Matematika 4

2. vaja

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Formule

- $F(s) = \mathcal{L}(f(t)) = \int_0^\infty f(t)e^{-st}dt.$
- $f(t) = \mathcal{L}^{-1}(F(s)) = \frac{1}{2\pi i}\lim_{T\to\infty}\int_{\gamma-iT}^{\gamma+iT} e^{st}F(s)ds,$
  kjer je $\gamma > \Re(s)$.
- S pomočjo Chauchyjevega izreka o residuih:
  $f(t) = \sum_k \mathrm{Res}(e^{st}F(s), s_k).$
- Računanje residuov v polu n-te stopnje s_k:
  $\mathrm{Res}(e^{st}F(s), s_k) = \frac{1}{(n-1)!}\lim_{s\to s_k}\frac{d^{n-1}}{ds^{n-1}}\left((s-s_k)^n e^{st}F(s)\right).$
- Računanje residuov v polu prve stopnje v s_k, če je
  $F(s) = \frac{P(s)}{Q(s)}$, $\mathrm{Res}(e^{st}F(s), s_k) = e^{s_k t}\frac{P(s_k)}{Q'(s_k)}.$

Pravila: $a > 0,\ b \in \mathbb{R}$

I $\mathcal{L}\left(b_1 f(t) + b_2 g(t)\right) = b_1 F(s) + b_2 G(s)$

II $\mathcal{L}\left((f * g)(t)\right) = F(s)G(s)$

III $\mathcal{L}\left(H(t-a)f(t-a)\right) = e^{-as}F(s)$

IV $\mathcal{L}\left(e^{bt}f(t)\right) = F(s-b)$

V $\mathcal{L}\left(f(a\,t)\right) = \frac{1}{a}F\left(\frac{s}{a}\right)$

VI $\mathcal{L}\left(f^{(n)}(t)\right) = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$

VII $\mathcal{L}\left(t^n f(t)\right) = (-1)^n F^{(n)}(s)$

VIII $\mathcal{L}\left(\frac{f(t)}{t}\right) = \int_s^\infty F(\sigma)d\sigma$

IX $\mathcal{L}\left(\int_0^t f(\tau)d\tau\right) = \frac{F(s)}{s}$

X $\lim_{t \searrow 0} f(t) = \lim_{s\to\infty}(sF(s))$

XI $\lim_{t\to\infty} f(t) = \lim_{s\to 0}(sF(s))$

Laplaceova transformacija nekaterih funkcij

Konstanta $a > 0$, medtem ko je $b \in \mathbb{R}$.

1. $\mathcal{L}(H(t)) = \frac{1}{s}$
2. $\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$
3. $\mathcal{L}\left(e^{bt}\right) = \frac{1}{s-b}$
4. $\mathcal{L}(H(t-a)) = \frac{1}{s}e^{-at}$
5. $\mathcal{L}(\sin(bt)) = \frac{b}{s^2+b^2}$
6. $\mathcal{L}(\cos(bt)) = \frac{s}{s^2+b^2}$
7. $\mathcal{L}(t\sin(bt)) = \frac{2bs}{(s^2+b^2)^2}$
8. $\mathcal{L}(t\cos(at)) = \frac{s^2-b^2}{(s^2+b^2)^2}$
9. $\mathcal{L}(\delta(t)) = 1$
10. $\mathcal{L}(\delta(t-a)) = e^{-as}$

Poišči Laplaceovo transformacijo funkcije

$f(t) = H(t)$.

- $\int_0^\infty H(t)e^{-st}\,dt = \int_0^\infty e^{-st}\,dt \to$
- Ker mora biti $\lim\limits_{t\to\infty} e^{-st} = 0$, je $\Re(s) > 0$. $\to$
- $\frac{1}{s}e^{-st}\Big|_0^\infty = \frac{1}{s}$.
- $\mathcal{L}\left(H(t)\right) = \frac{1}{s}$, $\Re(s) > 0$.

Poišči Laplaceovo transformacijo funkcije

$f(t) = e^t$.

- $\int_0^\infty e^t e^{-st}\, dt = \int_0^\infty e^{(1-s)t}\, dt \to$
- Ker mora biti $\lim_{t\to\infty} e^{(1-s)t} = 0$, je $\Re(s) > 1$. $\to$
- $\frac{1}{1-s} e^{(1-s)t} \Big|_0^\infty = \frac{1}{s-1}$.
- $\mathcal{L}\left(e^t\right) = \frac{1}{s-1}$, $\Re(s) > 1$.

