MATEMATIKA IV

Fourierjeva separacija – rešene kolokvijske naloge 99 – 06

Šolsko leto	2008 / 2009
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UREJANJE DOKUMENTA

VERZIJA	01	**REVIZIJA**	01
DATUM	29. 5. 2009		
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PREGLEDAL			

OPOMBE

Vkolikor je kdo reševal tudi ostale kolokvijske naloge, se priporočamo:
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POPRAVKI

1999

1. (4 točke) S Fourierovo metodo reši parcialno diferencialno enačbo

$$\begin{aligned} u_{xx} &= u_t + u \\ u(0,t) &= 0 \\ u(\pi,t) &= 0 \\ u(x,0) &= \sin 3x \end{aligned}$$

na območju $0 < x < \pi$, $t > 0$!

2000

1. (4 točke) Na območju $0 < x < 2,\ t > 0$ reši parcialno diferencialno enačbo

$$\begin{aligned} &u_t = 4u_{xx} \\ &u_x(0,t) = 0 \\ &u_x(2,t) = 0 \\ &u(x,0) = \cos^2 \tfrac{\pi x}{2} \end{aligned}$$

2001

1. (4 točke) S Fourierovo metodo separacije spremenljivk poišči rešitev $u(x,t)$ parcialne diferencialne enačbe

$$\begin{aligned} u_{xx} &= u_{tt} \\ u_x(0,t) &= 0 \\ u_x(\pi,t) &= 0 \\ u(x,0) &= \cos(4x) \\ u_t(x,0) &= 1 \end{aligned}$$

2002

1. (4 točke) Na območju $0 < x < \pi,\ t > 0$ reši parcialno diferencialno enačbo

$$\begin{aligned} &u_t = u_{xx} \\ &u_x(0,t) = 0 \\ &u_x(\pi,t) = 0 \\ &u(x,0) = \cos^2 x \end{aligned}$$

2003

1. (4 točke) Na območju $0 < x < 2\pi,\ t > 0$ reši parcialno diferencialno enačbo

$$\begin{aligned} u_t + u &= u_{xx} \\ u_x(0,t) &= 0 \\ u_x(2\pi,t) &= 0 \\ u(x,0) &= \cos 2x \end{aligned}$$

2004

1. (4 točke) Reši Dirichletovo nalogo v polarnih koordinatah

$$\begin{aligned} \triangle u(r,\varphi) &= 0 \quad , \quad \text{za } r < 1 \\ u(1,\varphi) &= 4\sin^3\varphi \end{aligned}$$

2005

1. (4 točke) Poišči rešitev $u(x,t)$ diferencialne enačbe

$$\begin{aligned} u_t &= u_{xx} \quad , 0 < x < \pi\,,\ t > 0 \\ u_x(0,t) &= 0 \\ u_x(\pi,t) &= 0 \\ u(x,0) &= \cos 4x \end{aligned}$$

2006

1. (4 točke) Poišči rešitev $u(x,t)$ diferencialne enačbe

$$\begin{aligned} u_{xx} &= u_t \qquad 0 < x < \pi\,,\ 0 < t \\ u(0,t) &= 0 \\ u(\pi,t) &= 0 \\ u(x,0) &= x \end{aligned}$$

$\boxed{1999/1}$

$u_{xx} = u_t + u$

$F''(x)G(t) = F(x)G'(t) + F(x)G(t)$

$\frac{F''(x)}{F(x)} = \frac{G'(t)}{G(t)} + 1 = k$

$u(\emptyset, t) = \emptyset$
$u(\pi, t) = \emptyset$
$u(x, \emptyset) = \sin 3x$

1) $F''(x) - kF(x) = \emptyset \qquad \lambda^2 - k = \emptyset \qquad \lambda_{1,2} = \pm\sqrt{k}$

1.3) homogeni, se vidi gor $\qquad k = -\ell^2 \qquad \lambda_{1,2} = \pm i\ell$

$F_\ell(x) = A_\ell \cos \ell x + B_\ell \sin \ell x$

• $\emptyset = A_\ell \cos \ell \emptyset + B_\ell \sin \ell \emptyset \rightarrow A_\ell = \emptyset$

