NUMERIČNE METODE
rešene izpitne naloge 1998 - 2008

Šolsko leto	2008 / 2009
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UREJANJE DOKUMENTA

VERZIJA	01	**REVIZIJA**	01
DATUM	29. 6. 2009		
ZADNJI POPRAVLJAL	/		
PREGLEDAL	/		

OPOMBE

POPRAVKI

Izpit iz Numeričnih metod

17. januar 2008

1. Sestavite formulo za približno računanje integralov oblike

$$\int_0^1 f(x)\,dx \approx \omega_1 f(0) + \omega_2 f(\xi) + \omega_3 f(1)$$

Določite koeficiente, ω_1, ω_2, ω_3 in vozlišče ξ, $0 < \xi < 1$ tako, da bo formula točna za polinome $p_0(x) = 1$, $p_1(x) = x$, $p_2(x) = x^2$ in $p_3(x) = x^3$. S pomočjo dobljene formule določi približno vrednost integrala

$$\int_0^1 \frac{\sin x}{\sqrt{x}}\,dx$$

2. Poišči X, tako da bo imela razlika $AX - B$ minimalno evklidsko normo.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

3. Na intervalu $[0, 2]$ ležita dva korena enačbe

$$\sqrt{x}e^{-x} - \frac{1}{4} = 0$$

Poišči oba korena. Določi, katerega od njiju lahko poiščemo s pomočjo naslednje iteracijske sheme

$$x_{n+1} = \sqrt{x_n}e^{-x_n} - \frac{1}{4} + x_n$$

pri izbiri primernega začetnega približka.

17.1.2008

$$\int_0^1 f(x)dx = w_1 f(\emptyset) + w_2 f(\xi) + w_3 f(1)$$

$f(x) = 1, x, x^2, x^3$ – za te bo formula točna

ξ $\boxed{0 < \xi 1}$

$f(x) = 1$: $\int_0^1 dx = w_1 + w_2 + w_3 = 1$

$f(x) = x$: $\int_0^1 x\,dx = w_1 \cdot \emptyset + w_2 \xi + w_3 \cdot 1 = \frac{x^2}{2} = \frac{1}{2}$

$f(x) = x^2$: $\int_0^1 x^2 dx = w_1 \cdot \emptyset + w_2 \xi^2 + w_3 = 1/3$

$f(x) = x^3$: $\int_0^1 x^3 dx = w_1 \cdot \emptyset + w_2 \xi^3 + w_3 \cdot 1 = 1/4$

$w_1 + w_2 + w_3 = 1$

$w_2 \xi + w_3 = 1/2$

$w_2 \xi^2 + w_3 = 1/3$

$w_2 \xi^3 + w_3 = 1/4$

$\boxed{w_2 (\xi - \xi^2) = \frac{1}{2} - 1/3 = \frac{1}{6}}$

$\boxed{w_2 (\xi^2 - \xi^3) = 1/3 - 1/4 = \frac{1}{12}}$

Enačbi delimo:

$$\frac{\xi - \xi^2}{\xi^2 - \xi^3} = \frac{\frac{1}{6}}{\frac{1}{12}} = 2 = \frac{\xi(1-\xi)}{\xi^2(1-\xi)} = \frac{1}{\xi}$$

$\frac{2-1}{4}$

$2 = \frac{1}{\xi}$

$\boxed{\xi = 1/2}$

$w_2 \left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{6}$

$w_2 \frac{1}{4} = \frac{1}{6}$

$w_2 = \frac{1}{6} \cdot 4 = \frac{2}{3}$

$w_1 = 1 - w_3 - w_2$

$= 1 - \frac{2}{3} - \frac{1}{6}$

$= \frac{6-4-1}{6} = \frac{1}{6}$

$\frac{2}{3} \cdot \frac{1}{2} + w_3 = \frac{1}{2}$

$w_3 = \frac{1}{2} - \frac{2}{3} \cdot \frac{1}{2}$

$= \frac{1}{2} - \frac{2}{6} = \frac{6-4}{12} = \frac{2}{12} = \frac{1}{6}$

$w_2 = \frac{2}{3} = \frac{4}{6}$

$w_3 = \frac{1}{6}$

$$\int_0^1 f(x)dx = \frac{1}{6} f(\emptyset) + \frac{4}{6} f\left(\frac{1}{2}\right) + \frac{1}{6} f(1)$$

$$= \frac{1}{6}\left[f(\emptyset) + 4 f\left(\frac{1}{2}\right) + f(1)\right]$$

$$\int_0^1 \frac{\sin x}{\sqrt{x}}\,dx = \frac{1}{6}\left[f(\emptyset) + 4f\left(\tfrac{1}{2}\right) + f(1)\right]$$

$$= \frac{1}{6}\left[\emptyset + 4\cdot\frac{\sin\frac{1}{2}}{\sqrt{1/2}} + \frac{\sin 1}{\sqrt{1}}\right]$$

$\sin x$ (v $\emptyset$) $= \emptyset$, ker je tudi imenovalec $\emptyset$ je treba zračunat limito!

$$\lim_{x\to 0}\frac{\sin x}{\sqrt{x}} = \lim_{x\to\emptyset}\frac{\sin x}{x}\cdot\frac{x}{\sqrt{x}} = \emptyset$$

$\frac{\sin x}{x} \to 1$, $\frac{x}{\sqrt{x}} = \sqrt{x}$ (vstaviš $\emptyset$ in je $\emptyset$)

$\approx$ 0.592251896

) Poišči X tako, da bo imela razlika AX − B minimalno evklidsko normo.

evklidska $\|Ax-B\|_2$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$A^T\cdot A\cdot X = A^T\cdot B$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$1\cdot1 + 2\cdot2 + 1\cdot1$

$$\begin{bmatrix} 6 & 9 \\ 9 & 14 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix} \qquad \begin{bmatrix} \frac{122}{6} \\ -13 \end{bmatrix}$$

$2\cdot1 + 3\cdot2 + 1\cdot1$

$6x_1 + 9(-13) = 5$

$x_1 = \frac{5-9(-13)}{6}$

$x_1 = \frac{122}{6}$

$6x_1 + 9x_2 = 5$

$9x_1 + 14x_2 = 7$

$3x_1 + \frac{9}{2}x_2 = \frac{5}{2}$

$9x_1 + \frac{9\cdot3}{2}x_2 = \frac{15}{2}$

$14x_2 - \frac{27}{2}x_2 = 7 - \frac{15}{2}$

$0.5x_2 = -6.5$

$x_2 = -\frac{6.5}{0.5}$

$x_2 = -13$

$6x_1 + 9x_2 = 5 \quad : 6$

$9x_1 + 14x_2 = 7$

$x_1 + \frac{9}{6}x_2 = \frac{5}{6} \quad / \cdot 9$

$9x_1 + \frac{81}{6}x_2 = \frac{9 \cdot 5}{6}$

$$-\begin{cases} 9x_1 + \frac{81}{6}x_2 = \frac{45}{6} \\ 9x_1 + 14x_2 = 7 \end{cases}$$

$\frac{81}{6}x_2 - 14x_2 = \frac{45}{6} - 7$

$-0.5x_2 = 0.5$

$\underline{\underline{x_2 = -1}}$

$6x_1 - 9 = 5$

$6x_1 = 5 + 9$

$x_1 = \frac{5+9}{6} = \ldots$

Rezultat:

$$\begin{bmatrix} \frac{14}{6} \\ -1 \end{bmatrix}$$

③ Na intervalu $[0,2]$ ležita dva korena enačbe $\boxed{\sqrt{x}\,e^{-x} - \frac{1}{4} = 0}$. Poišči oba korena. Določi katerega od njiju lahko poiščemo s pomočjo:

$$x_{n+1} = \sqrt{x_n}\,e^{-x_n} - \frac{1}{4} + x_n$$

$f(x) = \sqrt{x}\,e^{-x} - \frac{1}{4}$

$f'(x) = \frac{1}{2\sqrt{x}}e^{-x} - \sqrt{x}\,e^{-x} = \underline{\underline{\frac{e^{-x}(1-2x)}{2\sqrt{x}}}}$

$$x_{n+1} = x_n - \frac{\left(\sqrt{x_n}\,e^{-x_n} - \frac{1}{4}\right)\left(2\sqrt{x_n}\right)}{e^{-x_n}(1-2x_n)}$$ Newtonova metoda

0 1 2

$f(0) = -1/4$ $f(2) = -0.0586$

$f(1) = 0.11787$

\+ −

2. korena (najmanj!)

4

$x_1 = 0,07221$ }
$x_2 = 1,63084$ } izračunani

$$x_{n+1} = g(x_n)$$!