Poišči Laplaceovo transformacijo funkcije

$f(t) = e^{i\omega t}$.

- $\int_0^\infty e^{i\omega t} e^{-st}\, dt = \int_0^\infty e^{(i\omega - s)t}\, dt \to$
- Ker mora biti $\lim_{t\to\infty} e^{(i\omega - s)t} = 0$, je $\Re(s) > 0$. $\to$
- $\frac{1}{i\omega - s} e^{(i\omega - s)t}\Big|_0^\infty = \frac{1}{i\omega - s}$.
- $\mathcal{L}\left(e^t\right) = \frac{1}{s - i\omega}$, $\Re(s) > 0$.

Poišči Laplaceovo transformacijo funkcije

$f(t) = \cos(\omega t)$.

- $\cos(\omega t) = \dfrac{e^{i\omega t} + e^{-i\omega t}}{2} \rightarrow$
- $\mathcal{L}\left(e^{i\omega t}\right) = \dfrac{1}{s - i\omega} \rightarrow$
- $\mathcal{L}\left(e^{-i\omega t}\right) = \dfrac{1}{s + i\omega} \rightarrow$
- $\mathcal{L}\left(\cos(\omega t)\right) = \dfrac{1}{2}\left(\dfrac{1}{s - i\omega} + \dfrac{1}{s + i\omega}\right) = \dfrac{s}{s^2 + \omega^2}$.
- $\mathcal{L}\left(\cos(\omega t)\right) = \dfrac{s}{s^2 + \omega^2}$, $\Re(s) > 0$.

Poišči $\mathcal{L}(f(t))$ z uporabo pravil.

$f(t) = e^{-\lambda t}\cos(\omega t)$.

- Uporabimo (6)→(IV).
- $\mathcal{L}(\cos(\omega t)) = \dfrac{s}{s^2+\omega^2} \rightarrow$
- $\mathcal{L}\left(e^{-\lambda t}\cos(\omega t)\right) = \dfrac{s+\lambda}{(s+\lambda)^2+\omega^2}$.
- $\mathcal{L}\left(e^{-\lambda t}\cos(\omega t)\right) = \dfrac{s+\lambda}{(s+\lambda)^2+\omega^2}$.

Poišči $\mathcal{L}(f(t))$ z uporabo pravil.

$f(t) = t^2 e^{-\lambda t}$.

- Lahko uporabimo (VII) ali pa (2)→(IV).
- $\mathcal{L}\left(t^2\right) = \frac{2}{s^3} \rightarrow$
- $\mathcal{L}\left(t^2 e^{-\lambda t}\right) = \frac{2}{(s+\lambda)^3}$.

- $\mathcal{L}\left(t^2 e^{-\lambda t}\right) = \frac{2}{(s+\lambda)^3}$.

Poišči $\mathcal{L}(f(t))$ z uporabo pravil.

$f(t) = t^2 \sin(\omega t)$.

- Uporabimo (VII).
- $\mathcal{L}(\sin(\omega t)) = \frac{\omega}{s^2+\omega^2} \rightarrow$
- $\mathcal{L}\left(t^2 \sin(\omega t)\right) = \left(\frac{\omega}{s^2+\omega^2}\right)^{(2)} = -\frac{2\omega\left(\omega^2-3s^2\right)}{\left(s^2+\omega^2\right)^3}$
- $\mathcal{L}\left(t^2 \sin(\omega t)\right) = -\frac{2\omega\left(\omega^2-3s^2\right)}{\left(s^2+\omega^2\right)^3}$.

Poišči $\mathcal{L}(f(t))$ z uporabo pravil.

$f(t) = \sin(t - \frac{\pi}{3})$.

- Ne moremo uporabiti (III), ker je $f(t) = H(t)\sin(t - \frac{\pi}{3})$.
- Uporabimo adicijski izrek $\sin t \cos\frac{\pi}{3} - \sin\frac{\pi}{3}\cos t \rightarrow$.
- Uporabimo (I)$\rightarrow$ (5, 6)$\rightarrow$.
- $\mathcal{L}(f(t)) = \dfrac{1 - \sqrt{3}s}{2s^2 + 2}$.

