \mathcal{C} \triangle \mathcal{A} = \mathcal{B}$

8. Izrazi operacije $\cap$, $\cup$ in $-$ z:

(a) $\triangle$, $\cap$

(b) $\triangle$, $\cup$

(c) $-$, $\triangle$

(d) $\downarrow$, kjer je $\mathcal{A} \downarrow \mathcal{B} = \overline{\mathcal{A}} \cap \overline{\mathcal{B}}$

9. Dokaži:

(a) $\emptyset \neq \{\emptyset\}$

(b) $\mathcal{A} \subseteq \mathcal{B} \iff \mathcal{A} \cup \mathcal{B} = \mathcal{B} \iff \mathcal{A} \cap \mathcal{B} = \mathcal{A} \iff \mathcal{A} - \mathcal{B} = \emptyset \iff \overline{\mathcal{A}} \cup \mathcal{B} = \mathcal{U}$

(c) $\{\{1,2\},\{2,3\}\} \neq \{1,2,3\}$

10. Dokaži, da za poljubno množico $\mathcal{A}$ velja:

(a) $\mathcal{A} \subseteq \emptyset \Rightarrow \mathcal{A} = \emptyset$

(b) $\mathcal{U} \subseteq \mathcal{A} \Rightarrow \mathcal{A} = \mathcal{U}$

(c) $\mathcal{A} \cup \emptyset = \mathcal{A}$

(d) $\mathcal{A} \cap \emptyset = \emptyset$

(e) $\mathcal{A} \cup \mathcal{U} = \mathcal{U}$

(f) $\mathcal{A} \cap \mathcal{U} = \mathcal{A}$

11. Poišči množico $\mathcal{X}$, za katero velja:

    (a) $(\mathcal{A} - \mathcal{X} = \mathcal{B}) \wedge (\mathcal{A} \cup \mathcal{X} = \mathcal{C}) \wedge (\mathcal{B} \subseteq \mathcal{A} \subseteq \mathcal{C})$

    (b) $(\mathcal{A} \cap \mathcal{X} = \mathcal{B}) \wedge (\mathcal{A} \cup \mathcal{X} = \mathcal{C}) \wedge (\mathcal{B} \subseteq \mathcal{A} \subseteq \mathcal{C})$

    (c) $(\mathcal{A} - \mathcal{X} = \mathcal{B}) \wedge (\mathcal{X} - \mathcal{A} = \mathcal{C}) \wedge (\mathcal{B} \subseteq \mathcal{A} \wedge \mathcal{A} \cap \mathcal{C} = \emptyset)$

12. Poišči največjo množico $\mathcal{X}$, za katero velja $\left((\overline{\mathcal{A}} \cup \mathcal{X}) \cap \mathcal{B} = \emptyset\right) \wedge (\mathcal{B} \subseteq \mathcal{A})$.

13. Za poljubno podmnožico $\mathcal{X}$ univerzalne množice $\mathcal{U}$, $\mathcal{X} \subset \mathcal{U}$ definiramo karakteristično funkcija te podmnožice takole:

$$\chi_{\mathcal{X}} : \mathcal{U} \to \{0, 1\},$$
$$\chi_{\mathcal{X}}(x) = \begin{cases} 1; & x \in \mathcal{X} \\ 0; & \text{sicer} \end{cases}$$

    Izrazi karakteristično funkcijo $\chi_{\mathcal{X}}$ s karakterističnimi funkcijami $\chi_{\mathcal{A}}$ in $\chi_{\mathcal{B}}$, oziroma $\chi_{\mathcal{C}}$.

    (a) $\mathcal{X} = \mathcal{A} \cap \mathcal{B}$

    (b) $\mathcal{X} = \mathcal{A} \cup \mathcal{B}$

    (c) $\mathcal{X} = \overline{\mathcal{A}}$

    (d) $\mathcal{X} = \mathcal{A} - \mathcal{B}$

    (e) $\mathcal{X} = \mathcal{A} \triangle \mathcal{B}$

    (f) $\mathcal{X} = (\mathcal{A} \cup \mathcal{B}) \cap \mathcal{C}$

    (g) $\mathcal{X} = \mathcal{A} - (\mathcal{A} - \mathcal{B})$

    (h) $\mathcal{X} = \mathcal{A} \cap \overline{\mathcal{A}} = \emptyset$

# 2 Realna števila in matematična indukcija

1. Dokaži, da iz aksiomov množice realnih števil sledi:

   (a) $a < b \Longrightarrow -a > -b$

   (b) kvadrat kateregakoli števila je nenegativen

   (c) za vsaki števili $a$ in $b$ velja $0 \leq a^2 - 2ab + b^2$

   (d) za vsaki števili $a$ in $b$ velja $2ab \leq a^2 + b^2$

2. Dokaži neenakosti:

   (a) $\frac{1}{a} \geq \frac{1}{b}$ če $b \geq a > 0$

   (b) $a^2 < b^2$ če $b > a > 0$

   (c) $\sqrt{a} < \sqrt{b}$ če $b > a > 0$

   (d) $a + a^{-1} \geq 2, \quad a > 0$

   (e) $\frac{a+b}{2} \geq \sqrt{ab}, \quad a, b > 0$

   (f) $a^2 \geq 2a - 1$

3. Katere od naslednjih neenakosti veljajo za vse $0 < a < b < 1$:

   (a) $ab > 1$

   (b) $\frac{1}{a} < \frac{1}{b}$

   (c) $\frac{1}{b} < \frac{1}{a}$

   (d) $\frac{1}{b} < 1$

   (e) $\frac{1}{a} > 1$

   (f) $a + b < 1$

   (g) $a + b > 1$

   (h) $a - b < 1$

   (i) $a^2 < 1$

   (j) $a^2 + b^2 < 1$

   (k) $a^2 + b^2 > 1$

   (l) $a^2 + b^2 > 2$

   (m) $a^2 + b^2 < 2$

   (n) $|a - b| < 1$

   (o) $|a - b| > 1$

   (p) $|b - a| > 1$

4. Dokaži:

   (a) $|a| = \sqrt{a^2}$

   (b) $|-a| = a \Rightarrow a \geq 0$

   (c) $-|a| \leq a \leq |a|$

   (d) $|ab| = |a||b|$

   (e) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

   (f) $|a + b| \leq |a| + |b|$

   (g) $|a| = 0 \iff a = 0$

   (h) $|a| - |b| \leq \big||a| - |b|\big| \leq |a - b|$

5. Dokaži:

   (a) $\max\{a, b\} = \frac{1}{2}(a + b + |a - b|)$

   (b) $\min\{a, b\} = \frac{1}{2}(a + b - |a - b|)$

   (c) če velja $(a > b)$ in $c > 0$, potem $c\,a > c\,b$

6. Poišči napako v naslednjih razmislekih:

(a) $-1 = \sqrt{-1} \cdot \sqrt{-1} = \sqrt{(-1)(-1)} = \sqrt{1} = 1$

(b) $a < -3 \Leftrightarrow a^2 < 9 \Leftrightarrow (-a)^2 < 9 \Leftrightarrow -a < 3 \Leftrightarrow a > -3.$

(c) $a^2 < 4 \Leftrightarrow a < 2 \Leftrightarrow -a > -2 \Leftrightarrow a^2 > 4$

(d) $\sqrt{a^2} > -1 \Leftrightarrow a^2 > 1 \Leftrightarrow a > 1$

(e) $\sqrt{a^2} > 1 \Leftrightarrow a^2 > 1 \Leftrightarrow a > 1$

(f) $b^2 - a^2 > 0 \Leftrightarrow a^2 < b^2 \Leftrightarrow a < b \Leftrightarrow b - a > 0 \Leftrightarrow (b-a)^2 > 0$

(g) $a > a - 1 \Leftrightarrow \dfrac{a}{a-1} > 1 \Leftrightarrow \dfrac{1}{1-\frac{1}{a}} > 1 \Leftrightarrow 1 > 1 - \dfrac{1}{a} \Leftrightarrow 0 > -\dfrac{1}{a} \Leftrightarrow 0 > -1$

(h) $a > a - 1 \Leftrightarrow 1 > 1 - \dfrac{1}{a} \Leftrightarrow a > 0$

(i) $a < 1 \Leftrightarrow \pm\sqrt{a} < 1$

7. Določi podmnožice realnih števil:

(a) $\{x : \ 6\,x - 1 > 2\,x + 3\}$

(b) $\{x : \ 2\,x + 3 \leq 3\,x + 4\}$

(c) $\{x : \ (x-3)^2 - 2\,x\,(x-5) > x\,(7-x)\}$

(d) $\{x : \ 3\,x + 6 \geq 0 \wedge 2x - 6 < 0\}$

(e) $\{x : \ 2\,x < x + 1 < 2\,x - 1\}$

(f) $\{x : \ x - 5 < 2\,x - 3 \leq x + 2\}$

(g) $\{x : \ x^2 < 4\}$

(h) $\{x : \ x\,(x-5) < -6\}$

(i) $\{x : \ \dfrac{x+1}{x-1} > 0\}$

(j) $\{x : \ x + \dfrac{1}{x} \leq 2\}$

8. Določi podmnožice realnih števil:

(a) $\{x : \ x + |x+1| = 3\}$

(b) $\{x : \ |x+1| + |x-1| = 2\}$

(c) $\{x : \ |x+2| = |x| + 2\}$

(d) $\{x : \ |x| + x < |x+2|\}$

(e) $\{x : \ |x-1| > |x|\}$

(f) $\{x : \ \big|2\,|x| - 4\big| < 2\}$

(g) $\{x : \ |-x^2 + 2x - 3| = 1\}$

(h) $\{x : \ \left|\dfrac{2x-1}{x+1}\right| = 1\}$

(i) $\{x : \ x^2 + 2|x+3| - 2 \leq 0\}$

(j) $\{x : \ |x^2 - 7\,x + 12| > x^2 - 7\,x + 12\}$

(k) $\{x : \ |x+2| - |x| > 1\}$

(l) $\{x : \ \left|\dfrac{x}{x+4}\right| < 1\}$

(m) $\{x : \ |2\,x - 3| \leq |4\,x - 3|\}$

(n) $\{x : \ |x^3 - x^2| < |x^2 + x|\}$

9. Določi podmnožice realnih števil:

   (a) $\{x:\ a\,(x+2) < x + 2\,a^2\,;\, a \in \mathbb{R}\}$

   (b) $\{x:\ a\,(x+2\,b^2) > (x+2\,a^2)b\,;\, a,\, b \in \mathbb{R}\}$

   (c) $\{x:\ x\,(a^2-6) + 2 > a\,(1-x)\,;\, a \in \mathbb{R}\}$

   (d) $\{x:\ |x-a| + |x+a| = 4\,a \,\wedge\, a > 0\}$

10. Določi podmnožice realne ravnine:

    (a) $\{(x,y):\ x + 7\,y + 14 \leq 0\}$

    (b) $\{(x,y):\ -\dfrac{x}{2} + y - 1 > 0\}$

    (c) $\{(x,y):\ 2\,y - 3 < 0\}$

    (d) $\{(x,y):\ 2\,x - y \leq 4 \wedge -4\,x + 2\,y \leq 5\}$

    (e) $\{(x,y):\ x - 2\,y + 4 \geq 0 \wedge 2\,x + 3\,y + 6 \geq 0 \wedge y - 2 \leq 2 \wedge y + 2 \geq 0 \wedge x - 5 \leq 0\}$

    (f) $\{(x,y):\ x + 2\,y - 6 \geq 0 \vee 2\,x + y - 6 \leq 0\}$

11. Določi podmnožice realne ravnine:

    (a) $\{(x,y):\ y \leq |x+3| - |x-2|\}$

    (b) $\{(x,y):\ y \geq x + |x|\}$

    (c) $\{(x,y):\ |x| + |y| \leq 1\}$

    (d) $\{(x,y):\ \big|2\,y + |x|\big| + \big|2\,|y| + x\big| < 8\}$

    (e) $\{(x,y):\ |x-y| \geq |x + 2\,y|\}$

    (f) $\{(x,y):\ |x+2| \geq |y-2|\}$

    (g) $\{(x,y):\ |x| - |y| < 2\}$

    (h) $\{(x,y):\ \Big||x| - |y|\Big| < 3\}$

12. Določi podmnožice realne ravnine:

    (a) $\{(x,y):\ 1 \leq x^2 + y^2 \leq 4\}$

    (b) $\{(x,y):\ 5\,x\,y + 5\,x + y^2 + y \geq 0\}$

    (c) $\{(x,y):\ x^2 + 2\,x\,y + y^2 \leq 4\}$

    (d) $\{(x,y):\ (x^2 - y^2)\,(y+2) \geq 0\}$

    (e) $\{(x,y):\ y^2 \leq 2\,x + 1\}$

    (f) $\{(x,y):\ \max\{|x|, |y|\} \leq 1$

13. Reši naslednje enačbe in neenačbe:

    (a) $\dfrac{(1+x^2)}{(1-x^2)} \leq 1$

    (b) $\sqrt{x} + \sqrt{x+1} > 3$

    (c) $\left|\dfrac{x}{x+4}\right| < 1$

    (d) $|x^2 - x| - |x| < 1$

    (e) $\sqrt{5\,x+1} - \sqrt{2\,x+3} = \sqrt{7\,x-20}$

    (f) $\sqrt{x + \sqrt{2\,x-1}} + \sqrt{x - \sqrt{2\,x-1}} = \sqrt{2}$

    (g) $\left|1 + \sqrt{4-x^2}\right| + \left|1 - \sqrt{4-x^2}\right| = 2$

    (h) $\sqrt{19-x} - \sqrt{x+1} > 2$

    (i) $\sqrt{-x} < \sqrt{\sqrt{x+2} + 2}$

    (j) $ax - b > 3 - 2x$

(k) $x^2 + ax + 1 > 0$

(l) $\dfrac{(x^2 + a\,x + 1)}{(x^2 + x + 1)} < 5$

(m) $x^2 - 2(4\,a - 1)x + 15\,a^2 - 2\,a - 7 > 0$

(n) $\sqrt{3\,x^2 - 7\,x + 3} = 1 - x$

14. Dokaži z matematično indukcijo:

(a) $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + n(n+1) = \dfrac{n(n+1)(n+2)}{3}$

(b) $1 + 2 + 2^2 + \ldots + 2^{n-1} = 2^n - 1$

(c) $1 - 2 + 3 - \ldots + (-1)^{n-1} n = \dfrac{(1 + (-1)^{n-1}(2n+1))}{4}$

(d) $\dfrac{1}{(1 \cdot 3)} + \dfrac{1}{(3 \cdot 5)} + \ldots + \dfrac{1}{((2n-1)(2n+1)} = \dfrac{n}{(2n+1)}$

(e) $1^2 + 2^2 + 3^2 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

(f) $1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\dfrac{n(n+1)(2n+1)}{6}\right)^2$

(g) $1 + q + q^2 + \ldots + q^{n-1} = \dfrac{q^n - 1}{q - 1},$ če je $q \neq 1$

(h) $1^2 - 2^2 + 3^2 - \ldots + (-1)^{n-1} n^2 = (-1)^{n-1} \dfrac{n(n+1)}{2}$

(i) $1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \ldots + n \cdot 2^n = (2n-2)2^n + 2$

(j) $\cos(n \cdot \pi) = (-1)^n$

(k) $(\cos x + i \sin x)^n = \cos(nx) + i \sin(nx)$

(l) $\dfrac{1}{\sqrt{1}} + \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{3}} + \ldots + \dfrac{1}{\sqrt{n}} \geq \sqrt{n}$

(m) $n < 2^n$

(n) $n^2 < 2^n,$ če je $n \geq 5$

(o) $(1 + \beta)^n \geq 1 + n\beta, \quad$ za $\quad \beta > -1$

(p) $\max\{x_1, .., x_n\} = \max\{\max\{x_1, .., x_{n-1}\}, x_n\}$

(q) $\min\{x_1, .., x_n\} = \min\{\min\{x_1, .., x_{n-1}\}, x_n\}$

(r) če je $(\sum_{i=1}^{n} x_i = 0) \wedge (x_i \geq 0 \;\; i = 1..n), \;\Rightarrow\; (x_i = 0, \;\; i = 1..n)$

(s) vsota kubov treh zaporednih naravnih števil je deljiva z 9

(t) vsota $11^{n+1} + 12^{2n-1}$ je deljiva s 133

(u) končna množica z $n$ elementi ima $2^n$ različnih podmnožic

(v) vseh različnih razporedb $n$ različnih elementov je $n!$

(w) za vsak $n \geq 3$ je izraz $n^5 - 5n^3 + 4n$ deljiv s 120

(x) za vsako končno zaporedje $n$ števil $a_1, \ldots, a_n$ velja
$|a_1 + a_2 + \ldots + a_n| \leq |a_1| + \ldots + |a_n|$

15. Določi število diagonal v $n$-kotniku, kjer je $(n > 3)$, ter dobljeno formulo dokaži z matematično indukcijo.

16. Dokaži, da $\sqrt{2}$ ni racionalno število.

# 3 Kompleksna števila

1. Izračunaj:

   (a) $(1+2\,i)^3$

   (b) $(1+i)^4$

   (c) $\dfrac{5}{-3+4\,i}$

   (d) $\left(\dfrac{1+i}{3-2\,i}\right)^2$

   (e) $\dfrac{2-3\,i}{3-i}-\dfrac{4+i}{3+i}$

   (f) $(1+i)^2+\dfrac{2}{1+i}-i^{15}$

   (g) $\left(-\dfrac{1}{2}+\dfrac{i}{2}\right)^8$

   (h) $\dfrac{(1+i)^5}{(1-i)^3}$

   (i) $\left(\dfrac{4}{-1+i\sqrt{3}}\right)^6$

   (j) $\left(\dfrac{1+i\sqrt{3}}{1-i}\right)^{20}$

   (k) $\left(\dfrac{1+i\sqrt{3}}{2}\right)^6+\left(\dfrac{1-i\sqrt{3}}{2}\right)^3$