~~$\frac{e^{-x}(1-2x)}{2\sqrt{x}}$~~

$$g(x) = \sqrt{x}e^{-x} - \frac{1}{4} + x$$

$$g'(x) = \frac{1}{2\sqrt{x}}e^{-x} - \sqrt{x}e^{-x} + 1$$

$$|g'(x)| < 1$$

< 1 je privlačna
> 1 je odbojna

x_1 je odbojna, ker je
$g'(x) = 2,48105\ldots > 1$

x_2 je privlačna, ker je
$g'(x) = 0,82664 < 1$

Izpit iz Numeričnih metod

20. junij 2007

1. Ali lahko rešimo sistem $Ax = b$ z Gauss-Seidlovo iteracijo?

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Izračunaj prve tri korake iteracije. Začetni približek izberi $x_0 = [0; 0; 0]$. Kdaj bi se lahko zgodilo, da bi nas iteracijska metoda pripeljala do rešitve v končno korakih?

2. Pokažite, da leži na intervalu $I = [0.1, 1]$ natanko en koren enačbe

$$x + \log x = 0.$$

Poišči ta koren z Newtonovo metodo.
Naslednje tri funkcije

$$f(x) = -\log x, \quad f(x) = e^{-x}, \quad f(x) = \frac{1}{2}(x + e^{-x})$$

imajo natanko eno fiksno točko $x = f(x)$ na intervalu I, ki se ujema z rešitvijo zgornje enačbe. V katerih primerih je ta točka privlačna in v katerih odbojna. Z drugimi besedami, v katerih primerih lahko poiščemo začetni približek, različen od fiksne točke, tako da iteracija konvergira k fiksni točki in kdaj to ni mogoče?

$$x_{k+1} = -\log x_k, \quad x_{k+1} = e^{-x_k}, \quad x_{k+1} = \frac{1}{2}(x_k + e^{-x_k})$$

3. Reši integral s popravljeno Simpsonovo kvadraturno formulo za singularne integrale in primerjaj rezultat z rešitvijo, točno na tri decimalna mesta, ki jo dobiš s pomočjo razvoja v Taylorjevo vrsto.

$$\int_0^{1/2} \frac{\exp(-x^2)}{\sqrt(x)} dx$$

Navodilo: $\int_{x}^{1/2} f(x)/\sqrt{x}\, dx \approx w_1 f(0) + w_2 f(1/4) + w_3 f(1/2)$

20. junij 2007

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Ali lahko rešimo sistem? Ni strogo diagonalno dominantn
$|2| \not> |2|$. Ni nujno!

$$2x_1 + 2x_2 = 1 \qquad x_1 = \frac{1}{2}[1 - 2x_2] = \frac{1}{2} - x_2$$

$$2x_2 + x_3 = 2 \qquad x_2 = \frac{1}{2}[2 - x_3] = 1 - \frac{x_3}{2}$$

$$x_3 = 1 \qquad x_3 = 1$$

$$\vec{x_0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{x_1} = \begin{bmatrix} 0.5 \\ 1 \\ 1 \end{bmatrix} \qquad \vec{x_2} = \begin{bmatrix} -0.5 \\ 0.5 \\ 1.0 \end{bmatrix}$$

$x_1 = 0.5 - 1.0 = -0.5$
$x_2 = 1 - 0.5 = 0.5$
$x_3 = 1.0$

$$\vec{x_3} = \begin{bmatrix} 0 \\ 0.5 \\ 1 \end{bmatrix}$$

$x_1 = 0.5 - 0.5 = \emptyset$
$x_2 = 1 - 0.5 = 0.5$
$x_3 = 1$

$$x_1^{n+1} = \frac{1}{2} - x_2^n = x_1^n$$

$$x_2^{n+1} = 1 - \frac{x_3^n}{2} = x_2^n$$

$$x_3^{n+1} = 1 = x_3^n$$

2. del (ko bi bil x_1^{n+1} vrednost $= x_1^n \rightarrow$ ko se ne bi več spreminjale; vsaka naslednja bi bila enaka prejšnji)

$$x_1^{n+1} = \frac{1}{2} - x_2^n = x_1^n$$

$$x_2^{n+1} = 1 - \frac{1}{2} = x_2^n = 0.5 \qquad x_3^{n+1} = x_3^n = 1$$

$$x_1^{n+1} = \frac{1}{2} - \frac{1}{2} = \emptyset$$

V našem primeru pridemo do rešitve v končno korakih!

2) $I = [0.1, 1]$ $\boxed{x + \ln x = 0}$

Newtonova metoda!

izračunamo robne vrednosti:

$f(0.1) = 0.1 + \ln(0.1) = -2.2025\ldots$
$f(1) = 1 + \ln 1 = \underline{\underline{1}}$ } različno predznačeni

$f'(x) = \underbrace{1 + \frac{1}{x}}_{\text{povsod pozitiven odvod}} > 0$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$$x_{n+1} = x_n - \frac{x_n + \ln x_n}{1 + \frac{1}{x_n}}$$

$$= x_n - \frac{x_n + \ln x_n}{\frac{x_n + 1}{x_n}}$$

$$= x_n - \frac{x_n(x_n + \ln x_n)}{1 + x_n}$$

$\underline{x_0 = 1}$

n	x_n
0	1,000
1	0,5
2	0,56438 | 2393
3	0,56714 | 898
4	$\underline{0,56714}$ | 329

) Simpsonova kvadraturna formula

Taylorjeva vrsta

$$\int_0^{1/2} \frac{e^{-x^2}}{\sqrt{x}}\,dx$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \dots$$

$$\int_0^{1/2} \frac{e^{-x^2}}{\sqrt{x}}\,dx = \int_0^{1/2} \frac{1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \dots}{\sqrt{x}}\,dx$$

$$= \int_0^{1/2} \left(x^{-1/2} - x^{3/2} + \frac{1}{2}x^{7/2} - \frac{1}{6}x^{11/2} + \dots\right)dx$$

$$= \left.\frac{x^{1/2}}{1/2}\right|_0^{1/2} - \left.\frac{x^{5/2}}{5/2}\right|_0^{1/2} + \left.\frac{1}{2}\frac{x^{9/2}}{9/2}\right|_0^{1/2} - \left.\frac{1}{6}\frac{x^{13/2}}{13/2}\right|_0^{1/2}$$

$$= \left.2\sqrt{x}\right|_0^{1/2} - \left.\frac{2}{5}x^{5/2}\right|_0^{1/2} + \left.\frac{1}{9}x^{9/2}\right|_0^{1/2} - \dots$$ (naslednji členi so že premajhni)

na 3 mesta

$$= 1.4142 - 0.0707 + 0.0049 \approx 1.348|4$$

$$\int_0^{1/2} \frac{f(x)}{\sqrt{x}}\,dx \approx w_1 f(\emptyset) + w_2 f(1/4) + w_3 f(1/2)$$

$$f(x) = 1, x, x^2$$

$$\int_0^{1/2} \frac{dx}{\sqrt{x}} = w_1 \cdot 1 + w_2 \cdot 1 + w_3 \cdot 1 = \left.2\sqrt{x}\right|_0^{1/2} = 1.4142 = \sqrt{2}$$

$$\int_0^{1/2} \sqrt{x}\,dx = w_1 \cdot \emptyset + \frac{1}{4} w_2 + \frac{1}{2} w_3 = \left.\frac{2}{3} x\sqrt{x}\right|_0^{1/2}$$

$$\int_0^{1/2} x^{3/2}\,dx = w_1 \cdot \emptyset + \frac{1}{16} w_2 + \frac{1}{4} w_3 = \left.\frac{2}{5} x^{5/2}\right|_0^{1/2}$$

$w_1 + w_2 + w_3 = 1.4142$

~~$\frac{1}{4} w_2 + \frac{1}{2} w_3 = 0.2357$~~ $| \cdot 2$

$\frac{1}{16} w_2 + \frac{1}{4} w_3 = 0.0707$

$w_2 + 2w_3 = 0{,}2357 \cdot 4$

$w_2 + 4w_3 = 0{,}0707 \cdot 16$

~~$2w_3 = 16 \cdot 0{,}0707 - 4 \cdot 0{,}2357$~~

~~$w_3 = 8 \cdot 0{,}0707 - 2 \cdot 0{,}2357 =$~~

$w_3 = 0{,}0942$

~~$4w_3 = 0{,}0707 \cdot 16 - w_2$~~

~~$w_2 = 0{,}0707 \cdot 16 - 4w_3$~~

~~$w_2 = 1{,}2163$~~

$w_2 = 0.7544$

$w_1 = 0.5656$

$$\int_0^{\frac{1}{2}} \frac{f(x)}{\sqrt{x}} dx = 0{,}5656 f(0) + 0{,}7544 f\left(\tfrac{1}{4}\right) + 0{,}0942 \cdot f\left(\tfrac{1}{2}\right)$$

$f(x) = e^{-x^2}$

$$\int_0^{\frac{1}{2}} \frac{e^{-x^2}}{\sqrt{x}} = \text{~~}0{,}5656 + 0{,}7087 + 0{,}0734 \approx 1{,}348\text{~~}$$

Izpit iz Numeričnih metod

5. september 2007

1. Ali lahko rešimo sistem $Ax = b$ z Gauss-Seidlovo iteracijo?

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & -3 \end{bmatrix} \quad b = \begin{bmatrix} 3 \\ -3 \\ -1 \end{bmatrix}.$$

   Izračunaj prvi korak Jacobijeve in Gauss-Seidlove iteracije. Začetni približek je enak $x_0 = [0; 0; 0]$. Kolika je prva norma razlike posameznega približka in točne rešitve.