Poišči $\mathcal{L}(f(t))$ z uporabo pravil.

$f(t) = \sin^2 t$.

- Uporabimo formulo $\sin^2 t = \frac{1}{2}(1 - \cos(2t)) \rightarrow$.
- Uporabimo (I)$\rightarrow$ (6)$\rightarrow$.
- $\mathcal{L}(f(t)) = \dfrac{2}{s^3 + 4s}$.

Poišči $\mathcal{L}(f(t))$ z uporabo pravil.

$$f(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & drugod \end{cases}.$$

- Lahko zapišemo $f(t) = H(t) - H(t-1)$.
- Uporabimo (1) $\rightarrow$ (III).
- $\mathcal{L}(f(t)) = \frac{1}{s}\left(1 - e^{-t}\right)$.

Poišči $\mathcal{L}(f(t))$ z uporabo pravil.

$$f(t) = \int_0^t \frac{1-e^{-\tau}}{\tau} d\tau.$$

- Uporabimo (IX)$\rightarrow$(VIII) $\rightarrow$(2,3).
- $F(s) = \frac{1}{s}\mathcal{L}\left(\frac{1-e^{-\tau}}{\tau}\right) =\rightarrow$
- $= \frac{1}{s}\int_s^{\infty}\left(\frac{1}{\sigma} - \frac{1}{\sigma+1}\right) d\sigma =\rightarrow$
- $= \int_s^{\infty} \frac{d\sigma}{\sigma(\sigma+1)} = \ln \frac{\sigma}{\sigma+1}\Big|_s^{\infty} = -\frac{1}{s}\ln\frac{s}{s+1}$
- $\mathcal{L}\left(\int_0^t \frac{1-e^{-\tau}}{\tau} d\tau\right) = \frac{1}{s}\ln\frac{s+1}{s}$, $\Re(s) > 0.$

Poišči $\mathcal{L}\left(f(t)\right)$ z uporabo pravil.

$$f(t) = \int_0^t e^{2\tau}\cos(t-\tau).$$

- Uporabimo (II)$\rightarrow$(3,6)$\rightarrow$.
- $F(s) = \mathcal{L}\left(e^{2t}\right)\mathcal{L}\left(\cos t\right) = \rightarrow$
- $= \dfrac{1}{s-2}\dfrac{s}{s^2+1} = \dfrac{s}{(s-2)(s^2+1)}.$
- $\mathcal{L}\left(\int_0^t e^{2\tau}\cos(t-\tau)\right) = \dfrac{s}{(s-2)(s^2+1)}.$

Poišči $\mathcal{L}\left(f(t)\right)$ z uporabo pravil.

$$f(t) = \begin{cases} \sin t & \frac{\pi}{2} \leq t \\ 0 & drugod \end{cases}.$$

- Lahko zapišemo $f(t) = H(t - \frac{\pi}{2}) \sin\left((t - \frac{\pi}{2}) + \frac{\pi}{2}\right)$.
- Uporabimo (III) $\to$ adicijski izrek $\to$ (6)$\to$
- $\mathcal{L}\left(f(t)\right) = e^{-\frac{\pi}{2}s} \dfrac{s}{s^2 + 1}$.

Poišči $\mathcal{L}^{-1}(F(s))$ z uporabo pravil.

$$F(s) = \frac{s}{s^2+2s+2}.$$

- Lahko zapišemo
  $$F(s) = \frac{s}{(s+1)^2+1} = \frac{s+1}{(s+1)^2+1} - \frac{1}{(s+1)^2+1} \rightarrow$$
- (IV) $\rightarrow$ (5, 6)$\rightarrow$
- $\mathcal{L}^{-1}(F(s)) = e^{-t}(\cos t - \sin t)$.

Poišči $\mathcal{L}^{-1}(F(s))$ z uporabo pravil.

$$F(s) = \frac{s^2 - 4}{s^3 + 2s^2 - 3s}.$$

- Lahko zapišemo $F(s) = \dfrac{s^2 - 4}{s(s+3)(s-1)} \rightarrow$.
- Razcepimo na parcialne ulomke
  $\rightarrow F(s) = \dfrac{4}{3s} + \dfrac{5}{12(s+3)} - \dfrac{3}{4(s-1)}$.
- Uporabimo (I)$\rightarrow$(1)$\rightarrow$(IV)$\rightarrow$
- $\mathcal{L}^{-1}(F(s)) = \dfrac{5e^{-3t}}{12} - \dfrac{3e^{t}}{4} + \dfrac{4}{3}$.

