Ime, priimek ....

Naloga	točke
1.	
2.	
3.	
4.	
5.	
Skupaj	

IZPIT IZ MATEMATIKE IV - UNI

14. junij 2007

1. Poiščite Laplaceovo transformiranko funkcije
$$f(t) = \frac{e^{-at} - e^{-bt}}{t}.$$
2. Poiščite prvih pet členov v razvoju rešitve diferencialne enačbe
$$xy'' + y = 0, \qquad y(0) = 0, \quad y'(0) = 1.$$
3. Poiščite rešitev parcialne diferencialne enačbe
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$
na pravokotniku $[0,a] \times [0,b]$ pri pogojih
$$u(0,y) = u(x,0) = u(x,b) = 0, \quad u(a,y) = A \sin \frac{\pi y}{b}.$$
4. Poiščite ekstremalo funkcionala
$$I[y] = \int_a^b y'(1 + x^2 y')\,\mathrm{d}x.$$
5. Dva strelca ustrelita proti cilju in ta je pri tem enkrat zadet. Kolika je verjetnost, da ga je zadel prvi strelec, če prvi strelec zadeva z verjetnostjo $0,8$, drugi pa z verjetnostjo $0,6$.

1. $f(t) = \frac{e^{-at}}{t} - \frac{e^{-bt}}{t}$

POZOR!!! REŠITVE NISO NUJNO PRAVILNE. ČE IMA KDO BOLJŠE NAJ OBJAVI.

$\int \frac{1}{u+a} du = \int \frac{1}{t} dt = \ln t = \ln(u+a)$

$t = u+a$
$dt = du$

$\ln(1) = 0$
$a \ln(x) = \ln x^a$

$\mathcal{L}[e^{-at}] = \frac{1}{s+a} = F(s)$

$\mathcal{L}\left[\frac{e^{-at}}{t}\right] = \int_s^\infty F(u)du = \int_s^\infty \frac{1}{u+a} du = \ln|u+a| \Big|_s^\infty$

$\mathcal{L}\left[\frac{e^{-bt}}{t}\right] = \ln|u+b| \Big|_s^\infty$

$\mathcal{L}[f(t)] = \left[\ln|u+a| - \ln|u+b|\right]\Big|_s^\infty = \ln\frac{u+a}{u+b}\Big|_s^\infty = \ln\frac{1+\frac{a}{u}}{1+\frac{b}{u}}\Big|_s^\infty =$

$= \ln 1 - \ln\frac{1+\frac{a}{s}}{1+\frac{b}{s}} = \ln\frac{1+\frac{b}{s}}{1+\frac{a}{s}}$

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2. $xy'' + y = 0 \quad y(0) = 0 \quad y'(0) = 1 \Rightarrow y = \sum_{m=0}^{\infty} c_m x^m \quad y' = \sum_1 c_m x^{m-1} \cdot m$

$c_0 = 0 \quad c_1 = 1$

$y'' = \sum_2^\infty c_m (m-1) \cdot m \cdot x^{m-2}$

$\sum_2^\infty m(m-1)c_m x^{m-1} + \sum_0^\infty c_m x^m = 0$

$+ \sum_1^\infty c_{m-1} x^{m-1} = 0$

$c_0 + \sum_2^\infty x^{m-1} \cdot \underbrace{\left[c_{m-1} + c_m m(m-1)\right]}_{=0} = 0$

$c_m = -\frac{c_{m-1}}{m(m-1)}$

$c_0 = 0 \quad c_5 = -\frac{c_4}{5\cdot 4} = \frac{1}{144\cdot 20}$

$c_1 = 1$

$c_2 = -\frac{c_1}{2\cdot 1} = -\frac{1}{2}$

$c_3 = -\frac{c_2}{3\cdot 2} = \frac{1}{12}$

$c_4 = -\frac{c_3}{4\cdot 3} = -\frac{1}{144}$

$y(x) = x - \frac{1}{2}x^2 + \frac{1}{12}x^3 - \frac{1}{144}x^4 + \frac{1}{144\cdot 20}x^5 \ldots$

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4. $I[y(x)] = \int_a^b \underbrace{y'(1+x^2y')}_{f = f(x,y')} dx \Rightarrow f - y' f_{y'} = C_1$

$y' + x^2y'^2 - y'(1+2x^2y') = C_1$

$x^2 y' = -C_1$

$y' = -\frac{C_1}{x^2}$

$\frac{dy}{dx} = -\frac{C_1}{x^2}$

$\int dy = -C_1 \int \frac{dx}{x^2}$

$y = +C_1 x^{-1} + C_2$

$y = \frac{C_1}{x} + C_2$

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5. 2 strelca ustrelita:

MOŽNI DOGODKI

oba zgrešita $P(H_{00}) = 0{,}2 \cdot 0{,}4$

1. zadane 2. zgreši $P(H_{10}) = 0{,}8 \cdot 0{,}4$

2. zadane 1. zgreši $P(H_{01}) = 0{,}2 \cdot 0{,}6$

oba zadeneta $P(H_{11}) = 0{,}8 \cdot 0{,}6$

$\sum P(H_i) = 1$

$P(H_{10}|A) = ?$

A: le eden zadene

$P(A|H_{00}) = 0$
$P(A|H_{10}) = 1$
$P(A|H_{01}) = 1$
$P(A|H_{11}) = 0$

$P(H_{10}|A) = \frac{P(A|H_{10}) \cdot P(H_{10})}{P(A)} \quad ; \quad P(A) = \sum_1^4 P(A|H_i) \cdot P(H_i)$

$= \frac{1 \cdot 0{,}32}{0 + 1\cdot 0{,}32 + 1 \cdot 0{,}12 + 0 \cdot 0{,}48} = \frac{0{,}32}{0{,}44} = 0{,}727$

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3. ker se za spr. y robni pogoji homogeni začnemo z njo $(G(y))$

$u(x,y) = F(x) \cdot G(y) \quad u_{xx} = \ldots \quad u_{yy} = \ldots$

$\frac{\ddot{G}}{G} = -\frac{F''}{F} = -k^2 \Rightarrow G(y) = A\cos ky + B\sin ky \quad F(x) = C e^{+kx} + D e^{-kx} \quad u = \sum_0^\infty \sin\frac{n\pi}{b}y\,(C_n e^{k_n x} + D_n e^{-k_n x})$

$G(0) = A = 0$

$G(b) = B\sin kb = 0 \Rightarrow k_n = \frac{n\pi}{b}$

Itd....