• $\emptyset = B_\ell \sin \ell \pi \rightarrow \ell \in \mathbb{N} \qquad B_\ell \neq \emptyset$

$F_\ell(x) = B_\ell \sin \ell x$

2.3) $\frac{G'(t)}{G(t)} + 1 = k \qquad k = -\ell^2 \qquad \frac{dG(t)}{G(t)} = (-\ell^2 - 1)\,dt \rightarrow G_\ell(t) = C_\ell e^{(-\ell^2-1)t}$

3) $u(x,t) = \sum_{\ell=1}^{\infty} B_\ell \sin \ell x\, C_\ell e^{-(\ell^2+1)t} = \sum_{\ell=1}^{\infty} \vartheta_\ell \sin \ell x\, e^{-(\ell^2+1)t}$

4) $u(x, \emptyset) = \sin 3x$

$\sin 3x = \sum_{\ell=1}^{\infty} \vartheta_\ell \sin \ell x \cancel{e^{\ell}} \rightarrow \vartheta_\ell = 1$ za $\ell = 3$

5) $\boxed{u(x,t) = \sin(3x)\, e^{-10t}}$

$\boxed{2000/1}$

$u_t = 4u_{xx}$

$u_x(0,t)=0 \qquad u_x(2,t)=0 \qquad u(x,0)=\cos^2\frac{\pi x}{2}$

$F(x)G'(t)=4F''(x)G(t)$

$\frac{G'(t)}{4G(t)}=\frac{F''(x)}{F(x)}=k$

1) $F''(x)-kF(x)=0 \qquad \lambda^2-k=0 \qquad \lambda_{1,2}=\pm\sqrt{k}$

1.1) $k=l^2 \qquad F_l(x)=A_l e^{lx}+B_l e^{-lx} \qquad F_l'(x)=lA_l e^{lx}-lB_l e^{-lx}$

$\bullet\ 0=A_l-B_l \to A_l=B_l \qquad \bullet\ 0=lA_l(e^{2l}-e^{-2l}) \to 2l=-2l \to l=0$

trivialna rešitev

1.2) $F_0(x)=A_0+B_0x \qquad F_0'(x)=B_0 \qquad \bullet\ 0=B_0 \qquad A_0$ poljuben

$\underline{F_0(x)=A_0}$

1.3) $F_l(x)=A_l\cos lx+B_l\sin lx \qquad F_l'(x)=-lA_l\sin lx+lB_l\cos lx$

$\bullet\ 0=-l\cancel{A_l\sin l0}+lB_l\cos l0 \to B_l=0$

$\bullet\ 0=-lA_l\sin l2 \to 2l=n\pi \to l=n\pi/2$

$\underline{F_n(x)=A_l\cos(\frac{n\pi}{2}x)}$

2.2) $\frac{G'(t)}{4G(t)}=0 \to \dots \to \underline{G_0(t)=C_0}$

2.3) $\frac{G'(t)}{\cancel{4}G(t)}=-\frac{n^2\pi^2}{\cancel{4}} \to \frac{dG}{G}=(dt)(n^2\pi^2) \to \log G=-n^2\pi^2t+\log C$

$\underline{G_n(t)=C_n e^{-n^2\pi^2t}}$

3) $u(x,t)=F_0(x)G_0(t)+\sum_{n=1}^{\infty}F_n(x)G_n(t)=A_0C_0+\sum_{n=1}^{\infty}A_l\cos(\frac{n\pi}{2}x)e^{-n^2\pi^2t}C_n$

$=\alpha_0+\sum_{n=1}^{\infty}\alpha_n\cos(\frac{n\pi}{2}x)e^{-n^2\pi^2t}=*$

4) $u(x,0)=\underbrace{\cos^2\frac{\pi x}{2}}=*=\alpha_0+\sum_{n=1}^{\infty}\alpha_n\cos(\frac{n\pi}{2}x)\cancel{e^0}$