   (l) $\left(\dfrac{-\sqrt{2}+i\sqrt{2}}{\sqrt{3}+i}\right)^6$

   (m) $\dfrac{y+i\,x}{x+i\,y}+\dfrac{x+i\,y}{x-i\,y}$

   (n) $\dfrac{\sqrt{1+x^2}+i\,x}{x-i\sqrt{1+x^2}}$

2. Določi $\mathrm{Re}(w)$, $\mathrm{Im}(w)$, $|w|$ in $\overline{w}$ ($z=x+i\,y$):

   (a) $w=(1-i)\,(1+3\,i)$

   (b) $w=(1-3\,i)^2$

   (c) $w=\dfrac{1}{i}$

   (d) $w=\dfrac{1}{(1+2\,i)^2}$

   (e) $w=\dfrac{1+i}{i}$

   (f) $w=\dfrac{2}{1+i}$

   (g) $w=\dfrac{(3+i)\,(1+i)}{2-i}$

   (h) $w=\dfrac{1-3\,i}{1+i}-\dfrac{i}{i+2}$

   (i) $w=\left(1+i\sqrt{3}\right)^3$

   (j) $w=|z|$

   (k) $w=i\,z$

   (l) $w=z^2$

   (m) $w=\overline{z}^2$

   (n) $w=\dfrac{1}{z}$

   (o) $w=\dfrac{|z|}{z}$

   (p) $w=z\,|z|$

   (q) $w=z\,\overline{z}$

   (r) $w=\dfrac{\overline{z}}{z}$

3. Določi $\mathrm{Re}(w)$ in $\mathrm{Im}(w)$ ($z=x+i\,y$):

   (a) $w=\dfrac{z+\overline{z}}{2}$

   (b) $w=\dfrac{z-\overline{z}}{2}$

   (c) $w=(z+2\,i)\,\overline{(z+2\,i)}$

   (d) $w=z\,(1+z)$

   (e) $w=\dfrac{z}{z+1}$

   (f) $w=z+\dfrac{1}{z}$

   (g) $w=z^2+\dfrac{1}{z^2}$

   (h) $w=z^2\,\overline{z}$

   (i) $w=z\,\overline{z}^2$

   (j) $w=z^3$

   (k) $w=z^4$

   (l) $w=z^3-i\,z$

4. Zapiši kompleksna števila v polarni obliki:

(a) $z = -2$

(b) $z = 2\,i$

(c) $z = -\sqrt{2} + \sqrt{2}\,i$

(d) $z = 2 + 2\,i$

(e) $z = -i$

(f) $z = -1 + \sqrt{3}\,i$

(g) $z = -2 + 2\,\sqrt{3}\,i$

(h) $z = -1 - \sqrt{3}\,i$

(i) $z = -5 - 5\,i$

(j) $z = 3 - \sqrt{3}\,i$

(k) $z = 4 + 3\,i$

(l) $z = -7 - i$

(m) $z = -5 + 3\,i$

(n) $z = \dfrac{1-i}{1+i}$

5. Poišči vse rešitve enačb:

(a) $2\,z^2 - 3\,\overline{z}^2 = 10\,i$

(b) $\overline{z} - i\,z^2 = 0$

(c) $z^3 - i\,z = 0$

(d) $\dfrac{\overline{z}^2 + 1}{z+1} = 1$

(e) $i + \dfrac{1}{z} = 2$

(f) $z^2 - \dfrac{1}{\overline{z}} = 0$

(g) $\dfrac{1}{z-i} + \dfrac{2+i}{1+i} = 2$

(h) $(2\,z + 3\,\overline{z})\;(2\,\overline{z} + 3\,z) = -1$

(i) $z\,\overline{z} + 1 = 0$

(j) $|z| + z = 2 + i$

(k) $z^2 = i$

(l) $z^2 = 3 + 4\,i$

(m) $z^2 + (2\,i - 3)\;z + 5 - i = 0$

(n) $z^2 = 2 - 2\,i\,\sqrt{3}$

6. Poišči vse rešitve enačb:

(a) $z^3 + 1 = 0$

(b) $z^3 - 1 = 0$

(c) $z^3 = -1 + i$

(d) $z^3 = -2 + 2\,i$

(e) $(z+1)^3 = -i$

(f) $z^3 - \dfrac{1+i}{1-i} = 0$

(g) $z^4 + 1 = 0$

(h) $z^4 - 16 = 0$

(i) $z^4 = -8 + 8\,i\,\sqrt{3}$

(j) $\left(1 + i\,\sqrt{3}\right)\;z^4 + 32 = 0$

(k) $z^4 + 18\,z^2 + 81 = 0$

(l) $z^4 + 6\,z^3 + 9\,z^2 + 100 = 0$

(m) $z^6 - 1 = 0$

(n) $z^8 - 1 = 0$

(o) $z^8 - z^2 = 0$

(p) $2\,z^3 = \left(-1 - i\,\sqrt{3}\right)^6$

7. Poišči vse rešitve sistema enačb:

(a) $\left|\dfrac{z}{z+1}\right| = 1,$ $\qquad \dfrac{z}{\overline{z}} = i$

(b) $|z| - |1 - z| = 0,$ $\qquad |z| - \dfrac{1}{|z|} = 0$

(c) $(1+i)\ z+(1-i)\ \overline{z}+3=0$, $\quad i\,z-i\,\overline{z}-1=0$

(d) $z\,\overline{z}+(2+i)\ z+(2-i)\ \overline{z}+4=0$, $\quad (1-i)\ z+(1+i)\ \overline{z}+4=0$

(e) $|z-2|=3$, $\quad |z+1|=3$

(f) $z\,\overline{z}+(2+i)\ z+(2-i)\ \overline{z}+4=0$, $\quad z\,\overline{z}+(1-i)\ z+(1+i)\ \overline{z}=0$

(g) $(2+i)z_1+(2-i)z_2=6$, $\quad (3+2\,i)z_1+(3-2\,i)z_2=8$

(h) $(3-i)z_1+(4+2\,i)z_2=1+3\,i$, $\quad (4+2\,i)z_1-(2-i)z_2=3$

(i) $z_1\overline{z_2}=2\sqrt{2}$, $\quad \dfrac{z_1}{\overline{z_2}}=i\sqrt{2}$

8. Določi in nariši podmnožice kompleksne ravnine:

(a) $\Re(z)+\Im(z^2)=2$

(b) $\Im(\dfrac{1}{z})=1$

(c) $z\,\overline{z}+(1-i)\ z+(1+i)\ \overline{z}=4$

(d) $2\,z\,\overline{z}+(2+i)\ z+(2-i)\ \overline{z}=2$

(e) $(1+i)\ z+(1-i)\ \overline{z}+4=0$

(f) $\Re(z^2+\overline{z})=0$

(g) $3\,\Re(z)^2-\Im(z-i)^2=0$

(h) $|z|+\Re(z)=1$

(i) $|z-1|-|z-i|=0$

(j) $|z-1|+|z+1|=3$

(k) $|z-1|+|z-i|=3$

(l) $\left|\dfrac{z+1}{z-1}\right|=4$

9. Določi eno od linearnih funkcij kompleksne spremenljivke $w=a\,z+b$, ki preslika območje $\mathcal{Z}$ ravnine $(z)$ v območje $\mathcal{W}$ ravnine $(w)$.

(a) $\mathcal{Z}=\{z: z=\lambda_1+i\,\lambda_2;\ \lambda_1,\lambda_2>0\ \wedge\ \lambda_1+\lambda_2<1\}$
$\mathcal{W}=\{w: w=2\,\mu_1+(i+1)\,\mu_2;\ \mu_1,\mu_2>0\ \wedge\ \mu_1+\mu_2<1\}$

(b) $\mathcal{Z}=\{z: z=\lambda_1+i\,\lambda_2+(i+1)\,\lambda_3;\ \lambda_1,\lambda_2,\lambda_3>0\ \wedge\ \lambda_1+\lambda_2+\lambda_3<1\}$
$\mathcal{W}=\{w: w=\mu_1+i\,\mu_2-\mu_3-i\,\mu_4;\ \mu_1,\mu_2,\mu_3,\mu_4>0\ \wedge\ \mu_1+\mu_2+\mu_3+\mu_4=1\}$

(c) $\mathcal{Z}=\{z: |z+1-i|<3\}$
$\mathcal{W}=\{w: |w|<1\}$

(d) $\mathcal{Z}=\{z: \Re z>0\ \wedge\ \Im(z)<0\}$
$\mathcal{W}=\{w: \Re w>1\ \wedge\ \Im(z)<-1\}$

(e) $\mathcal{Z}=\{z: z=\lambda_1+i\,\lambda_2;\ \lambda_1+\lambda_2=1\}$
$\mathcal{W}=\{w: w=2\,\mu_1+(i+1)\,\mu_2;\ \mu_1+\mu_2=1\}$

(f) $\mathcal{Z} = \{z : z = \lambda_1 + i\,\lambda_2;\ \lambda_1 + \lambda_2 > 1\}$
$\mathcal{W} = \{w : w = 2\,\mu_1 + (i+1)\,\mu_2;\ \mu_1 + \mu_2 < 2\}$

10. Določi in nariši podmnožice kompleksne ravnine:

(a) $0 \le \Im(z) \le 1$

(b) $\Im(z^2) \le 2$

(c) $\frac{1}{4} \le \Re(\frac{1}{\overline{z}}) + \Im(\frac{1}{\overline{z}}) \le \frac{1}{2}$

(d) $z\,\overline{z} - i\,\overline{z} + i\,z - 8 < 0$

(e) $1 \le |z - 1 - 2\,i| \le 2$

(f) $\frac{\pi}{4} \le \arg(z) \le \frac{3\pi}{4}$

(g) $|z+1| < |z+i|$

(h) $|z| \ge 1 - \Re(z)$

11. Nariši krivuljo, ki jo v kompleksni ravnini opiše točka $z$, ko parameter $t$ preteče vsa realna števila:

(a) $z = (2-i)\ t$

(b) $z = -i + t + 3\,i\,t^2$

(c) $z = \frac{1+i\,t}{1-i\,t}$

(d) $z = e^{-i\,t} + 2\,e^{i\,t}$

(e) $z = 1 + i + e^{i\,t}$

(f) $z = 2 + i + e^{i\,t}\,t$

12. Določi pare družin krivulj $\Re(w) = a$ in $\Im(w) = b$ v kompleksni ravnini:

(a) $w = z$

(b) $w = (i+1)\ z$

(c) $w = \left(i + \frac{1}{2}\right)\ z$

(d) $w = z^2$

(e) $w = z + \left(\frac{1}{2} + i\right)\ z^2$

(f) $w = \frac{1}{z}$

(g) $w = \frac{i + 2\,z}{1 - z}$

(h) $w = \frac{1}{z^2}$

(i) $w = z + \frac{1}{z}$

(j) $w = e^z$

# 4 Zaporedja

1. Določi monotonost zaporedij:

(a) $a_n = \dfrac{1}{n+1}$

(b) $a_n = \dfrac{n^2}{2^n}$

(c) $a_n = \left(1 + \dfrac{1}{n}\right)^n$

(d) $a_n = \dfrac{1+(-1)^n}{2}$

(e) $a_n = \sin(\pi\, n)$

(f) $a_n = \dfrac{\sqrt{n}}{100+n}$

(g) $a_n = n^2 \left(\dfrac{9}{10}\right)^n$

(h) $a_n = -n^2 + 9n + 100$

(i) $a_n = \dfrac{3^n}{n!}$

(j) $a_n = n^2$

(k) $a_n = n^2 - 20\, n + 100$

(l) $a_n = \dfrac{n-1}{n+1}$

(m) $a_n = \dfrac{n+2}{n}$

(n) $a_n = n \left(\dfrac{95}{100}\right)^n$

(o) $a_n = \ln n$

(p) $a_n = e^{-n}$

(q) $a_n = \sqrt{n}$

(r) $a_n = \sin n$

2. Določi največji in najmanjši člen zaporedja, če obstaja:

(a) $a_n = \dfrac{n}{n+1}$

(b) $a_n = \dfrac{(-1)^n}{n}$

(c) $a_n = \dfrac{n+2}{n}$

(d) $a_n = \dfrac{2^n}{n!}$

(e) $a_n = \dfrac{1000^n}{n!}$

(f) $a_n = \dfrac{n}{1+\left(\frac{n}{10}\right)^2}$

(g) $a_n = \dfrac{2n-1}{3n+1}$

(h) $a_n = \dfrac{n^2-2n-1}{n^2+3n+5}$

(i) $a_n = n^2 - 9n - 1$

(j) $a_n = n + \dfrac{100}{n}$

(k) $a_n = n^{(-1)^{n+1}}$

(l) $a_n = 3 - \dfrac{1}{n}$

(m) $a_n = \dfrac{1000^n}{n}$

(n) $a_n = (n-1)(-1)^n$

(o) $a_n = \sin(n\,\pi)$

(p) $a_n = \sin n$

(q) $a_n = \dfrac{(-1)^n}{1+n^2}$

(r) $a_n = n \left(\dfrac{95}{100}\right)^n$

(s) $a_n = \sin \dfrac{1}{n}$

(t) $a_n = \sin \dfrac{n\pi}{2}$

3. Dokaži:

(a) $\lim\limits_{n\to\infty} a = a$

(b) $\lim\limits_{n\to\infty} \dfrac{1}{n} = 0$

(c) $\lim\limits_{n\to\infty} \dfrac{n}{1+n} = 1$

(d) $\lim\limits_{n\to\infty} \dfrac{1}{2^n} = 0$

(e) $\lim\limits_{n\to\infty} \dfrac{1}{n!} = 0$

(f) $\lim\limits_{n\to\infty} n^{-n} = 0$

(g) $\lim\limits_{n\to\infty} \dfrac{n}{2^n} = 0$

(h) $\lim\limits_{n\to\infty} \dfrac{2^n}{n!} = 0$

(i) $\lim\limits_{n\to\infty} \dfrac{n^k}{2^n} = 0$

(j) $\lim\limits_{n\to\infty} \sqrt[n]{2} = 1$

(k) $\lim\limits_{n\to\infty} \sqrt[n]{n} = 1$

(l) $\lim\limits_{n\to\infty} \sqrt[n]{1+a^n} = a, \quad a \geq 1$

(m) $\lim\limits_{n\to\infty} \dfrac{\ln n}{n} = 0$

(n) $\lim\limits_{n\to\infty} \dfrac{\sin n}{n} = 1$

4. Dokaži:

(a) $\lim\limits_{n\to\infty} q^n = 0,\ |q| < 1$

(b) $\lim\limits_{n\to\infty} n\,q^n = 0,\ |q| < 1$

(c) $\lim\limits_{n\to\infty} n^k\,q^n = 0,\ |q| < 1$

(d) $\lim\limits_{n\to\infty} \sqrt[n]{q} = 1,\ q > 0$

5. Izračunaj limito zaporedja, če obstaja:

(a) $\lim\limits_{n\to\infty} \dfrac{1-n}{1+n}$

(b) $\lim\limits_{n\to\infty} \dfrac{1-n}{1+n^2}$

(c) $\lim\limits_{n\to\infty} \dfrac{\sqrt{n+1}}{2+n}$

(d) $\lim\limits_{n\to\infty} \dfrac{\sqrt{n^2+1}}{1+2\,n}$

(e) $\lim\limits_{n\to\infty} \sqrt{n+1} - \sqrt{n}$

(f) $\lim\limits_{n\to\infty} (\sqrt{n+1} - \sqrt{n})\sqrt{n-1}$

(g) $\lim\limits_{n\to\infty} \sqrt[3]{n+1} - \sqrt[3]{n}$

(h) $\lim\limits_{n\to\infty} (\sqrt[3]{n+1} - \sqrt[3]{n})\sqrt[3]{1+n^2}$

(i) $\lim\limits_{n\to\infty} \dfrac{2^{n+1}+3^{n+1}}{2^n+3^n}$

(j) $\lim\limits_{n\to\infty} \dfrac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}$

(k) $\lim\limits_{n\to\infty} \sqrt{n}\,(\sqrt{n^3+n+1} - \sqrt{n^3})$

(l) $\lim\limits_{n\to\infty} n\,\big(\ln(n+1) - \ln n\big)$

(m) $\lim\limits_{n\to\infty} \Big(1+\dfrac{1}{n}\Big)^{n+6}$

(n) $\lim\limits_{n\to\infty} \Big(\dfrac{n}{n+1}\Big)^{n}$

(o) $\lim\limits_{n\to\infty} \Big(\dfrac{n^2+2\,n}{n^2+1}\Big)^{n}$

(p) $\lim\limits_{n\to\infty} \Big(1+\dfrac{1}{n^2}\Big)^{n}$

(q) $\lim\limits_{n\to\infty} \Big(\dfrac{2\,n^2+6}{2\,n^2+5}\Big)^{4\,n^2+3}$

(r) $\lim\limits_{n\to\infty} \Big(\dfrac{n^2+4\,n}{n^2-n+1}\Big)^{n}$

6. Izračunaj limito zaporedja, če obstaja:

(a) $\lim\limits_{n\to\infty} \Big(\dfrac{1}{n^2}\sum\limits_{k=1}^{n}(k-1)\Big)$

(b) $\lim\limits_{n\to\infty} \Big|\dfrac{1}{n}\sum\limits_{k=1}^{n} k\,(-1)^{k-1}\Big|$

(c) $\lim\limits_{n\to\infty} \Big(\dfrac{1}{n^3}\sum\limits_{k=1}^{n}(k-1)^2\Big)$

(d) $\lim\limits_{n\to\infty} \Big(\dfrac{1}{n^3}\sum\limits_{k=1}^{n}(2\,k-1)^2\Big)$

(e) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \dfrac{2\,k-1}{2^k}$

(f) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \dfrac{1}{k\,(k+1)}$

(g) $\lim\limits_{n\to\infty} \Big(\dfrac{1}{n^3}\sum\limits_{k=1}^{n} k\,(k+1)\Big)$

(h) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \dfrac{1}{k}$

7. Ugotovi, od katerega člena dalje, se vsi členi zaporedja $a_n$ razlikujejo od limite za manj kot $\varepsilon$:

(a) $a_n = \dfrac{n^2+n}{n^2+1}$ $\quad \varepsilon = \dfrac{1}{10}$

(b) $a_n = 0,\underbrace{11\ldots1}_{n\ enic}$ $\quad \varepsilon = \dfrac{1}{10^6}$

(c) $a_n = \dfrac{5^n-1}{5^n+2}$ $\quad \varepsilon = 25^{-25}$

(d) $a_n = \dfrac{3^n-9}{3^n+9}$ $\quad \varepsilon = 3^{-50}$

(e) $a_n = \sqrt[n]{c},\ c>1$ $\quad \varepsilon > 0$

(f) $a_n = \dfrac{n^2+(-1)^n 2n}{n^2-2n+3}$ $\quad \varepsilon = \dfrac{1}{5}$

8. Določi stekališča zaporedja:

(a) $\{\frac{1}{2},\ \frac{1}{2},\ \frac{1}{4},\ \frac{3}{4},\ \frac{1}{8},\ \frac{7}{8},\ \ldots\}$

(b) $a_n = (-1)^n\left(2-\dfrac{1}{n}\right)$

(c) $a_n = n^{(-1)^n}$

(d) $a_n = 1+\dfrac{n}{n+1}\cos\dfrac{n\pi}{2}$

(e) $a_n = \cos^n\dfrac{2n\pi}{3}$

(f) $\{\frac{1}{2},\ \frac{1}{3},\ \frac{2}{3},\ \frac{1}{4},\ \frac{2}{4},\ \frac{3}{4},\ \frac{1}{5},\ \ldots\}$

(g) $a_n = \dfrac{n-1}{n+1}\sin^2\dfrac{n\pi}{4}$

(h) $a_n = (-1)^n\dfrac{n-1}{n+1}$

(i) $a_n = \sin\dfrac{n\,\pi}{3}$

(j) $a_n = \sin\dfrac{n\,\pi}{3}\cos\dfrac{n\,\pi}{4}$

9. Določi konvergenco zaporedja:

(a) $a_n = \dfrac{2^n}{n!}$

(b) $a_n = \dfrac{3^n}{n}$

(c) $a_n = \dfrac{(-1)^n n}{n-5}$

(d) $a_n = \dfrac{(-1)^n n}{n^2+1}$

(e) $a_n = n^{(-1)^n}$

(f) $a_n = \dfrac{n+\cos(n\pi)}{n+2}$

(g) $a_n = \displaystyle\sum_{k=1}^{n}\frac{1}{2\,k}$

(h) $a_n = \displaystyle\sum_{k=1}^{n}\frac{1}{2^k}$

(i) $a_n = \sqrt[n]{n}$

(j) $a_n = \sin n$

(k) $a_n = \sin\dfrac{1}{n}$

(l) $a_n = \sin(n\,\pi)$

(m) $a_n = \sin\dfrac{n\,\pi}{2}$

(n) $a_n = \dfrac{\sin n}{n}$

(o) $a_n = \cos(n\,\pi)$

(p) $a_n = \cos(n\,\pi)\,\sin\dfrac{1}{n}$

10. Dokaži, da je zaporedje monotono in omejeno, ter izračunaj limito:

(a) $a_1 = 1$ $\quad a_{n+1} = \dfrac{a_n}{2}+1$

(b) $a_1 = 3$ $\quad a_{n+1} = 3-\dfrac{2}{a_n}$

(c) $a_1 = \sqrt{c}$ $\quad a_{n+1} = \sqrt{c+a_n}\quad c>0$

(d) $a_1 = \sqrt{2}$ $\quad a_{n+1} = \sqrt{2a_n}$

(e) $a_1 = 1$ $\qquad a_{n+1} = \frac{1}{2}\left(a_n + \frac{3}{a_n}\right)$

11. Izračunaj limito zaporedja:

(a) $a_n = 1 + \underbrace{\sqrt{1+\sqrt{1+\cdots+\sqrt{2}}}}_{n\ korenov}$

(b) $a_n = \underbrace{1 + \cfrac{1}{1+\cfrac{1}{1+\cdots\frac{1}{2}}}}_{n\ krat\ 1+}$

12. Obravnavaj konvergenco rekurzivno podanega zaporedja $\{x_n\}_{n\in\mathbb{N}}$ v odvisnosti od vrednosti prvega člena $x_1$.

