Ploskovni integral

Vektorska enota normale:

$\vec{\nu} = \cos\alpha\vec{i} + \cos\beta\vec{j} + \cos\gamma\vec{k}$

Element površine:

$d\omega = \sqrt{EG - F^2}\,dudv$

Parametri:

$E = \vec{r}_u\vec{r}_u$

$F = \vec{r}_u\vec{r}_v$

$G = \vec{r}_v\vec{r}_v$

Ploskovni integral:

-parametrična oblika $\vec{r}(u,v)$

$$\iint_S f(x,y,z)d\omega = \iint_D f[x(u,v),y(u,v),z(u,v)]\sqrt{EG-F^2}\,dudv$$

-eksplicitna oblika $z = g(x,y)$

$$\iint_S f(x,y,z)d\omega = \iint_S f[x,y,g(x,y)]\sqrt{1+p^2+q^2}\,dxdy = \iint_S f[x,y,g(x,y)]\frac{dxdy}{\cos\gamma}$$

Ploskovni integral vektorskega polja $\vec{v}$ po ploskvi *S*:

$$\iint_S (u_1dydz + u_2dxdz + u_3dxdy) = \iint_S (u_1\cos\alpha + u_2\cos\beta + u_3\cos\gamma)d\omega = \iint_S \vec{u}\,\vec{\nu}d\omega$$

Vektorska enota normale:

$$\vec{\nu} = \frac{(p,q,-1)}{\sqrt{p^2+q^2+1}}$$

Element površine:

$d\omega = \sqrt{1+p^2+q^2}\,dxdy$

Parametri:

$p = \frac{\partial u}{\partial x}$

$q = \frac{\partial u}{\partial y}$

Pretok polja $\vec{v}$ skozi ploskev *S*:

$$\iint_S \vec{v}\,\vec{\nu}dS = \iint_S \vec{v}d\vec{S}$$

-parametrična oblika:

$$P = \iint_S \vec{u}\,\vec{\nu}d\omega = \iint_S u_1dydz + u_2dxdz + u_3dxdy = \pm\iint_S \vec{u}(\vec{r}_u \times \vec{r}_v)dudv = \pm\iint_S \begin{vmatrix} u_1 & u_2 & u_3 \\ x_u & y_u & z_u \\ x_v & y_v & z_v \end{vmatrix} dudv$$

-eksplicitna oblika $z = z(x,y)$, $\vec{r}_u \times \vec{r}_v = (-p,-q,1)$:

$$P = \iint_S \vec{u}\,\vec{\nu}d\omega = \pm\iint_S \begin{vmatrix} u_1 & u_2 & u_3 \\ 1 & 0 & p \\ 0 & 1 & q \end{vmatrix} dxdy = \pm\iint_S (-pu_1 - qu_2 + u_3)dxdy$$

Gaussov izrek:

$$\iiint_V \left(\frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}\right)dxdydz = \iint_S (v_1dydz + v_2dzdx + v_3dxdy)$$

$$\iiint_V div\vec{v}dxdydz = \iint_S \vec{v}\,\vec{\nu}d\omega$$

$\iiint_V \Delta f dxdydz = \iint_S \frac{\partial f}{\partial n}d\omega$, $\frac{\partial f}{\partial n}$ predstavlja odvod funkcije *f* v smeri $\vec{\nu}$

Volumen telesa, ki ga omejuje sklenjena ploskev S:

$$V = \frac{1}{3}\iint_S \vec{r}\,\vec{\nu} d\omega$$

Prvi Greenov izrek:

$$\int_S g\nabla f \vec{\nu} d\omega = \int_V [g\nabla f + \nabla g \nabla f] dxdydz$$

Drugi Greenov izrek:

$$\int_S (f\nabla g - g\nabla f)\vec{\nu} d\omega = \int_V (f\Delta g - g\Delta f) dxdydz$$

Stokesov izrek:

$$\iint_S rot\vec{v}\,\vec{\nu} d\omega = \oint_C \vec{v} d\vec{r}$$

Normala na ploskev:

$$\vec{\nu} = \frac{gradF}{|gradF|}$$

Če imamo vektorsko funkcijo $\vec{v}(x,y,z) = (v_1(x,y,z), v_2(x,y,z), v_3(x,y,z))$, normalni vektor na ploskev $\vec{\nu} = (\cos\alpha, \cos\beta, \cos\gamma)$ krivuljo C $\vec{r} = (x(s), y(s), z(s))$ in njen tangentni vektor $\vec{t} = \frac{d\vec{r}}{ds} = \left(\frac{dx}{ds}, \frac{dy}{ds}, \frac{dz}{ds}\right)$ dobimo:

$$\iint_S [((v_3)_y - (v_2)_y)\cos\alpha + ((v_1)_z - (v_3)_x)\cos\beta + ((v_2)_x - (v_1)_y)\cos\gamma] d\omega = \oint_C (v_1 dx + v_2 dy + v_3 dz)$$

Brezkoordinatna definicija divergence:

$$div\vec{v} = \lim_{r\to 0} \frac{1}{V_r} \iint_{S_r} \vec{v}\,\vec{\nu} d\omega$$

Brezkoordinatna definicija rotorja:

$$[rot\vec{v}(P)]_n = \lim_{r\to 0} \frac{1}{\omega_r} \oint_{C_r} v_t ds$$