   Rešitev:
   $x_\infty = [1, -1, 1]$, $x_J = [0.75, -1, 0.33]$, $x_{GS} = [0.75, 1.25, 1]$.

2. Pokažite, da leži na intervalu $[0, 2]$ natanko en koren enačbe

$$x^2 e^{-x} - 1/3 = 0.$$

   Poišči ta koren z Newtonovo metodo.
   Rešitev: x=0.910008

3. Reši integral z Gaussovo kvadraturno formulo in primerjaj rezultat s točno rešitvijo.

$$\int_0^1 \exp(-x)\sin(x)\,dx$$

   Navodilo Gauss: $\int_0^1 f(x)\,dx \approx wf(u) + wf(1-u)$
   Rešitev:
   Točna: `1/(2e)(e-Cos[1]-Sin[1])=0.245837,`
   Gauss: `w=1/2, u=1/2(1-1/Sqrt[3]) I=0.246096`

5.9.2007

Preverimo, ali je matrika diagonalno dominantna po vrsticah

$|4| > |2| + |1|$

$|3| > |1| + |-1|$

$|-3| > |1| + |-1|$

$4x_1 + 2x_2 + x_3 = 3$

$x_1 + 3x_2 - x_3 = -3$

$x_1 - x_2 - 3x_3 = -1$

~~(Izrazimo x_1 iz prve, x_2 iz druge, x_3 iz tretje)~~

$4x_1 + 2x_2 + x_3 = 3$

$$x_1 = \frac{1}{4}(3 - 2x_2 - x_3)$$

~~$x_1 + 3x_2 - x_3 = -3$~~

$$x_2 = \frac{1}{3}(-3 - x_1 + x_3)$$

$x_1 - x_2 - 3x_3 = -1$

$$x_3 = -\frac{1}{3}(-1 - x_1 + x_2)$$

Prvi korak Jacobijeve iteracije

Začetni približek je $\vec{x}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Vstavimo ga v desno stran enačb in dobimo nov približek

$$\vec{x}_1 = \begin{bmatrix} 3/4 \\ -1 \\ 1/3 \end{bmatrix}$$

$x_1 = 1/4(3 - 2 \cdot \emptyset - \emptyset) = 3/4$

Prvi korak Gauss Seidlove iteracije

$$\vec{x}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

~~$x_1 = 1/4(3 - \emptyset - \emptyset) = 3/4$~~

~~Ko računamo x_2, kot x_1 uporabimo~~ že novo vrednost $- 3/4$, za x_3 še vedno $\emptyset$!

$$\vec{x}_1 = \begin{bmatrix} 3/4 \\ -5/4 \\ 1 \end{bmatrix}$$

$x_2 = \frac{1}{3}(-3 - \frac{3}{4} + \emptyset)$

$x_2 = -1 - 1/4 = -5/4 = -1.25$

$x_3 = -\frac{1}{3}\left(-1 - \frac{3}{4} + \left(-\frac{5}{4}\right)\right)$

$x_3 = +\frac{1}{3} + \frac{1}{4} + \frac{5}{4}\cdot\frac{1}{3} =$

$= \frac{4+3+5}{12} = \underline{\underline{1}}$

Če hočemo izračunati zadnji del naloge potrebujemo točno rešitev.

$12x_1 + 6x_2 + 3x_3 = 9$

$4x_1 + 2x_2 + x_3 = 3$

amo 3 $\quad x_1 + 3x_2 - x_3 = -3$

štejemo zadnji $\quad x_1 - x_2 - 3x_3 = -1$

$5x_1 + 5x_2 = 0$

$13x_1 + 5x_2 = 8$

$x_1 = -x_2$

$13x_1 - 5x_1 = 8 \qquad \underline{\underline{x_2 = -1}}$

$8x_1 = 8$

$\underline{\underline{x_1 = 1}}$

$1 - (-1) - 3x_3 = -1$

$2 - 3x_3 = -1$

$-3x_3 = -3$

$\underline{\underline{x_3 = 1}}$

$\underline{\underline{\vec{x}_{točna}}} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

rva norma razlike posameznega približka in točne rešitve

$\boxed{\text{Jacobi}}$

$\vec{x} = (\vec{x_1} - \vec{x}_{točna}) = \begin{bmatrix} 3/4 \\ -1 \\ 1/3 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/4 \\ 0 \\ -2/3 \end{bmatrix}$

$\|\vec{x}\|_1 = \left|-\frac{1}{4}\right| + |0| + \left|-\frac{2}{3}\right| = \frac{1}{4} + \frac{2}{3} = \frac{3+8}{12} = \underline{\underline{\frac{11}{12}}}$

prvo normo dobimo tako, da seštejemo absolutne vrednosti

$\boxed{\text{auss-Seidel}}$

$\vec{x} = (\vec{x_1} - \vec{x}_{točna}) = \begin{bmatrix} 3/4 \\ -5/4 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/4 \\ -1/4 \\ 0 \end{bmatrix}$

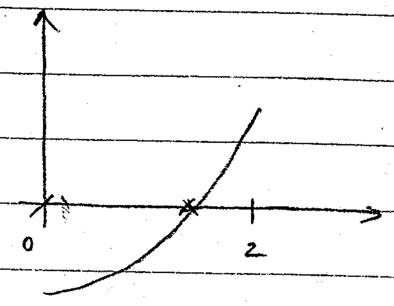
$\|\vec{x}\|_1 = \left|-\frac{1}{4}\right| + \left|-\frac{1}{4}\right| + |0| = \underline{\underline{\frac{1}{2}}}$

1) Interval $[0,2]$

Newtonova metoda

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f(x) = x^2 \cdot e^{-x} - \frac{1}{3} = \emptyset$$



Izračunamo mejni vrednosti

$f(\emptyset) = -1/3 \quad < \emptyset$

$f(2) = 0{,}2\ldots \quad > 0$

vidimo da sta različno predznačeni, zato leži na tem int. najmanj en koren!

$$f'(x) = 2x \cdot e^{-x} - x^2 e^{-x}$$
$$= e^{-x} x(2-x)$$

Pogledamo kakšne vrednosti zavzame odvod na tem intervalu

$e^{-x} > \emptyset$

x [med $\emptyset$ in 2] $\geq \emptyset$

$(2-x) \geq \emptyset$

povsod razen v krajiščih je $> \emptyset$
(Funkcija nima prevoja, torej je le ena rešitev!)

$$x_{n+1} = x_n - \frac{x_n^2 \cdot e^{-x_n} - 1/3}{e^{-x_n} x_n (2-x_n)}$$

Za začetni približek vzamemo npr. $x_0 = 1$

n	x_n
$\emptyset$	1,00
1	0.906094
2	0.91000135
⋮	

3.) $\int_0^1 e^{-x} \sin(x)\,dx$

Navodilo:

$$\int_0^1 f(x)\,dx \approx w f(u) + w f(1-u)$$

Točna rešitev: (integral iz priročnika)

$$\int_0^1 e^{-x}\sin(x)\,dx = \frac{e^{-x}}{2}\left[-\sin x - \cos x\right]\Big|_0^1$$

$$= \frac{e^{-1}}{2}\left[-\sin 1 - \cos 1\right] - \frac{1}{2}\left[-1\right] =$$

$$= \frac{e^{-1}}{2}\left[-\sin 1 - \cos 1\right] + 1/2$$

$$= \frac{1}{2e}\left[-\sin 1 - \cos 1 + e\right] \approx \underline{0.245837}$$

$$\int_0^1 f(x)\,dx = w_0 f(u) + w_1 f(1-u)$$

$f(x) = 1, x, x^2$

$f(x) = 1$: $\int_0^1 1\,dx = w_0 \cdot 1 + w_1 \cdot 1 = 2w = 1$ $\quad \boxed{w = 1/2}$