(a) $x_{n+1} = 2\,x_n(1-x_n)$

(b) $x_{n+1} = \frac{1}{4}(2\,x_n^3 + 5x_n^2 - 3)$

(c) $x_{n+1} = \frac{11\,x_n^2+1}{6(x_n^2+1)}$

(d) $x_{n+1} = \frac{4\,x_n^2 + 5\,x_n - 2}{3}$

13. Pri katerih pozitivnih $\alpha$ zaporedje konvergira?

(a) $x_{n+1} = \alpha\,x_n(1-x_n), \quad x_1 = \frac{1}{2}$

(b) $x_{n+1} = \frac{1}{\alpha}(x_n^2+1), \quad x_1 = 0$

14. Podani sta števili $a$ in $b$, kjer je $a < b$. Definirajmo zaporedji $a_n$ in $b_n$ s predpisi:

$$a_1 = a,\ b_1 = b,\ a_{n+1} = \sqrt{a_n b_n},\ b_{n+1} = \frac{a_n + b_n}{2}.$$

Dokaži:

(a) za vsak $n$ velja $a_n \le b_n$

(b) zaporedje $a_n$ je naraščajoče, zaporedje $b_n$ pa padajoče

(c) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n$.
To limito imenujemo aritmetično-geometrična sredina števil $a$ in $b$.

15. Določi natančno zgornjo in natančno spodnjo mejo zaporedja $\{a_n\}_{n\in\mathbb{N}}$:

(a) $a_n = 1 + (-1)^n 2^{-n}$

(b) $a_n = \frac{1+2\,(-1)^n}{n}$

(c) $a_n = 1 + n\sin\left(\frac{n\pi}{2}\right)$

(d) $a_n = n^2\left(\frac{9}{10}\right)^n$

(e) $a_n = (-1)^n\frac{3n+1}{n+3}$

(f) $a_n = \frac{n^2}{n^2+4}$

(g) $a_n = n^2 2^{-n}$

(h) $a_n = (-1)^n\frac{n+1}{n}$

# 5 Številske vrste

- Vrsta $\sum_{n=1}^{\infty} a_n$ *konvergira*, če je konvergira zaporedje delnih vsot $S_N = \sum_{n=1}^{N} a_n$.
- Vsota $S = \sum_{n=1}^{\infty} a_n$ je limita $\lim_{N\to\infty} S_N$.
- Vsota *geometrijske vrste* $\sum_{n=1}^{\infty} a\, q^{n-1} = \frac{a}{1-q}\,, \quad |q| < 1$
- Vrsta *absolutno konvergira*, če konvergira vrsta $\sum_{n=1}^{\infty} |a_n|$.
- Vrsta *pogojno konvergira*, če konvergira, ne konvergira pa absolutno.
- Če vrsta $\sum_{n=1}^{\infty} a_n$ konvergira, je $\lim_{n\to\infty} a_n = 0$

**Kriteriji za konvergenco vrst s pozitivnimi členi**

1. *Prvi primerjalni kriterij*

   Če je $0 \le a_n \le b_n$ za vsak $n$ večji od nekega $n_0$, potem

   (a) vrsta $\sum_{n=1}^{\infty} a_n$ konvergira, če konvergira vrsta $\sum_{n=1}^{\infty} b_n$

   (b) vrsta $\sum_{n=1}^{\infty} b_n$ divergira, če divergira vrsta $\sum_{n=1}^{\infty} a_n$

2. *Drugi primerjalni kriterij*

   Če $\lim_{n\to\infty} \frac{a_n}{b_n}$ obstaja in je različna od 0, potem vrsti $\sum_{n=1}^{\infty} a_n$ in $\sum_{n=1}^{\infty} b_n$ konvergirata ali divergirata hkrati.

3. *Kvocientni kriterij:* Naj bo $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = q$ :

   (a) $q < 1$, vrsta $\sum_{n=1}^{\infty} a_n$ konvergira

   (b) $q > 1$, vrsta $\sum_{n=1}^{\infty} a_n$ divergira

   (c) $q = 1$, vrsta lahko konvergira ali pa divergira

4. *Korenski (Cauchyjev) kriterij:* Naj bo $\lim_{n\to\infty} a_n^{\frac{1}{n}} = q$ :

   (a) $q < 1$, vrsta $\sum_{n=1}^{\infty} a_n$ konvergira

   (b) $q > 1$, vrsta $\sum_{n=1}^{\infty} a_n$ divergira

   (c) $q = 1$, vrsta lahko konvergira ali pa divergira

Vrsta je *alternirajoča*, če se da izraziti kot
$\pm\sum_{n=1}^{\infty}(-1)^n a_n$, kjer je $a_n > 0$ za vsak $n$.

**Kriterij za konvergenco alternirajoče vrste:**
Vrsta $\sum_{n=1}^{\infty}(-1)^n a_n$ konvergira, če zaporedje $\{a_n : \ n \geq n_0\}$ monotono pada proti 0.

1. Seštej vrsto tako, da izračunaš limito zaporedja delnih vsot $S_N = \sum_{n=1}^{N} a_n$:

   (a) $\sum_{n=1}^{\infty} \frac{1}{n\,(n+1)}$

   (b) $\sum_{n=1}^{\infty} \frac{1}{(3n-2)\,(3n+1)}$

   (c) $\sum_{n=1}^{\infty} \frac{1}{(n+k)\,(n+k+1)}$

   (d) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$

   (e) $\sum_{n=1}^{\infty} \frac{1}{n^k} - \frac{1}{(n+1)^k}$ $\ k \in \mathcal{Z}^+$

   (f) $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$

   (g) $\sum_{n=1}^{\infty} \frac{1}{9n^2+3n-2}$

   (h) $\sum_{n=1}^{\infty} \sqrt{n+2} - 2\sqrt{n+1} + \sqrt{n}$

   (i) $\sum_{n=1}^{\infty} \frac{n+2}{n^3+3\,n^2+2\,n}$

   (j) $\sum_{n=1}^{\infty} q^n, \quad |q| < 1$

2. Izračunaj vsoto geometrijske vrste:

   (a) $\sum_{n=1}^{\infty} \frac{3}{4^{n-1}}$ (b) $\sum_{n=1}^{\infty} \frac{37}{100^n}$ (c) $\sum_{n=1}^{\infty} \frac{6}{10^n}$ (d) $\sum_{n=1}^{\infty}(-1)^n$

3. Izračunaj vsoto vrste:

   (a) $\sum_{n=1}^{\infty} q^n \cos(n\,\alpha)$

   (b) $\sum_{n=1}^{\infty} q^n \sin(n\,\alpha)$

*Navodilo: Pomagaj si z geometrijsko vrsto s kompleksnim kvocientom.*

4. Zapiši decimalno število v obliki ulomka:

   (a) $0.2323232323\ldots$ (c) $3.2394394394\ldots$ (e) $0.1111111111\ldots$

   (b) $5.146146146\ldots$ (d) $2.7182871828\ldots$ (f) $0.9999999999\ldots$

5. S pomočjo kvocientnega kriterija določi konvergenco vrste:

   (a) $\sum_{n=1}^{\infty} \frac{4 \cdot 7 \cdot 10 \ldots (3n+1)}{2 \cdot 6 \cdot 10 \ldots (4n-2)}$ (e) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ (i) $\sum_{n=1}^{\infty} \frac{e^{2n}}{(2n-1)!}$

   (b) $\sum_{n=1}^{\infty} \frac{1}{n!}$ (f) $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$ (j) $\sum_{n=1}^{\infty} \frac{n!}{\ln(n+1)}$

   (c) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ (g) $\sum_{n=1}^{\infty} \frac{3^n n!}{n^n}$ (k) $\sum_{n=1}^{\infty} \frac{1000^n}{n!}$

   (d) $\sum_{n=1}^{\infty} \frac{(3n)!}{(2n)!}$ (h) $\sum_{n=1}^{\infty} \frac{n \cdot n!}{(2n)!}$ (l) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

6. S pomočjo korenskega kriterija določi konvergenco vrste:

   (a) $\sum_{n=1}^{\infty} \left(\frac{n-1}{n+1}\right)^{n(n-1)}$ (d) $\sum_{n=1}^{\infty} \frac{3^{2n+1}}{n\,5^n}$ (g) $\sum_{n=1}^{\infty} \left(\frac{n}{3n-1}\right)^{2n-1}$

   (b) $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{n^2}$ (e) $\sum_{n=1}^{\infty} \frac{2n-1}{2^n}$ (h) $\sum_{n=1}^{\infty} n^{\frac{1}{n}}$

   (c) $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{n^2}$ (f) $\sum_{n=1}^{\infty} \frac{n^2}{(2+\frac{1}{n})^n}$

7. S pomočjo minorante ali majorante določi konvergenco vrste:

   (a) $\sum_{n=1}^{\infty} \frac{1}{\ln(n!)}$ (d) $\sum_{n=1}^{\infty} \operatorname{tg} \frac{1}{n}$ (g) $\sum_{n=1}^{\infty} \frac{1! + 2! + \ldots + n!}{(2n)!}$

   (b) $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n-1}}$ (e) $\sum_{n=1}^{\infty} \sin \frac{1}{n^2}$ (h) $\sum_{n=1}^{\infty} \frac{n + \cos n}{n^3+1}$

   (c) $\sum_{n=1}^{\infty} \frac{3n^2+5n}{2^n(n^2+1)}$ (f) $\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{2^n}\right)$

8. Razišči konvergenco vrste s pozitivnimi členi:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{n+\frac{1}{n}}}{\left(n+\frac{1}{n}\right)^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(3n-1)^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2n-1}{2^{\frac{n}{2}}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+1}{2n-1}\right)^n$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{(n+1)\sqrt{n}}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+a}\ \ a>0$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n+1)^3}{(n^3+1)^2}$

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{2+\cos n}{n^2}$

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^n}$

(k) $\displaystyle\sum_{n=1}^{\infty} \frac{n+\ln n}{n^3+n+1}$

(l) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

(m) $\displaystyle\sum_{n=1}^{\infty} e^{-n^2}$

(n) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2-1}}$

(o) $\displaystyle\sum_{n=1}^{\infty} n^2 \sin^2 \frac{1}{n}$

(p) $\displaystyle\sum_{n=1}^{\infty} \frac{\operatorname{arctg} n}{\sqrt{1+n^2}}$

9. Določi konvergenco vrste s pozitivnimi členi:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{(n^3)}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\operatorname{arctg} n}{1+n^2}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$

(f) $\displaystyle\sum_{n=1}^{\infty} q^n,\ \ q>0$

(g) $\displaystyle\sum_{n=1}^{\infty} \sin^2(\pi\, n)$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

10. Določi konvergenco alternirajoče vrste:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n\,(n+1)}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^\alpha}\ \ \alpha>0$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\ln a}}\ \ a>0$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\,3^n}$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^n}$

11. Določi konvergenco vrste:

(a) $\displaystyle\sum_{n=1}^{\infty} \sin n$

(b) $\displaystyle\sum_{n=1}^{\infty} \sin \frac{n\,\pi}{12}\ \frac{1}{\ln(n+1)}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos \frac{n\,\pi}{4}}{n^\alpha}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}+(-1)^n}$

(e) $\sum_{n=1}^{\infty}(-1)^{\frac{n^2+n}{2}}\frac{n}{2^n}$

(f) $\sum_{n=1}^{\infty}\frac{(-1)^{\lfloor\sqrt{n}\rfloor}}{n}$

(g) $\sum_{n=1}^{\infty}\sin n\,\frac{\sin n^2}{n^2}$

(h) $\sum_{n=1}^{\infty}\frac{(-1)^{\lfloor \lg_2 n\rfloor}}{n}$

12. Določi konvergenco alternirajoče vrste:

(a) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n+1}-1}$

(b) $\sum_{n=1}^{\infty}\frac{1+(-1)^n}{2^n}+\frac{-1+(-1)^n}{3^n}$

(c) $\sum_{n=1}^{\infty}\frac{1+(-1)^n}{n}+\frac{-1+(-1)^n}{\sqrt{n}}$

(d) $\sum_{n=1}^{\infty}\frac{1+(-1)^n}{2^n}+\frac{-1+(-1)^n}{n}$

13. Ali je vsota vrst $\sum_{n=1}^{\infty}\frac{1+n}{3^n}$ in $\sum_{n=1}^{\infty}\frac{(-1)^n-n}{3^n}$ konvergentna vrsta?

14. Ali je razlika vrst $\sum_{n=1}^{\infty}\frac{1}{2n-1}$ in $\sum_{n=1}^{\infty}\frac{1}{2n}$ konvergentna vrsta?

15. Med krivuljama $y=\frac{1}{x^3}$, $y=\frac{1}{x^2}$ desno od presečišča konstruiramo zaporedje enako oddaljenih daljic, ki so vzporedne $y$ osi. Ali je vsota dolžin vseh daljic končno število?

# 6 Funkcije

1. Naj bosta $\mathcal{A}$ in $\mathcal{B}$ končni množici z $m$ in $n$ elementi:
   (a) koliko je vseh funkcij, ki množico $\mathcal{A}$ preslikajo v množico $\mathcal{B}$?
   (b) koliko je injektivnih?
   (c) koliko je surjektivnih?
   (d) koliko je bijektivnih?

2. Ali predstavlja množica urejenih parov $\mathcal{G}$ graf funkcije?
   (a) $\mathcal{G} = \{(2,3),(2,4),(2,1),(3,2),(4,4)\}$
   (b) $\mathcal{G} = \{(1,3),(2,4),(5,1),(3,2),(4,4)\}$
   (c) $\mathcal{G} = \{(1,1),(2,1),(3,1),(4,1),(5,1)\}$
   (d) $\mathcal{G} = \{(1,3),(1,4),(1,1),(1,2),(4,4)\}$
   (e) $\mathcal{G} = \{(2,4),(2,3),(3,1),(3,2),(3,3)\}$
   (f) $\mathcal{G} = \{(1,1),(2,2),(3,3),(4,4),(5,5)\}$

3. Katere od naslednjih funkcij, ki preslikajo množico $\mathcal{D} = \{1,2,3,4,5\}$ vase, predstavljene z grafom $\mathcal{G}$, so *bijektivne*?
   (a) $\mathcal{G} = \{(1,1),(2,2),(3,3),(4,4),(5,5)\}$
   (b) $\mathcal{G} = \{(1,3),(2,4),(3,1),(4,2),(5,4)\}$
   (c) $\mathcal{G} = \{(1,3),(2,4),(3,1),(4,2),(5,4)\}$
   (d) $\mathcal{G} = \{(1,1),(2,1),(3,1),(4,1),(5,1)\}$
   (e) $\mathcal{G} = \{(1,3),(2,4),(3,1),(4,2),(5,5)\}$
   (f) $\mathcal{G} = \{(1,4),(2,5),(3,1),(4,2),(5,3)\}$

4. Dve funkciji, $f$ in $g$, ki preslikata množico $\mathcal{D} = \{1,2,3,4,5\}$ vase, imamo predstavljeni z grafoma $\mathcal{G}_f$ in $\mathcal{G}_g$. Poišči kompozitume $f(g)$, $g(f)$, $f(f)$ in $g(g)$.
   (a) $\mathcal{G}_f = \{(1,3),(2,4),(3,1),(4,2),(5,4)\}$ in $\mathcal{G}_g = \{(1,3),(2,2),(3,1),(4,3),(5,4)\}$
   (b) $\mathcal{G}_f = \{(1,3),(2,3),(3,1),(4,1),(5,1)\}$ in $\mathcal{G}_g = \{(1,2),(2,2),(3,1),(4,1),(5,3)\}$
   (c) $\mathcal{G}_f = \{(1,3),(2,4),(3,1),(4,5),(5,2)\}$ in $\mathcal{G}_g = \{(1,3),(2,2),(3,5),(4,4),(5,1)\}$
   (d) $\mathcal{G}_f = \{(1,3),(2,1),(3,2),(4,2),(5,1)\}$ in $\mathcal{G}_g = \{(1,3),(2,3),(3,3),(4,3),(5,3)\}$
   (e) $\mathcal{G}_f = \{(1,2),(2,4),(3,5),(4,2),(5,1)\}$ in $\mathcal{G}_g = \{(1,1),(2,2),(3,1),(4,2),(5,5)\}$
   (f) $\mathcal{G}_f = \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$ in $\mathcal{G}_g = \{(1,1),(2,2),(3,3),(4,4),(5,5)\}$

**V naslednji nalogah bo $f : \mathcal{D} \to \mathbb{R}$, kjer je $\mathcal{D} \subset \mathbb{R}$.**

5. Določi definicijsko območje funkciji $f(x)$:

(a) $f(x) = \dfrac{x-2}{2x-1}$

(b) $f(x) = \sqrt{1-2x}$

(c) $f(x) = \sqrt{4-x^2} - \dfrac{1}{x}$

(d) $f(x) = \ln x^2$

(e) $f(x) = 2\ln x$

(f) $f(x) = \ln \dfrac{x-1}{x+1}$

(g) $f(x) = \ln(x-1) - \ln(x+1)$

(h) $f(x) = \ln\Big(\ln \dfrac{1-x}{1+x}\Big)$

(i) $f(x) = \lg_2(\lg_3(\lg_4 x))$

(j) $f(x) = \sqrt{x - |1-2x|}$

(k) $f(x) = \ln(1 - 2\cos x)$

(l) $f(x) = \sqrt{\dfrac{1+x}{1-x}}$

(m) $f(x) = \sqrt{\sqrt{2x-1} - x}$

(n) $f(x) = \sqrt{\sqrt{2x-1} + x}$

(o) $f(x) = \dfrac{1}{1+\frac{1}{x}}$

(p) $f(x) = \arccos(\sin x)$

(q) $f(x) = \cos(\arcsin x)$

(r) $f(x) = \ln(x + \sqrt{1+x^2})$

(s) $f(x) = \arccos(1-x) + \ln(\ln x)$

(t) $f(x) = \ln(x - \sqrt{1+x^2})$

(u) $f(x) = \ln \frac{1+\sqrt{1+\frac{1}{x^2}}}{x}$

(v) $f(x) = \ln\left(\ln\left(\frac{1}{x} + \frac{1}{2}\right)\right)$

6. Določi definicijsko območje in zalogo vrednosti funkciji $f(x)$:

(a) $f(x) = 3 - |x|$

(b) $f(x) = 1 - x^2$

(c) $f(x) = x^2 + 2x + 2$

(d) $f(x) = \dfrac{1}{1+x^2}$

(e) $f(x) = \dfrac{2\,x}{1+x^2}$

(f) $f(x) = \dfrac{1}{1+\frac{1}{x^2}}$

(g) $f(x) = \sqrt{x^2}$

(h) $f(x) = (\sqrt{x})^2$

(i) $f(x) = \dfrac{x}{|x|}$

(j) $f(x) = \sqrt{1+x^2}$

(k) $f(x) = \sqrt{x^4 + x^2}$

(l) $f(x) = x\sqrt{x^2+1}$

(m) $f(x) = e^{-\frac{1}{x^2}}$

(n) $f(x) = e^{\ln x}$

(o) $f(x) = \ln e^x$

(p) $f(x) = \arcsin e^x$

(q) $f(x) = \dfrac{1}{1+e^{-\frac{1}{x^2}}}$

(r) $f(x) = e^{\ln|x|}$

(s) $f(x) = \arcsin \dfrac{1}{x}$

(t) $f(x) = \operatorname{arctg} \dfrac{1}{x}$

7. Določi definicijsko območje in zalogo vrednosti funkciji $f(x)$:

   (a) $f(x) = |x-1| + |2-3x| + x$

   (b) $f(x) = |x^2-2| - 2\,|x-2|$

   (c) $f(x) = \sqrt{2+x-x^2}$

   (d) $f(x) = \dfrac{\sqrt{x^4+x^2}}{x}$

   (e) $f(x) = \ln(1+|\sin x|)$

   (f) $f(x) = \dfrac{|x+1|-|x-1|}{2x}$

   (g) $f(x) = 2 + \arcsin\dfrac{3x-1}{2}$

   (h) $f(x) = \arccos(2\sin x)$

   (i) $f(x) = \arcsin\dfrac{2x}{1+x}$

   (j) $f(x) = e^{-\frac{1}{x^2}}$

8. Ugotovi injektivnost, surjektivnost in bijektivnost funkcije $f(x)$ z definicijskim območjem in zalogo vrednosti v množici realnih števil:

   (a) $f(x) = 3+3x$

   (b) $f(x) = 8x - 2x^2$

   (c) $f(x) = \arctan x$

   (d) $f(x) = 3 - 2x - x^2 + x^3$

   (e) $f(x) = 1 + x^3$

   (f) $f(x) = \dfrac{2x}{1+x^2}$

   (g) $f(x) = \dfrac{x}{\sqrt{1+x^2}}$

   (h) $f(x) = x\log|x|$

   (i) $f(x) = \sin x$

   (j) $f(x) = 1 - x\,|x|$

   (k) $f(x) = \dfrac{x}{1+x^4}$

   (l) $f(x) = e^x$

   (m) $f(x) = 4x + |1-x| - |1-2x|$

   (n) $f(x) = x + \big|x + |1-3x| + |2x-3|\big|$

   (o) $f(x) = x\sqrt{|1+x|}$

   (p) $f(x) = \dfrac{1}{1+e^{\frac{1}{x}}}$

9. Poišči primer funkcije $f$ in par takih množic $\mathcal{A}$ in $\mathcal{B}$ iz definicijskega območja funkcije $f$, kjer velja $\mathcal{B} \subset \mathcal{A}$, da bo $f(\mathcal{A}-\mathcal{B}) \neq f(\mathcal{A}) - f(\mathcal{B})$.

