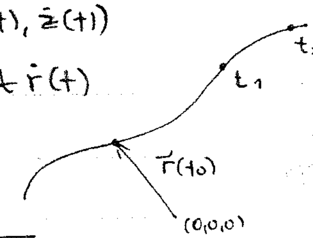
DIFERENCIALNA GEOMETRIJA

KRIVULJE $\vec{r}(t) = (x(t), y(t), z(t))$

TANGENTA NA KRIVULJO — smerni vektor $\dot{\vec{r}}(t) = (\dot{x}(t), \dot{y}(t), \dot{z}(t))$

tang. premica $\vec{r}_T(t) = \vec{r}(t_0) + t\,\dot{\vec{r}}(t)$



NORMALNA RAVNINA $(\vec{r}(t) - \vec{r}(t_0)) \cdot \vec{n} = 0$

ima $\vec{r}_T(t)$ za normalo $(\vec{r}(t) - \vec{r}(t_0)) \cdot \dot{\vec{r}}(t_0) = 0$

LOČNA DOLŽINA KRIVULJE (dolžina $\dot{\vec{r}}(t)$) $s = \int_{t_1}^{t_2} \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\, dt$

NARAVNI PARAMETER ($|\dot{\vec{r}}(s)| = 1$) $s = \int_c^t \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\, dt'$ zračunaš s in izra t ter vneseš v začetno enačbo.

PARAMETRIZACIJA DALJICE $\vec{r}(t) = \vec{r}_A + t\,\overrightarrow{AB}$ $(0 \le t \le 1)$

PLOSKEV V PROSTORU $\vec{r}(u,v) = (x(u,v), y(u,v), z(u,v))$

stožec $z = \sqrt{x^2 + y^2}$

valj $x^2 + y^2 = R^2$

paraboloid $z = x^2 + y^2$

"cev" $y = x^2,\ z = z$

sfera $x^2 + y^2 + z^2 = R^2$

deteljica $(x^2 + y^2)^2 = 2(x^2 - y^2)$

srčnica $\sqrt{x^2+y^2} \le 1 + \frac{x}{\sqrt{x^2+y^2}}$

NORMALA NA PLOSKEV $\vec{r}(u,v) = (x(u,v), y(u,v), z(u,v))$ $\vec{n} = \vec{r}_u \times \vec{r}_v$

$F(x,y,z) = 0$ $\vec{n} = (F_x, F_y, F_z)$

$z = f(x,y)$ $\vec{n} = (z_x, z_y, -1)$

TANGENTNA RAVNINA ima $\vec{n}$ za normalo

NORMALNA PREMICA ima $\vec{n}$ za smerni vektor

$\cos\alpha = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| \cdot |\vec{n}_2|}$

KOORDINATNE KRIVULJE en parameter je fiksiran

$\vec{r}(u,v) = (x(u,v), y(u,v), z(u,v))$ $\vec{r}(C,v)$ in $\vec{r}(u,D)$

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	P
$\sin\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	
$\cos\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	
$\tan\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	

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INTEGRALI S PARAMETROM

$$F(x)=\int_{u(x)}^{v(x)} f(x,y)\,dy$$

$$F'(x)=\int_{u(x)}^{v(x)} \frac{\partial F(x,y)}{\partial x}\,dy + f(x,v(x))\,v'(x) - f(x,u(x))\cdot u'(x)$$

Če je EK, lahko vrstni red integriranja zamenjamo.
Če je EK in parcialno zvezno odvedljiva Fja, jo lahko odvajamo.

DVOJNI IN DVAKRATNI INTEGRALI

$$\iint_D f(x,y)\,dx\,dy \Rightarrow \int_a^b dx \int_{g(x)}^{h(x)} f\,dy \qquad \int_c^d dy \int_{k(y)}^{l(y)} f\,dx$$

UVEDBA NOVIH SPREMENLJIVK

$$\iint_D f(x,y)\,dx\,dy = \iint_\omega f(x(u,v),y(u,v))\,|J|\,du\,dv$$

$$J=\begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$$

Sferične koordinate
(r - oddaljenost od (0,0,0)
· ϑ - kot glede na xy $(-\frac{\pi}{2},\frac{\pi}{2})$
· φ - polarni kot od osi +x $(0,2\pi)$)

$x=r\cos\vartheta\cos\varphi$
$y=r\cos\vartheta\sin\varphi$
$z=r\sin\vartheta$
$J=r^2\cos\vartheta$

Cilindrične koordinate
(· r - oddaljenost od osi z
· φ - kot glede na +x $(0,2\pi)$
· z - koordinata z)