$f(x) = x$: $\int_0^1 x\,dx = \frac{1}{2}\left[u + (1-u)\right] = \frac{x^2}{2} \Rightarrow \frac{1}{2}$

$f(x) = x^2$: $\int_0^1 x^2\,dx = \frac{1}{2}\left[u^2 + (1-u)^2\right] = \frac{1}{3}$

$$u^2 + 1 - 2u + u^2 = 2/3$$

$$2u^2 - 2u + 1 - 2/3 = 0$$

$$6u^2 - 6u + 1 = 0$$

$$u_{1,2} = \frac{6 \pm \sqrt{36-24}}{12} = \frac{6 \pm \sqrt{12}}{12}$$

$u_1 = 0.788675 \qquad u_2 = 0.211132$

$$\int_0^1 f(x)\,dx = \frac{1}{2}\left[f(u_1) + f(1-u_1)\right] = \frac{1}{2}\left[f(u_1) + f(u_2)\right]$$

$f(x) = e^{-x}\sin x \qquad = \frac{1}{2}\left[0.322393 + 0.169679\right]$

Izpit iz Numeričnih metod

29. junij 2004

1. Sestavite formulo za približno računanje singularnih integralov oblike

$$\int_0^1 \frac{f(x)\,dx}{\sqrt{x}} \approx a_1 f(x_1) + a_2 f(x_2)$$

Določite koeficienta, a_1 in a_2 ter vozlišči x_1 in x_2 tako, da bo formula točna za polinome $p_0(x) = 1$, $p_1(x) = x$, $p_2(x) = x^2$ in $p_3(x) = x^3$. S pomočjo dobljene formule določi približno vrednost za integral

$$\int_0^1 \frac{\sin x}{\sqrt{x}}\,dx$$

Pomoč: koeficienta a_i, $i = 1, 2$ lahko pišemo $a_i = 1 \pm b$, medtem ko za vozlišči x_i, $i = 1, 2$ velja $x_i = \frac{3}{7} \pm y$

2. Poišči X, tako da bo imela razlika $AX - B$ minimalno evklidsko normo.

$\vec{X} = \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

3. Na intervalu $[0, 2]$ ležita dva korena enačbe

$$\sqrt{x}e^{-x} - \frac{1}{3} = 0$$

Poišči oba korena. Določi, katerega od njiju lahko poiščemo s pomočjo naslednje iteracijske sheme

$$x_{n+1} = \sqrt{x_n}e^{-x_n} - \frac{1}{3} + x_n$$

pri izbiri primernega začetnega približka. Določi maksimalni podinterval intervala $[0, 2]$ iz katerega lahko izbiramo začetni približek tako, da bo gornja shema konvergirala. Zakaj drugega korena ne moremo poiskati na na ta način?

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29. junij 2004

2. Poišči tako, da bo imela razlika $AX-B$ minimalno evklidsko normo.

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$A^T \cdot A \cdot X = A^T \cdot B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$1\cdot1 + 1\cdot1 + 1\cdot1 = 3$ $\quad 2\cdot1 + 3\cdot1 + 1\cdot1 = 6$

$$\begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

$1\cdot2 + 1\cdot3 + 1\cdot1 = 6$

$4 + 9 + 1 =$

$1\cdot1 + 1\cdot1 + 1\cdot1 = 3$

$2\cdot1 + 3\cdot1 + 1\cdot2 = 7$

$3x_1 + 6x_2 = 3 \quad /2 \Rightarrow$

$6x_1 + 14x_2 = 7$

$6x_1 + 12x_2 = 6$

$6x_1 + 14x_2 = 7$] −

$-2x_2 = -1$

$x_2 = 1/2$

$3\cdot x_1 + 6\cdot\frac{1}{2} = 3$

$3x_1 + 3 = 3$

$3x_1 = 0$

$x_1 = 0$

$$\vec{x} = \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$$

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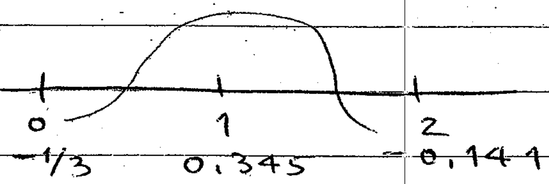
Na intervalu $[0,2]$ ležita dva korena enačbe $\boxed{\sqrt{x}\, e^{-x} - \frac{1}{3} = \varnothing}$. Poišči oba korena. Določi, katerega od njiju lahko poiščemo s pomočjo naslednje iteracijske sheme

$$\boxed{x_{n+1} = \sqrt{x_n}\, e^{-x_n} - \frac{1}{3} + x_n}$$

$f(\varnothing) = -1/3$

$f(2) = -0{,}14194034$

$f(1) = 0{,}34546$



$$f(x) = \sqrt{x}\, e^{-x} - 1/3$$

$$f'(x) = -\frac{1}{2} x^{-1/2} e^{-x} - \sqrt{x} e^{-x}$$

$$= +\frac{e^{-x}}{2\sqrt{x}} - \sqrt{x}\, e^{-x} = e^{-x}\left[\frac{1}{2\sqrt{x}} - \sqrt{x}\right]$$

$$= \frac{e^{-x}(1-2x)}{2\sqrt{x}}$$

$$\boxed{x_{n+1} = x_n - \frac{\left(\sqrt{x_n}\, e^{-x_n} - \frac{1}{3}\right)(2\sqrt{x_n})}{e^{-x_n}(1-2x_n)}} \quad \text{Formula}$$

Približek $\boxed{x_0 = 0{,}1}$

$x_1 = 0{,}141238108$

$x_2 = 0{,}149705382$

$x_3 = 0{,}14997589$

$\boxed{x_4 = 0{,}1499\boxed{7589}}$

ENA REŠITEV >1 ODBOJNA

$\boxed{x_0 = 1{,}2}$

$x_1 = 1{,}18237816$

$x_2 = 1{,}182374744$

$\boxed{x_3 = 1{,}1823\boxed{7}}$

DRUGA REŠITEV <1 PRIVLAČNA

0,807625

$$\boxed{x_{n+1} = g(x)}$$

$$g'(x) = \frac{1}{2}\frac{e^{-x_n}}{\sqrt{x_n}} + 1 - \sqrt{x_n}\, e^{-x_n}$$

VSTAVIMO TOČKE V ODVOD IN VIDIMO ALI JE >1 (ODB.) ALI <1 (PRIVL.)

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Izpit iz numeričnih metod.

17. junij 2003

1. Poiščite stevilo m tako, da bo iteracaija

$$x = g(x), \quad g(x) = x - m\,\frac{f(x)}{f'(x)}$$

kjer je

$$f(x) = x^2 - 2x + 1$$

kar najhitreje konvergirala h korenu enačbe $f(x) = 0$.

2. Izračunajte integral

$$\int_0^1 \frac{e^x}{\sqrt{x}}\,dx$$

3. Izračunaj neskončno normo matrike A, $\|A\|_\infty$.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -1 \\ 5 & -1 & 1 \end{bmatrix}$$

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17.6.2003

1.) Poišcite število m tako, da bo iteracija $x = g(x)$, $g(x) = x - m\frac{f(x)}{f'(x)}$
kjer je $f(x) = x^2 - 2x + 1$ kar najhitreje
konvergirala h korenu enačbe $f(x) = 0$.

$$|g'(x)| < 1$$

$$g(x) = x - m\frac{x^2 - 2x + 1}{2x - 2} = x - m\frac{(x-1)^2}{2(x-1)} =$$

$$= x - m\frac{x-1}{2}$$

$$g'(x) = 1 - \frac{m}{2}$$

Iteracija konvergira, ko je $|g'(x)| < 1$; čim manjša je $g'(x)$ tem hitreje konvergira $g'(x) = 0$.

$$g'(x) = 1 - \frac{m}{2} = 0$$

$$-\frac{m}{2} = -1$$

$$\frac{m}{2} = 1$$

$$\boxed{m = 2}$$

3.) Izračunaj neskončno normo matrike A, $\|A\|_\infty$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -1 \\ 5 & -1 & 1 \end{bmatrix}$$

Izračunamo absolutne vrednosti po vrsticah, največja je neskončna norma!