$=\frac{1}{2}+\frac{1}{2}\cos\pi x \qquad // \qquad \alpha_0=\frac{1}{2} \qquad \alpha_n=\frac{1}{2} \qquad \frac{n\pi}{2}=\pi \to n=2$

5) $\boxed{u(x,t)=\frac{1}{2}+\frac{1}{2}\cos(\pi x)\,e^{-4\pi^2t}}$

I

2001/1

$u_{xx} = u_{tt}$ $\quad u_x(0,t) = 0$

$F''(x)G(t) = F(x)G''(t) \; /:()$ $\quad u_x(\pi,t) = 0$

$\frac{F''(x)}{F(x)} = \frac{G''(t)}{G(t)} = k$ $\quad u(x,0) = \cos 4x$

$u_t(x,0) = 1$

1) $F''(x) - kF(x) = 0 \quad \lambda^2 - k = 0 \quad \lambda = \pm\sqrt{k}$

1.1) $k = l^2 \quad \lambda_{1,2} = \pm l \quad F_l(x) = A_l e^{lx} + B_l e^{-lx}$

$F_l'(x) = l A_l e^{lx} - l B_l e^{-lx}$

$\cdot\, 0 = l A_l - l B_l \quad A_l = B_l$

$\cdot\, 0 = A_l(l e^{\pi l} - l e^{-\pi l}) \rightarrow \pi l = -\pi l \rightarrow l = 0 \quad \times \; l > 0$

$A_l = 0 \rightarrow B_l = 0 \quad \times$ triv

1.2) $k = 0 \quad \lambda_{1,2} = 0 \quad F_0(x) = A_0 + B_0 x \quad F_0'(x) = B_0$

$\cdot\, 0 = B_0 \rightarrow A$ poljuben $\quad \underline{F_0(x) = A_0}$

1.3) $k = -l^2 \quad \lambda_{1,2} = \pm i l \quad F_l(x) = A_l \cos lx + B_l \sin lx$

$F_l'(x) = -l A_l \sin lx + l B_l \cos lx$

$\cdot\, 0 = -l \cdot \cancel{A_l \sin l0} + l B_l \cos l0 \rightarrow B_l = 0$

$\cdot\, 0 = -l A_l \sin l\pi \rightarrow \cdot\, A_l = 0 \quad \times$ triv

$\cdot\, l \in \cancel{\mathbb{Z}\setminus\{0\}}\ \mathbb{N}, \; l > 0$

$\underline{\underline{F_l(x) = A_l \cos lx \;, \; l \in \mathbb{N}}}$

2.2) $\frac{G''(t)}{G(t)} = 0 \quad (k = 0) \quad \lambda_{1,2} = 0$

$G_0(t) = C_0 + D_0 t$

2.3) $G''(t) - kG(t) = 0 \quad \lambda^2 - k = 0$

$\lambda_{1,2} = \pm i l$

$G_l(t) = C_l \cos lt + D_l \sin lt$

3) $u(x,t) = \sum_k F_k(x) G_k(t) = F_0(x)G_0(t) + \sum_{l=1}^{\infty} F_l(x) G_l(t) =$

$= A_0(C_0 + D_0 t) + \sum_{l=1}^{\infty} A_l \cos lx (C_l \cos lt + D_l \sin lt)$

$= \gamma_0 + \delta_0 t + \sum_{l=1}^{\infty} \cos lx (\gamma_l \cos lt + \delta_l \sin lt)$

4) $\cos 4x = \gamma_0 + \cancel{\delta_0 0} + \sum_{l=1}^{\infty} \gamma_l \cos lx \quad \underline{\underline{\gamma_l = 1 \quad \gamma_0 = 0 \quad l = 4}}$ za $l = 1$

II

$u_t'(x,t) = \sigma_0 + \sum_{l=1}^{\infty} \cos lx \, \sigma_l \cos lt - \cos lx \, \gamma_l \sin lt$

$u_t(x,\emptyset) = 1$

$1 = \sigma_0 + \sum_{l=1}^{\infty} \cos lx \, \sigma_l \qquad \sigma_0 = 1 \qquad \sigma_l = \emptyset$

