

Rešitve kolokvija MATEMATIKE IV

31.5.2007

1. naloga.

$$\begin{aligned}\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \varphi^2} &= 4 \\ (ru')' &= 4r \\ ru' &= 2r^2 + A \\ u' &= 2r + \frac{A}{r} \\ u &= r^2 + A \ln r + B \\ r = 1 &\rightarrow B = 0 \\ r = 3 &\rightarrow A = -1 \\ u &= r^2 - \ln r\end{aligned}$$

2. naloga.

$$\begin{aligned}0 - \frac{d}{dx} \left(\frac{y'}{x\sqrt{1+y'^2}} \right) &= 0 \\ \frac{y'}{x\sqrt{1+y'^2}} &= A \\ y'^2 &= A^2 x^2 (1 + y'^2) \\ y'^2 (1 - A^2 x^2) &= A^2 x^2 \\ y' &= \frac{Ax}{\sqrt{1 - A^2 x^2}} \\ y &= -\frac{1}{A} \sqrt{1 - A^2 x^2} + B \\ (y - B)^2 &= \frac{1}{A^2} - x^2 \\ x^2 + (y - B)^2 &= R^2 \\ (3 - B)^2 &= R^2 \\ 4 + (1 - B)^2 &= R^2 \\ (3 - B)^2 - (1 - B)^2 &= 4 \\ 8 - 4B &= 4 \\ B &= 1 \\ R^2 &= 4 \\ x^2 + (y - 1)^2 &= 4\end{aligned}$$

3. naloga.

Za posamezno partijo označimo dogodke:

B = zmaga beli

C = zmaga črni

R = remi

P (dvoboj se konča v največ treh partijah) =

$$P(\overline{R} \cup R\overline{R} \cup RR\overline{R}) =$$

$$\frac{3}{5} + \frac{2}{5} \frac{3}{5} + \left(\frac{2}{5}\right)^2 \frac{3}{5} = \frac{75+30+12}{125} = \frac{117}{125}$$

P (zmaga tisti, ki ima prvi bele figure) =

P (zmaga z belimi \cup zmaga s črnimi) =

$$P(B \cup RRB \cup RRRRB \cup \dots) + P(RC \cup RRRC \cup RRRRRC \cup \dots) =$$

$$\frac{2}{5} + \left(\frac{2}{5}\right)^2 \frac{2}{5} + \left(\frac{2}{5}\right)^4 \frac{2}{5} + \dots + \frac{2}{5} \frac{1}{5} + \left(\frac{2}{5}\right)^3 \frac{1}{5} + \left(\frac{2}{5}\right)^5 \frac{1}{5} + \dots =$$

$$\frac{\frac{2}{5}}{1 - \frac{4}{25}} + \frac{\frac{2}{25}}{1 - \frac{4}{25}} = \frac{10+2}{21} = \frac{12}{21} = \frac{4}{7}$$