Poišči $\mathcal{L}^{-1}(F(s))$ z uporabo pravil.

$$F(s) = \frac{s^2+4}{s^4+2s^3+2s^2}.$$

- Lahko zapišemo $F(s) = \dfrac{s^2+4}{s^2(s^2+2s+2)} \to .$
- Razcepimo na parcialne ulomke
  $\to \dfrac{2s+3}{s^2+2s+2} + \dfrac{2}{s^2} - \dfrac{2}{s} = \dfrac{2(s+1)+1}{(s+1)^2+1} + \dfrac{2}{s^2} - \dfrac{2}{s}.$
- Uporabimo (I)→(2,5,6)→(IV) →
- $\mathcal{L}^{-1}(F(s)) = 2(t-1) + e^{-t}(\sin t + 2\cos t).$

Poišči $\mathcal{L}^{-1}(F(s))$ z uporabo pravil.

$F(s) = \dfrac{s}{(s^2+1)^2}$.

- Uporabimo (II)→(5,6)→.
- $\mathcal{L}^{-1}\left(\dfrac{1}{s^2+1}\right)\mathcal{L}^{-1}\left(\dfrac{s}{s^2+1}\right) = \int_0^t \sin\tau\cos(t-\tau)\,d\tau \rightarrow$
- $\cos t\int_0^t \cos\tau\sin\tau\,d\tau + \sin t\int_0^t \sin^2\tau\,d = \tau \rightarrow$
- $\dfrac{1}{2}t\sin t$
- $\mathcal{L}^{-1}\left(\dfrac{s}{(s^2+1)^2}\right) = \dfrac{1}{2}t\sin t$

Poišči $\mathcal{L}^{-1}(F(s))$ z uporabo pravil.

$$F(s) = \frac{s^2+4}{s^6+2s^4+s^2}.$$

- Lahko zapišemo $F(s) = \dfrac{s^2+4}{s^2(s^2+1)^2} \rightarrow$.
- Razcepimo na parcialne ulomke $\rightarrow \dfrac{4}{s^2} - \dfrac{4}{s^2+1} - \dfrac{3}{(s^2+1)^2}$.
- Uporabimo (I)$\rightarrow$(2,5,6)$\rightarrow$(II) $\rightarrow$
- $\mathcal{L}\left(\int_0^t \sin(\tau)\sin(t-\tau)d\tau\right) = \dfrac{1}{s^2+1}\dfrac{1}{s^2+1} \rightarrow$
- $\mathcal{L}^{-1}(F(s)) = \dfrac{1}{2}(8t - 11\sin(t) + 3t\cos(t))$.

Poišči $\mathcal{L}^{-1}(F(s))$ s pomočjo residuov.

$$F(s) = \frac{s^2+4}{s^6+2s^4+s^2}.$$

- Lahko zapišemo $F(s) = \frac{s^2+4}{s^2(s^2+1)^2} \to$.
- Singularne točke 0, i in $-i$ so poli druge stopnje.
- $\mathrm{Res}(e^{st}F(s), i) = \lim_{s \to i} \left(e^{st}(s-i)^2 F(s)\right)' = \to$
- $\left(e^{st}\frac{s^2+1}{s^2(s+i)^2}\right)'\Big|_{s=i} = -\frac{1}{8}e^{it}(-6t-22i)$.
- $\mathrm{Res}\left(e^{st}, 0\right) = 4t$, $\mathrm{Res}\left(e^{st}, -i\right) = -\frac{1}{8}e^{-it}(-6t+22i)$.
- Vsota residuov je $\frac{1}{2}(8t - 11\sin(t) + 3t\cos(t))$.

- $\mathcal{L}^{-1}(F(s)) = \dfrac{1}{2}(8t - 11\sin(t) + 3t\cos(t))$.

Reši diferencialno enačbo

$\dot{x}(t) + x(t) = e^{-t}$, $x(0) = 1$.

- $s\left(\mathcal{L}_t[x(t)](s)\right) + \mathcal{L}_t[x(t)](s) - x(0) = \frac{1}{s+1}$,
- $\mathcal{L}_t[x(t)](s) = \frac{sx(0) + x(0) + 1}{(s+1)^2}$,
- $x(t) \to e^{-t}(t+1)$.