5) $u(x,t) = \boxed{1 \cdot t + \cos 4x \cos 4t}$

$\boxed{2002/1}$

$u_t = u_{xx}$ $\qquad u_x(\emptyset, t) = \emptyset$

$F(x)G'(t) = F''(x)G(t)$ $\qquad u_x(\pi, t) = \emptyset$

$\frac{F''(x)}{F(x)} = \frac{G'(t)}{G(t)} = k$ $\qquad u(x, \emptyset) = \cos^2 x$

1) $F''(x) - kF(x) = \emptyset \qquad \lambda^2 - k = \emptyset \qquad \lambda = \pm\sqrt{k}$

1.1) $k = l^2 \qquad F_l(x) = A_l e^{lx} + B_l e^{-lx} \qquad F_l'(x) = lA_l e^{lx} - lB_l e^{-lx}$

$\bullet\ \emptyset = A_l - B_l \rightarrow A_l = B_l \qquad \bullet\ \emptyset = lA_l(e^{l\pi} - e^{-l\pi}) \rightarrow l\pi = -l\pi \rightarrow l = \emptyset \ \otimes$

$A_l = B_l = \emptyset \ \otimes$

1.2) $k = \emptyset \qquad F_0(x) = A_0 + B_0 x \qquad F_0'(x) = B_0$

$\bullet\ \emptyset = B_0$, A poljuben

1.3) $k = -l^2 \qquad \lambda_{1,2} = \pm il \qquad F_l(x) = A_l \cos lx + B_l \sin lx$

$F_l'(x) = -lA_l \sin lx + lB_l \cos lx$

$\bullet\ \emptyset = -l\cancel{A_l} \sin l\emptyset + lB_l \cos l\emptyset \rightarrow B_l = \emptyset$

$\bullet\ \emptyset = -lA_l \sin l\pi \rightarrow l \in \mathbb{N} \qquad A_l = \emptyset \ \otimes$

$F_l(x) = A_l \cos lx, \ l \in \mathbb{N}$

2.2) $\frac{dG}{dtG} = \emptyset \qquad \log(G) = \log(C) \qquad G_0(t) = C_0$

2.3) $\frac{dG}{dtG} = -l^2 \qquad \log G = -l^2 t + \log C \qquad G_l(t) = C_l e^{-l^2 t}$

3) $u(x,t) = F_0(x)G_0(x) + \sum_{l=1}^{\infty} F_l(x)G_l(t) = B_0C_0 + \sum_{l=1}^{\infty} A_l \cos lx \, C_l e^{-l^2 t}$

$= \xi_0 + \sum_{l=1}^{\infty} \xi_l \cos(lx) e^{-l^2 t}$

4) $u(x, \emptyset) = \cos^2 x \qquad \cos^2 x = \xi_0 + \sum_{l=1}^{\infty} \xi_l \cos lx$

$\cos^2\alpha = \frac{1}{2}(1 + \cos 2\alpha)$

$\frac{1}{2} + \frac{1}{2}\cos 2\cancel{x} = \xi_0 + \sum_{l=1}^{\infty} \xi_l \cos lx \qquad l = 2 : \xi_l = 1/2$

$\xi_0 = 1/2$

5) $\boxed{u(x,t) = 1/2 + 1/2 \cos(2x) e^{-4t}}$

I

$\boxed{2003/1}$

$u_t + u = u_{xx}$ $\quad u_x(0,t)=0$, $u_x(2\pi,t)=0$ } to ni homogeno

$0<x<2\pi$

$t>0$ $\quad u(x,0)=\cos 2x$

$$F(x)G'(t)+F(x)G(t)=F''(x)G(t)$$

$$\frac{G'(t)}{G(t)}+1=\frac{F''(x)}{F(x)}=k$$

1) $F''(x)-kF(x)=\emptyset \qquad \lambda^2-k=\emptyset \qquad \lambda=\pm\sqrt{k}$

