Izpit MATEMATIKA II

19. junij 2008

rešitve

1. naloga

$$\vec{n} = \begin{bmatrix} i & j & k \\ 3 & 2 & -2 \\ 3 & 4 & -3 \end{bmatrix} = (2, 3, 6)$$

$$((x, y, z) - (2, 4, -2)) \cdot (2, 3, 6) = 0$$

enačba ravnine : $2x + 3y + 6z - 4 = 0$

razdalja točke : $d = |\frac{2x+3y+6z-4}{\sqrt{4+9+36}}| \Big|_{(1,0,-2)} = |\frac{2-12-4}{7}| = 2$

2. naloga

$$XA = B$$

$$A^T X^T = B^T$$

$$\begin{bmatrix} 2 & 4 & -1 & 1 & -2 \\ 1 & 2 & 0 & 2 & 1 \\ 1 & 1 & 1 & 5 & 4 \end{bmatrix} \quad v_1 \longleftrightarrow v_3$$

$$\begin{bmatrix} 1 & 1 & 1 & 5 & 4 \\ 1 & 2 & 0 & 2 & 1 \\ 2 & 4 & -1 & 1 & -2 \end{bmatrix} \quad v_2 - v_1 \ , \ v_3 - 2 * v_1$$

$$\begin{bmatrix} 1 & 1 & 1 & 5 & 4 \\ 0 & 1 & -1 & -3 & -3 \\ 0 & 2 & -3 & -9 & -10 \end{bmatrix} \quad v_1 - v_2 \ , \ v_3 - 2 * v_2$$

$$\begin{bmatrix} 1 & 0 & 2 & 8 & 7 \\ 0 & 1 & -1 & -3 & -3 \\ 0 & 0 & -1 & -3 & -4 \end{bmatrix} \quad v_2 - v_3 \ , \ v_1 + 2 * v_3 \ , \ -v_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 & 4 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 0 & 3 \\ -1 & 1 & 4 \end{bmatrix}$$

3. naloga

$f_x = \sqrt{y} - 2x + 6 = 0$

$f_y = \frac{x}{2\sqrt{y}} - 1 = 0 \longrightarrow \sqrt{y} = \frac{x}{2}$ vstavim v prvo enačbo

$\frac{x}{2} - 2x + 6 = 0$

$x = 4 \quad , \quad y = 4$

$D(x,y) = f_{xx}f_{yy} - f_{xy}^2 = (-2)(-\frac{x}{4\sqrt{y^3}}) - (\frac{1}{2\sqrt{y}})^2$

$D(4,4) = 2 \cdot \frac{1}{8} - \frac{1}{16} > 0 \quad \text{in} \quad f_{xx} < 0 \longrightarrow \textbf{maximum}$

4. naloga

Prva rešitev kot **homogena** enačba

$y' = \frac{y}{x} - (\frac{y}{x})^2$

nova spremenljivka $y = xz \quad , \quad y' = z + xz'$

$z + xz' = z - z^2$

$-\int \frac{dz}{z^2} = \int \frac{dx}{x}$

$\frac{1}{z} = \ln x + \ln C$

$\frac{x}{y} = \ln(Cx)$

$y = \frac{x}{\ln(Cx)}$

$y(1) = 1 \longrightarrow C = e$

$y = \frac{x}{\ln(ex)}$

Druga rešitev kot **Bernoullijeva** enačba

$$\frac{y'}{y^2}x^2 + 1 = \frac{x}{y}$$

nova spremenljivka $\frac{1}{y} = z$

$$-x^2 z' + 1 = xz$$

Homogena enačba

$$-x^2 \frac{dz}{dx} = xz$$

$$\frac{dz}{z} = -\frac{dx}{x}$$

$$\ln z = \ln C - \ln x$$

$$z_h = \frac{C}{x}$$

Variacija konstante

$$z = \frac{C(x)}{x}$$

$$-x^2(\frac{C'}{x} - \frac{C}{x^2}) + 1 = C$$

$$-xC' = -1$$

$$C(x) = \ln x$$

$$z_p = \frac{\ln x}{x}$$

$$y = \frac{1}{z_h + z_p} = \frac{x}{\ln x + C}$$

$$y(1) = 1 \longrightarrow C = 1$$

$$y = \frac{x}{\ln x + 1}$$

5. naloga

$$2\lambda^2 - 5\lambda + 2 = 0$$
$$2(\lambda - 2)(\lambda - \frac{1}{2}) = 0$$
$$y_h = C_1 e^{2x} + C_2 e^{x/2}$$

$$y_p = A + Be^{2x}x$$
$$y' = Be^{2x}(2x + 1)$$
$$y'' = Be^{2x}(4x + 2 + 2)$$

$$2Be^{2x}(4x + 4) - 5Be^{2x}(2x + 1) + 2A + 2Be^{2x}x = 1 + 6e^{2x}$$
$$Be^{2x}(8x + 8 - 10x - 5 + 2x) + 2A = 1 + 6e^{2x}$$
$$A = \frac{1}{2} \quad , \quad B = 2$$

$$y = C_1 e^{2x} + C_2 e^{x/2} + \frac{1}{2} + 2xe^{2x}$$