Reši diferencialno enačbo

$\ddot{x}(t) + \dot{x}(t) = te^{-t}$, $x(0) = 0$, $\dot{x}(0) = 0$.

- $s^2 \left(\mathcal{L}_t[x(t)](s)\right) + s \left(\mathcal{L}_t[x(t)](s)\right) = \frac{1}{(s+1)^2}$,
- $\mathcal{L}_t[x(t)](s) = \frac{1}{s(s+1)^3}$,
- $x(t) = 1 - \frac{1}{2}e^{-t}(t(t+2)+2)$.

Reši diferencialno enačbo

$\ddot{x}(t) + x(t) = 0$, $x(0) = 0$, $\dot{x}(0) = 1$.

- $s^2\mathcal{L}_t[x(t)](s) + \mathcal{L}_t[x(t)](s) = sx(0) + \dot{x}(0)$
- $\mathcal{L}_t[x(t)](s) = \dfrac{1}{s^2+1}$
- $x(t) = \sin t$.

Reši diferencialno enačbo

$\ddot{x}(t) + x(t) = \delta(t)$, $x(0) = 0$, $\dot{x}(0) = 0$.

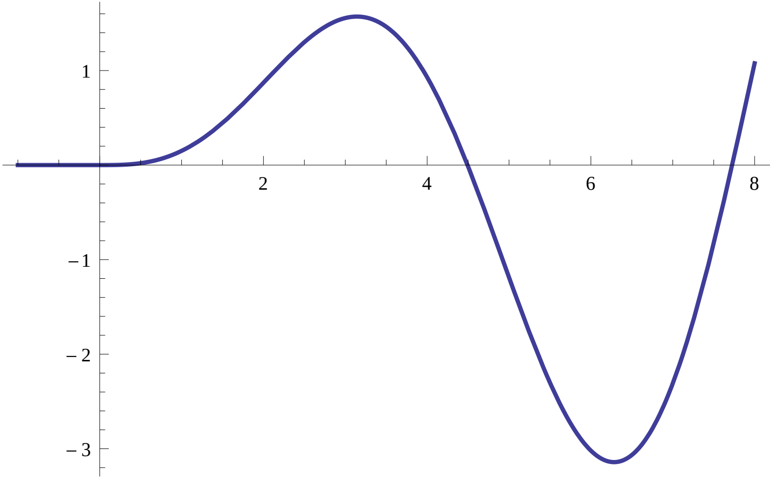
- $s^2 \mathcal{L}_t[x(t)](s) + \mathcal{L}_t[x(t)](s) = 1 + sx(0) + \dot{x}(0)$,
- $\mathcal{L}_t[x(t)](s) = \dfrac{1}{s^2 + 1}$,
- $x(t) = \sin t$.

Reši diferencialno enačbo

$\ddot{x}(t) + x(t) = \sin t$, $x(0) = 0$, $\dot{x}(0) = 0$.

- $s^2\mathcal{L}_t[x(t)](s)) + \mathcal{L}_t[x(t)](s) = sx(0) + \dot{x}(0) + \frac{1}{s^2+1}$,
- $\mathcal{L}_t[x(t)](s) = \frac{1}{(s^2+1)^2}$,
- $x(t) = \frac{1}{2}(\sin(t) - t\cos(t))$.

Graf funkcije $x(t)$

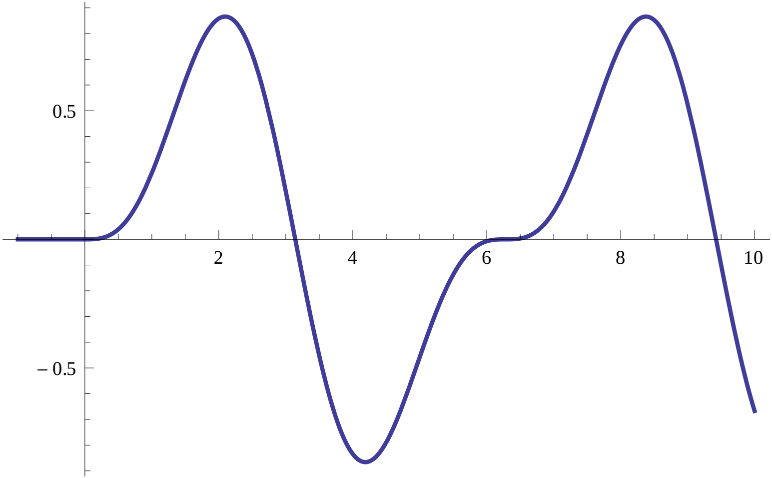


Reši diferencialno enačbo

$\ddot{x}(t) + x(t) = \sin(2t)$, $x(0) = 0$, $\dot{x}(0) = 0$.

