Fourierov transform

Fourierova transformacija in inverzna transformacija: Eksistenca:

$$F(\omega)=\int_{-\infty}^{\infty} f(t)e^{i\omega t}dt \qquad f(t)=F^{-1}[F(\omega)]=\int_{-\infty}^{\infty}F(\omega)e^{-i\omega t}d\omega \qquad \int_{-\infty}^{\infty}|f(t)|dt<\infty$$

Lastnosti: Konvolucija:

$$F\left[af(t)+bg(t)\right]=aF(f)+bF(g) \qquad h(t)=\int_{-\infty}^{\infty} f(t-u)g(u)du$$

$$F\left[f(at)\right]=\frac{1}{|a|}F\left(\frac{\omega}{a}\right) \qquad F\left[h(t)\right]=F\left[f(t)\right]\cdot F\left[g(t)\right]$$

$$F\left[\overline{f(t)}\right]=\overline{F(-\omega)}$$

$$F\left[f(t-a)\right]=e^{ia\omega}F\left[f(t)\right]=e^{ia\omega}F(\omega)$$

Parsevalova enačba:

$$F\left[e^{iat}f(t)\right]=F(\omega+a) \qquad \int_{-\infty}^{\infty}|f(t)|^2dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}|F(\omega)|^2d\omega$$

Odvod:

$$F^{(n)}(\omega)=F\left[(it)^n f(t)\right] \qquad F\left[f^{(n)}(t)\right]=(-i\omega)^n F\left[f(t)\right]$$

Sinusna transformacija in inverzna transformacija:

$$F_S\left[f(t)\right]=\int_0^{\infty}f(t)\sin\omega t\,dt \qquad F^{-1}\left[F_S(\omega)\right]=\frac{2}{\pi}\int_0^{\infty}F(\omega)\sin\omega t\,d\omega$$

Kosinusna transformacija in inverzna transformacija:

$$F_C\left[f(t)\right]=\int_0^{\infty}f(t)\cos\omega t\,dt \qquad F^{-1}\left[F_C(\omega)\right]=\frac{2}{\pi}\int_0^{\infty}F(\omega)\cos\omega t\,d\omega$$

Parsevalova enačba za sinusno in kosinusno transformacijo

$$\int_0^{\infty}|f(t)|^2dt=\frac{2}{\pi}\int_0^{\infty}|F(\omega)|^2d\omega$$

Laplaceov transform

Laplaceova transformacija: Eksistenca: Izrek o premiku:

$$F(s)=\int_0^{\infty}e^{-st}f(t)dt \qquad |f(t)|\le Me^{\alpha} \qquad L\left[e^{at}f(t)\right]=F(s-a)$$

Transformacija odvoda: Transformacija integrala:

$$L\left[f^{(n)}\right]=s^nL(f)-s^{n-1}f(0)-s^{n-2}f'(0)-\ldots-f^{(n-1)}(0) \qquad L\left[\int_0^t f(\tau)d\tau\right]=\frac{1}{s}L\left[f(t)\right]=\frac{F(s)}{s}$$

Kompleksna inverzna ali *Bromwicheva* formula : Inverzna transformacija in residui:

$$f(t)=\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}e^{st}F(s)ds \qquad f(t)=\sum res\ e^{st}F(s)$$

-osnovna formula:

$$\operatorname*{res}_{z=z_0} f(z)=\lim_{z\to z_0}f(z)(z-z_0)$$

-pol *m*-te stopnje:

$$\operatorname*{res}_{z=z_0} f(z)=\frac{1}{(m-1)!}\lim_{z\to z_0}\frac{d^{m-1}}{dz^{m-1}}\left[(z-z_0)^m f(z)\right]$$

Odvod transformiranke: Integral transformiranke: Eksistenca integrala:

$$F'(s)=-L\left[tf(t)\right] \qquad L\left[\frac{f(t)}{t}\right]=\int_s^{\infty}F(\sigma)d\sigma \qquad \lim_{t\to 0+}\frac{f(t)}{t}\ \text{obstaja}$$

