Izpit Matematika IV

5.julij 2011

Rešitve

1. naloga

Z uporabo pravila $\mathcal{L}[f(t-a)u_a(t)] = F(s)e^{-as}$ upoštevamo faktor $e^{-\pi s}$ nazadnje.
Prej poiščemo $\mathcal{L}^{-1}$ funkcij $G(s)$ in $H(s)$.

$$G(s) = \frac{s+1}{s^2+s+1} = \frac{(s+\frac{1}{2})+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}} = H(s+\frac{1}{2}) \quad , \quad H(s) = \frac{s+\frac{1}{2}}{s^2+\frac{3}{4}}$$

Poiščemo najprej $\mathcal{L}^{-1}\Big[H(s)\Big]$ in uporabimo pravilo $\mathcal{L}^{-1}\Big[H(s-a)\Big] = e^{-at}h(t)$.

$$h(t) = \mathcal{L}^{-1}\Big[\frac{s+\frac{1}{2}}{s^2+\frac{3}{4}}\Big] = \mathcal{L}^{-1}\Big[\frac{s}{s^2+(\sqrt{3}/2)^2}+\frac{1}{2}\frac{2}{\sqrt{3}}\frac{\sqrt{3}/2}{s^2+(\sqrt{3}/2)^2}\Big] = \cos\Big(\frac{\sqrt{3}}{2}t\Big)+\frac{1}{\sqrt{3}}\sin\Big(\frac{\sqrt{3}}{2}t\Big)$$

$$g(t) = \mathcal{L}^{-1}\Big[H(s+\frac{1}{2})\Big] = e^{-t/2}\Big[\cos\Big(\frac{\sqrt{3}}{2}t\Big)+\frac{1}{\sqrt{3}}\sin\Big(\frac{\sqrt{3}}{2}t\Big)\Big]$$

$$\boxed{f(t) = e^{-(t-\pi)/2}\Big[\cos\Big(\frac{\sqrt{3}}{2}(t-\pi)\Big)+\frac{1}{\sqrt{3}}\sin\Big(\frac{\sqrt{3}}{2}(t-\pi)\Big)\Big]\cdot u_\pi(t)}$$

2. naloga

$$s^2Y - s + 2 + Y = \frac{1}{s^2}$$
$$(s^2+1)Y = \frac{1}{s^2} + s - 2$$
$$Y = \frac{s^3 - 2s^2 + 1}{s^2(s^2+1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{Cs}{s^2+1} + \frac{D}{s^2+1}$$
$$A(s^2+1) + Bs(s^2+1) + Cs^3 + Ds^2 = s^3 - 2s^2 + 1$$

$$k \quad \rightarrow \quad A = 1$$
$$ks \quad \rightarrow \quad B = 0$$
$$ks^2 \quad \rightarrow \quad A + D = -2 \quad \rightarrow \quad D = -3$$
$$ks^3 \quad \rightarrow \quad B + C = 1 \quad \rightarrow \quad C = 1$$

$$Y = \frac{1}{s^2} + \frac{s}{s^2+1} - \frac{3}{s^2+1}$$

$$\boxed{y(t) = t + \cos t - 3\sin t}$$

3. naloga

$$u_x = u_v + 3u_z$$
$$u_{xx} = (u_v + 3u_z)_v + 3(u_v + 3u_z)_z = u_{vv} + 6u_{vz} + 9u_{zz}$$
$$u_{xy} = (u_v + 3u_z)_v + (u_v + 3u_z)_z = u_{vv} + 4u_{vz} + 3u_{zz}$$
$$u_y = u_v + u_z$$
$$u_{yy} = (u_v + u_z)_v + (u_v + u_z)_z = u_{vv} + 2u_{vz} + u_{zz}$$

$$u_{vv} + 6u_{vz} + 9u_{zz} - 4(u_{vv} + 4u_{vz} + 3u_{zz}) + 3(u_{vv} + 2u_{vz} + u_{zz}) = 0$$
$$u_{vv} + 6u_{vz} + 9u_{zz} - 4u_{vv} - 16u_{vz} - 12u_{zz} + 3u_{vv} + 6u_{vz} + 3u_{zz} = 0$$
$$-4u_{vz} = 0$$

$$u_v = f_1(v)$$
$$u = \int f_1(v)dv = f(v) + g(z)$$

$$\boxed{u(x,y) = f(x+y) + g(x+3y)}$$

4. naloga

$$-(1+x^2 2y')' = 0$$
$$1 + x^2 2y' = C = 1 - 2A$$
$$y' = -\frac{A}{x^2}$$

$$\boxed{y = \frac{A}{x} + B}$$

5. naloga

$p = P(\text{ pade stran A }) = \frac{2}{3}$

Poskus ponovimo n=3 krat, najti je treba verjetnost, da se dogodek zgodi k=1 krat.

$$P = \binom{n}{k} p^k (1-p)^{n-k} = \binom{3}{1} \frac{2}{3} (\frac{1}{3})^2 = \boxed{\frac{2}{9}}$$