- $s^2 \mathcal{L}_t[x(t)](s) + \mathcal{L}_t[x(t)](s) - sx(0) - \dot{x}(0) = \dfrac{2}{s^2+4}$,
- $\mathcal{L}_t[x(t)](s) = \dfrac{2}{(s^2+1)(s^2+4)}$,
- $x(t) = \dfrac{2}{3}\sin t - \dfrac{1}{3}\sin(2t)$.

Graf funkcije $x(t)$



Reši diferencialno enačbo

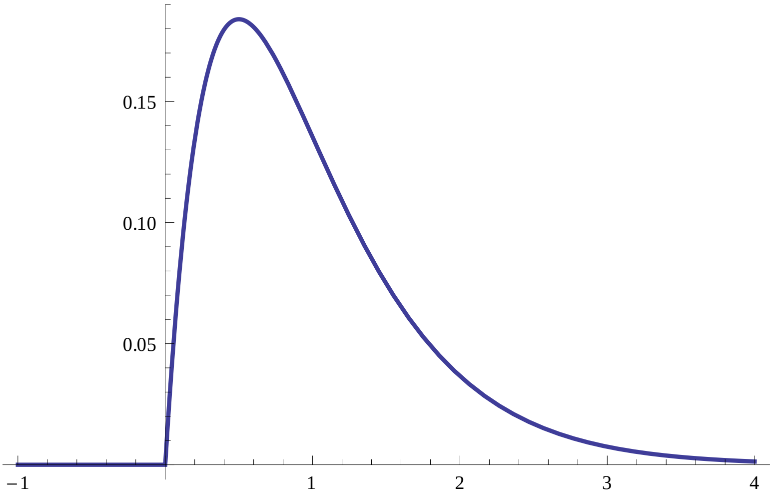
$\ddot{x}(t) + 4\dot{x}(t) + 4x(t) = 0,\ x(0) = 1,\ \dot{x}(0) = 4.$

- $s^2\left(\mathcal{L}_t[x(t)](s)\right) + 4\left(\mathcal{L}_t[x(t)](s)\right) + 4\left(s\left(\mathcal{L}_t[x(t)](s)\right) - 1\right) - s - 4 = 0$
- $\mathcal{L}_t[x(t)](s) = \dfrac{s+8}{(s+2)^2}$
- $x(t) = e^{-2t}(6t+1)$

Reši diferencialno enačbo

$\ddot{x}(t) + 4\dot{x}(t) + 4x(t) = 0,\ x(0) = 0,\ \dot{x}(0) = 1.$

- $s^2\mathcal{L}_t[x(t)](s) + 4\mathcal{L}_t[x(t)](s) + 4s\mathcal{L}_t[x(t)](s) = 4x(0) + sx(0) + \dot{x}(0),$
- $\mathcal{L}_t[x(t)](s) = \dfrac{1}{(s+2)^2},$
- $x(t) = e^{-2t}t.$