$$|1| + |2| + |-1| = 4$$
$$|0| + |3| + |-1| = 4$$
$$|5| + |-1| + |1| = 7$$

$$\boxed{\|A\|_\infty = 7}$$

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$$\int_0^1 \frac{e^x}{\sqrt{x}}\,dx \qquad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\int_0^1 \frac{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots}{\sqrt{x}}\,dx \qquad \frac{x^4}{4!} - 24$$

$$= \int_0^1 \left(x^{-1/2} + x^{1/2} + \frac{1}{2}x^{3/2} + \frac{1}{6}x^{5/2} + \dots\right)dx$$

$$= \left(\frac{x^{1/2}}{1/2}\Big|_0^1 + \frac{x^{3/2}}{3/2}\Big|_0^1 + \frac{1}{2}\frac{x^{5/2}}{5/2}\Big|_0^1 + \frac{1}{6}\frac{x^{7/2}}{7/2} + \Big|_0^1\dots\right) \qquad \frac{1}{24}\frac{x^{9/2}}{9}$$

$$= 2x^{1/2}\Big|_0^1 + 2\frac{x^{3/2}}{3}\Big|_0^1 + \frac{1}{2}\frac{2x^{5/2}}{5}\Big|_0^1 + \frac{2}{6}\frac{x^{7/2}}{7}\Big|_0^1 + \dots$$

$$= 2{,}925304$$

5.6.2003

(3)

	x	$f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$
x_0	0	4			
			−1		
	1	3		13	
			12		−22
	2	15		−9	
			3		
	3	18			

$$r = \frac{x - x_0}{h} = \frac{x-0}{1} = x$$

~~$P_3 = 4 + (-1)$~~

$$P = f(x_0) + r\Delta f_0 + \frac{r(r-1)}{2!}\Delta^2 f_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 f_0 +$$

$$= 4 + r(-1) + \frac{r(r-1)}{2} 13 + \frac{r(r-1)(r-2)}{6}(-22)$$

$x = 1{,}5$; $r = \frac{1{,}5-0}{h} = 1{,}5$

Preveri ✓

(Tako, da vstavimo v dobljeno npr $x=1$. Če dobimo 3 je pravilno)

1

Izpit iz numeričnih metod.

5. junij 2003

1. Poiščite $\sqrt[3]{25}$ z napako manjšo od 10^{-4}. Uporabite eno od metod za numerično reševanje enačb.

2. Poiščite w_1 in w_2, tako, da bo integracijska formula

$$\int_{-1}^{1} f(x)\,dx \approx w_1 f(-\frac{1}{3}) + w_2 f(\frac{2}{3})$$

točna za polinome najvišje možne stopnje.

3. Poiščite polinom, za katerega velja:

x	0	1	2	3
f(x)	4	3	15	18

Newtonova za tiste ki so ekvidistantni, za ostale Lagrange ali deljene diference

Pregej

(Tako, da vstavimo v dobljeno npr x=1. Če dobimo 3 je pravilno)

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5. junij 2003

2.924017738 je točka

1.

$\sqrt[3]{25} \Rightarrow$ $\boxed{x^3 - 25 = 0}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= \frac{x_n}{1} - \frac{x_n^3 - 25}{3x_n^2} = \frac{3x_n^3 - x_n^3 + 25}{3x_n^2}$$

$$= \frac{2x_n^3 + 25}{3x_n^2}$$

Za začetni približek

$x_0 =$ vseeno kaj vzamemo
ker po določenem št. iteracij
pridemo do istega rezultata
ne glede na to kje začnemo

Rezultat: 2.924017738 / lahko izračunamo direkt s kalkulatorjem

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5. junij 2003

(3.) Poišate polinom za katerega velja:

	x	f(x)	Δ	Δ^2	Δ^3
x_0	0	4			
			−1		
x_1	1	3		13	
			12		−22
x_2	2	15		−9	
			3		
x_3	3	18			

$$\frac{3-4}{1} = \frac{-1}{1} = -1$$

$$f(x) = f(x_0) + \Delta \cdot 1 (x-x_0) + \Delta^2 (x-x_0)(x-x_1) + \Delta^3 (x-x_0)(x-x_1)(x-x_2)$$

$$= 4 - 1(x-\emptyset) + 13(x)(x-1) - 22(x)(x-1)(x-2)$$

$$= 4 - x + 13x(x-1) - 22x(x-1)(x-2)$$

(2.)

$$\int_{-1}^{1} f(x)dx \approx w_1 f(-1/3) + w_2 f(2/3)$$

$$\int_{-1}^{1} f(x) = \frac{-2}{3} f(-1/3) + \frac{8}{3} f(2/3)$$

$$\int_{-1}^{1} 1dx \approx w_1 + w_2 = x\Big|_{-1}^{1} = 1-(-1) = 2$$

$$\int_{-1}^{1} xdx \approx w_1(-1/3) + 2/3 w_2 = \frac{x^2}{2}\Big|_{-1}^{1} = \frac{1}{2} + \frac{1}{2} = 1$$

$$w_1 + w_2 = 2 \quad /$$

$$-\frac{1}{3}w_1 + 2/3 w_2 = 2 \quad /3$$

$$-w_1 + 2w_2 = 6$$

$$w_1 + w_2 = 2 \quad /-1$$

$$-w_1 - w_2 = -2$$

$$-w_1 + 2w_2 = 6 \quad -$$

$$-3w_2 = -8$$

$$w_2 = \frac{-8}{-3} = \frac{8}{3}$$

$$w_1 = 2 - w_2 = 2 - 8/3 = -2/3$$

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(diferencialni izpit)
64980293

Ime, priimek PETER ROGELJ

Naloga	točke
1.	
2.	
3.	
Skupaj	

NUMERIČNE METODE

7. februar 2003

~~Kako vemo, da smo našli vse?~~

1. Poiščite vse ničle funkcije $f(x) = x^2 + 10\cos x$. Napaka naj bo manjša od 10^{-3}.

2. Izračunajte približno vrednost integrala

$$\int_0^1 \frac{e^x}{\sqrt{x}} dx$$

3. S pomočjo Taylorjeve metode drugega reda poiščite približno rešitev enačbe

$$y' = 1 + t\sin(ty), 0 \leq t \leq 0.5, y(0) = 0.$$

s korakom $h = 0.1$!

7.2.2003

$f(x) = x^2 + 10\cos x$

Če damo not 0 je 10
če damo not 2 je -0.1
(-1) je 6.40?

$$x^2 + 10\cos x = 0$$

$f(x) = x^2 + 10\cos x$

$f'(x) = 2x - (10\sin x)$

$$x_{n+1} = x_n - \frac{x_n^2 + 10\cos x_n}{2x_n - 10\sin x_n}$$

$x_0 = 4$

3.16195

$x_0 = 1$

$x_1 = 1.99817$

$x_2 = 1.96836$

$x_3 = 1.96887$

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7.2.2003

3.) S pomočjo T. metode 2. reda poišči približno rešitev enačbe

$$y' = 1 + x\sin(xy); \qquad 0 \le x \le 0{,}5 \qquad y(\emptyset) = \emptyset$$

$$y'' = \sin(xy) + x[\cos(xy) \cdot y]$$

↳ ker odvajamo po $\underline{\underline{x}}$

$$y(x_0+h) = \underbrace{y(x_0)}_{\emptyset} + h(1 + x_0 \sin(x_0 y_0)] + \frac{h^2}{2}[\sin(x_0y_0) + x_0 y_0 \cos(x_0 y_0)]$$

0 0.1 0.2 0.3 0.4 0.5

$x_0 = \emptyset \qquad y_0 = \emptyset$

x_0	y_0	y'	y''
∅	∅	1	∅
0.1	0.1	1.00099	0.01999
0.2			
⋮			

itd.

Izpit iz numeričnih metod.

12. september 2003

1. Z uporabo deljenih diferenc izračunajte $f(8.2)$.

x	8.0	8.1	8.3
$f(x)$	16.6355	17.6155	17.5649

2. Določite prve tri zaporedne približke Gauss-Seidlove iteracije za sistem enačb:

$$\begin{aligned} 4x_1 + 3x_2 &= 24 \\ 3x_1 + 4x_2 - x_3 &= 30 \\ -x_2 + 4x_3 &= -24 \end{aligned}$$

Uporabite začetni približek $\vec{x} = [1, 1, 1]^T$.