1.1) $k>0 \quad \lambda_{1,2}=\pm l$

$F_l(x)=A_l e^{lx}+B_l e^{-lx} \qquad F_l'(x)=lA_l e^{lx}+lB_l e^{-lx}$

- dobimo $A_l=\emptyset$, $B_l=\emptyset \rightarrow F_l(x)=0$

1.2) $k=0 \quad \lambda_{1,2}=\emptyset$

$F_0(x)=A_0+B_0x \qquad F_0'(x)=B_0$

A_0 poljuben, $B_0=\emptyset$

$F_0(x)=A_0$

1.3) $\lambda_{1,2}=\pm il$

$F_l(x)=A_l\cos lx+B_l\sin lx \qquad F_l'(x)=-A_l\cdot l\cdot\sin lx+B_l\cdot l\cdot\cos lx$

- $B_l\cdot l=\emptyset \rightarrow B_l=\emptyset$ (dobimo z vstavitvijo pogojev)
- $A_l=\emptyset$ ne gre
- $\sin 2\pi l=\emptyset \qquad 2l\pi=n\cdot\pi$, $n\in\mathbb{N}$

$l=n/2$

$F_n(x)=A_l\cos\frac{nx}{2}$, $n\in\mathbb{N}$

2) $\frac{G'(t)}{G(t)}+1=k$

2.1) X

2.2) $k=\emptyset \qquad \frac{G'(t)}{G(t)}=-1 \qquad G_0(t)=C_0e^{-t}$

2.3) $k=-l^2 \quad l>\emptyset$

$\frac{G'(t)}{G(t)}+1=-l^2 \qquad \frac{G'(t)}{G(t)}=-\frac{n^2}{4}-1 \qquad G_n(t)=C_ne^{\left(-\frac{n^2}{4}-1\right)\cdot t}$

II

3) $u(x,t) = F_0(x) G_0(t) + \sum_{n=1}^{\infty} F_n(x) G_n(t) =$

$= A_0 C_0 e^{-t} + \sum_{n=1}^{\infty} A_n \cos\frac{nx}{2} \cdot C_n e^{\left(-\frac{n^2}{4} - 1\right)t}$

4) $\cos 2x = D_0 + \sum_{n=1}^{\infty} D_n \cos\frac{nx}{2}$ $\quad D_0 = 0,\ D_n = 1$ za $n = 4$

5) $u(x,t) = \cos 2x\, e^{(-4-1)t} = \boxed{\cos 2x\, e^{-5t}}$

(1)

$\boxed{2004/1}$

$\Delta u(r,\varphi) = \emptyset$ $\quad r < 1$

$u(1,\varphi) = 4\sin^3\varphi$

$\Delta u = u_{xx} + u_{yy}$

$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\varphi\varphi} = \emptyset$

$F''(r)G(\varphi) + \frac{1}{r}F'(r)G(\varphi) + \frac{1}{r^2}F(r)G''(\varphi) = \emptyset$

$F''(r)G(\varphi) + \frac{1}{r}F'(r)G(\varphi) = -\frac{1}{r^2}F(r)G''(\varphi) \qquad /:(-F(r)G(\varphi))/r^2$

$-\frac{F''(r)r^2}{F(r)} - \frac{F'(r)r}{F(r)} = \frac{G''(\varphi)}{G(\varphi)} = k$

① $G''(\varphi) - kG(\varphi) = \emptyset \qquad \lambda^2 - k = \emptyset \qquad \lambda_{1,2} = \pm\sqrt{k}$

1.1) $k = \ell^2 \quad G_\ell(\varphi) = A_\ell e^{\ell\varphi} + B_\ell e^{-\ell\varphi}$

$G_\ell(\varphi) = \emptyset$ (periodično)

1.2) $k = \emptyset \quad G_0(\varphi) = A_\ell + B_0\varphi$

$G(\varphi) = G(\varphi + 2\pi) \rightarrow A_0 + B_0\varphi = A_0 + B_0(\varphi + 2\pi) \rightarrow B_0 = \emptyset$