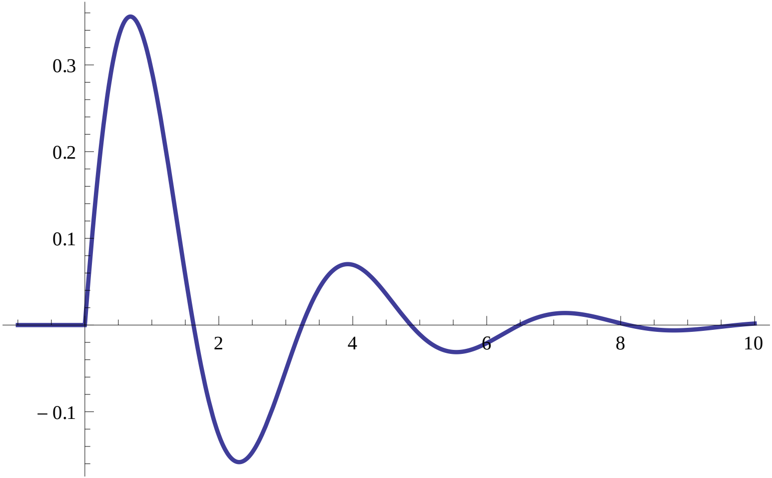

0.15
0.10
0.05
−1
1
2
3
4

Reši diferencialno enačbo

$\ddot{x}(t) + \dot{x}(t) + 4x(t) = 0,\ x(0) = 0,\ \dot{x}(0) = 1.$

- $s^2\mathcal{L}_t[x(t)](s) + s\mathcal{L}_t[x(t)](s) + 4\mathcal{L}_t[x(t)](s) = sx(0) + \dot{x}(0) + x(0),$
- $\mathcal{L}_t[x(t)](s) \to \dfrac{1}{s^2 + s + 4},$
- $x(t) = \dfrac{2}{\sqrt{15}} e^{-t/2} \sin\left(\dfrac{\sqrt{15}t}{2}\right).$

Graf funkcije $x(t)$



Reši diferencialno enačbo

$\ddot{x}(t) + 4x(t) = \sin(2t),\ x(0) = 0,\ \dot{x}(0) = 1.$

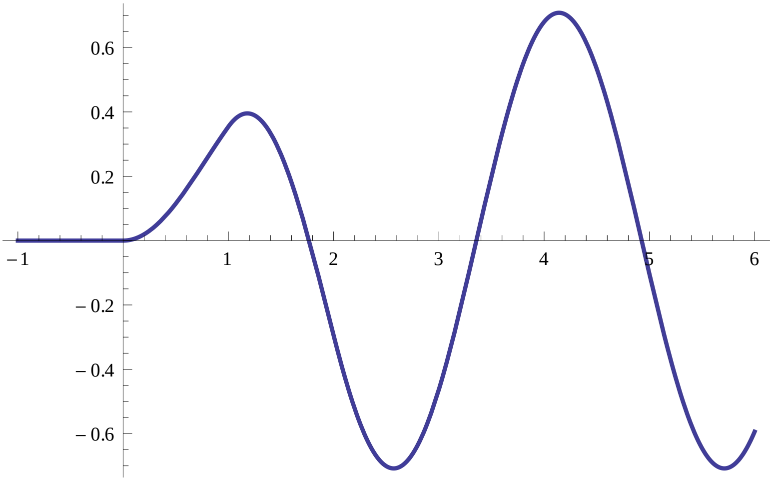
- $s^2 (\mathcal{L}_t[x(t)](s)) + 4 (\mathcal{L}_t[x(t)](s)) - sx(0) - \dot{x}(0) = \dfrac{2}{s^2+4},$
- $\mathcal{L}_t[x(t)](s) = \dfrac{s^2+6}{(s^2+4)^2},$
- $x(t) = \dfrac{1}{8}(5\sin(2t) - 2t\cos(2t))$

Reši diferencialno enačbo $\ddot{x}(t) + 4x(t) = f(t)$, kjer je

$$x(0) = 0,\ \dot{x}(0) = 0 \text{ in } f(t) \begin{cases} 1, & 0 \leq t \leq 1 \\ -1, & 1 < t \leq 2 \\ 0, & \text{drugod} \end{cases}.$$

- $(s^2 + 4)\left(\mathcal{L}_t[x(t)](s)\right) = s\,x(0) + \dot{x}(0) + \frac{1}{s}\left(1 - 2e^{-s} + e^{-2s}\right)$,
- $\mathcal{L}_t[x(t)](s) = \dfrac{e^{-2s}\left(e^s - 1\right)^2}{s\left(s^2 + 4\right)}$,
- $x(t) =$
  $\frac{1}{2}\left(\theta(t-2)\sin^2(2-t) + \theta(t-1)(\cos(2-2t) - 1) + \sin^2(t)\right)$.