10. Bodi $f$ funkcija, ki preslika $\mathcal{X}$ v $\mathcal{Y}$. Pokaži, da so naslednje trditve ekvivalentne:

    (a) funkcija $f$ je injektivna

    (b) za poljubno množico $\mathcal{A} \subseteq \mathcal{X}$ velja $f^{-1}(f(\mathcal{A})) = \mathcal{A}$

    (c) za poljuben par množic $\mathcal{A}, \mathcal{B} \subseteq \mathcal{X}$ velja $f(\mathcal{A} \cap \mathcal{B})) = f(\mathcal{A}) \cap f(\mathcal{B})$

    (d) za poljuben par množic $\mathcal{A}, \mathcal{B} \subseteq \mathcal{X}$, takih da je $f(\mathcal{A} \cap \mathcal{B}) = \emptyset$,
    velja $f(\mathcal{A}) \cap f(\mathcal{B}) = \emptyset$

    (e) za poljuben par množic $\mathcal{A}, \mathcal{B} \subseteq \mathcal{X}$, takih da je $\mathcal{B} \subset \mathcal{A}$,
    velja $f(\mathcal{A}-\mathcal{B}) = f(\mathcal{A}) - f(\mathcal{B})$

11. Katere funkcije $f : \mathcal{D} \to \mathbb{R}$, $\mathcal{D} \subset \mathbb{R}$, na sliki so sode, lihe, injektivne, bijektivne ali surjektivne:

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)

(l)

(m) (n)

12. Katere od funkcij so sode oziroma lihe, če je $x \in \mathbb{R}$ in $f(x) \in \mathbb{R}$:

(a) $f(x) = \dfrac{x}{1+x^2}$

(b) $f(x) = \ln(x + \sqrt{1+x^2})$

(c) $f(x) = \ln(x - \sqrt{x^2-1})$

(d) $f(x) = \dfrac{\sin x}{x}$

(e) $f(x) = x \cos x$

(f) $f(x) = \sin(\cos x)$

(g) $f(x) = \sin(2x)$

(h) $f(x) = \sin^2 x$

(i) $f(x) = \dfrac{\sin^2 x}{x}$

(j) $f(x) = \arcsin \dfrac{2x}{1+x^2}$

(k) $f(x) = \operatorname{arctg} \dfrac{x-1}{x+1}$

(l) $f(x) = \operatorname{arctg} \left|\dfrac{x-1}{x+1}\right| - \dfrac{\pi}{4}$

(m) $f(x) = \dfrac{1+\sin x}{x}$

(n) $f(x) = \ln \left|\dfrac{1-x}{1+x}\right|$

(o) $f(x) = \ln \dfrac{1}{|x|}$

(p) $f(x) = \ln \dfrac{x}{1+x^2}$

(q) $f(x) = \ln \sqrt{1+x^2}$

(r) $f(x) = (\sqrt{x})^2$

(s) $f(x) = \dfrac{\sin x}{x^2}$

(t) $f(x) = x + \sqrt{x^4+x^6}$

(u) $f(x) = x + x\sqrt{x^2+x^4}$

(v) $f(x) = \sqrt[3]{x^3+x}$

(w) $f(x) = \sin(x^2)$

(x) $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(y) $f(x) = \dfrac{x}{\operatorname{tg} x}$

(z) $f(x) = \sqrt[3]{x^3-x}$

13. Dokaži, da lahko vsako funkcijo, definirano na simetrični množici okrog izhodišča, zapišemo v obliki vsote sode in lihe funkcije.

14. Ali je soda funkcija lahko monotona na intervalu, ki vsebuje izhodišče?

15. Dani sta funkciji, ki sta obe naraščajoči ali podajoči. Dokaži, da je njun kompozitum naraščajoč.

16. Dokaži, da je kompozitum naraščajoče in padajoče funkcije padajoča funkcija.

17. Dokaži, da gre graf lihe funkcije, definirane na intervalu, ki vsebuje izhodišče, skozi izhodišče.

18. Koliko je $f(x)g(x)$, če je $f(x) = |x| - x$ in $g(x) = |x| + x$?

19. Ali sta funkciji $f(x)$ in $g(x)$ so enaki?

| | | |
|---|---|---|
| (a) | $f(x) = \sqrt{x^2}$ | $g(x) = x$ |
| (b) | $f(x) = (\sqrt{x})^2$ | $g(x) = \sqrt{x^2}$ |
| (c) | $f(x) = (\sqrt{\lvert x\rvert})^2$ | $g(x) = \sqrt{x^2}$ |
| (d) | $f(x) = \sqrt{x^2}$ | $g(x) = \lvert x\rvert$ |
| (e) | $f(x) = \ln(x^2)$ | $g(x) = 2\ln x$ |
| (f) | $f(x) = \ln\sqrt{x}$ | $g(x) = \frac{1}{2}\ln x$ |
| (g) | $f(x) = \ln(x^2)$ | $g(x) = 2\ln\lvert x\rvert$ |
| (h) | $f(x) = \sqrt{x^3 + x^2}$ | $g(x) = x\sqrt{x+1}$ |
| (i) | $f(x) = \ln\frac{1-x}{1+x}$ | $g(x) = \ln(1-x) - \ln(1+x)$ |
| (j) | $f(x) = \ln\frac{x-1}{x+1}$ | $g(x) = \ln(x-1) - \ln(x+1)$ |
| (k) | $f(x) = \frac{x}{\lvert x\rvert}$ | $g(x) = \operatorname{sign} x$ |
| (l) | $f(x) = \frac{x}{x}$ | $g(x) = 1$ |
| (m) | $f(x) = \frac{1}{1+\frac{1}{x^2}}$ | $g(x) = \frac{x^2}{1+x^2}$ |
| (n) | $f(x) = \frac{1}{1+\frac{1}{1+x^2}}$ | $g(x) = 1 - \frac{1}{2+x^2}$ |

20. Določi definicijsko območje dane funkcije. V točkah, kjer funkcija ni definirana, določi levo in desno limito, če obstajata:

| | | | |
|---|---|---|---|
| (a) | $\lvert x\rvert$ | (g) | $e^{\frac{x}{4-x^2}}$ |
| (b) | $x^2$ | (h) | $e^{-\frac{1}{x^2}}$ |
| (c) | $\frac{x}{\lvert x\rvert}$ | (i) | $\frac{e^{\frac{1}{x-1}} - 1}{e^{\frac{1}{x-1}} + 1}$ |
| (d) | $\frac{1}{1+\frac{1}{x}}$ | (j) | $\frac{1}{1+e^{\frac{1}{x}}}$ |
| (e) | $\frac{x-2}{x-3}$ | (k) | $\operatorname{arctg}\left\lvert\frac{x}{1-x}\right\rvert$ |
| (f) | $x\sin\frac{1}{x}$ | (l) | $\operatorname{arctg}\frac{x}{1-x}$ |

21. V točkah, kjer funkcija ni zvezna, določi levo in desno limito, če obstajata:

(a) $f(x) = \begin{cases} \frac{(1+x)^n-1}{x}; & x \neq 0 \wedge n \in \mathcal{R} \\ 0; & x = 0 \end{cases}$

(b) $f(x) = \begin{cases} 2\sqrt{x}; & 0 \leq x < 1 \\ 4 - 2x & 1 < x < \frac{5}{2} \\ 2x - 7; & 2.5 \leq x < 4 \end{cases}$

(c) $f(x) = \begin{cases} x^2 \sin \frac{1}{x}; & x \leq 0 \\ 0; & 0 \leq x \end{cases}$

(d) $f(x) = \begin{cases} \ln |x| \,; & x \neq 0 \\ 1; & x = 0 \end{cases}$

(e) $f(x) = \begin{cases} 0; & x \text{ je racionalno število} \\ 1; & \text{drugod} \end{cases}$

(f) $f(x) = \begin{cases} \frac{1}{n}; & x \text{ je okrajšan ulomek } \frac{m}{n} \\ 0; & \text{drugod} \end{cases}$

22. Določi parameter $a$ tako, da bo funkcija $f(x)$ zvezna:

(a) $f(x) = \begin{cases} \frac{1}{\sqrt{1+\frac{1}{x^2}}}; & x \neq 0 \\ a; & x = 0 \end{cases}$

(b) $f(x) = \begin{cases} \frac{x^2+x-2}{x-1}; & x \neq 1 \\ a; & x = 1 \end{cases}$

(c) $f(x) = \begin{cases} e^x; & x < 0 \\ a + x; & 0 \leq x \end{cases}$

(d) $f(x) = \begin{cases} \frac{1}{1-e^{1/x}}; & x < 0 \\ a; & 0 \leq x \end{cases}$

(e) $f(x) = \begin{cases} \frac{1}{1-e^{-1/x^2}}; & x \neq 0 \\ a; & x = 0 \end{cases}$

(f) $f(x) = \begin{cases} x \sin \frac{1}{x}; & x \neq 0 \\ a; & x = 0 \end{cases}$

23. Poišči maksimum in minimum funkcije $f(x)$ na danem intervalu $\mathcal{I}$:

(a) $x^2$ $\mathcal{I} = [-1, 4]$

(b) $1 + x^3$ $\mathcal{I} = [-1, 3]$

(c) $\sqrt{x - x^2}$ $\mathcal{I} = [0, 1]$

(d) $\dfrac{x}{x-2}$ $\mathcal{I} = [0, 1]$

(e) $|x^2 - 2| - 2|x - 2|$ $\mathcal{I} = [-3, 3]$

(f) $4x + |1 - x| - |1 + 2x|$ $\mathcal{I} = [-1, 3]$

(g) $-1 + x^2 - |x + 1|$ $\mathcal{I} = [-2, 2]$

(h) $x + \big|x + |1 - 3x| + |2x - 3|\big|$ $\quad \mathcal{I} = [-1, 3]$

(i) $\dfrac{1}{1 + x^2}$ $\quad \mathcal{I} = [-1, 3]$

(j) $x\sqrt{|1 + x|}$ $\quad \mathcal{I} = [-2, 1]$

(k) $\dfrac{x}{1 + x^2}$ $\quad \mathcal{I} = [-1, 2]$

(l) $x\sqrt{\dfrac{1}{1 + x^2}}$ $\quad \mathcal{I} = [-1, 2]$

(m) $x|x|$ $\quad \mathcal{I} = [-1, 2]$

24. Dana je podmnožica $\mathcal{A}$ definicijskega območja funkcije $f(x)$. Poišči njeno sliko $\mathcal{A}' = f(\mathcal{A})$.

(a) $f(x) = x^2$ $\quad \mathcal{A} = [-1, 1)$

(b) $f(x) = \dfrac{2x}{1 + x^2}$ $\quad \mathcal{A} = (-1, 2)$

(c) $f(x) = -x^2 - x^3 + x^4$ $\quad \mathcal{A} = (-\dfrac{1}{2}, 1]$

(d) $f(x) = x + |1 - 3x| + |2x - 3|$ $\quad \mathcal{A} = (-1, 2)$

(e) $f(x) = x^2 e^{-x}$ $\quad \mathcal{A} = (0, 3)$

(f) $f(x) = 1 + x^3$ $\quad \mathcal{A} = (-1, 1)$

(g) $8x - 2x^2$ $\quad \mathcal{A} = [-1, 4]$

(h) $3 - 2x - x^2 + x^3$ $\quad \mathcal{A} = (-2, 1)$

(i) $\big|x^2 - 1\big| + |x + 2| + x$ $\quad \mathcal{A} = (-2, 2]$

(j) $x\sqrt{|1 + x|}$ $\quad \mathcal{A} = (-1, 1)$

(k) $\sqrt{|x^2 + x^3|}$ $\quad \mathcal{A} = (-2, 1]$

(l) $\begin{cases} \arctan\big|\frac{1-x}{1+x}\big|, & x \neq -1 \\ \frac{\pi}{2}, & x = -1 \end{cases}$ $\quad \mathcal{A} = (-2, 2)$

25. Dana je podmnožica $\mathcal{A}'$ zaloge vrednosti funkcije $f(x)$.
Poišči njen original $\mathcal{A} = f^{-1}(\mathcal{A}')$.

(a) $f(x) = x^2$ $\quad \mathcal{A}' = [1, 4)$

(b) $f(x) = -1 + x^2 - |1 + x|$ $\quad \mathcal{A}' = (0, 2)$

(c) $f(x) = x + \dfrac{1}{x}$ $\quad \mathcal{A}' = (2, 4]$

(d) $f(x) = \dfrac{1}{1 + x^2}$ $\quad \mathcal{A}' = (0, 1)$

(e) $f(x) = x + |1 - 3x| + |2x - 3|$ $\quad \mathcal{A}' = (4, 6)$

(f) $f(x) = 4x + |1 - x| - |1 + 2x|$ $\quad \mathcal{A}' = [0, 2)$

26. Določi kompozitume $f(f(x))$, $f(g(x))$, $g(f(x))$ in $g(g(x))$:

(a) $f(x) = a\,x$ $\quad g(x) = b\,x$

(b) $f(x) = x^a$ $\quad g(x) = x^b$

(c) $f(x) = -1 + 2\,x$ $\quad g(x) = 2 - 3\,x$

(d) $f(x) = x^2$ $\quad g(x) = \sqrt{x}$

(e) $f(x) = \dfrac{1}{1+x}$ $\quad g(x) = x^2$

(f) $f(x) = \dfrac{x+1}{x-1}$ $\quad g(x) = \dfrac{x+2}{x-3}$

(g) $f(x) = \dfrac{1}{x}$ $\quad g(x) = \operatorname{sign}(x)$

(h) $f(x) = 1 + \dfrac{1}{x}$ $\quad g(x) = \dfrac{1+x}{1-x}$

27. Določi kompozituma $f(g(x))$ in $g(f(x))$:

(a) $f(x) = \dfrac{1}{2+x}$ $\quad g(x) = 3 + x$

(b) $f(x) = 1 + x$ $\quad g(x) = 1 + 3\,x + x^2$

(c) $f(x) = \dfrac{3+x}{x-2}$ $\quad g(x) = \dfrac{4+x}{x-1}$

(d) $f(x) = 1 + \dfrac{1}{x}$ $\quad g(x) = \dfrac{1}{1-x}$

(e) $f(x) = \dfrac{1}{1+x}$ $\quad g(x) = 1 + x^2$

(f) $f(x) = \dfrac{1+x}{x-1}$ $\quad g(x) = x^2$

(g) $f(x) = \dfrac{1}{x} + x$ $\quad g(x) = -\dfrac{1}{x} + x$

(h) $f(x) = \dfrac{1}{1+x^2}$ $\quad g(x) = 1 + \dfrac{1}{x}$

(i) $f(x) = x^2$ $\quad g(x) = e^x$

(j) $f(x) = 0$ $\quad g(x) = \ln(x + \sqrt{1+x^2})$

(k) $f(x) = \dfrac{1}{1-x^2}$ $\quad g(x) = \dfrac{1+x}{1-x}$

(l) $f(x) = \dfrac{1}{x}$ $\quad g(x) = x + \sqrt{1+x^2}$

(m) $f(x) = x + \sqrt{1+x^2}$ $g(x) = \dfrac{x^2-1}{2\,x}$

(n) $f(x) = 1 + \dfrac{1}{x}$ $g(x) = \dfrac{1+x}{1-x}$

(o) $f(x) = \dfrac{x}{1+x}$ $g(x) = x^2$

(p) $f(x) = x^{-2}$ $g(x) = \dfrac{1+x}{1-x}$

(q) $f(x) = x\,|1+x|$ $g(x) = \dfrac{\sqrt{1-4\,x}-1}{2}$

(r) $f(x) = \dfrac{1}{1+x^2}$ $g(x) = \sqrt{\dfrac{1-x}{x}}$

(s) $f(x) = \dfrac{x}{1+x^2}$ $g(x) = \dfrac{1-\sqrt{1-4\,x^2}}{2\,x}$

28. Določi funkcijo $f(x)$, če je:

(a) $f(1+x) = 2 - 3\,x + x^2$

(b) $f(1+\dfrac{1}{x}) = -\dfrac{1}{x} + x$

(c) $f(\dfrac{1}{x}) = x + \sqrt{1+x^2}$

(d) $f(\dfrac{x}{1+x}) = x^2$

(e) $f(\dfrac{1+x}{1-x}) = \sqrt{x}$

(f) $f(3+2\,x) = 1 - \dfrac{1}{x}$

(g) $f(1+x) = \dfrac{x}{1+2\,x+x^2}$

(h) $f(\dfrac{1+x}{x-1}) = \dfrac{x-1}{1+x}$

(i) $f(\dfrac{1}{x}) = \sqrt{\dfrac{1+x}{1-x}}$

(j) $f(x\ |x|) = x^2 + 2\,x + 1$

29. Poišči inverzno funkcijo $f^{-1}(x)$ k funkciji $f(x)$ in definicijsko območje tako dobljene funkcije:

(a) $f(x) = 1 + 3\,x$

(b) $f(x) = 3 - \dfrac{x}{2}$

(c) $f(x) = 1 + x^3$

(d) $f(x) = \dfrac{2\,x+3}{x-2}$

(e) $f(x) = 1 + \dfrac{1}{x}$

(f) $f(x) = |x-2| + 2\,x - 1$

(g) $f(x) = |1-x| + 4\,x - |1+2\,x|$

(h) $f(x) = x\,|x|$

(i) $f(x) = 1 - x\,|x|$

(j) $f(x) = \dfrac{x}{\sqrt{1+x^2}}$

(k) $f(x) = 1 + e^{-\frac{x}{2}}$

(l) $f(x) = e^x - e^{-x}$

(m) $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(n) $f(x) = \log\dfrac{x}{2}$

(o) $f(x) = 1 + \arctan(3\,x)$

(p) $f(x) = \dfrac{1}{1+e^{\frac{1}{x}}}$

30. Poišči inverzno funkcijo k funkciji $f(x)$ na danem intervalu in določi definicijsko območje dobljene funkcije:

(a) $f(x) = x^2$ $(-\infty, 0)$

(b) $f(x) = 1 - x^2$ $(0, \infty)$

(c) $f(x) = \sqrt{2 + 2\,x}$ $(-1, \infty)$

(d) $f(x) = \dfrac{1 - x}{1 + x}$ $(-1, 1)$

(e) $f(x) = \dfrac{1 - x}{1 + x}$ $(1, \infty)$

(f) $f(x) = \sin(x)$ $(\frac{\pi}{2}, \pi)$

(g) $f(x) = e^{-x} + e^{x}$ $(-\infty, 0)$

(h) $f(x) = e^{-x} + e^{x}$ $(0, \infty)$

(i) $f(x) = \dfrac{x^2 - 1}{1 + x^2}$ $(-\infty, 0)$

(j) $f(x) = \dfrac{x^2 - 1}{1 + x^2}$ $(0, \infty)$

(k) $f(x) = \dfrac{2\,x}{1 + x^2}$ $(-\infty, -1)$

(l) $f(x) = \dfrac{2\,x}{1 + x^2}$ $(-1, 1)$

(m) $f(x) = \dfrac{2\,x}{1 + x^2}$ $(1, \infty)$

(n) $f(x) = \dfrac{x}{\sqrt{1 + x^4}}$ $(-\infty, -1)$

(o) $f(x) = \dfrac{x}{\sqrt{1 + x^4}}$ $(-1, 1)$

(p) $f(x) = \dfrac{x}{\sqrt{1 + x^4}}$ $(1, \infty)$

(q) $f(x) = \log\left|\dfrac{1 - x}{1 + x}\right|$ $(-1, 1)$

(r) $f(x) = \log\left|\dfrac{1 - x}{1 + x}\right|$ $(1, \infty)$

31. Izračunaj limite funkcij:

(a) $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{2\,x^2 - x - 1}$

(b) $\lim\limits_{x \to 2} \dfrac{x^2 + 5}{x^2 - 3}$

(c) $\lim\limits_{x \to -1} \dfrac{x^2 - x - 2}{x^3 + 1}$

(d) $\lim\limits_{x \to 1} \dfrac{x^4 + 2\,x^2 - 3}{x^2 - 3\,x + 2}$

(e) $\lim\limits_{x \to 1} \left(\dfrac{1}{x - 1} - \dfrac{2}{x^2 - 1}\right)$

(f) $\lim\limits_{x \to 1} \dfrac{x^m - 1}{x^n - 1}$

(g) $\lim\limits_{x \to 0} \dfrac{(1 + m\,x)^n - (1 + n\,x)^m}{x^2}$

(h) $\lim\limits_{x \to \infty} \dfrac{x^3 + x}{x^4 - 3\,x + 1}$

(i) $\lim\limits_{x \to \infty} \left(\dfrac{x^3}{2\,x^2 - 1} - \dfrac{x^2}{2\,x + 1}\right)$

