Krivuljni integral

Krivuljni integral skalarne funkcije:

$$\int_C f(x,y,z)ds = \int_a^b f(x(s),y(s),z(s))ds\,, \quad \dot{s} = \sqrt{\dot{\vec{r}}\dot{\vec{r}}} = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

Krivuljni integral vektorske funkcije:

$$\int_C (v_1 dx + v_2 dy + v_3 dz) = \int_C \vec{v}\frac{d\vec{r}}{ds}ds = \int_C \vec{v}d\vec{r}\,, \quad \vec{v}(x,y,z) = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$$

Greenova formula:

$$\iint_D \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right)dxdy = \oint_C (fdx + gdy)$$

Ploščina lika:

$$P = \frac{1}{2}\oint_C (xdy - ydx)$$

$$\iint_D \Delta w dxdy = \oint_C \frac{\partial w}{\partial n}ds\,, \quad \vec{n} = \frac{dy}{ds}\vec{i} - \frac{dx}{ds}\vec{j}\,, \quad w = w(x,y)$$

Eksaktnost izraza:

izraz $fdx + gdy + hdz$ je eksakten, če velja $du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}du$

ali $\vec{v} = f\vec{i} + g\vec{j} + h\vec{k}$ in $\vec{v} = \text{grad}\, u = \frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial u}\vec{k}$

Če je izraz $fdx + gdy + hdz$ velja, da krivuljni integral ni odvisen od poti:

$$\int_C (fdx + gdy + hdz) = \int_P^Q \left(\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz\right) = \int_{t_0}^{t_1} \frac{du}{dt}dt = u(x(t),y(t),z(t))\Big|_{t_0}^{t_1} = u(Q) - u(P)$$

potreben in zadosten pogoj za neodvisnost je izpolnjen tudi, če velja: $\text{rot}\,\vec{v} = 0$