3. Z uvedbo nove spremenljivke izračunajte integral:

$$I = \int_1^\infty x^{-3/2} \sin\left(\frac{1}{x}\right) dx$$

Gauss-Seidlova iteracija (3. zaporedne približke)

začetni približek

$$x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$4x_1 + 3x_2 = 24$$

$$3x_1 + 4x_2 - x_3 = 30$$

$$-x_2 + 4x_3 = -24$$

$$x_1 = \frac{1}{4}(24 - 3x_2)$$

$$x_2 = \frac{1}{4}(30 - 3x_1 + x_3)$$

$$x_3 = \frac{1}{4}(-24 + x_2)$$

$$x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad x_1 = \begin{bmatrix} 5.25 \\ 3.8125 \\ -5.046875 \end{bmatrix}$$

$\frac{1}{4}(30 - 3 \cdot 5.25 + 1)$

$15\frac{1}{4} : 4 = \frac{61}{16}$

$$x_2 = \begin{bmatrix} 3.140625 \\ 3.8828125 \\ -5.0292968875 \end{bmatrix} \qquad x_3 = \begin{bmatrix} 3.087890625 \\ 3.926757813 \\ -5.018310547 \end{bmatrix} \rightarrow \begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix}$$

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[illegible]2003

DELJENE DIFERENCE

x	f(x)	Δ	Δ^2
x_0 8.0	16.6355		
		9.8	
8.1	17.6155		−33.51
		−0.253	
8.3	17.5649		

$$\Delta_1 = \frac{17.6155-16.6355}{8.1-8.0} = \frac{0.98}{0.1} = 9.8$$

$$\Delta_2 = \frac{17.5649-17.6155}{0.2} = \frac{-0.0506}{0.2}$$

$$\Delta^2 = \frac{-0.253-9.8}{0.3} = -33.51$$

$$f(8.2) = 16.6355 + 9.8(8.2-8.0) + (-33.51)(x-8.0)(x-$$

(annotations: Δ, x − x_0, Δ^2 $(x-x_0)(x-x_1)$)

$$= 16.6355 + 9.8\cdot 0.2 - 33.51(0.2)(0.1)$$

$$= 17.9253$$

(2.) $$I = \int_1^\infty x^{-\frac{3}{2}} \sin\left(\frac{1}{x}\right) dx = \int_1^\infty \frac{\sin\left(\frac{1}{x}\right)}{x^{3/2}} dx$$

$\frac{1}{x} = u$

$x = \frac{1}{u}$

$dx = \frac{-du}{u^2}$

$$= -\int_1^0 \frac{\sin u}{\frac{1}{u^{3/2}}} \frac{du}{u^2}$$

$$= \int_0^1 \frac{\sin u \cdot du}{\sqrt{u}}$$

$2^{-3/2} =$

$= \frac{4}{2} - \frac{3}{2} = \frac{1}{2}$

Naprej računamo npr. z visto

Izpit iz numeričnih metod.

1. september 2003

1. S pomočjo Taylorjeve metode drugega reda poiščite približno rešitev diferencialne enačbe *Posredni odvod*

$$y' = \left(\frac{y}{x}\right)^2 + \frac{y}{x}, \quad 1 \leq x \leq 1.2, \quad y(1) = 1, \quad h = 0.1$$

(nisem delala)

2. Poiščite koeficiente a, b in c tako, da boste dobili aproksimacijo odvoda

$$y'(x_0) = ay(x_0 - h) + by(x_0) + cy(x_0 + h)$$

najvišjega možnega reda.

3. Z uporabo Newtonove metode poiščite pozitivno rešitev enačbe

$$x^2 - 10\cos(x) = 0$$

1.9.2003

$f(x) = x^2 - 10\cos x$

$f'(x) = 2x + 10\sin x$

$$x_{n+1} = x_n - \frac{x_n^2 - 10\cos x_n}{2x_n + 10\sin x_n}$$

$f(0) = -10$

$f(1) = -4.4$

$f(4) = 22.5$

$x_0 = 0.5$

1.37936

(1.) $y' = \left(\frac{y}{x}\right)^2 + \frac{y}{x}$ $\qquad 1 \leq x \leq 1.2$

$y(1) = 1 \qquad h = 0.1$

$$y(x_0+h) = \underbrace{y(x_0) + h y'(x_0) + \frac{h^2}{2!} y''(x_0)}_{\text{2. reda}} + \frac{h^3}{3!} y'''(x_0)$$

$\left(y \cdot \frac{1}{x}\right)^2 \;\downarrow\; 2\left(y \cdot \frac{1}{x}\right)$

$f(x,y) = \left(\frac{y}{x}\right)^2 + \left(\frac{y}{x}\right)$

~~$y'' = \frac{df}{dx} = f_y \cdot y' + f_x$~~ — posredni odvod

~~$= \left[\left(2\frac{y}{x} \cdot \frac{1}{x}\right)\right] y' + \left[\left(2\frac{y}{x}\right) \ldots x^2\right]$~~

$= \left[2\frac{y}{x} \cdot \frac{1}{x} + \frac{1}{x}\right] y' + \left[2\frac{y}{x} \cdot \frac{-y}{x^2} - \frac{y}{x^2}\right]$

$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2} = \frac{x \cdot \emptyset - y \cdot 1}{x^2}$

$x^{-1} \cdot y$

$= -1x^{-2} \cdot y + x^{-1} \cdot \emptyset$

$= \left[2\frac{y}{x^2} + \frac{1}{x}\right] y' + \left[-2\frac{y^2}{x^3} - \frac{y}{x^2}\right]$

$y_0 = 1 \quad x_0 = 1 \quad h = 0.1$

$y(1.1) = 1 + h \cdot 2 + \frac{h^2}{2} \cdot 3$

$y'(x_0) = y'(1) = 2$

$y''(1) = 3 \cdot 2 + (-2-1) = 6 - 3 = 3$

$x = 1$

$y(1.1) = 1 + 2 \cdot 0.1 + \frac{0.1^2 \cdot 3}{2} = 1.215$

$y(1.2) = \ldots$ Izračunaš! $\quad 1.46525$

I / 4 $\quad$ I / 1.1 $\quad$ 35 / 1.2

18.6.2002

1.) $x = f(x)$ ... negibna točke

$$x = x^2 e^{-x} + \frac{1}{2}$$

$$f(x) = x^2 e^{-x} - x + \frac{1}{2} = 0$$

$$f'(x) = 2x e^{-x} - x^2 e^{-x} - 1$$

$|f'(x)| < 1$ ... pravilna

2.) $$\begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\left.\begin{array}{l} 2x_1 + x_2 = 2 \\ 2x_1 + 2x_2 = 2 \end{array}\right\} \begin{array}{l} x_1 = 1 - \frac{x_2}{2} \\ x_2 = 1 - x_1 \end{array}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & -\frac{1}{2} \\ -1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -\lambda & -\frac{1}{2} \\ -1 & -\lambda \end{vmatrix} = \lambda^2 - \frac{1}{2} = 0$$

$$\lambda_{1,2} = \pm\sqrt{\frac{1}{2}} < 1$$

3.) a) $$\int_0^{0,1} \frac{1 - x + \frac{x^2}{2} - \frac{x^3}{3}}{\sqrt{x}}$$

b) $$\int_0^{0,1} \frac{f(x)}{\sqrt{x}} dx = w_0 f(0) + w_1 f(0,1)$$

c) ; $\sqrt{x} = u$; $x = u^2$; $dx = 2u\,du$

$$\int_0^{\sqrt{0,1}} \frac{e^{-u^2}}{u} 2u\,du = 2\int_0^{\sqrt{0,1}} e^{-u^2} du$$ { trapez. simp. pr. ...