(j) $\lim\limits_{x \to 1} \left(\dfrac{m}{1 - x^m} - \dfrac{n}{1 - x^n}\right)$

(k) $\lim\limits_{x \to \infty} \dfrac{\sqrt{x - 1} - 3}{2 + \sqrt{x}}$

(l) $\lim\limits_{x \to 0} \dfrac{\sqrt[m]{x + 1} - 1}{x}$

(m) $\lim\limits_{x \to 1} \dfrac{(x - 1)\,\sqrt{2 - x}}{x^2 - 1}$

(n) $\lim\limits_{x \to 4} \dfrac{\sqrt{1 + 2\,x} - 3}{\sqrt{x} - 2}$

(o) $\lim\limits_{x \to 0} \dfrac{-1 - x + \sqrt{1 - 2\,x - x^2}}{x}$

(p) $\lim\limits_{x \to \infty} \dfrac{\sqrt{x^2 + 1} - \sqrt[3]{x^2 + 1}}{\sqrt[4]{x^4 + 1} - \sqrt[5]{x^4 + 1}}$

(q) $\lim\limits_{x \to 1} \dfrac{x^2 - \sqrt{x}}{\sqrt{x} - 1}$

(r) $\lim\limits_{x \to -\infty} \left(\sqrt{x^2 - 1} - x\right)$

(s) $\lim\limits_{x \to \infty} \left(\sqrt{x^2 - 1} - x\right)$

(t) $\lim\limits_{x \to \infty} \left(\sqrt[3]{(x + 1)^2} - \sqrt[3]{(x - 1)^2}\right)$

32. Izračunaj limite funkcij:

(a) $\lim\limits_{x\to 0} \dfrac{\sin(3\,x)}{x}$

(b) $\lim\limits_{x\to \infty} \dfrac{\sin(3\,x)}{x}$

(c) $\lim\limits_{x\to 0} \dfrac{\sin(m\,x)}{\sin(n\,x)}$

(d) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$

(e) $\lim\limits_{x\to 0} \dfrac{\sin(a\,x)}{\sin(b\,x)}$

(f) $\lim\limits_{x\to 0} \dfrac{\tan x - \sin x}{\sin^3 x}$

(g) $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{1-\sin x}{\left(\frac{\pi}{2}-x\right)^2}$

(h) $\lim\limits_{x\to 0} \left(\dfrac{1}{\sin x} - \dfrac{1}{\tan x}\right)$

(i) $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{\tan(3\,x)}{\tan x}$

(j) $\lim\limits_{x\to 0} \dfrac{\frac{x}{\tan x}-1}{x^2}$

(k) $\lim\limits_{x\to 0} \dfrac{\cos(\sin x)-\cos x}{x^4}$

(l) $\lim\limits_{x\to 0} \dfrac{1-\cos^2 x}{x^2\,\sin(x^2)}$

(m) $\lim\limits_{x\to 0} \left(\cot x - \dfrac{1}{x}\right)$

(n) $\lim\limits_{x\to 0} \dfrac{2\,\arcsin x}{3\,x}$

(o) $\lim\limits_{x\to 0} \dfrac{\arcsin(2\,x) - 2\,\arcsin x}{x^2}$

(p) $\lim\limits_{x\to 0} \dfrac{a^x-1}{x}$

(q) $\lim\limits_{x\to 0} \left(\dfrac{1}{x} - \dfrac{1}{e^x-1}\right)$

(r) $\lim\limits_{x\to 0} \dfrac{x\,(e^x+1) - 2\,(e^x-1)}{x^3}$

(s) $\lim\limits_{x\to 0} (x\,\log x)$

(t) $\lim\limits_{x\to 1} (\log x\,\log(1-x))$

(u) $\lim\limits_{x\to 0} \dfrac{\log(1+x)}{x}$

(v) $\lim\limits_{x\to 1} \left(\dfrac{1}{\log x} - \dfrac{1}{x}\right)$

33. Izračunaj limite funkcij:

(a) $\lim\limits_{x\to 1} x^{\frac{1}{1-x}}$

(b) $\lim\limits_{x\to 0} \left(\dfrac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}$

(c) $\lim\limits_{x\to -\infty} \left(1+\dfrac{3}{x}\right)^x$

(d) $\lim\limits_{x\to 0} \left(1+\dfrac{x}{4}\right)^{\frac{2}{x}}$

(e) $\lim\limits_{x\to 0} x^{\sin x}$

(f) $\lim\limits_{x\to 0} x^x$

(g) $\lim\limits_{x\to 0} x^{x^{x-1}}$

(h) $\lim\limits_{x\to 0} x^{x^x-1}$

(i) $\lim\limits_{x\to \infty} \log x^{\frac{1}{x}}$

(j) $\lim\limits_{x\to \infty} \left(\dfrac{x^2+1}{x^2-1}\right)^{x^2}$

(k) $\lim\limits_{x\to \infty} \left(\dfrac{x+1}{2\,x-1}\right)^x$

(l) $\lim\limits_{x\to \infty} \left(\dfrac{2\,x+1}{x-1}\right)^x$

(m) $\lim\limits_{x\to \infty} \left(\dfrac{x^2-1}{x^2+1}\right)^{x^2}$

(n) $\lim\limits_{x\to 0} (1+\tan x)^{\cot x}$

(o) $\lim\limits_{x\to 0} \left(\dfrac{\tan x}{x}\right)^{\frac{1}{x^2}}$

(p) $\lim\limits_{x\to \infty} \left(\dfrac{2\,\arctan x}{\pi}\right)^x$

34. Nariši grafe funkcij:

(a) $y = 3 + 2\,x$

(b) $y = 8\,x - 2\,x^2$

(c) $y = -1 + 2\,x - x^2$

(d) $y = 2 - 3\,x + x^2$

(e) $y = 1 + x + \dfrac{x^2}{2}$

(f) $y = 1 + x^3$

(g) $y = x^2 - x^4$

(h) $y = -x^2 - x^3 + x^4$

(i) $y = 3 - 2\,x - x^2 + x^3$

(j) $y = -x^2 + x^4$

(k) $y = 3 - |x|$

(l) $y = x - |1 - 2x|$

(m) $y = |x - 1| + |2 - 3x| + x$

(n) $y = 4\,x + |1 - x| - |1 + 2\,x|$

(o) $y = x^2 - 1 - |1 + x|$

(p) $y = x + \big|x + |1 - 3x| + |2x - 3|\big|$

(q) $y = \min\{x, 3 + x, |x - 2|\}$

(r) $y = \max\{2\,|x|, |1 + x|\}$

(s) $y = |x^2 - 2| - 2|x - 2|$

(t) $y = |x^2 - 1| + |x + 2| + x$

35. Nariši grafe funkcij:

(a) $y = \dfrac{1}{x} + x$

(b) $y = x^{-2} + x$

(c) $y = \dfrac{2\,x}{1 + x^2}$

(d) $y = \dfrac{1}{1 + x^2}$

(e) $y = \dfrac{2\,x - 1}{1 + x^3}$

(f) $y = \dfrac{1 + x^3}{x - x^2}$

(g) $y = \dfrac{4\,\left(x^2 - x^4\right)}{1 - 4\,x^2}$

(h) $y = \dfrac{(1 - x)\,x^3}{1 - 2\,x^2}$

(i) $y = \dfrac{|x + 1| - |x - 1|}{2x}$

(j) $y = x + \dfrac{1 + x}{|x - 1|}$

(k) $y = \dfrac{x + x^3}{2\,|x|}$

(l) $y = \dfrac{4 - x^2}{|4 - x^3|}$

36. Nariši grafe funkcij:

(a) $y = \sqrt{\left|\dfrac{1 - x}{1 + x}\right|}$

(b) $y = x\,\sqrt{|1 + x|}$

(c) $y = \sqrt{|x^2 + x^3|}$

(d) $y = \sqrt{|x^2 - x^4|}$

(e) $y = \sqrt{\dfrac{|x^4 - 1|}{1 + x^2}}$

(f) $y = \sqrt[3]{x^2 + x^3}$

(g) $y = \sqrt{\dfrac{|x|}{1 + x^2}}$

(h) $y = \dfrac{x}{\sqrt{1 + x^4}}$

(i) $y = \dfrac{x - 2}{\sqrt{1 + x^2}}$

(j) $y = \dfrac{x}{\sqrt[3]{1 - x^2}}$

37. Nariši grafe funkcij:

(a) $y = \arcsin \dfrac{2\,x}{1+x^2}$

(b) $y = \arcsin(\sin\, x)$

(c) $y = \arcsin \dfrac{1}{1+x^2}$

(d) $y = \arcsin(0.9\, \cos\, x)$

(e) $y = \sin(2\, \arcsin\, x)$

(f) $y = \sin(3\, \arcsin\, x)$

(g) $y = \arctan(\dfrac{1}{x})$

(h) $y = x + \arctan x$

(i) $y = x\, \arctan x$

(j) $y = x\, \arctan \dfrac{1}{x}$

(k) $y = \arctan \dfrac{1-x}{1+x}$

(l) $y = \arctan\Big|\dfrac{1-x}{1+x}\Big|$

38. Nariši grafe funkcij:

(a) $y = e^{-\frac{x}{8}}\, \sin x$

(b) $y = e^{-\frac{1}{x^2}}$

(c) $y = e^{\frac{x}{1-x^2}}$

(d) $y = e^{-x^2}$

(e) $y = \dfrac{1}{1+e^{\frac{1}{x}}}$

(f) $y = x^2\, e^{-x}$

(g) $y = e^{\frac{0.1}{1-x^2}}\, \dfrac{1}{1+x^2}$

(h) $y = x + \sin x$

(i) $y = x\, \sin \dfrac{2}{x}$

(j) $y = x^2\, \sin \dfrac{2}{x}$

(k) $y = \dfrac{\sin\, x}{x}$

(l) $y = \sin(x^2)$

(m) $y = \dfrac{1}{\ln x}$

(n) $y = x\, \ln\, x$

(o) $y = x^x$

(p) $y = x^{\frac{1}{x}}$

(q) $y = (1+x)^{\frac{1}{x}}$

(r) $y = \dfrac{1}{1+x}\left(1+\dfrac{1}{x}\right)^x$

(s) $y = \ln\Big|\dfrac{1-x}{1+x}\Big|$

(t) $y = \ln \dfrac{|3\,x|}{1+x^2}$

(u) $y = \ln(x + \sqrt{1+x^2})$

(v) $y = \ln(1+x^2)$

39. Nariši grafe funkcij:

(a) $y = \dfrac{\sin x^2}{x}$

(b) $y = \dfrac{\cos x\, \sin x}{x}$

(c) $y = \arctan\Big|\dfrac{1-x}{1+x}\Big|$

(d) $y = -\dfrac{\pi}{4} + \arctan\Big|\dfrac{1-x}{1+x}\Big|$

(e) $y = \sqrt{\dfrac{x^2}{1+x^2}}$

(f) $y = x\sqrt{\dfrac{1}{1+x^2}}$

(g) $y = x - 3\,x^3 + x^5$

(h) $y = 1 - x\,|x|$

# 7 Rešitve

## 7.1 Množice

8. (a) $\mathcal{A} \cup \mathcal{B} = \mathcal{A} \triangle \mathcal{B} \triangle (\mathcal{A} \cap \mathcal{B})$ $\quad \mathcal{A} - \mathcal{B} = \mathcal{A} \triangle (\mathcal{A} \cap \mathcal{B})$

   (b) $\mathcal{A} \cap \mathcal{B} = (\mathcal{A} \cup \mathcal{B}) \triangle \mathcal{A} \triangle \mathcal{B}$ $\quad \mathcal{A} - \mathcal{B} = (\mathcal{A} \cup \mathcal{B}) \triangle \mathcal{A}$

   (c) $\mathcal{A} \cap \mathcal{B} = \mathcal{A} - (\mathcal{A} - \mathcal{B})$ $\quad \mathcal{A} \cup \mathcal{B} = (\mathcal{A} \triangle \mathcal{B}) \triangle \big(\mathcal{A} - (\mathcal{A} - \mathcal{B})\big)$

   (d) $\mathcal{A} \cap \mathcal{B} = (\mathcal{A} \downarrow \mathcal{A}) \downarrow (\mathcal{B} \downarrow \mathcal{B})$ $\quad \mathcal{A} \cup \mathcal{B} = (\mathcal{A} \downarrow \mathcal{B}) \downarrow (\mathcal{A} \downarrow \mathcal{B})$

11. Rešitev sistema

    (a) $\mathcal{X} = \mathcal{C} - \mathcal{B}$ $\quad$ (c) $\mathcal{X} = (\mathcal{A} - \mathcal{B}) \cup \mathcal{C}$

    (b) $\mathcal{X} = (\mathcal{C} - \mathcal{A}) \cup \mathcal{B}$

12. $\mathcal{X} = \overline{\mathcal{B}}$

13. Karakteristične funkcije množic:

    (a) $\chi_{\mathcal{X}}(x) = \chi_{\mathcal{A}}(x) \cdot \chi_{\mathcal{B}}(x)$

    (b) $\chi_{\mathcal{X}}(x) = \chi_{\mathcal{A}}(x) + \chi_{\mathcal{B}}(x) - \chi_{\mathcal{A}}(x) \cdot \chi_{\mathcal{B}}(x)$

    (c) $\chi_{\mathcal{X}}(x) = 1 - \chi_{\mathcal{A}}(x)$

    (d) $\chi_{\mathcal{X}}(x) = \chi_{\mathcal{A}}(x) - \chi_{\mathcal{A}}(x) \cdot \chi_{\mathcal{B}}(x)$

    (e) $\chi_{\mathcal{X}}(x) = \chi_{\mathcal{A}}(x) + \chi_{\mathcal{B}}(x) - 2\,\chi_{\mathcal{A}}(x) \cdot \chi_{\mathcal{B}}(x)$

    (f) $\chi_{\mathcal{X}}(x) = \chi_{\mathcal{A}}(x) \cdot \chi_{\mathcal{C}}(x) + \chi_{\mathcal{B}}(x) \cdot \chi_{\mathcal{C}}(x) - \chi_{\mathcal{A}}(x) \cdot \chi_{\mathcal{B}}(x) \cdot \chi_{\mathcal{A}}(x)$

    (g) $\chi_{\mathcal{X}}(x) = \chi_{\mathcal{A}}(x) \cdot \chi_{\mathcal{B}}(x)$

    (h) $\chi_{\mathcal{X}}(x) = 0$

## 7.2 Realna števila in matematična indukcija

3. Neenakosti, ki veljajo za vse $0 < a < b < 1$

   (a) ne
   (b) ne
   (c) da
   (d) ne
   (e) da
   (f) ne
   (g) ne
   (h) da
   (i) da
   (j) ne
   (k) ne
   (l) ne
   (m) da
   (n) da
   (o) ne
   (p) ne

7. Podmnožice realnih števil

   (a) $(1, \infty)$
   (b) $[-1, \infty)$
   (c) $(-\infty, 3)$
   (d) $[2, 3)$
   (e) $\emptyset$
   (f) $(-2, 5]$
   (g) $(-2, 2)$
   (h) $(2, 3)$
   (i) $(-\infty, -1) \bigcup (1, \infty)$
   (j) $(-\infty, 0) \bigcup \{1\}$

8. Podmnožice realnih števil

   (a) $\{1\}$
   (b) $[-1, 1]$
   (c) $[0, \infty)$
   (d) $(-\infty, -2) \bigcup (-2, 2)$
   (e) $(-\infty, \frac{1}{2})$
   (f) $(-3, -1) \bigcup (1, 3)$
   (g) $\emptyset$
   (h) $\{0, 2\}$
   (i) $\emptyset$
   (j) $(3, 4)$
   (k) $(-\frac{1}{2}, \infty)$
   (l) $(-2, \infty)$
   (m) $(0, 1)$
   (n) $(\frac{1}{2}, 0) \bigcup (0, \infty)$

9. Podmnožice realnih števil

(a) $\begin{cases} (-\infty, 2a) & a > 1, \\ \emptyset & a = 1, \\ (2a, \infty) & a < 1. \end{cases}$

(b) $\begin{cases} (2ab, \infty) & a > b, \\ \emptyset & a = b, \\ (-\infty, 2ab) & a < b. \end{cases}$

(c) $\begin{cases} \left(\frac{1}{a+3}, \infty\right) & a > 2 \vee a < -3, \\ \emptyset & a = 2, \\ (-\infty, \infty) & a = -3, \\ \left(-\infty, \frac{1}{a+3}\right) & -3 < a < 2. \end{cases}$

(d) $\{-2a, 2a\}$

10. Podmnožice realne ravnine:

(a)

(c)

(b)

(d)

(e)

(f)

11. Podmnožice realne ravnine:

(a)

(c)

(b)

(d)

(e)

(g)

(f)

(h)

12. Podmnožice realne ravnine:

(a)

(b)

(c)

(d)

(e)

(f)

13. Neenačbe in enačbe

(a) $(-\infty, -1) \cup (1, \infty) \cup \{0\}$

(b) $(\frac{16}{9}, \infty)$

(c) $(-2, \infty)$

(d) $(-1, 1 + \sqrt{2})$

(e) $\{3\}$

(f) $[\frac{1}{2}, 1]$

(g) $\{2, -2\}$

(h) $[-1, 3)$

(i) $(-2, 0]$

(j) $\begin{cases} \left(-\infty, \frac{b+3}{a+2}\right) & a < -2 \\ \left(\frac{b+3}{a+2}, \infty\right) & a > -2 \\ \emptyset & a = -2 \wedge b \geq -3 \\ (-\infty, \infty) & a = -2 \wedge b < -3 \end{cases}$

(k) $\begin{cases} (-\infty, \infty) & |a| < 2 \\ x \neq -a & |a| = 2 \\ (\infty, -a - \sqrt{a^2 - 1}) \cup (-a + \sqrt{a^2 - 1}, \infty) & |a| > 2 \end{cases}$

(l) $x^2 + 2\,\dfrac{5 - a}{8} + 1 > 0$

(m) $(-\infty, \infty), \quad 2 < a < 4$

(n) $\{\frac{1}{2}\}$

## 7.3 Kompleksna števila

1. Reziltati izračunaj

   (a) $-11-2\,i$

   (b) $-4$

   (c) $-\dfrac{3}{5}-\dfrac{4\,i}{5}$

   (d) $-\dfrac{24}{169}+\dfrac{10\,i}{169}$

   (e) $-\dfrac{2}{5}-\dfrac{3\,i}{5}$

   (f) $1+2\,i$

   (g) $\dfrac{1}{16}$

   (h) $2$

   (i) $64$

   (j) $512\left(1-i\sqrt{3}\right)$

   (k) $0$

   (l) $-i$

   (m) $\dfrac{(1+i)\left(x^2+2\,x\,y-y^2\right)}{x^2+y^2}$

   (n) $i$

2. $\mathrm{Re}(w)$, $\mathrm{Im}(w)$, $|w|$ in $\overline{w}$

   (a) $\mathrm{Re}(w)=4$ $\quad \mathrm{Im}(w)=2$
   $|w|=2\sqrt{5}$ $\quad \overline{w}=4-2\,i$

