

Dolžina krivulje:

$$s(t) = \int_{\alpha}^{\beta} \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} dt$$

Normalna ravnina:

-vektorska oblika

$$(\vec{r} - \vec{r}_0) \cdot \dot{\vec{r}}_0 = 0$$

-izpis po komponentah

$$(x - x_0)\dot{x}_0 + (y - y_0)\dot{y}_0 + (z - z_0)\dot{z}_0 = 0$$

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Funkcionalne determinante (če vsaj ena od njih ni 0 ploskev ne preide v krivuljo):

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_u & y_u & z_u \\ x_v & y_v & z_v \end{vmatrix}, \quad \frac{\partial(y, z)}{\partial(u, v)} = \begin{vmatrix} y_u & z_u \\ y_v & z_v \end{vmatrix}, \quad \frac{\partial(z, x)}{\partial(u, v)} = \begin{vmatrix} z_u & x_u \\ z_v & x_v \end{vmatrix}, \quad \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} x_u & y_u \\ x_v & y_v \end{vmatrix}$$

Tangentna ravnina:

-vektorska oblika:

$$(\vec{r} - \vec{r}_0) \cdot (\vec{r}_u \times \vec{r}_v) = 0 \quad (\vec{r} - \vec{r}_0, \vec{r}_u, \vec{r}_v) = 0$$

-izpis po komponentah

$$\frac{\partial(y, z)}{\partial(u, v)}(x - x_0) + \frac{\partial(z, x)}{\partial(u, v)}(y - y_0) + \frac{\partial(x, y)}{\partial(u, v)}(z - z_0) = 0$$

-eksplicitna oblika

$$z - z_0 = p(x - x_0) + q(y - y_0), \quad p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}$$

-implicitna oblika

$$(x - x_0) \frac{\partial F}{\partial x} + (y - y_0) \frac{\partial F}{\partial y} + (z - z_0) \frac{\partial F}{\partial z} = 0$$

Normala na ploskev:

-vektorska oblika

$$\vec{r} - \vec{r}_0 = \lambda(\vec{r}_u \times \vec{r}_v)$$

-izpis po komponentah

$$\frac{x - x_0}{p} = \frac{y - y_0}{q} = \frac{z - z_0}{-1}$$

-implicitna oblika

$$\frac{x - x_0}{F_x} = \frac{y - y_0}{F_y} = \frac{z - z_0}{F_z}$$

Tangenta na krivuljo:

-vektorska oblika

$$\vec{r} = \vec{r}_0 + \lambda \dot{\vec{r}}_0$$

-izpis po komponentah

$$\frac{x - x_0}{\dot{x}_0} = \frac{y - y_0}{\dot{y}_0} = \frac{z - z_0}{\dot{z}_0}$$

Enotni vektor normale:

-vektorska oblika

$$\vec{v} = \frac{\vec{r}_u \times \vec{r}_v}{|\vec{r}_u \times \vec{r}_v|}$$

-eksplicitna oblika

$$\vec{v} = \left(-\frac{p}{\sqrt{1+p^2+q^2}}, -\frac{q}{\sqrt{1+p^2+q^2}}, \frac{1}{\sqrt{1+p^2+q^2}} \right)$$

Prva fundamentalna forma:

$$ds^2 = Edu^2 + 2Fdudv + Gdv^2, \quad E = \vec{r}_u \cdot \vec{r}_u, \quad F = \vec{r}_u \cdot \vec{r}_v, \quad G = \vec{r}_v \cdot \vec{r}_v$$

Integral s parametrom:

$$F(y) = \int_a^b f(x, y) dx$$

$$\frac{dF(y)}{dy} = \frac{\partial}{\partial y} \int_a^b f(x, y) dx = \int_a^b \frac{\partial f}{\partial y} dx$$

$$F(y) = \int_{u(y)}^{v(y)} f(x, y) dx$$

$$\frac{dF}{dy} = \int_{u(y)}^{v(y)} \frac{\partial f}{\partial y} dx + f(v, y) \frac{dv}{dy} - f(u, y) \frac{du}{dy}$$

$$\int_a^b dx \int_c^d f(x, y) dy = \int_c^d dy \int_a^b f(x, y) dx$$

Dvojni integral:

$$\iint_D f(x, y) dx dy = \int_a^b dx \int_{g(x)}^{h(x)} f(x, y) dy = \int_c^d dy \int_{p(y)}^{q(y)} f(x, y) dx$$

Vpeljava novih spremenljivk:

$$x = x(u, v), \quad y = y(u, v)$$

-Jacobijeva determinanta

$$J(u, v) = \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} \neq 0$$

-formula za vpeljavo

$$\iint_D f(x, y) dx dy = \iint_{\Delta} f[x(u, v), y(u, v)] |J(u, v)| du dv$$

-element ploščine v krivočrtnih koordinatah

$$d\omega = |J(u, v)| du dv$$

Posplošeni integral:

$$\iint_D f(x, y) dx dy = \lim_{\varepsilon \rightarrow 0} \iint_{D'} f(x, y) dx dy$$