Konvolucija: Enotina stopnica: Izrek o pomiku:

$$h(t)=\int_0^t f(\tau)g(t-\tau)d\tau \qquad u_a=\begin{cases}0, t<a\\1, t>a\end{cases} \qquad e^{-as}F(s)=L\left[f(t-a)u_a(t)\right]$$

$$H(s)=F(s)\cdot G(s)$$

Transformacija periodičnih funkcij:

$$f(t+p)=f(t) \qquad L\left[f(t)\right]=\frac{1}{1-e^{-sp}}\int_0^p e^{-st}f(t)dt$$

Specialne funkcije

Gama funkcija: Beta funkcija:

$$\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt\,,\ x>0 \qquad B(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}dt\,,\ x>0\,,\ y>0$$

$$\Gamma(x)=\frac{1}{x}\Gamma(x+1) \qquad B(x,y)=\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

Lastnosti: Lastnosti:

$$\Gamma(n+1)=n! \qquad B(x,y)=B(y,x)$$

$$\lim_{x\to 0}\Gamma(x)=\lim_{x\to 0}\frac{\Gamma(x+1)}{x}=\infty$$

Legendrova diferencialna enačba: $n=0$:

$$\lim_{x\to\infty}\Gamma(x)=\infty \qquad (1-x)^2y''-2xy'+n(n+1)y=0 \qquad (1-x)^2y''-2xy'=0$$

$$\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$$

Besselova diferencialna enačba: $n=0$:

$$x^2y''+xy'+\left(x^2-\nu^2\right)y=0 \qquad xy''+y'+xy=0$$

Legendrovi polinomi:

$$P_n(x)=\sum_{k=0}^{N}(-1)^k\frac{(2n-2k)!}{2^n k!(n-k)!(n-2k)!}x^{n-2k} \qquad N=n/2 \text{ za sode } n \qquad N=(n-1)/2 \text{ za lihe } n$$

Rodriguesova formula:

$$P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}\left[\left(x^2-1\right)^n\right]$$

Besselova funkcija prve vrste reda ν : če velja $\nu=n\in N$:

$$J_\nu(x)=x^\nu\sum_{m=0}^{\infty}\frac{(-1)^m x^{2m}}{2^{2m+\nu}m!\Gamma(m+\nu+1)} \qquad J_n(x)=x^n\sum_{m=0}^{\infty}\frac{(-1)^m x^{2m}}{2^{2m+n}m!(n+m)!}$$

$$J_{-\nu}(x)=x^{-\nu}\sum_{m=0}^{\infty}\frac{(-1)^m x^{2m}}{2^{2m-\nu}m!\Gamma(m-\nu+1)} \qquad J_{-n}(x)=(-1)^nJ_n(x) \quad n=1,2,\dots$$

Lastnosti:

$$J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin x \qquad J_{-\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\cos x$$

$$\frac{d}{dx}\left[x^\nu J_\nu(x)\right]=x^\nu J_{\nu-1}(x) \qquad \frac{d}{dx}\left[x^{-\nu}J_\nu(x)\right]=-x^{-\nu}J_{\nu+1}(x)$$

Rekurtančni formuli

$$J_{\nu-1}(x)+J_{\nu+1}(x)=\frac{2\nu}{x}J_\nu(x) \qquad J_{\nu-1}(x)-J_{\nu+1}(x)=2J_\nu'(x)$$

Eulerjeva konstanta:

$$\gamma=\lim_{s\to\infty}\left(1+\frac{1}{2}+\dots+\frac{1}{s}-\ln s\right)$$

Besselova funkcija druge vrste reda 0: Lastnosti:

$$Y_0(x)=\frac{2}{\pi}\left[J_0(x)\left(\ln\frac{x}{2}+\gamma\right)+\sum_{m=1}^{\infty}\frac{(-1)^{m-1}h_m}{2^{2m}(m!)^2}x^{2m}\right] \qquad \lim_{n\to 0}Y_0(x)=-\infty$$

Besselova funkcija druge vrste reda ν : Lastnosti:

$$Y_\nu(x)=\frac{1}{\sin\nu\pi}\left[J_\nu(x)\cos\nu\pi-J_{-\nu}(x)\right] \qquad Y_n(x)=\lim_{\nu\to n}Y_\nu(x)$$