   (b) $\mathrm{Re}(w)=-8$ $\quad \mathrm{Im}(w)=-6$
   $|w|=10$ $\quad \overline{w}=-8+6\,i$

   (c) $\mathrm{Re}(w)=0$ $\quad \mathrm{Im}(w)=-1$
   $|w|=1$ $\quad \overline{w}=i$

   (d) $\mathrm{Re}(w)=-\dfrac{3}{25}$ $\quad \mathrm{Im}(w)=-\dfrac{4}{25}$
   $|w|=\dfrac{1}{5}$ $\quad \overline{w}=-\dfrac{3}{25}+\dfrac{4\,i}{25}$

   (e) $\mathrm{Re}(w)=1$ $\quad \mathrm{Im}(w)=-1$
   $|w|=\sqrt{2}$ $\quad \overline{w}=1+i$

   (f) $\mathrm{Re}(w)=1$ $\quad \mathrm{Im}(w)=-1$
   $|w|=\sqrt{2}$ $\quad \overline{w}=1+i$

   (g) $\mathrm{Re}(w)=0$ $\quad \mathrm{Im}(w)=2$
   $|w|=2$ $\quad \overline{w}=-2\,i$

   (h) $\mathrm{Re}(w)=-\dfrac{6}{5}$ $\quad \mathrm{Im}(w)=-\dfrac{12}{5}$
   $|w|=\dfrac{6}{\sqrt{5}}$ $\quad \overline{w}=-\dfrac{6}{5}+\dfrac{12\,i}{5}$

(i) $\mathrm{Re}(w) = -8$ $\quad \mathrm{Im}(w) = 0$

$|w| = 8$ $\quad \overline{w} = -8$

(j) $\mathrm{Re}(w) = \sqrt{x^2 + y^2}$ $\quad \mathrm{Im}(w) = 0$

$|w| = \sqrt{x^2 + y^2}$ $\quad \overline{w} = \sqrt{x^2 + y^2}$

(k) $\mathrm{Re}(w) = -y$ $\quad \mathrm{Im}(w) = x$

$|w| = \sqrt{x^2 + y^2}$ $\quad \overline{w} = -i\,x - y$

(l) $\mathrm{Re}(w) = x^2 - y^2$ $\quad \mathrm{Im}(w) = 2\,x\,y$

$|w| = \left(x^2 + y^2\right)$ $\quad \overline{w} = -(i\,x + y)^2$

(m) $\mathrm{Re}(w) = x^2 - y^2$ $\quad \mathrm{Im}(w) = -2\,x\,y$

$|w| = \left(x^2 + y^2\right)$ $\quad \overline{w} = -(-i\,x + y)^2$

(n) $\mathrm{Re}(w) = \dfrac{x}{x^2 + y^2}$ $\quad \mathrm{Im}(w) = -\dfrac{y}{x^2 + y^2}$

$|w| = \sqrt{\dfrac{1}{x^2 + y^2}}$ $\quad \overline{w} = \dfrac{i}{i\,x + y}$

(o) $\mathrm{Re}(w) = \dfrac{x}{\sqrt{x^2 + y^2}}$ $\quad \mathrm{Im}(w) = -\dfrac{y}{\sqrt{x^2 + y^2}}$

$|w| = 1$ $\quad \overline{w} = \dfrac{x + i\,y}{\sqrt{x^2 + y^2}}$

(p) $\mathrm{Re}(w) = x\,\sqrt{x^2 + y^2}$ $\quad \mathrm{Im}(w) = y\,\sqrt{x^2 + y^2}$

$|w| = x^2 + y^2$ $\quad \overline{w} = (x - i\,y)\,\sqrt{x^2 + y^2}$

(q) $\mathrm{Re}(w) = x^2 + y^2$ $\quad \mathrm{Im}(w) = 0$

$|w| = x^2 + y^2$ $\quad \overline{w} = x^2 + y^2$

(r) $\mathrm{Re}(w) = \dfrac{x^2 - y^2}{x^2 + y^2}$ $\quad \mathrm{Im}(w) = \dfrac{-2\,x\,y}{x^2 + y^2}$

$|w| = 1$ $\quad \overline{w} = \dfrac{i\,x - y}{i\,x + y}$

3. $\mathrm{Re}(w)$ in $\mathrm{Im}(w)$

(a) $\mathrm{Re}(w) = x$ $\quad \mathrm{Im}(w) = 0$

(b) $\mathrm{Re}(w) = 0$ $\quad \mathrm{Im}(w) = y$

(c) $\mathrm{Re}(w) = 4 + x^2 + 4\,y + y^2$ $\quad \mathrm{Im}(w) = 0$

(d) $\mathrm{Re}(w) = x + x^2 - y^2$ $\quad \mathrm{Im}(w) = y + 2\,x\,y$

(e) $\mathrm{Re}(w) = \dfrac{x + x^2 + y^2}{1 + 2\,x + x^2 + y^2}$ $\quad \mathrm{Im}(w) = \dfrac{y}{1 + 2\,x + x^2 + y^2}$

(f) $\mathrm{Re}(w) = x + \dfrac{x}{x^2+y^2}$ $\quad \mathrm{Im}(w) = y - \dfrac{y}{x^2+y^2}$

(g) $\mathrm{Re}(w) = x^2 - y^2 + \dfrac{x^2-y^2}{\left(x^2+y^2\right)^2}$ $\quad \mathrm{Im}(w) = 2\,x\,y - \dfrac{2\,x\,y}{\left(x^2+y^2\right)^2}$

(h) $\mathrm{Re}(w) = x^3 + x\,y^2$ $\quad \mathrm{Im}(w) = x^2\,y + y^3$

(i) $\mathrm{Re}(w) = x^3 + x\,y^2$ $\quad \mathrm{Im}(w) = -\left(y\,\left(x^2+y^2\right)\right)$

(j) $\mathrm{Re}(w) = x^3 - 3\,x\,y^2$ $\quad \mathrm{Im}(w) = 3\,x^2\,y - y^3$

(k) $\mathrm{Re}(w) = x^4 - 6\,x^2\,y^2 + y^4$ $\quad \mathrm{Im}(w) = 4\,x\,y\,\left(x^2-y^2\right)$

(l) $\mathrm{Re}(w) = x^3 + y - 3\,x\,y^2$ $\quad \mathrm{Im}(w) = -x + 3\,x^2\,y - y^3$

4. Kompleksna števila v polarni obliki

(a) $z = 2\,e^{i\pi}$

(b) $z = 2\,e^{i\frac{\pi}{2}}$

(c) $z = 2\,e^{i\frac{3\,\pi}{4}}$

(d) $z = 2\,\sqrt{2}e^{i\frac{\pi}{4}}$

(e) $z = e^{i\frac{-\pi}{2}}$

(f) $z = 2\,e^{i\frac{2\,\pi}{3}}$

(g) $z = 4\,e^{i\frac{2\,\pi}{3}}$

(h) $z = 2\,e^{-i\frac{2\,\pi}{3}}$

(i) $z = 5\,\sqrt{2}e^{-i\frac{3\,\pi}{4}}$

(j) $z = 2\,\sqrt{3}e^{-i\frac{\pi}{6}}$

(k) $z = 5\,e^{i\,\arctan\frac{3}{4}}$

(l) $z = 5\,\sqrt{2}e^{i(\pi+\arctan 7)}$

(m) $z = \sqrt{34}e^{i(\pi-\arctan\frac{3}{5})}$

(n) $z = e^{-i\frac{\pi}{2}}$

5. Rešitve enačb

(a) $\{-1+i, -1+i\}$

(b) $\{0, i, \dfrac{-i-\sqrt{3}}{2}, \dfrac{-i+\sqrt{3}}{2}\}$

(c) $\{0, \dfrac{-1-i}{\sqrt{2}}, \dfrac{1+i}{\sqrt{2}}\}$

(d) $\{0, 1, -\dfrac{1}{2} - \dfrac{i}{2}\,\sqrt{3}, -\dfrac{1}{2} + \dfrac{i}{2}\,\sqrt{3}\}$

(e) $\{\dfrac{2}{5} + \dfrac{i}{5}\}$

(f) $\{1\}$

(g) $\{1\}$

(h) { ni rešitve }

(i) { ni rešitve }

(j) $\{\dfrac{3}{4} + i\}$

(k) $\{\dfrac{1+i}{\sqrt{2}}, \dfrac{-1-i}{\sqrt{2}}\}$

(l) $\{2+i, -2-i\}$

(m) $\{1+i, 2-3\,i\}$

(n) $\{(-1+i)\,\sqrt{3}, (1-i)\,\sqrt{3}\}$

6. Rešitve enačb

(a) $\{-1, \dfrac{1-i\,\sqrt{3}}{2}, \dfrac{1+i\,\sqrt{3}}{2}\}$

(b) $\{1, \frac{-1-i\sqrt{3}}{2}, \frac{-1+i\sqrt{3}}{2}\}$

(c) $\{\sqrt[3]{2}e^{i\,(\frac{\pi}{4}+\frac{k\,2\pi}{3})}; k=0,1,2\}$

(d) $\{\sqrt{2}\,e^{i\,(\frac{\pi}{4}+\frac{k\,2\pi}{3})}; k=0,1,2\}$

(e) $\{-1+i, \frac{-2-i-\sqrt{3}}{2}, \frac{-2-i+\sqrt{3}}{2}\}$

(f) $\{-i, \frac{i-\sqrt{3}}{2}, \frac{i+\sqrt{3}}{2}\}$

(g) $\{\frac{-1-i}{\sqrt{2}}, \frac{-1+i}{\sqrt{2}}, \frac{1-i}{\sqrt{2}}, \frac{1+i}{\sqrt{2}}\}$

(h) $\{-2, 2, -2\,i, 2\,i\}$

(i) $\{-1+i\sqrt{3}, 1-i\sqrt{3}, -i-\sqrt{3}, i+\sqrt{3}\}$

(j) $\{-1+i\sqrt{3}, 1-i\sqrt{3}, -i-\sqrt{3}, i+\sqrt{3}\}$

(k) $\{-3\,i, -3\,i, 3\,i, 3\,i, -3\,i, -3\,i, 3\,i, 3\,i\}$

(l) $\{-4-2\,i, 1-2\,i, -4+2\,i, 1+2\,i\}$

(m) $\{-1, \frac{-1-i\sqrt{3}}{2}, \frac{-1+i\sqrt{3}}{2}, \frac{1-i\sqrt{3}}{2}, \frac{1+i\sqrt{3}}{2}, 1\}$

(n) $\{-1, 1, \frac{-1-i}{\sqrt{2}}, \frac{-1+i}{\sqrt{2}}, \frac{1-i}{\sqrt{2}}, \frac{1+i}{\sqrt{2}}, -i, i\}$

(o) $\{-1, -\frac{1}{2}-\frac{i}{2}\sqrt{3}, \frac{i}{2}\left(i+\sqrt{3}\right), 0, 0, \frac{1-i\sqrt{3}}{2}, \frac{1+i\sqrt{3}}{2}, 1\}$

(p) $\{2\sqrt[3]{4}, -i\sqrt[3]{4}\left(-i+\sqrt{3}\right), i\sqrt[3]{4}\left(i+\sqrt{3}\right)\}$

7. Rešitve sistema enačb

(a) $\{-\frac{1}{2}-\frac{i}{2}\}$

(b) $\{\frac{1}{2}+\frac{i\sqrt{3}}{2}, \frac{1}{2}-\frac{i\sqrt{3}}{2}\}$

(c) $\{-2-\frac{i}{2}\}$

(d) $\{-3+i, -2\}$

(e) $\{\frac{1}{2}+\frac{3\sqrt{3}\,i}{2}, \frac{1}{2}-\frac{3\sqrt{3}\,i}{2}\}$

(f) $\{-2, \frac{6}{5}+\frac{2\,i}{5}\}$

(g) $\{2+2\,i, 1-i\}$

(h) $\{1, 1\}$

(i) $\{\{(1+i)\sqrt{2}, 1+i\}, \{-(1+i)\sqrt{2}, -(1+i)\}\}$

8. Podmnožice kompleksne ravnine

(a)

$$x - 2 + 2\,x\,y = 0$$

(b)

$$x^2 + y^2 + y = 0 \wedge x \neq 0$$

(c)

$$x^2 + y^2 + 2\,x + 2\,y = 4$$

(d)

$$x^2 + y^2 + 4\,x - 2\,y = 2$$

(e)

$$4 + 2\,x - 2\,y = 0$$

(f)

$$x^2 - y^2 + x = 0$$

(g)

$\sqrt{3}\,x + y = 1 \vee \sqrt{3}\,x - y = -1$

(h)

$y^2 = 1 - 2\,x$

(i)

$x = y$

(j)

*elipsa*

(k)

*elipsa*

(l)

$x^2 + y^2 - \frac{34}{15}\,x + 1 = 0$

9. Preslikave z linearno funkacjo:

(a) $w = -(1+i)\,z + 1 + i$

(b) $w = \dfrac{i\sqrt{2}}{2}\,(2\,z - 1 - i)$

(c) $w = \dfrac{1}{3}\,(z - 1 + i)$

(d) $w = -z + 1 - i$

(e) $w = 2\,z$

(f) $w = -z + 5$

10. Podmnožice kompleksne ravnine

(a)

$0 \leq y \leq 1$

(b)

$0 \leq x\,y \leq 1$

(c)

$\frac{x^2+y^2}{4} \leq x + y \leq \frac{x^2+y^2}{2}$

(d)

$x^2 + (y-1)^2 < 9$

(e)

$$1 \leq (x-1)^2 + y - 2)^2 \leq 4$$

(f)

$$\arg(x + i\,y)$$

(g)

$$x < y$$

(h)

$$y^2 \geq 1 - 2\,x$$

11. Krivulje v kompleksni ravnini

(a)

$$\begin{aligned} x &= 2\,t \\ y &= -t \end{aligned}$$

(b)

$$\begin{aligned} x &= t \\ y &= -1 + 3\,t^2 \end{aligned}$$

(c)

$$\begin{aligned} x &= \tfrac{1}{1+t^2} - \tfrac{t^2}{1+t^2} \\ y &= \tfrac{2\,t}{1+t^2} \end{aligned}$$

(d)

$$\begin{aligned} x &= 3\,\cos t \\ y &= \sin t \end{aligned}$$

(e)

$$\begin{aligned} x &= 1 + \cos t \\ y &= 1 + \sin t \end{aligned}$$

(f)

$$\begin{aligned} x &= 2 + t\,\cos t \\ y &= 1 + t\,\sin t \end{aligned}$$

12. Družine krivulj v kompleksni ravnini

(a)

$$\begin{aligned}\mathrm{Re}(w) &= x\\ \mathrm{Im}(w) &= y\end{aligned}$$

(b)

$$\begin{aligned}\mathrm{Re}(w) &= x - y\\ \mathrm{Im}(w) &= x + y\end{aligned}$$

(c)

$$\begin{aligned}\mathrm{Re}(w) &= \tfrac{x}{2} - y\\ \mathrm{Im}(w) &= x + \tfrac{y}{2}\end{aligned}$$

(d)

$$\begin{aligned}\mathrm{Re}(w) &= x^2 - y^2\\ \mathrm{Im}(w) &= 2\,x\,y\end{aligned}$$

(e)

$$\begin{aligned}\mathrm{Re}(w) &= x + \tfrac{x^2}{2} - 2\,x\,y - \tfrac{y^2}{2}\\ \mathrm{Im}(w) &= x^2 + y + x\,y - y^2\end{aligned}$$

(f)

$$\begin{aligned}\mathrm{Re}(w) &= \tfrac{x}{x^2+y^2}\\ \mathrm{Im}(w) &= -\tfrac{y}{x^2+y^2}\end{aligned}$$

(g)

$$\begin{aligned}\mathrm{Re}(w) &= \frac{2\,x-2\,x^2-y-2\,y^2}{1-2\,x+x^2+y^2}\\ \mathrm{Im}(w) &= \frac{1-x+2\,y}{1-2\,x+x^2+y^2}\end{aligned}$$

(h)

$$\begin{aligned}\mathrm{Re}(w) &= \frac{x^2-y^2}{(x^2+y^2)^2}\\ \mathrm{Im}(w) &= \frac{-2\,x\,y}{(x^2+y^2)^2}\end{aligned}$$

(i)

$$\begin{aligned}\mathrm{Re}(w) &= x+\frac{x}{x^2+y^2}\\ \mathrm{Im}(w) &= y-\frac{y}{x^2+y^2}\end{aligned}$$

(j)

$$\begin{aligned}\mathrm{Re}(w) &= e^x\,\cos y\\ \mathrm{Im}(w) &= e^x\,\sin y\end{aligned}$$

## 7.4 Zaporedja

1. Monotonost zaporedij

   (a) padajoče

   (b) padajoče $n \geq 4$

   (c) naraščajoče

   (d) ni monotono

   (e) konstantno

   (f) padajoče $n \geq 100$

   (g) padajoče $n \geq 19$

   (h) padajoče $n \geq 100$

   (i) padajoče $n \geq 2$

   (j) naraščajoče

   (k) naraščajoče $n \geq 20$

   (l) naraščajoče

   (m) padajoče

   (n) padajoče $n \geq 19$

   (o) naraščajoče

   (p) padajoče

   (q) naraščajoče

   (r) ni monotono

2. Največji in najmanjši člen zaporedja

   (a) $\max\{a_n\} = a_1$

   (b) $\max\{a_n\} = a_2\,, \quad \min\{a_n\} = a_1$

   (c) $\max\{a_n\} = a_1$

   (d) $\max\{a_n\} = a_1 = a_2$

   (e) $\max\{a_n\} = a_{1000}$

   (f) $\max\{a_n\} = a_{10}$

   (g) $\min\{a_n\} = a_1$

   (h) $\min\{a_n\} = a_1$

   (i) $\min\{a_n\} = a_4 = a_5$

   (j) $\min\{a_n\} = a_{10}$

   (k) nima

   (l) $\min\{a_n\} = a_1$

   (m) $\min\{a_n\} = a_1$

   (n) nima

   (o) $a_n = 0$

   (p) nima

   (q) $\max\{a_n\} = a_2\,, \quad \min\{a_n\} = a_1$

   (r) $\max\{a_n\} = a_{19}$

   (s) $\max\{a_n\} = a_1$

   (t) $\max\{a_n\} = a_{4k+1}\,, \quad \min\{a_n\} = a_{4k+3}\,, \; k \in \mathbb{N}$

5. Limite zaporedij

(a) $-1$
(b) $0$
(c) $0$
(d) $\frac{1}{2}$
(e) $0$
(f) $\frac{1}{2}$
(g) $0$
(h) $\frac{1}{3}$
(i) $3$
(j) $1$
(k) $\frac{1}{2}$
(l) $1$
(m) $e$
(n) $\frac{1}{e}$
(o) $e^2$
(p) $1$
(q) $e^2$
(r) $e^5$

6. Limite zaporedij

(a) $\frac{1}{2}$
(b) $\frac{1}{2}$
(c) $\frac{1}{3}$
(d) $\frac{4}{3}$
(e) $3$
(f) $1$
(g) $\frac{1}{3}$
(h) $\infty$

7. $\varepsilon$-okolica limite

(a) $a_8$
(b) $a_6$
(c) $a_{51}$
(d) $a_{53}$
(e) $n > \lg_c(1+\varepsilon)$
(f) $a_{20}$

8. Stekališča zaporedij

(a) $\{0, 1\}$
(b) $\{-2, 2\}$
(c) $\{0\}$
(d) $\{0, 1, 2\}$
(e) $\{0, 1\}$
(f) $[0, 1]$
(g) $\{0, \frac{1}{2}, 1\}$
(h) $\{-1, 1\}$
(i) $\{0, \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}\}$
(j) $\{0, \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{\sqrt{8}}, -\frac{\sqrt{3}}{\sqrt{8}}\}$

9. Konvergentnost zaporedij

(a) konvergira
(b) ne konvergira
(c) ne konvergira
(d) konvergira
(e) ne konvergira
(f) konvergira
(g) ne konvergira
(h) konvergira
(i) konvergira
(j) ne konvergira
(k) konvergira
(l) konvergira
(m) ne konvergira
(n) konvergira
(o) ne konvergira
(p) konvergira

