Rešitve 2. Kolokvija iz Matematike IV

30.5.2005

1. naloga

$$\begin{aligned}
u &= F(x)G(t)\\
FG' &= F''G\\
\tfrac{G'}{G} &= \tfrac{F''}{F} = -\lambda^2
\end{aligned}$$

$$\begin{aligned}
F'' + \lambda^2 F &= 0\\
F(x) &= A\cos\lambda x + B\sin\lambda x\\
F'(x) &= (-A\sin\lambda x + B\cos\lambda x)\lambda\\
x = 0 &\Rightarrow B = 0\\
x = \pi &\Rightarrow \lambda = n\\
F_n(x) &= A_n\cos nx
\end{aligned}$$

$$\begin{aligned}
\tfrac{1}{G}\tfrac{dG}{dt} &= -n^2\\
\int \tfrac{dG}{G} &= \int -n^2 dt\\
\ln G &= -n^2 t + \ln C\\
G_n(t) &= C_n e^{-n^2 t}
\end{aligned}$$

$$\begin{aligned}
u(x,t) &= \sum_{n=1}^{\infty} a_n e^{-n^2 t}\cos nx\\
t = 0 &\Rightarrow \sum_{n=1}^{\infty} a_n \cos nx = \cos 4x\\
& \quad a_4 = 1\,,\ a_n = 0
\end{aligned}$$

$$u(x,t) = e^{-16t}\cos 4x$$

2. naloga

$$
\begin{array}{lll}
X=2 & GG & p_2=\frac{1}{2}\frac{1}{2}=\frac{1}{4}\\
X=3 & CGG & p_3=\frac{1}{2}\frac{1}{2}\frac{1}{2}=\frac{1}{8}\\
X=4 & YCGG\,,\ Y=\text{karkoli} & p_4=1\frac{1}{2}\frac{1}{2}\frac{1}{2}=\frac{1}{8}\\
X=5 & YYCGG\,,\ YY\neq GG & p_5=\frac{3}{4}\frac{1}{2}\frac{1}{2}\frac{1}{2}=\frac{3}{32}\\
X=7 & YYYYCGG & p_7=\frac{1}{8}(1-(\frac{1}{4}+\frac{1}{8}+\frac{1}{8}))=\frac{1}{16}\\
 & YYYY=\text{vse razen} & \\
 & GGYY\,,\ CGGY\,,\ YCGG & \\
X=n & Y\ldots YCGG & p_n=\frac{1}{8}(1-(p_2+p_3+\cdots+p_{n-3}))\\
 & Y\ldots Y=\text{vse razen} & \\
 & (X=2)\cup(X=3)\cup\ldots(X=n-3) &
\end{array}
$$

3. naloga

$$
\begin{array}{ll}
\text{premica}: & y=\operatorname{tg}\alpha\, x+1 \quad , \quad 0<\alpha<\pi\\
\text{ničla}: & \operatorname{tg}\alpha\, x+1=0\\
 & x=-\operatorname{ctg}\alpha
\end{array}
$$

$$P(X<1)=P(-\operatorname{ctg}\alpha<1)=P(\operatorname{ctg}\alpha>-1)=P(\alpha<\operatorname{arcctg}(-1))=P(\alpha<\frac{3\pi}{4})=\frac{3}{4}$$

$$F(x)=P(X<x)=P(-\operatorname{ctg}\alpha<x)=P(\operatorname{ctg}\alpha>-x)=P(\alpha<\operatorname{arcctg}(-x))=\frac{1}{\pi}\operatorname{arcctg}(-x)$$