Kompleksna analiza

Zveznost funkcije v točki: $\lim_{z\to z_0} f(z) = f(z_0)$

Oblika funkcije kompleksne spremenljivke: $w = u(x,y) + iv(x,y)$

Analitičnost in odvedljivost funkcije (Cauchy-Riemann): $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

Lastnost analitične funkcije: $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$

Elementarne kompleksne funkcije:

$z^\alpha = e^{\alpha[\ln|z| + i(\phi + 2n\pi)]} = e^{\alpha \ln|z|} e^{\alpha\phi i} e^{2n\alpha\pi i}$, $n = 0, \pm1, \pm2, ...$

$e^z = e^{x+iy} = e^x(\cos y + i \sin y)$ $\ln z = \ln|z| + i(\phi + 2n\pi)$, $n = \pm1, \pm2, \pm3, ...$

$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$ $\arcsin z = -i \ln(iz \pm \sqrt{1 - z^2})$

$\cos z = \frac{e^{iz} + e^{-iz}}{2}$ $\arccos z = -i \ln(z \pm \sqrt{z^2 - 1})$

$\tan z = \frac{1}{i} \frac{e^{iz} - e^{-iz}}{e^{iz} + e^{-iz}}$ $\arctan z = \frac{i}{2} \ln\left(\frac{i + z}{i - z}\right)$

$\sinh z = \frac{e^z - e^{-z}}{2}$ $\cosh z = \frac{e^z + e^{-z}}{2}$

Cauchyjeva integralska formula: $f(z_0) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z - z_0} dz$

n-ti odvod funkcije v točki: $f^{(n)}(z_0) = \frac{n!}{2\pi i} \oint_C \frac{f(z)}{(z - z_0)^{n+1}} dz$

Kriterij za konvergenco: $\lim_{n\to\infty} \frac{|f_{n+1}(z)|}{|f_n(z)|} = r(z)$

-absolutno konvergira: $0 \le r(z) < 1$

-divergira: $r(z) < 1$

-ne vemo: $r(z) = 1$

Vrste:

Taylorjeva vrsta: $f(z)=\sum_{n=0}^{\infty}\frac{f^{(n)}(z_0)}{n!}(z-z_0)^n$

$\sin z=\sum_{n=0}^{\infty}(-1)^n\frac{z^{2n+1}}{(2n+1)!}$

$\cos z=\sum_{n=0}^{\infty}(-1)^n\frac{z^{2n}}{(2n)!}$

Potenčna vrsta: $f(z)=\sum_{n=0}^{\infty}a_n(z-z_0)^n$

-koeficienti: $a_n=\frac{1}{n!}f^n(z_0)$, $a_n=\frac{1}{2\pi i}\oint_C\frac{f(z)}{(z-z_0)^{n+1}}dz$

Laurentova vrsta: $f(z)=\sum_{n=0}^{\infty}a_n(z-z_0)^n+\sum_{n=1}^{\infty}\frac{c_n}{(z-z_0)^n}$

-koeficienti: $a_n=\frac{1}{2\pi i}\oint_C\frac{f(z^*)}{f(z^*-z_0)^{n+1}}dz^*$ $c_n=\frac{1}{2\pi i}\oint_C(z^*-z_0)^{n-1}f(z^*)dz^*$

Če določimo c_n je a_{-n}: $f(z)=\sum_{n=-\infty}^{\infty}a_n(z-z_0)^n$

-koeficienti: $a_n=\frac{1}{2\pi i}\oint_C\frac{f(z^*)}{f(z^*-z_0)^{n+1}}dz^*$

Residui:

Definicija residuuma (Laurentova vrsta): $c_1=\frac{1}{2\pi i}\oint_C f(z)dz=res_{z=a}f(z)$

Residuum v polu stopnje m pri $z=a$: $res_{z=a}f(z)=\frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left[(z-a)^m f(z)\right]$

Če je pol je enostaven: $res_{z=a}f(z)=\frac{p(a)}{q'(a)}$

Reševanje realnih integralov s pomočjo residuov:

$\oint_{C_j}f(z)dz=2\pi i\sum_{j=1}^{m}res_{z=a_j}f(z)$, $j=1,2,...,m$

$\int_{-\infty}^{\infty}f(x)dx=2\pi i\sum res\ f(z)$