Vektorska analiza

Gradient

$$\text{grad}\, f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

Divergenca

$$\text{div}\,\vec{v} = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}$$

Rotor

$$\text{rot}\,\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_1 & v_2 & v_3 \end{vmatrix}$$

Operator nabla:

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

Laplace-ov operator:

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla \circ \nabla = \nabla^2 = \Delta$$

Zapis vektorskih funkcij z operatorjem nabla:

grad $f = \nabla f$ div $f = \nabla \circ f$ rot $f = \nabla \times f$

$$\text{grad}\, f(r) = f'(r)\frac{\vec{r}}{r}$$

$$\text{div grad}\; f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = \Delta f$$

$$\text{div}(f\vec{v}) = f\,\text{div}\,\vec{v} + \vec{v}\,\text{grad}\, f$$

$$\text{div}(\lambda\vec{v}) = \lambda\,\text{div}\,\vec{v}$$

$$\text{div}(\vec{u} + \vec{v}) = \text{div}\,\vec{u} + \text{div}\,\vec{v}$$

$$\text{div}(f\nabla g) = f\Delta g + \nabla f \nabla g$$

$$\text{div}(f\nabla g - \text{div}(g\nabla f)) = f\Delta g - g\Delta f$$

$$\text{rot}(\text{grad}\, f) = 0$$

$$\text{div}(\text{rot}\,\vec{v}) = 0$$

$$\text{rot}(f\vec{v}) = \text{grad}\, f \times \vec{v} + f\,\text{rot}\,\vec{v}$$

$$\text{rot rot}\,\vec{v} = \text{grad div}\,\vec{v} - \Delta\vec{v}$$

Laplace-ov operator v kartezičnih koordinatah:

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

Laplace-ov operator v cilindričnih koordinatah:

$$\Delta u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2} + \frac{\partial^2 u}{\partial z^2}$$

Laplace-ov operator v sferičnih koordinatah:

$$\Delta u = \frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r}\frac{\partial^2 u}{\partial \varphi^2} + \frac{\cot\varphi}{r^2}\frac{\partial u}{\partial \varphi} + \frac{1}{r^2 \sin^2\varphi}\frac{\partial^2 u}{\partial \vartheta^2}$$

Vrste polj:

Centralno polje: $u = f(r),\ \ r = |\vec{r}|$

Solenoidalno polje: $\text{div}\,\vec{v} = 0$

Potencialno polje (nevrtinčno): $\text{rot}\,\vec{v} = 0$