Izpit Matematika IV

15.9.2008

Rešitve

1. naloga

$sY - 0 - 2aY + (a^2 + 1)\frac{Y}{s} = \frac{1}{s}$

$\left(s^2 - 2as + a^2 + 1\right)Y = 1$

$Y = \frac{1}{(s-a)^2+1}$

$\mathcal{L}^{-1}\left[\frac{1}{s^2+1}\right] = \sin t$

$$\boxed{y(t) = e^{at}\sin t}$$

2. naloga

Funkcija je liha, zato nastavimo samo lihe *Legendreove* polinome:

$5x^3 + x = C_1 P_1(x) + C_3 P_3(x)$

$5x^3 + x = C_1 x + C_3 \frac{1}{2}(5x^3 - 3x)$

$C_3 = 2$, $x = C_1 x - 3x$

$C_1 = 4$

$$\boxed{y = 4P_1(x) + 2P_3(x)}$$

3. naloga

$u = F(x)G(y)$

$xF'(x)G(x) = yF(x)G'(y)$

$x\frac{F'(x)}{F(x)} = y\frac{G'(y)}{G(y)} = A$

$\frac{x}{F}\frac{dF}{dx} = A$

$\frac{dF}{F} = A\frac{dx}{x}$

$\ln F = A \ln x + \ln C$

$F(x) = Cx^A$

Na isti način $G(y) = Dy^A$

$$\boxed{u(x,y) = B(xy)^A}$$

4. naloga

$2y - 2\cosh x - (-2y')' = 0$

$y'' + y = \cosh x$

$\lambda^2 + 1 = 0$

$\lambda_{1,2} = \pm i$

$y_h = A\cos x + B\sin x$

$y_p = C\cosh x + D\sinh x$

$C\cosh x + D\sinh x + C\cosh x + D\sinh x = \cosh x$

$C = \frac{1}{2}$, $D = 0$

$y_p = \frac{1}{2}\cosh x$

$$\boxed{y = A\cos x + B\sin x + \frac{1}{2}\cosh x}$$

5. naloga

$P(A) = 1 - P(\text{družina ima 3deč}) - P(\text{družina ima 3dek}) = 1 - \frac{1}{8} - \frac{1}{8} = \frac{3}{4}$
$P(B) = P(\text{ima enage deč}) + P(\text{ima tri dek}) = \frac{3}{8} + \frac{1}{8} = \frac{1}{2}$
$P(AB) = P(\text{ima enega deč}) = \frac{3}{8}$
$P(B/A) = \frac{P(AB)}{P(A)} = \frac{3/8}{3/4} = \frac{1}{2}$

Vidimo, da je $P(B/A) = P(B)$
B je neodvisen od A, neodvisnost pa je vzajemna.