Graf funkcije $x(t)$



Reši sistem diferencialnih enačb

$$\begin{aligned} \dot{x}(t) &= y(t) - x(t) + z(t), & x(0) &= 0, \\ \dot{y}(t) &= x(t) - y(t), & y(0) &= 0, \\ \dot{z}(t) &= -z(t), & z(0) &= 1. \end{aligned}$$

- $$\begin{aligned} s\mathcal{L}_t[x(t)](s) &= -\mathcal{L}_t[x(t)](s) + \mathcal{L}_t[y(t)](s) + \mathcal{L}_t[z(t)](s) \\ s\mathcal{L}_t[y(t)](s) &= \mathcal{L}_t[x(t)](s) - \mathcal{L}_t[y(t)](s) \\ s\mathcal{L}_t[z(t)](s) &= -\mathcal{L}_t[z(t)](s) + 1 \end{aligned}$$
- $x(t) = \frac{1}{2}e^{-2t}\left(e^{2t} - 1\right), y(t) = \frac{1}{2}e^{-2t}\left(e^{t} - 1\right)^2, z(t) = e^{-t}$

Reši integralsko enačbo

$x(t) = t^2 + \int_0^t x(\tau) d\tau.$

- $\mathcal{L}_t[x(t)](s) = \frac{\mathcal{L}_t[x(t)](s)}{s} + \frac{2}{s^3},$
- $\mathcal{L}_t[x(t)](s) = \frac{2}{(s-1)s^2},$
- $x(t) \to 2\left(-t + e^t - 1\right).$

Reši integralsko enačbo

$x(t) = t + 2 - 2\cos(t) - \int_0^t (t-\tau)x(\tau)d\tau.$

- $\mathcal{L}_t[x(t)](s) = -\frac{\mathcal{L}_t[x(t)](s)}{s^2} - \frac{2s}{s^2+1} + \frac{1}{s^2} + \frac{2}{s},$
- $\mathcal{L}_t[x(t)](s) = \frac{s^2+2s+1}{(s^2+1)^2},$
- $x(t) = t\sin(t) + \sin(t).$

Reši integralsko enačbo

$x(t) = t - \int_0^t x(\tau)\cos(t-\tau)d\tau.$

- $\mathcal{L}_t[x(t)](s) = \frac{1}{s^2} - \frac{s\left(\mathcal{L}_t[x(t)](s)\right)}{s^2+1}$
- $\mathcal{L}_t[x(t)](s) = \frac{s^2+1}{s^2\left(s^2+s+1\right)}$
- $x(t) = -1 + t + \frac{e^{-t/2}}{\sqrt{3}}\left(\sqrt{3}\cos\left(\frac{\sqrt{3}t}{2}\right) - \sin\left(\frac{\sqrt{3}t}{2}\right)\right).$

Reši integro-diferencialno enačbo

$\dot{x}(t) = t + \int_0^t x(\tau)\cos(t-\tau)d\tau,\ x(0) = 1$

- $s\left(\mathcal{L}_t[x(t)](s)\right) - 1 = \frac{s\left(\mathcal{L}_t[x(t)](s)\right)}{s^2+1} + \frac{1}{s^2}$
- $\mathcal{L}_t[x(t)](s) = \frac{\left(s^2+1\right)^2}{s^5}$
- $x(t) = \frac{t^4}{24} + t^2 + 1.$

Reši integro-diferencialno enačbo

$$\ddot{x}(t) + \int_0^t (x(\tau) + \ddot{x}(\tau)) \sin(t-\tau) d\tau = 2\cos(t),$$

$x(0) = 0$, $\dot{x}(0) = 0$.

- $s^2 (\mathcal{L}_t[x(t)](s)) + \mathcal{L}_t[x(t)](s) = \dfrac{2s}{s^2+1}$,
- $\mathcal{L}_t[x(t)](s) = \dfrac{2s}{(s^2+1)^2}$,
- $x(t) = t\sin(t)$.

Reši sistem integralskih enačb

$x(t) = t + \int_0^t y(\tau)d\tau,\ y(t) = 1 + \int_0^t x(\tau)d\tau$

- $\mathcal{L}_t[x(t)](s) = \dfrac{\mathcal{L}_t[y(t)](s)}{s} + \dfrac{1}{s^2}$

  $\mathcal{L}_t[y(t)](s) = \dfrac{\mathcal{L}_t[x(t)](s)}{s} + \dfrac{1}{s}$
- $\mathcal{L}_t[x(t)](s) = \dfrac{2}{s^2 - 1}, \quad \mathcal{L}_t[y(t)](s) = \dfrac{2s}{s^2 - 1} - \dfrac{1}{s}$
- $x(t) = e^{-t}\left(e^{2t} - 1\right), \quad y(t) = e^{-t} + e^{t} - 1$