$x=r\cos\varphi$
$y=r\sin\varphi$
$z=z$
$J=r$

UPORABA V GEOMETRIJI

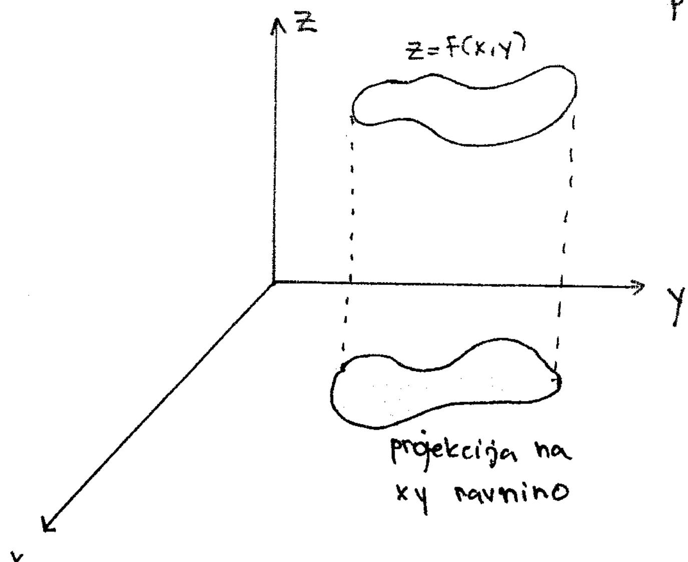
PROSTORNINA POD PLOSKVIJO

$$V=\iint_D f(x,y)\,dx\,dy$$

$$V=\iint_D (f(x,y)-g(x,y))\,dx\,dy$$

POVRŠINA PLOSKVE

$$P=\iint_D \sqrt{1+z_x^2+z_y^2}\,dx\,dy$$



PLOŠČINA OBMOČJA $\iint_D dx\,dy$

PLOŠČINA ELIPSE πab

skalarno polje $u = f(x,y,z)$

vektorsko polje $\vec{v} = (v_1, v_2, v_3)$

$$\operatorname{grad} u = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right)$$

$$\operatorname{div} \vec{v} = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}$$

$$\operatorname{rot} \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_1 & v_2 & v_3 \end{vmatrix}$$

potencialno polje $\operatorname{rot} \vec{v} = 0$ in $\vec{v} = \operatorname{grad} u$

Krivuljni integral

$$\int_C f(x,y,z)\, ds = \int_{t_{min}}^{t_{max}} g(t)\, dt \qquad ds = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\, dt$$

$$\int_C \vec{v} \cdot d\vec{r} = \int_C v_1\, dx + v_2\, dy + v_3\, dz = \int_{t_{začetek}}^{t_{konec}} g(t)\, dt$$

Ploskovni integral

$$\iint_S f(x,y,z)\, dS = \iint f(u,v) \sqrt{EG - F^2}\, du\, dv$$

$$\iint_S \vec{v}\, d\vec{S} = \iint \vec{v}(u,v) \cdot (\vec{r}_u \times \vec{r}_v)\, du\, dv$$

$$E = \vec{r}_u \cdot \vec{r}_u \,; \quad G = \vec{r}_v \cdot \vec{r}_v \,; \quad F = \vec{r}_u \cdot \vec{r}_v \,; \quad dS = \sqrt{EG - F^2}\, du\, dv$$

Gaussova f.

$$\iint_S \vec{v}\, d\vec{S} = \iiint_V \operatorname{div} \vec{v}\, dx\, dy\, dz$$

cilindrične koordinate

$x = r \cos \varphi$

$y = r \sin \varphi$

$z = z$

$J = r$

$[x^2 + y^2 = r^2]$

Stokesova f.

$$\int_C \vec{v}\, d\vec{r} = \iint_S \operatorname{rot} \vec{v}\, d\vec{S}$$

sferične koordinate

$x = r \cos \varphi \cos \vartheta$

$y = r \sin \varphi \cos \vartheta$

$z = r \sin \vartheta$

$J = r \cos \vartheta$

$[x^2 + y^2 + z^2 = r^2]$

Greenova f.

$$\int_C [f(x,y)\, dx + g(x,y)\, dy] = \iint_D \left[\frac{\partial g}{\partial x}(x,y) - \frac{\partial f}{\partial y}(x,y)\right] dx\, dy$$

Integracija $\mathbb{C}$

$$\int_C f(z)\,dz = \int_C \big(u(x,y) + i\,v(x,y)\big)\big(dx + i\,dy\big) =$$

$$= \int_C u(x,y)\,dx - v(x,y)\,dy + i\int_C v(x,y)\,dx + u(x,y)\,dy$$

Kompleksna analiza

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ in $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ analitična

$z = x + iy$

$$\operatorname*{Res}_{z=z_0} f(z) = \lim_{z\to z_0} (z - z_0) f(z)$$ 1. stopnje

$$\operatorname*{Res}_{z=z_0} f(z) = \frac{1}{(m-1)!} \lim_{z\to z_0} \frac{d^{m-1}}{dz^{m-1}}\left[(z-z_0)^m f(z)\right]$$ m-te stopnje

$$\oint f(z)\,dz = 2\pi i \sum_{i=1}^{n} \operatorname*{Res}_{z=z_i} f(z)$$

smerni odvod skalarnega polja

$$\frac{\partial u}{\partial \vec{e}}(x,y,z) = \operatorname{grad} u(x,y,z) \cdot \frac{\vec{e}}{|\vec{e}|}$$

$$\vec{r}(t) = T_1 + t\,\overline{T_1T_2}$$

$$e^{ix} = \cos x + i \sin x$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\log z = \log|z| + i\,(\varphi + 2k\pi)$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\sin(x+iy) = \sin x \operatorname{ch} y + \cos x \operatorname{sh} y$$

stožec: $z = \sqrt{x^2+y^2}$

valj: $x^2 + y^2 = R^2$

paraboloid: $z = x^2 + y^2$

sfera: $x^2 + y^2 + z^2 = R^2$

cev: $y = z^2 \quad z = z$