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17.6.2003

$\|A\|_\infty \rightarrow$

$$\begin{cases} 4 = |1| + |2| + |-1| \\ 4 = |0| + |3| + |-1| \\ 7 = |5| + |-1| + |1| \end{cases}$$

(3) $\|A\|$

$\|A\|_\infty = 7$

(1) $|g'(x)| < 1$

$$g(x) = x - m\,\frac{x^2 - 2x + 1}{2x - 2} = x - m\,\frac{(x-1)^2}{2(x-1)} =$$

$$= x - m\,\frac{x-1}{2}$$

$$g'(x) = 1 - \frac{m}{2}$$

$$g'(x) = 0 \Rightarrow \boxed{m = 2}$$

najhitreje konvergira

$$x^2 + 5x + 6$$

$x = 2$; $y = 20$

$4^{-1/2} = 1/2$

$x^{1-1/2}$

$3^{-1/2}$

$x^{2-1/2}$

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(17.1.2008) 2. $\|Ax - B\|_2$

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 - 1 \\ 2x_1 + 3x_2 - 1 \\ x_1 + x_2 - 2 \end{bmatrix}$$

$$y_1 = x_1 + 2x_2 - 1$$
$$y_2 = 2x_1 + 3x_2 - 1 \qquad \sqrt{y_1^2 + y_2^2 + y_3^2} = 4$$
$$y_3 = x_1 + x_2 - 2 \qquad y_1^2 + y_2^2 + y_3^2$$

$$(x_1 + 2x_2 - 1)^2 + (2x_1 + 3x_2 - 1)^2 + (x_1 + x_2 - 2)^2 = z$$

$$\frac{\partial z}{\partial x_1} = 0 \; ; \; \frac{\partial z}{\partial x_2} = 0$$

$$\frac{\partial z}{\partial x_1} = 2(x_1 + 2x_2 - 1) + 2(2x_1 + 3x_2 - 1)\,2 + 2(x_1 + x_2 - 2) = 0$$

$$\frac{\partial z}{\partial x_2} = 2(x_1 + 2x_2 - 1)\,2 + 2(2x_1 + 3x_2 - 1)\cdot 3 + 2(x_1 + x_2 - 2) = 0$$

$$\left.\begin{array}{l} 6x_1 + 9x_2 = 5 \\ 9x_1 + 14x_2 = 7 \end{array}\right\}$$

$$\boxed{A^T A x = A^T B}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 9 \\ 9 & 14 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$f(x)=\sqrt{x}e^{-x}-\frac{1}{4}=0$$

x	0	1	2
$f(x)$	$-\frac{1}{4}$	0,111	−0,0586
	<0	>0	<0

$$f'(x)=\frac{1}{2\sqrt{x}}e^{-x}-\sqrt{x}e^{-x}=\frac{e^{-x}(1-2x)}{2\sqrt{x}}$$

$$x_{n+1}=x_n-\frac{\left(\sqrt{x_n}e^{-x_n}-\frac{1}{4}\right)2\sqrt{x_n}}{e^{-x_n}(1-2x_n)}=$$

$$=x_n-\frac{2x_n e^{-x_n}-\frac{1}{2}\sqrt{x_n}}{e^{-x_n}(1-2x_n)}$$

$$=x_n-\frac{2x_n-\frac{1}{2}e^{x_n}\sqrt{x_n}}{1-2x_n}$$

$$=x_n-\frac{4x_n-e^{x_n}\sqrt{x_n}}{2(1-2x_n)}=\frac{2x_n-4x_n^2-4x_n+e^{x_n}\sqrt{x_n}}{2(1-2x_n)}$$

$$=\frac{-2x_n-4x_n^2+e^{x_n}\sqrt{x_n}}{2(1-2x_n)}$$

$x_0=0,1$; 0,0684

$x_0=1,6408$

$x_{n+1}=g(x_n)$; $g(x)$

$|g'(x)|<1$

$$g(x)=\sqrt{x}e^{-x}-\frac{1}{4}+x$$

$$g'(x)=\frac{1}{2\sqrt{x}}e^{-x}-\sqrt{x}e^{-x}+1$$

17.1.2008

$f(x) = x$

$f(\xi_1) = \xi$

1.
$$\int_0^1 f(x)\,dx \approx \omega_1 f(0) + \omega_2 f(\xi_1) + \omega_3 f(1)$$

$$f(x) = 1, x, x^2, x^3$$

$$f(x)=1: \int_0^1 dx = \omega_1 + \omega_2 + \omega_3 = 1$$

$$f(x)=x: \int_0^1 x\,dx = \omega_1 \cdot 0 + \omega_2 \xi + \omega_3 = \frac{1}{2}$$

$$f(x)=x^2: \int_0^1 x^2\,dx = \omega_1 \cdot 0 + \omega_2 \xi^2 + \omega_3 \cdot 1 = \frac{1}{3}$$

$$f(x)=x^3: \int_0^1 x^3\,dx = \omega \cdot 0 + \omega_2 \xi^3 + \omega_3 \cdot 1 = \frac{1}{4}$$

$$\omega_1 + \omega_2 + \omega_3 = 1$$

$$\omega_2 \xi + \omega_3 = \frac{1}{2}$$

$$\omega_2 \xi^2 + \omega_3 = \frac{1}{3}$$

$$\omega_2 \xi^3 + \omega_3 = \frac{1}{4}$$

$$\omega_2(\xi - \xi^2) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$\omega_2(\xi^2 - \xi^3) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$\frac{\xi - \xi^2}{\xi^2 - \xi^3} = \frac{\frac{1}{6}}{\frac{1}{12}} = \frac{12}{6} = 2 = \frac{\xi(1-\xi)}{\xi^2(1-\xi)} = \frac{1}{\xi}$$

$$\boxed{\xi = \frac{1}{2}}$$

$$\omega_2\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{6} = \omega_2 \frac{1}{4} = \frac{1}{6} \Rightarrow \boxed{\omega_2 = \frac{2}{3}}$$

$$\frac{2}{3} \cdot \frac{1}{2} + \omega_3 = \frac{1}{2} \Rightarrow \boxed{\omega_3 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}}$$

$$\omega_1 = 1 - \frac{2}{3} - \frac{1}{6} = \frac{6-4-1}{6} = \frac{1}{6}$$

$$\int_0^1 f(x)\,dx = \frac{1}{6} f(0) + \frac{4}{6} f\left(\frac{1}{2}\right) + \frac{1}{6} f(1) = \frac{1}{6}\left[f(0) + 4f\left(\frac{1}{2}\right)\right.$$

$$\int_0^1 \frac{\sin x}{\sqrt{x}}\,dx \approx \frac{1}{6}\left[f(0)+4f\left(\tfrac{1}{2}\right)+f(1)\right]=$$

$$=\frac{1}{6}\left[0+4\,\frac{\sin\left(\frac{1}{2}\right)}{\sqrt{\frac{1}{2}}}+\frac{\sin 1}{\sqrt{1}}\right]=$$

$$\lim_{x\to 0}\frac{\sin x}{\sqrt{x}}=\lim_{x\to 0}\frac{\sin x}{x}\,\frac{x}{\sqrt{x}}=0 \quad\Big|\quad =\frac{1}{6}\left[2{,}7120+\dots\right]$$

$$=\underline{\underline{0{,}5923}}$$

$$\int_0^1 \frac{x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\dots}{\sqrt{x}}\,dx=$$

$$=\int_0^1\left[\sqrt{x}-\frac{1}{6}x^{\frac{5}{2}}+\frac{1}{120}x^{\frac{9}{2}}-\frac{1}{5040}x^{\frac{13}{2}}+\dots\right]dx=$$

$$=\frac{2x^{\frac{3}{2}}}{3}\Big|_0^1-\frac{2}{6}\,\frac{x^{\frac{7}{2}}}{7}\Big|_0^1+\frac{2}{120}\,\frac{x^{\frac{11}{2}}}{11}\Big|_0^1-\frac{2}{5040}\,\frac{x^{\frac{15}{2}}}{15}\Big|_0^1-\dots=$$

$$=\frac{2}{3}-\frac{1}{21}+\frac{1}{11\cdot 60}-\frac{1}{2520\cdot 15}+\dots$$

$$=\underline{\underline{0{,}6205}}$$

NUMERIČNE METODE

18. junij 2002

1. Na intervalu $[0, 1]$ leži natanko ena negibna točka funkcije

$$f(x) = x^2 e^{-x} + \frac{1}{2}$$

   (a) Pošči jo z Newtonovo iteracijo, na 4 decimalna mesta natančno.

   (b) Ali je negibna točka privlačna ali odbojna?

2. Dan je sistem enačb.

$$\begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix} x = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

   (a) Ali sistem enačb lahko rešimo iterativno z Jacobijevo iteracijo?

   (b) Poišči rešitev.

3. Reši integral

$$\int_0^{0.1} \frac{e^{-x}}{\sqrt{x}}\, dx$$

na štiri decimalna mesta natančno.

$f(\emptyset) = 1/2$
$= 0{,}5$

$f(1) = e^{-1} + 1/2$
$= 0.86787944$

18.6.2002

Na intervalu $[0,1]$ leži natanko ena negibna točka funkcije
$f(x) = x^2 e^{-x} + 1/2$.

a.) Poišči jo z Newtonovo iteracijo, na 4 dec. mesta natančno
b.) Ali je negibna točka privlačna ali odbojna?