$G_0(\varphi) = A_0$

1.3) $k = -\ell^2 \qquad \lambda_{1,2} = \pm i\ell$

$G_\ell(\varphi) = A_\ell\cos\ell\varphi + B_\ell\sin\ell\varphi \qquad \ell \in \mathbb{N} \quad$ periodičnost

2

2.2) $\frac{-r^2 F''(r)}{F(r)} - \frac{r F'(r)}{F(r)} = \emptyset \qquad r^2 F''(r) + r F'(r) = \cancel{\emptyset}$

$r^\lambda: \lambda(\lambda-1) + \lambda = \emptyset$

$\lambda^2 - \lambda + \lambda = 0$

$\lambda^2 = 0 \qquad F_r(r) = C_0 + D_0 \log r$

2.3) $k = -\ell^2: \quad -r^2 F''(r) - rF'(r) = \ell^2 F(r)$

$r^2 F''(r) + r F'(r) - l^2 F(r) = 0$

$\lambda(\lambda-1) + \lambda - \ell^2 = \cancel{\emptyset} \qquad \lambda^2 = \ell^2 \qquad \lambda_{1,2} = \pm \ell$

$F_\ell(r) = C_\ell r^\ell + D_\ell r^{-\ell}$

3) $u(r,\varphi) = F_0(r) G_0(r) + \sum_{\ell=1}^{\infty} F_\ell(r) G_\ell(\varphi) =$

$= (C_0 + D_0 \log r) A_0 + \sum_{\ell=1}^{\infty} (C_\ell r^\ell + D_\ell r^{-\ell})(A_\ell \cos \ell\varphi + B_\ell \sin \ell\varphi)$

$r < 1: \quad A_0 D_0 = \emptyset$

$D_\ell = \emptyset$

$u(r,\varphi) = A_0 C_0 + \sum_{\ell=1}^{\infty} C_\ell r^\ell (A_\ell \cos \ell\varphi + B_\ell \sin \ell\varphi)$

$= E_0 + \sum_{\ell=1}^{\infty} r^\ell (E_\ell \cos \ell\varphi + H_\ell \sin \ell\varphi)$

4) $4 \sin^3 \varphi = E_0 + \sum_{\ell=1}^{\infty} (E_\ell \cos \ell\varphi + H_\ell \sin \ell\varphi)$

$\downarrow$

$3 \sin\varphi - \sin 3\varphi \qquad E_0 = 0$

$E_\ell = 0 \qquad 3 = H_1 \qquad H_3 = -1$

5) $\boxed{u(r,\varphi) = 3 \cdot r \cdot \sin\varphi - r^3 \cos 3\varphi}$

$\boxed{2005/11}$

 2005/30/5

1

$\boxed{u_t = u_{xx}}$

$u_x(0,t) = \emptyset$

$u_x(\pi,t) = 0$

$u(x,0) = \cos 4x$

/:

$F(x)G'(t) = F''(x)G(t)$

$\frac{G'(x)}{G(t)} = \frac{F''(x)}{F(x)} = k$

1) $F''(x) - kF(x) = \emptyset$

1.1) $\lambda_{1,2} = \pm l \quad F_l(x) = A_l e^{lx} + B_l e^{-lx}$

$F_l'(x) = l \cdot A_l e^{lx} - l \cdot B_l e^{-lx}$

$\emptyset = l \cdot A_l - l \cdot B_l \quad A_l = B_l$ X

2) $F_0(x) = A_0 + B_0 x \quad F_0'(x) = B_0 \quad B_0 = \emptyset \quad A_0$ karkoli

$F_0(x) = A_0$

3) $F_l(x) = A_l \cos lx + B_l \sin lx$

$F_l'(x) = -A_l l \cdot \sin lx + B_l \cdot l \cdot \cos lx$

$\emptyset = -A_l \cdot l \cdot \sin\emptyset + B_l \cdot l \cdot \cos\emptyset \rightarrow B_l = \emptyset$