10. Limite monotonih zaporedij

(a) 2

(b) 2

(c) $\frac{1+\sqrt{1+4\,c}}{2}$

(d) 2

(e) $\sqrt{3}$

11. Limite zaporedij:

(a) 3

(b) $\frac{1+\sqrt{5}}{2}$

12. Konvergenca rekurzivno podanih zaporedij v odvisnosti od prvega člena.

(a) $x_1 \in [0, 1]$

(b) $x_1 \in [-3, 1]$

(c) $x_1 \in [-\infty, \infty]$

(d) $x_1 \in \{-1, \frac{1}{2}\}$

13. Konvergenca v odvisnosti od parametra $\alpha$.

(a) $\alpha \in [0, 3]$

(b) $|\alpha| \geq 2$

15. Natančna zgornja in natančna spodnja meja

(a) $\inf\{a_n\} = \frac{1}{2}$ $\sup\{a_n\} = \frac{5}{4}$

(b) $\inf\{a_n\} = -1$ $\sup\{a_n\} = \frac{3}{2}$

(c) nima

(d) $\inf\{a_n\} = 0$ $\sup\{a_n\} = a_{19}$

(e) $\inf\{a_n\} = -3$ $\sup\{a_n\} = 3$

(f) $\inf\{a_n\} = \dfrac{1}{5}$ $\sup\{a_n\} = 1$

(g) $\inf\{a_n\} = 0$ $\sup\{a_n\} = \dfrac{9}{8}$

(h) $\inf\{a_n\} = -2$ $\sup\{a_n\} = \dfrac{3}{2}$

## 7.5 Številske vrste

1. Vsota vrste

   (a) 1
   (b) $\frac{1}{3}$
   (c) $\frac{1}{k+1}$
   (d) 1
   (e) 1
   (f) $\frac{3}{4}$
   (g) $\frac{1}{6}$
   (h) $1 - sqrt2$
   (i) 1
   (j) $\frac{q}{1-q}$

2. Vsota geometrijske vrste

   (a) 4
   (b) $\frac{37}{99}$
   (c) $\frac{2}{3}$
   (d) ne obstaja

3. Vsota vrste

   (a) $\frac{q\,(\cos\alpha - q)}{1 - 2\,q\,cos\alpha + q^2} \quad |q| < 1$
   (b) $\frac{q\,\sin\alpha}{1 - 2\,q\,cos\alpha + q^2} \quad |q| < 1$

4. Decimalno število v obliki ulomka

   (a) $\frac{23}{99}$
   (b) $\frac{5141}{999}$
   (c) $\frac{32362}{9999}$
   (d) $\frac{271826}{99999}$
   (e) $\frac{1}{9}$
   (f) 1

5. Kvocientni kriterij

   (a) konvergira
   (b) konvergira
   (c) konvergira
   (d) divergira
   (e) konvergira
   (f) konvergira
   (g) divergira
   (h) konvergira
   (i) konvergira
   (j) divergira
   (k) konvergira
   (l) konvergira

6. Korenski kriterij

   (a) konvergira
   (b) divergira
   (c) konvergira
   (d) divergira
   (e) konvergira
   (f) konvergira
   (g) konvergira
   (h) divergira

7. Konvergenca vrste z minoranto oziroma majoranto

(a) divergira
(b) divergira
(c) konvergira
(d) divergira
(e) konvergira
(f) konvergira
(g) konvergira
(h) konvergira

8. Konvergenco vrste s pozitivnimi členi

(a) divergira
(b) divergira
(c) konvergira
(d) konvergira
(e) konvergira
(f) konvergira
(g) divergira
(h) konvergira
(i) konvergira
(j) konvergira
(k) konvergira
(l) konvergira
(m) konvergira
(n) konvergira
(o) divergira
(p) divergira

9. Konvergenca vrste s pozitivnimi členi

(a) divergira
(b) konvergira
(c) konvergira
(d) konvergira
(e) konvergira
(f) konvergira za $q < 1$
(g) konvergira
(h) divergira

10. Konvergenca alternirajoče vrste

(a) pogojno
(b) absolutno konvergira
(c) $\alpha > 1$ absolutno $0 < \alpha \leq 1$ pogojno
(d) absolutno
(e) absolutno
(f) $a > e$ absolutno $1 < a \leq e$ pogojno
(g) absolutno
(h) absolutno

11. Konvergenca vrste

(a) divergira
(b) pogojno
(c) $\alpha > 1$ absolutno $0 < \alpha \leq 1$ pogojno
(d) divergira
(e) absolutno
(f) pogojno
(g) absolutno
(h) divergira

12. Konvergenco alternirajoče vrste:

(a) pogojno
(b) absolutno
(c) divergira
(d) divergira

13. $\sum_{n=1}^{\infty} \frac{1+(-1)^n}{3^n}$, absolutno konvergira.

14. $\sum_{n=1}^{\infty} \frac{1}{2\,n\,(2\,n-1)}$, absolutno konvergira.

15. je končno število.

## 7.6 Funkcije

1. Naj bosta $\mathcal{A}$ in $\mathcal{B}$ končni množici z $m$ in $n$ elementi:
   (a) vseh funkcij, ki množico $\mathcal{A}$ preslikajo v množico $\mathcal{B}$, je $n^m$
   (b) injektivnih je 0, če je $m > n$ in $\frac{n!}{(n-m)!}$, če je $m \leq n$
   (c) surjektivnih je 0, če je $m < n$ in
   $n^m - \binom{n}{1}(n-1)^m + \binom{n}{2}(n-2)^m - \ldots (-1)^{n-1}\binom{n}{n-1}$, če je $m \geq n$
   (d) bijektivnih je 0, če je $m \neq n$ in $m!$, če je $m = n$

2. Ali predstavlja množica urejenih parov $\mathcal{G}$ graf funkcije?

   (a) ne (b) da (c) da (d) ne (e) ne (f) da

3. Bijektivne funkcije

   (a) da (b) ne (c) ne (d) ne (e) da (f) da

4. Kompozitumi $f(g)$, $g(f)$, $f(f)$ in $g(g)$.
   (a) $\mathcal{G}_f(g) = \{(1,1)(2,4)(3,3)(4,1)(5,2)\}$ $\mathcal{G}_g(f) = \{(1,1)(2,3)(3,3)(4,2)(5,3)\}$
   $\mathcal{G}_f(f) = \{(1,1)(2,2)(3,3)(4,4)(5,2)\}$ $\mathcal{G}_g(g) = \{(1,1)(2,2)(3,3)(4,1)(5,3)\}$
   (b) $\mathcal{G}_f(g) = \{(1,3)(2,3)(3,3)(4,3)(5,1)\}$ $\mathcal{G}_g(f) = \{(1,1)(2,1)(3,2)(4,2)(5,2)\}$
   $\mathcal{G}_f(f) = \{(1,1)(2,1)(3,3)(4,3)(5,3)\}$ $\mathcal{G}_g(g) = \{(1,2)(2,2)(3,2)(4,2)(5,1)\}$
   (c) $\mathcal{G}_f(g) = \{(1,1)(2,4)(3,2)(4,5)(5,3)\}$ $\mathcal{G}_g(f) = \{(1,5)(2,4)(3,3)(4,1)(5,2)\}$
   $\mathcal{G}_f(f) = \{(1,1)(2,5)(3,3)(4,2)(5,4)\}$ $\mathcal{G}_g(g) = \{(1,5)(2,2)(3,1)(4,4)(5,3)\}$
   (d) $\mathcal{G}_f(g) = \{(1,2)(2,2)(3,2)(4,2)(5,2)\}$ $\mathcal{G}_g(f) = \{(1,3)(2,3)(3,3)(4,3)(5,3)\}$
   $\mathcal{G}_f(f) = \{(1,2)(2,3)(3,1)(4,1)(5,3)\}$ $\mathcal{G}_g(g) = \{(1,3)(2,3)(3,3)(4,3)(5,3)\}$
   (e) $\mathcal{G}_f(g) = \{(1,2)(2,4)(3,2)(4,4)(5,1)\}$ $\mathcal{G}_g(f) = \{(1,2)(2,2)(3,5)(4,2)(5,1)\}$
   $\mathcal{G}_f(f) = \{(1,4)(2,2)(3,1)(4,4)(5,2)\}$ $\mathcal{G}_g(g) = \{(1,1)(2,2)(3,1)(4,2)(5,5)\}$
   (f) $\mathcal{G}_f(g) = \{(1,5)(2,4)(3,3)(4,2)(5,1)\}$ $\mathcal{G}_g(f) = \{(1,5)(2,4)(3,3)(4,2)(5,1)\}$
   $\mathcal{G}_f(f) = \{(1,1)(2,2)(3,3)(4,4)(5,5)\}$ $\mathcal{G}_g(g) = \{(1,1)(2,2)(3,3)(4,4)(5,5)\}$

5. Definicijsko območje $\mathcal{D}_f$ funkcije $f(x)$

   (a) $\mathcal{D}_f = \{x : x \neq \frac{1}{2}\}$

   (b) $\mathcal{D}_f = \{x : x \leq \frac{1}{2}\}$

   (c) $\mathcal{D}_f = \{x : -2 \leq x < 0 \vee 0 < x \leq 2\}$

   (d) $\mathcal{D}_f = \{x : x \neq 0\}$

   (e) $\mathcal{D}_f = \{x : x > 0\}$

   (f) $\mathcal{D}_f = \{x : x < -1 \vee 1 < x\}$

(g) $\mathcal{D}_f = \{x : 1 < x\}$

(h) $\mathcal{D}_f = \{x : -1 < x < 0\}$

(i) $\mathcal{D}_f = \{x : x > 4\}$

(j) $\mathcal{D}_f = \{x : \frac{1}{3} \leq x \leq 1\}$

(k) $\mathcal{D}_f = \{x : \frac{\pi}{3} + k2\pi \leq x \leq \frac{5\pi}{3} + k2\pi; k = 0, \pm 1, \pm 2, \ldots\}$

(l) $\mathcal{D}_f = \{x : -1 \leq x < 1\}$

(m) $\mathcal{D}_f = \{x : x = 1\}$

(n) $\mathcal{D}_f = \{x : \frac{1}{2} \leq x\}$

(o) $\mathcal{D}_f = \{x : (x \neq 0) \wedge (x \neq -1)\}$

(p) $\mathcal{D}_f = \{x : -\infty < x < \infty\}$

(q) $\mathcal{D}_f = \{x : -1 \leq x \leq 1\}$

(r) $\mathcal{D}_f = \{x : -\infty < x < \infty\}$

(s) $\mathcal{D}_f = \{x : 1 < x \leq 2\}$

(t) $\mathcal{D}_f = \emptyset$

(u) $\mathcal{D}_f = \{x : x > 0\}$

(v) $\mathcal{D}_f = \{x : 0 < x < 2\}$

6. Definicijsko območje $\mathcal{D}_f$ in zaloga vrednosti $\mathcal{R}_f$ funkcije $f(x)$

(a) $\mathcal{D}_f = \{x : -\infty < x < \infty\}$ $\mathcal{R}_f = \{y : y \leq 3\}$

(b) $\mathcal{D}_f = \{x : -\infty < x < \infty\}$ $\mathcal{R}_f = \{y : y \leq 1\}$

(c) $\mathcal{D}_f = \{x : -\infty < x < \infty\}$ $\mathcal{R}_f = \{y : 1 \leq y\}$

(d) $\mathcal{D}_f = \{x : -\infty < x < \infty\}$ $\mathcal{R}_f = \{y : 0 < y \leq 1\}$

(e) $\mathcal{D}_f = \{x : -\infty < x < \infty\}$ $\mathcal{R}_f = \{y : -1 \leq y \leq 1\}$

(f) $\mathcal{D}_f = \{x : x \neq 0\}$ $\mathcal{R}_f = \{y : 0 < y < 1\}$

(g) $\mathcal{D}_f = \{x : -\infty < x < \infty\}$ $\mathcal{R}_f = \{y : 0 \leq y\}$

(h) $\mathcal{D}_f = \{x : 0 \leq x\}$ $\mathcal{R}_f = \{y : 0 \leq y\}$

(i) $\mathcal{D}_f = \{x : x \neq 0\}$ $\mathcal{R}_f = \{y : y = 1\}$

(j) $\mathcal{D}_f = \{x : -\infty < x < \infty\}$ $\mathcal{R}_f = \{y : 1 \leq y\}$

(k) $\mathcal{D}_f = \{x : -\infty < x < \infty\}$ $\mathcal{R}_f = \{y : 0 \leq y\}$

(l) $\mathcal{D}_f = \{x : -\infty < x < \infty\}$ $\mathcal{R}_f = \{y : -\infty < y < \infty\}$

(m) $\mathcal{D}_f = \{x : x \neq 0\}$ $\mathcal{R}_f = \{y : 0 < y < 1\}$

(n) $\mathcal{D}_f = \{x : 0 < x\}$ $\mathcal{R}_f = \{y : 0 < y < \infty\}$

(o) $\mathcal{D}_f = \{x : -\infty < x < \infty\}$ $\mathcal{R}_f = \{y : -\infty < y < \infty\}$

(p) $\mathcal{D}_f = \{x : x \leq 0\}$ $\mathcal{R}_f = \{y : 0 < y \leq \frac{\pi}{2}\}$

(q) $\mathcal{D}_f = \{x : x \neq 0\}$ $\mathcal{R}_f = \{y : 0 < y < 1\}$

(r) $\mathcal{D}_f = \{x : x \neq 0\}$ $\mathcal{R}_f = \{y : 0 < y < \infty\}$

(s) $\mathcal{D}_f = \{x : x \neq 0\}$ $\mathcal{R}_f = \{y : -\frac{\pi}{2} \leq y < 0 \vee 0 < y \leq \frac{\pi}{2}\}$

(t) $\mathcal{D}_f = \{x : x \neq 0\}$ $\mathcal{R}_f = \{y : -\frac{\pi}{2} < y < -1 \vee 1 < y < \frac{\pi}{2}\}$

7. Definicijsko območje $\mathcal{D}_f$ in zaloga vrednosti $\mathcal{R}_f$ funkcije $f(x)$

(a) $\mathcal{D}_f = \{x : -\infty < x < \infty\}$ $\mathcal{R}_f = \{y : 1 \leq y\}$

(b) $\mathcal{D}_f = \{x : -\infty < x < \infty\}$ $\mathcal{R}_f = \{y : -2\,(2 + \sqrt{2}) \leq y\}$

(c) $\mathcal{D}_f = \{x : -1 \leq x \leq 2\}$ $\mathcal{R}_f = \{y : 0 \leq y \leq \frac{\sqrt{3}}{2}\}$

(d) $\mathcal{D}_f = \{x : x \neq 0\}$ $\mathcal{R}_f = \{y : y \leq -1 \vee 1 \leq y\}$

(e) $\mathcal{D}_f = \{x : -\infty < x < \infty\}$ $\mathcal{R}_f = \{y : 0 \leq y \leq \ln 2\}$

(f) $\mathcal{D}_f = \{x : x \neq 0\}$ $\mathcal{R}_f = \{y : 0 < y \leq 1\}$

(g) $\mathcal{D}_f = \{x : -\frac{1}{3} \leq x \leq 1\}$ $\mathcal{R}_f = \{y : 2 - \frac{\pi}{2} \leq y \leq 2 + \frac{\pi}{2}\}$

(h) $\mathcal{D}_f = \{x : \frac{-\pi}{6} + k2\pi \leq x \leq \frac{\pi}{6} + k2\pi; k = 0, \pm 1, \pm 2, \ldots\}$ $\mathcal{R}_f = \{y : 0 \leq y \leq \pi\}$

(i) $\mathcal{D}_f = \{x : -\frac{1}{3} \leq x \leq 1\}$ $\mathcal{R}_f = \{y : -\frac{\pi}{2} \leq y < \frac{\pi}{2}\}$

(j) $\mathcal{D}_f = \{x : x \neq 0\}$ $\mathcal{R}_f = \{y : 0 < y < 1\}$

8. Injektivnost, surjektivnost in bijektivnost funkcije $f(x)$

(a) bijektivna

(b) nič od tega

(c) injektivna

(d) surjektivna

(e) bijektivna

(f) nič od tega

(g) injektivna

(h) surjektivna

(i) nič od tega

(j) bijektivna

(k) nič od tega

(l) injektivna

(m) bijektivna

(n) nič od tega

(o) surjektivna

(p) injektivna

9. Primer: $(\mathcal{B} \subset \mathcal{A}) \ \wedge \ (f(\mathcal{A}-\mathcal{B}) \neq f(\mathcal{A}) - f(\mathcal{B}))$
   $f(x) = \sin x \mathcal{A} = [0, 2\pi], \mathcal{B} = [\frac{\pi}{2}, \ \frac{3\pi}{2}]$

   $f(\mathcal{A}-\mathcal{B}) = (-1, 1)$, $f(\mathcal{A}) - f(\mathcal{B}) = \emptyset$

11. Sode in lihe funkcije

(a) bijektivna

(b) soda

(c) surjektivna

(d) soda

(e) surjektivna

(f) surjektivna

(g) liha

(h) nič od tega

(i) liha

(j) soda

(k) liha, injektivna

(l) liha, bijektivna

(m) liha, bijektivna

(n) liha, injektivna

12. Sode in lihe funkcije

(a) liha

(b) liha

(c) nič od tega

(d) soda

(e) liha

(f) soda

(g) liha

(h) soda

(i) liha

(j) liha

(k) nič od tega

(l) liha

(m) nič od tega

(n) liha

(o) soda

(p) nič od tega
(q) soda
(r) nič od tega
(s) liha
(t) nič od tega
(u) liha
(v) liha
(w) soda
(x) liha
(y) soda
(z) liha

14. Soda funkcija ne more biti padajoča ali naraščajoča na intervalu, ki vsebuje izhodišče.

18. $(|x| - x)(|x| + x) = 0$

19. Funkcija $f(x) = g(x)$

(a) ne
(b) ne
(c) da
(d) da
(e) ne
(f) da
(g) da
(h) ne
(i) da
(j) ne
(k) ne
(l) ne
(m) ne
(n) da

20. Leva in desna limita

(a) definirana in zvezna povsod

(b) definirana in zvezna povsod

(c) $\lim_{x \nearrow 0} f(x) = -1$ $\lim_{x \searrow 0} f(x) = 1$

(d) $\lim_{x \nearrow 0} f(x) = 0$ $\lim_{x \searrow 0} f(x) = 0$

$\lim_{x \nearrow -1} f(x) = -\infty$ $\lim_{x \searrow -1} f(x) = \infty$

(e) $\lim_{x \nearrow 3} f(x) = -\infty$ $\lim_{x \searrow 3} f(x) = \infty$

(f) $\lim_{x \nearrow 0} f(x) = 0$ $\lim_{x \searrow 0} f(x) = 0$

(g) $\lim_{x \nearrow -2} f(x) = \infty$ $\lim_{x \searrow -2} f(x) = 0$

$\lim_{x \nearrow 2} f(x) = \infty$ $\lim_{x \searrow 2} f(x) = 0$

(h) $\lim_{x \nearrow 0} f(x) = 0$ $\lim_{x \searrow 0} f(x) = 0$

(i) $\lim_{x \nearrow 1} f(x) = -1$ $\lim_{x \searrow 1} f(x) = 1$

(j) $\lim_{x \nearrow 0} f(x) = 1$ $\lim_{x \searrow 0} f(x) = 0$

(k) $\lim_{x \nearrow 1} f(x) = \frac{\pi}{2}$ $\lim_{x \searrow 1} f(x) = \frac{\pi}{2}$

(l) $\lim_{x \nearrow 1} f(x) = \frac{\pi}{2}$ $\lim_{x \searrow 1} f(x) = -\frac{\pi}{2}$

21. Nezveznosti

(a) $\lim\limits_{x \nearrow 0} f(x) = n$ $\lim\limits_{x \searrow 0} f(x) = n$

(b) $\lim\limits_{x \nearrow \frac{5}{2}} f(x) = -1$ $\lim\limits_{x \searrow \frac{5}{2}} f(x) = -2$

(c) povsod zvezna

(d) $\lim\limits_{x \nearrow 0} f(x) = -\infty$ $\lim\limits_{x \searrow 0} f(x) = -\infty$

