Izpit Matematika IV

29.junij.2009

Rešitve

1. naloga

$$Y(s) = \frac{2}{s^3} + Y(s)\frac{1}{s^2+1}$$

$$Y(s)\left(1 - \frac{1}{s^2+1}\right) = \frac{2}{s^3}$$

$$Y(s) = \frac{2(s^2+1)}{s^5} = \frac{2}{s^3} + \frac{2}{s^5}$$

$$\boxed{Y(t) = t^2 + \frac{t^4}{12}}$$

2. naloga

Prve štiri *Legendrove* polinome prepišemo iz priročnika.

$10x^3 + 2x^2 + x + 3 = a \cdot 1 + b \cdot x + c \cdot \frac{1}{2}(3x^2 - 1) + d\frac{1}{2}(5x^3 - 3x)$

$kx^3 \quad \rightarrow \qquad 10 = \frac{5}{2}d \quad \rightarrow \qquad d = 4$

$kx^2 \quad \rightarrow \qquad 2 = \frac{3}{2}c \quad \rightarrow \qquad c = \frac{4}{3}$

$kx \quad \rightarrow \qquad 1 = b - \frac{3}{2}d \quad \rightarrow \quad b = 7$

$k \quad \rightarrow \qquad 3 = a - \frac{1}{2}c \quad \rightarrow \quad a = \frac{11}{3}$

$$y = \frac{11}{3}P_0 + 7P_1 + \frac{4}{3}P_2 + 4P_3$$

3. naloga

$u = F(x)G(y)$

$F'G + FG' = 2(x+y)FG$

$\frac{F'}{F} + \frac{G'}{G} = 2(x+y)$

$\frac{F'}{F} - 2x = -\frac{G'}{G} + 2y = A$

$\frac{F'}{F} = A + 2x$

$\frac{dF}{F} = (A + 2x)dx$

$\ln F = Ax + x^2 + \ln B$

$F(x) = Be^{Ax+x^2}$

$\frac{G'}{G} = -A + 2y$

$\frac{dG}{G} = (-A + 2y)dy$

$\ln G = (-Ay + y^2) + \ln C$

$G(y) = Ce^{-Ay+y^2}$

$$\boxed{u = De^{A(x-y)+(x^2+y^2)}}$$

4. naloga

$2y - (2y')' = 0$
$y'' - y = 0$

$\lambda^2 - 1 = 0$
$\lambda_{1,2} = \pm 1$
$y = A \cosh x + B \sinh x$

$x = 0 \quad \rightarrow \quad A = 0$
$x = 1 \quad \rightarrow \quad B \sinh 1 = 1$

$$\boxed{y = \frac{\sinh x}{\sinh 1}}$$

5. naloga

$1.kocka$	$2.kocka$	$verjetnost$
1	X	$\frac{6}{36}$
2	< 6	$\frac{5}{36}$
3	< 5	$\frac{4}{36}$
4	< 4	$\frac{3}{36}$
5	< 3	$\frac{2}{36}$
6	< 2	$\frac{1}{36}$

$$P(\text{ vrgli bomo manj kot 8 pik }) = \frac{6}{36} + \frac{5}{36} + \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \boxed{\frac{7}{12}}$$