$\emptyset = -A_l \cdot l \cdot \sin l\pi \rightarrow l \in N$

$F_l(x) = A_l \cos lx$

2x) $\frac{G'(t)}{G(t)} = k \qquad \frac{dG}{dt\,G} = \emptyset \qquad G_0(t) = H_0$

2y) $\log G = -l^2 t \cdot \log \xi \rightarrow G_l(t) = \xi e^{-l^2 t}$

3) $u(x,t) = H_0 + \sum_{l=1}^{\infty} \mu \cdot \cos lx \cdot e^{-l^2 t}$

4) $\cos 4x = H_0 + \sum_{l=1}^{\infty} \mu \cdot \cos lx \cdot 1 \qquad H_0 = \emptyset$

$\cos 4x = \sum_{l=1}^{\infty} \mu \cos lx \rightarrow \mu = 1;\ l = 4$

5) $u(x,t) = \cos(4x)\, e^{-16t}$

2006/1

$u_{xx} = u_t \qquad 0 < x < \pi \qquad u(0,t) = 0$ } homogeni robni

$\qquad\qquad 0 < t \qquad u(\pi,t) = 0$ } pogoji (na robu enako 0)

$\qquad\qquad\qquad u(x,0) = x$

$u(x,t) = F(x)G(t)$

$F''(x)G(t) = F(x)G'(t) \quad /: F(x)G(t)$

$\frac{F''(x)}{F(x)} = \frac{G'(t)}{G(t)} = k$

razvoj v F.V. na $[-a, a]$:
$a_0 + \sum a_n \cos\frac{n\pi x}{a} + b_n \sin\frac{n\pi x}{a}$
- $2lx \rightarrow a = \pi/2$
- $lx/2 \rightarrow a = 2\pi$

⋮

1) $\frac{F''(x)}{F(x)} = k \rightarrow F''(x) - kF(x) = 0$

$\lambda^2 - k = 0 \qquad \lambda_{1,2} = \pm\sqrt{k}$

$k = -l^2$ ker homogeni, konst. negativna

$\lambda = \pm il$

$F_l(x) = A_l \cos lx + B_l \sin lx$

- $0 = A_l \cos 0 + B_l \sin 0 \rightarrow A_l = 0$
- $0 = B_l \sin l\pi \rightarrow$ a) $B_l = 0$ ker A_l $0 \rightarrow$ trivialna.
  b) $l \in \mathbb{Z} - \{0\}$, ker $l \neq 0$

$F_l(x) = B_l \sin lx \; ; \; l \in \mathbb{N}$

2) $\frac{G'(t)}{G(t)} = k \qquad k = -l^2$ upoštevamo le k, ki smo ga pod točko 1)

$\log G(t) = -l^2 t + \log C_l$

$G_l(t) = C_l e^{-l^2 t}$

3) $u(x,t) = \sum_l^\infty F_l(x) G_l(t) = \sum_l^\infty B_l \sin lx \, C_l e^{-l^2 t} = \qquad \alpha_l = B_l C_l$

$= \sum_l^\infty \alpha_l \sin lx \, e^{-l^2 t}$

4) Začetni pogoji: $u(x,0) = x$: $x = \sum_{l=1}^\infty \alpha_l \sin lx$ razvoj v vrsto

$a = \pi$

$\alpha_l = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cdot \sin lx \, dx = \begin{pmatrix} x = u \; ; \; \sin lx \, dx = dv \\ dx = du \; ; \; -\cos lx / l = v \end{pmatrix} = \frac{1}{\pi}\left[\frac{-\cos lx}{l}\Big|_{-\pi}^{\pi} + \int_{-\pi}^{\pi} \frac{\cos lx}{l} dx\right]$

$= \frac{1}{\pi}\left[\frac{-\pi}{l} \cdot (-1)^l - \frac{\pi}{l}(-1)^l + \frac{\sin lx}{l^2}\Big|_{-\pi}^{\pi}\right] = \frac{2 \cdot (-1)^{l+1}}{l}$

5) $\boxed{u(x,t) = \sum_{l=1}^{\infty} \frac{2(-1)^{l+1}}{l} \sin lx \, e^{-l^2 t}}$