(e) nikjer zvezna funkcija

(f) nezvezna v racionalnih točkah

22. Funkcija $f(x)$ je zvezna, če je

(a) $a = 0$ (c) $a = 1$ (e) $a = 1$

(b) $a = 3$ (d) $a = 1$ (f) $a = 0$

23. Maksimum in minimum funkcije $f(x)$ na danem intervalu

(a) maksimum $= 16$ minimum $= 0$

(b) maksimum $= 28$ minimum $= 0$

(c) maksimum $= \frac{1}{2}$ minimum $= 0$

(d) maksimum $= 0$ minimum $= -1$

(e) maksimum $= 5$ minimum $= -2\sqrt{2} - 4$

(f) maksimum $= 7$ minimum $= -3$

(g) maksimum $= 2$ minimum $= -\frac{9}{4}$

(h) maksimum $= 17$ minimum $= 3$

(i) maksimum $= 1$ minimum $= \frac{1}{10}$

(j) maksimum $= \sqrt{2}$ minimum $= -2$

(k) maksimum $= \frac{1}{2}$ minimum $= -\frac{1}{2}$

(l) maksimum $= \frac{1}{\sqrt{2}}$ minimum $= -\frac{1}{\sqrt{2}}$

(m) maksimum $= 4$ minimum $= -1$

24. $\mathcal{A}' = f(\mathcal{A})$:

(a) $\mathcal{A}' = [0, 1]$

(b) $\mathcal{A}' = (-1, 1]$

(c) $\mathcal{A}' = [0, 1]$

(d) $\mathcal{A}' = [\frac{8}{3}, 8)$

(e) $\mathcal{A}' = (0, \frac{4}{e^2}]$

(f) $\mathcal{A}' = (0, 2)$

(g) $\mathcal{A}' = [-10, 8]$

(h) $\mathcal{A}' = (-5, \frac{61+14\sqrt{17}}{27})$

(i) $\mathcal{A}' = [0, 9]$

(j) $\mathcal{A}' = (-\frac{2}{3\sqrt{3}}, \sqrt{2})$

(k) $\mathcal{A}' = [0, 2\sqrt{3})$

(l) $\mathcal{A}' = [-\arctan 3, -\frac{\pi}{2}) \cap [-\arctan \frac{1}{3}, \frac{\pi}{2}]$

25. $\mathcal{A} = f^{-1}(\mathcal{A}')$

(a) $\mathcal{A} = (-2, -1] \cup [1, 2)$

(b) $\mathcal{A} = (-2, -1) \cup (2, \dfrac{1+\sqrt{17}}{2})$

(c) $\mathcal{A} = [2 - \sqrt{3}, 2) \cup (2, 2 + \sqrt{3})$

(d) $\mathcal{A} = (-\infty, 0) \cup (0, \infty)$

(e) $\mathcal{A} = (-\dfrac{1}{2}, 0) \cup (1, \dfrac{5}{3})$

(f) $\mathcal{A} = [0, \dfrac{4}{3})$

26. Kompozitumi $f(f(x))$, $f(g(x))$, $g(f(x)$ in $g(g(x))$ :

(a) $f(f(x)) = a^2\, x$ $\quad f(g(x)) = a\, b\, x$

$g(f(x)) = a\, b\, x$ $\quad g(g(x)) = b^2\, x$

(b) $f(f(x)) = x^{a^2}$ $\quad f(g(x)) = x^{a\, b}$

$g(f(x)) = x^{a\, b}$ $\quad g(g(x)) = x^{b^2}$

(c) $f(f(x)) = -3 + 4\, x$ $\quad f(g(x)) = 3 - 6\, x$

$g(f(x)) = 5 - 6\, x$ $\quad g(g(x)) = -4 + 9\, x$

(d) $f(f(x)) = x^4$ $\quad f(g(x)) = x$

$g(f(x)) = \sqrt{x^2}$ $\quad g(g(x)) = \sqrt[4]{x}$

(e) $f(f(x)) = \dfrac{1}{1+\frac{1}{1+x}}$ $\quad f(g(x)) = \dfrac{1}{1+x^2}$

$g(f(x)) = (1+x)^{-2}$ $\quad g(g(x)) = x^4$

(f) $f(f(x)) = x$ $\quad f(g(x)) = \dfrac{-1+2\,x}{5}$

$g(f(x)) = \dfrac{1-3\,x}{2\,(x-2)}$ $\quad g(g(x)) = \dfrac{4-3\,x}{2\,x-11}$

(g) $f(f(x)) = x$ $\quad f(g(x)) = \operatorname{sign}(\dfrac{1}{x})$

$g(f(x)) = \operatorname{sign}(\dfrac{1}{x})$ $\quad g(g(x)) = \operatorname{sign}(x)$

(h) $f(f(x)) = 1 + \dfrac{1}{1+\frac{1}{x}}$ $\quad f(g(x)) = \dfrac{2}{1+x}$

$g(f(x)) = -1-2\,x$ $\quad g(g(x)) = -\dfrac{1}{x}$

27. Kompozituma $f(g(x))$ in $g(f(x)$

(a) $f(g(x)) = \dfrac{1}{5+x}$ $\quad g(f(x)) = 3 + \dfrac{1}{2+x}$

(b) $f(g(x)) = 2+3\,x+x^2$ $\quad g(f(x)) = 5+5\,x+x^2$

(c) $f(g(x)) = \dfrac{1+4\,x}{6-x}$ $\quad g(f(x)) = -1+x$

(d) $f(g(x)) = 2-x$ $\quad g(f(x)) = -x$

(e) $f(g(x)) = \dfrac{1}{2+x^2}$ $\quad g(f(x)) = 1+(1+x)^{-2}$

(f) $f(g(x)) = \dfrac{1+x^2}{x^2-1}$ $\quad g(f(x)) = \left(\dfrac{x+1}{x-1}\right)^2$

(g) $f(g(x)) = -\dfrac{1}{x} + x + \dfrac{1}{-\frac{1}{x}+x}$ $\quad g(f(x)) = \dfrac{1}{x} + x - \dfrac{1}{\frac{1}{x}+x}$

(h) $f(g(x)) = \dfrac{1}{2 + x^{-2} + \frac{2}{x}}$ $g(f(x)) = 2 + x^2$

(i) $f(g(x)) = e^{2\,x}$ $g(f(x)) = e^{x^2}$

(j) $f(g(x)) = 0$ $g(f(x)) = 0$

(k) $f(g(x)) = -\dfrac{(x-1)^2}{4\,x}$ $g(f(x)) = 1 - \dfrac{2}{x^2}$

(l) $f(g(x)) = \dfrac{1}{x + \sqrt{1 + x^2}}$ $g(f(x)) = \sqrt{1 + x^{-2}} + \dfrac{1}{x}$

(m) $f(g(x)) = \dfrac{x^2 - 1}{2\,x} + \dfrac{x^2 + 1}{2\,|x|}$ $g(f(x)) = x$

(n) $f(g(x)) = \dfrac{2}{1 + x}$ $g(f(x)) = -1 - 2\,x$

(o) $f(g(x)) = \dfrac{x^2}{1 + x^2}$ $g(f(x)) = \dfrac{x^2}{(1+x)^2}$

(p) $f(g(x)) = \dfrac{(-1+x)^2}{(1+x)^2}$ $g(f(x)) = \dfrac{1 + x^2}{-1 + x^2}$

(q) $f(g(x) = -x$ $g(f(x)) = -x$

(r) $f(g(x)) = x$ $g(f(x)) = \sqrt{x^2}$

(s) $f(g(x)) = x$ $g(f(x)) = \dfrac{1 + x^2 - |1 - x^2|}{2\,x}$

28. Funkcija $f(x)$

(a) $f(x) = 6 - 5\,x + x^2$

(b) $f(x) = 1 + \dfrac{1}{-1 + x} - x$

(c) $f(x) = \sqrt{1 + x^{-2}} + \dfrac{1}{x}$

(d) $f(x) = \dfrac{x^2}{(1-x)^2}$

(e) $f(x) = \sqrt{\dfrac{-1+x}{1+x}}$

(f) $f(x) = 1 - \dfrac{2}{-3 + x}$

(g) $f(x) = \dfrac{-1 + x}{x^2}$

(h) $f(x) = \dfrac{1}{x}$

(i) $f(x) = \sqrt{\dfrac{x+1}{x-1}}$

(j) $f(x) = |x| + 1 + 2\,\sqrt{|x|}\,\mathrm{sign}(x)$

29. Inverzna funkcija k dani funkciji $f(x)$ in njeno definicijsko območje

(a) $f^{-1}(x) = \dfrac{x-1}{3}$ $(-\infty, \infty)$

(b) $f^{-1}(x) = 6 - 2\,x$ $(-\infty, \infty)$

(c) $f^{-1}(x) = \sqrt[3]{x-1}$ $(-\infty, \infty)$

(d) $f^{-1}(x) = \dfrac{3+2\,x}{x-2}$ $(-\infty, 2)$

(e) $f^{-1}(x) = \dfrac{1}{x-1}$ $(-\infty, 1)$

(f) $f^{-1}(x) = \begin{cases} x-1, & x \le 3 \\ 1+\frac{x}{3}, & 3 < x \end{cases}$ $(-\infty, \infty)$

(g) $f^{-1}(x) = \begin{cases} \frac{x-2}{5}, & x \le -\frac{1}{2} \\ x & -\frac{1}{2} < x \le 1 \\ \frac{2+x}{3} & 1 < x \end{cases}$ $(-\infty, \infty)$

(h) $f^{-1}(x) = \begin{cases} -\sqrt{-x}, & x \le 0 \\ \sqrt{x}, & 0 < x \end{cases}$ $(-\infty, \infty)$

(i) $f^{-1}(x) = \begin{cases} -\sqrt{1-x}, & x \le 1 \\ \sqrt{x-1}, & 1 < x \end{cases}$ $(-\infty, \infty)$

(j) $f^{-1}(x) = \dfrac{x}{\sqrt{x^2-1}}$ $(-1, 1)$

(k) $f^{-1}(x) = -2\ln(x-1)$ $(1, \infty)$

(l) $f^{-1}(x) = \ln(x + \dfrac{\sqrt{4+x^2}}{2})$ $(-\infty, \infty)$

(m) $f^{-1}(x) = \ln\left|\dfrac{1+x}{1-x}\right|$ $(-1, 1)$

(n) $f^{-1}(x) = 2\,e^x$ $(0, \infty)$

(o) $f^{-1}(x) = \dfrac{\tan(x-1)}{3}$ $(1-\frac{\pi}{2}, 1+\frac{\pi}{2})$

(p) $f^{-1}(x) = \dfrac{1}{\ln\frac{1-x}{x}}$ $(0, \frac{1}{2}) \cup (\frac{1}{2}, 1)$

30. Inverzna funkcija k dani $f(x)$ na danem intervalu in njeno definicijsko območje

(a) $f^{-1}(x) = -\sqrt{x}$ $(0, \infty)$

(b) $f^{-1}(x) = -\sqrt{1-x}$ $(-\infty, 1)$

(c) $f^{-1}(x) = -1 + \frac{x^2}{2}$ $(0, \infty)$

(d) $f^{-1}(x) = \frac{1-x}{1+x}$ $(0, \infty)$

(e) $f^{-1}(x) = \frac{1-x}{1+x}$ $(-1, 0)$

(f) $f^{-1}(x) = \pi - \arcsin x$ $(-1, 1)$

(g) $f^{-1}(x) = \ln(\frac{x - \sqrt{-4+x^2}}{2})$ $(1, \infty)$

(h) $f^{-1}(x) = \ln(\frac{x + \sqrt{-4+x^2}}{2})$ $(1, \infty)$

(i) $f^{-1}(x) = -\sqrt{\frac{1+x}{1-x}}$ $(-1, 1)$

(j) $f^{-1}(x) = \sqrt{\frac{1+x}{1-x}}$ $(-1, 1)$

(k) $f^{-1}(x) = \frac{1+\sqrt{1-x^2}}{x}$ $(-1, 0)$

(l) $f^{-1}(x) = \frac{1-\sqrt{1-x^2}}{x}$ $(-1, 1)$

(m) $f^{-1}(x) = \frac{1+\sqrt{1-x^2}}{x}$ $(0, 1)$

(n) $f^{-1}(x) = -\frac{\sqrt{1+\sqrt{1-4\,x^4}}}{\sqrt{2}\,x}$ $(-\frac{1}{\sqrt{2}}, 0)$

(o) $f^{-1}(x) = -\frac{\sqrt{1-\sqrt{1-4\,x^4}}}{\sqrt{2}\,x}$ $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

(p) $f^{-1}(x) = \frac{\sqrt{1+\sqrt{1-4\,x^4}}}{\sqrt{2}\,x}$ $(0, \frac{1}{\sqrt{2}})$

(q) $f^{-1}(x) = \frac{1-e^x}{1+e^x}$ $(-\infty, \infty)$

(r) $f^{-1}(x) = \frac{1+e^x}{1-e^x}$ $(1, \infty)$

31. Limite funkcij

(a) $\frac{2}{3}$

(b) 9

(c) $-1$

(d) $-8$

(e) $\frac{1}{2}$

(f) $\frac{m}{n}$

(g) $\frac{m\,n\,(-m+n)}{2}$

(h) 0

(i) $\frac{1}{4}$

(j) $\frac{m-n}{2}$

(k) 1

(l) $\frac{1}{m}$

(m) $\frac{1}{2}$

(n) $\frac{4}{3}$

(o) $-2$

(p) 1

(q) 3

(r) $\infty$

(s) 0

(t) 0

32. Limite funkcij

(a) 3

(b) 0

(c) $\frac{m}{n}$

(d) $\frac{1}{2}$

(e) $\frac{a}{b}$

(f) $\frac{1}{2}$

(g) $\frac{1}{2}$

(h) 0

(i) $\frac{1}{3}$

(j) $-\frac{1}{3}$

(k) $\frac{1}{6}$

(l) $\infty$

(m) 0

(n) $\frac{2}{3}$

(o) 0

(p) $\log(a)$

(q) $\frac{1}{2}$

(r) $\frac{1}{6}$

(s) 0

(t) 0

(u) 1

(v) $\infty$

33. Limite funkcij

(a) $-1$

(b) $\frac{1}{2}$

(c) $e^3$

(d) $\sqrt{e}$

(e) 1

(f) 1

(g) 0

(h) 1

(i) 0

(j) $e^2$

(k) 0

(l) $\infty$

(m) $e^{-2}$

(n) $e$

(o) $e^{\frac{1}{3}}$

(p) $e^{-\frac{2}{\pi}}$

34. Grafi funkcij

(a)

$$y = 3 + 2\,x$$

(b)

$$y = 8\,x - 2\,x^2$$

(c)

$$y = -1 + 2\,x - x^2$$

(d)

$$y = 2 - 3\,x + x^2$$

(e)

$$y = 1 + x + \frac{x^2}{2}$$

(f)

$$y = 1 + x^3$$

(g)

$$y = x^2 - x^4$$

(h)

$$y = -x^2 - x^3 + x^4$$

(i)

$$y = 3 - 2\,x - x^2 + x^3$$

(j)

$$y = -x^2 + x^4$$

(k)

$$y = 3 - |x|$$

(l)

$$y = x - |1 - 2x|$$

(m)

$$y = |x - 1| + |2 - 3x| + x$$

(n)

$$y = 4\,x + |1 - x| - |1 + 2\,x|$$

(o)

$$y = x^2 - 1 - |1 + x|$$

(p)

$$y = x + \big|x + |1 - 3\,x| + |2\,x - 3|\big|$$

(q)

$$y = \min\{x, 3 + x, |x - 2|\}$$

(s)

$$y = |x^2 - 2| - 2|x - 2|$$

(r)

$$y = \max\{2|x|, |1 + x|\}$$

(t)

$$y = |x^2 - 1| + |x + 2| + x$$

35. Grafi funkcij

(a)

$$y = \frac{1}{x} + x$$

(b)

$$y = x^{-2} + x$$

(c)

$$y = \frac{2\,x}{1+x^2}$$

(d)

$$y = \frac{1}{1+x^2}$$

(e)

$$y = \frac{2\,x-1}{1+x^3}$$

(f)

$$y = \frac{1+x^3}{x-x^2}$$

(g)

$$y = \frac{4\,(x^2-x^4)}{1-4\,x^2}$$

(h)

$$y = \frac{(1-x)\,x^3}{1-2\,x^2}$$

(i)

$$y = \frac{|x+1|-|x-1|}{2x}$$

(k)

$$y = \frac{x+x^3}{2\,|x|}$$

(j)

$$y = x + \frac{1+x}{|x-1|}$$

(l)

$$y = \frac{4-x^2}{|4-x^3|}$$

36. Grafi funkcij

(a)

$$y = \sqrt{\left|\frac{1-x}{1+x}\right|}$$

(b)

$$y = x\,\sqrt{|1+x|}$$

(c)

$$y = \sqrt{|x^2 + x^3|}$$

(d)

$$y = \sqrt{|x^2 - x^4|}$$

(e)

$$y = \sqrt{\frac{|x^4-1|}{1+x^2}}$$

(f)

$$y = \sqrt[3]{x^2 + x^3}$$

(g)

$$y = \sqrt{\frac{|x|}{1+x^2}}$$

(h)

$$y = \frac{x}{\sqrt{1+x^4}}$$

(i)

$$y = \frac{x-2}{\sqrt{1+x^2}}$$

(j)

$$y = \frac{x}{\sqrt[3]{1-x^2}}$$

37. Grafi funkcij

(a)

$$y = \arcsin \frac{2\,x}{1+x^2}$$

(c)

$$y = \arcsin \frac{1}{1+x^2}$$

(b)

$$y = \arcsin(\sin x)$$

(d)

$$y = \arcsin(0.9\,\cos x)$$

(e)

$$y = \sin(2 \arcsin x)$$

(f)

$$y = \sin(3 \arcsin x)$$

(g)

$$y = \arctan \tfrac{1}{x}$$

(h)

$$y = x + \arctan x$$

(i)

$$y = x \arctan x$$

(j)

$$y = x \arctan \tfrac{1}{x}$$

(k)

$$y = \arctan \tfrac{1-x}{1+x}$$

(l)

$$y = \arctan\left|\tfrac{1-x}{1+x}\right|$$

38. Grafi funkcij

(a)

$$y = e^{-\frac{x}{8}} \sin x$$

(c)

$$y = e^{\frac{x}{1-x^2}}$$

(b)

$$y = e^{-\frac{1}{x^2}}$$

(d)

$$y = e^{-x^2}$$

(e)

$$y = \frac{1}{1+e^{\frac{1}{x}}}$$

(f)

$$y = x^2\, e^{-x}$$

(g)

$$y = e^{0.1/(1-x^2)}\, \frac{1}{1+x^2}$$

(h)

$$y = x + \sin x$$

(i)

$$y = x\, \sin(\tfrac{2}{x})$$

(j)

$$y = x^2\, \sin(\tfrac{2}{x})$$

(k)

$$y = \frac{\sin x}{x}$$

(l)

$$y = \sin(x^2)$$

(m)

$$y = \frac{1}{\ln x}$$

(n)

$$y = x \ln x$$

(o)

$$y = x^x$$

(p)

$$y = x^{\frac{1}{x}}$$

(q)

$$y = (1+x)^{\frac{1}{x}}$$

(r)

$$y = \frac{1}{1+x}\left(1+\frac{1}{x}\right)^{x}$$

(s)

$$y = \ln\left|\frac{1-x}{1+x}\right|$$

(t)

$$y = \ln\frac{3\,|x|}{1+x^2}$$

(u)

$$y = \ln(x+\sqrt{1+x^2})$$

(v)

$$y = \ln(1+x^2)$$

39. Grafi funkcij

(a)

$$y = \frac{\sin^2 x}{x}$$

(b)

$$y = \frac{\cos x \, \sin x}{x}$$

(c)

$$y = \arctan\left|\frac{1-x}{1+x}\right|$$

(d)

$$y = -\frac{\pi}{4} + \arctan\left|\frac{1-x}{1+x}\right|$$

(e)

$$y = \sqrt{\frac{x^2}{1+x^2}}$$

(f)

$$y = x\sqrt{\frac{1}{1+x^2}}$$

(g)

$$y = x - 3\,x^3 + x^5$$

(h)

$$y = 1 - x\,|x|$$