$x = f(x)$ negibna točka

$|f'(x)| < 1$ je točka privlačna

$x = x^2 e^{-x} + 1/2$

$f(x) = x^2 e^{-x} + 1/2 - x = \emptyset$

$f'(x) = 2xe^{-x} + x^2 e^{-x}(-1) - 1$
$= 2xe^{-x} - x^2 e^{-x} - 1$

1. del

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^2 e^{-x_n} + 1/2 - x_n}{2x_n e^{-x_n} - x_n^2 e^{-x_n} - 1}$$

$\approx 0{,}7780|4$

n	x_n
$\emptyset$	0.6
1	0,7810250[illegible]
2	0,7780396[illegible]
3	0,778037[illegible]

$f'(x) = 2 \cdot 0{,}7780 \cdot e^{-0{,}7780} - (0{,}7780)^2 e^{-0{,}7780} - 1$

$= -0{,}56331357$

$|f'(x)| < 1$

Točka je privlačna.

2.) Dan je sistem enačb

$$\begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix} x = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

a.) Ali lahko sistem enačb rešimo iterativno z Jacobijevo iteracijo?

b.) Poišči rešitev.

$$\boxed{\begin{aligned} 2x_1 + x_2 &= 2 \\ 2x_1 + 2x_2 &= 2 \end{aligned}} \qquad \begin{aligned} x_1 &= 1 \\ x_2 &= 0 \end{aligned} \qquad \begin{aligned} x_1 &= \frac{1}{2}(2 - x_2) = 1 - \frac{x_2}{2} \\ x_2 &= \frac{1}{2}(2 - 2x_1) = 1 - x_1 \end{aligned}$$

$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ začetni približek $\vec{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; $\vec{x}_2 = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}$; $\vec{x}_3 = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}$

$\vec{x}_4 = \begin{bmatrix} \frac{3}{4} \\ 0 \end{bmatrix}$; $\vec{x}_5 = \begin{bmatrix} 1 \\ \frac{1}{4} \end{bmatrix}$; $\vec{x}_6 = \begin{bmatrix} \frac{7}{8} \\ 0 \end{bmatrix}$; $\vec{x}_7 = \begin{bmatrix} 1 \\ \frac{1}{8} \end{bmatrix}$; $\vec{x}_8 = \begin{bmatrix} \frac{15}{16} \\ 0 \end{bmatrix}$

Ena gre proti 1 (x_1), x_2 pa proti 0

kar vidimo tudi iz iteracij

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3. Reši integral

$$\int_0^{0.1} \frac{e^{-x}}{\sqrt{x}}\,dx$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-x} = 1 - x + \frac{(-x)^2}{2!} + \frac{(-x)^3}{3!}$$

$$= 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

$$= \int_0^{0.1} \frac{1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots}{\sqrt{x}}\,dx \qquad \frac{x^4}{4!}$$

$$= \int_0^{0.1} x^{-1/2} - x^{1/2} + \frac{1}{2}x^{3/2} - \frac{1}{6}x^{5/2} + \dots \frac{1}{24}x^{7/2}$$

$$= \frac{x^{1/2}}{1/2}\Big|_0^{0.1} - \frac{x^{3/2}}{3/2} + \frac{1}{2}\frac{x^{5/2}}{5/2} - \frac{1}{6}\frac{x^{7/2}}{7/2} + \frac{1}{24}\frac{x^{9/2}}{9/2}$$

$$= 2x^{1/2}\Big|_0^{0.1} - \frac{2x^{3/2}}{3} + \frac{1}{1}\frac{x^{5/2}}{5} - \frac{1}{3}\frac{1\,x^{7/2}}{3\cdot 7} + \frac{1}{12}\frac{x^{9/2}}{9}$$

$$= 0,611903362$$

$$\cong 0.6119$$

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28.6.2004

$$\int_0^1 \frac{f(x)}{\sqrt{x}}\,dx = a_1 f(x_1) + a_2 f(x_2)$$

$$f(x) = 1, x, x^2, x^3$$

$f(x)=1$: $\int_0^1 \frac{dx}{\sqrt{x}} = a_1 \cdot 1 + a_2 \cdot 1 = 2\sqrt{x}\Big|_0^1 = 2$

$f(x)=x$: $\int_0^1 \sqrt{x}\,dx = \cancel{a_1 \cdot 0 + a_2 \cdot 1} =$

$a_1 \cdot x_1 + a_2 x_2 = \frac{2x^{\frac{3}{2}}}{3}\Big|_0^1 = \frac{2}{3}$

$f(x)=x^2$: $\int_0^1 x^{\frac{3}{2}}\,dx = a_1 x_1^2 + a_2 x_2^2 = \frac{2x^{\frac{5}{2}}}{5}\Big|_0^1 = \frac{2}{5}$

$f(x)=x^3$: $\int_0^1 x^{\frac{5}{2}}\,dx = a_1 x_1^3 + a_2 x_2^3 = \frac{2x^{\frac{7}{2}}}{7}\Big|_0^1 = \frac{2}{7}$

✓ $a_1 + a_2 = 2$ $\qquad a_1 = 1+b$

$a_1 x_1 + a_2 x_2 = \frac{2}{3}$ $\qquad a_2 = 1-b$

$a_1 x_1^2 + a_2 x_2^2 = \frac{2}{5}$ $\qquad x_1 = \frac{3}{7}+y$

$a_1 x_1^3 + a_2 x_2^3 = \frac{2}{7}$ $\qquad x_2 = \frac{3}{7}-y$

$$\boxed{1+b+1-b=2}$$

$$(1+b)\left(\frac{3}{7}+y\right) + (1-b)\left(\frac{3}{7}-y\right) = \frac{2}{3}$$

$$\frac{3}{7} + \cancel{\frac{3}{7}b} + \cancel{y} + by + \frac{3}{7} + by = \frac{2}{3}$$

$$2by + \frac{6}{7} = \frac{2}{3} \Rightarrow 2by = \frac{2}{3} - \frac{6}{7} = \frac{14-18}{21} = -\frac{4}{21}$$

$$\boxed{by = -\frac{2}{21}}$$

$1\cdot1 + 2\cdot2 + 1\cdot1 = 6$

$2\cdot1 + 3\cdot2 + 1\cdot1 = 2+6+1 =$

$1\cdot2 + 2\cdot3 + 1\cdot1 =$

$1\cdot2 + 2\cdot3 + 1\cdot1$

2 6 1

$2\cdot2 + 3\cdot3 + 1\cdot1 = 4+9+1$

$1\cdot1 + 2\cdot1 + 1\cdot2$

$1+2+2$

$2\cdot1 + 3\cdot1 + 2\cdot1$

$2+3+2$

NUMERIČNE METODE

6. junij 2002

1. Pokažite, da leži na intervalu [0, 2] natanko en koren enačbe

$$x^2e^{-x} - 1/3 = 0;$$

Koren želimo določiti z metodo bisekcije tako, da bo napaka $\|x - x_n\|$ za gotovo manjša od 10^{-3}. Najmanj koliko korakov iteracije je potrebnih?

2. Izračunajte število občutljivosti matrike A, cond(A) *condition number*.

$$\begin{bmatrix} 1 & 2 \\ 1+10^{-4} & 2 \end{bmatrix}$$

3. Določite uteži w_i kvadraturne formule

$$\int_0^1 \frac{f(x)}{\sqrt{1-x}}\,dx = \sum_{i=0}^{2} w_i f(x_i), \quad x_i = \frac{i}{2}$$

tako, da bo točna za polinome druge stopnje.

Preglej

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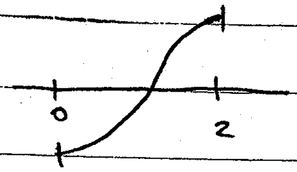
6.6. 2002

① $[0,2]$ $\quad x^2e^{-x} - 1/3 = \phi$



$f(\phi) = -1/3 < \phi$

$f(2) = 4e^{-2} - 1/3 = 0{,}2080\ldots > \phi$

$f'(x) = 2x\cdot e^{-x} + x^2e^{-x}(-1)$

$= 2xe^{-x} - x^2e^{-x}$ vedno pozitivno, zato samo ena ničla. (ni prevoja)

$v\ f'(2) = \phi$

* $\frac{1}{2^n} < \frac{1}{10^{+3}}$

$2^n > 10^{+3}$

$n \cdot \ln 2 > 3 \cdot \ln 10$

$n > \frac{3\cdot \ln 10}{\ln 2}$

$n > 9.9$

$n > 10$

$\|A\infty\|$ = izračunaš tako, da seštej̃eš ~~komponente, ki so vrsti po vrstici~~ v vrstici. Tista vrstica ki ima največjo abs. vrednost seštevka je $\|A\infty\|$

$\|A\infty\|$ ↓ 2. vrstica $= 3{,}0001$

② $\text{cond } A = \|A\|\|A^{-1}\|$

$\text{cond } A = 3{,}0001 \times 0{,}5 = 1{,}50005$

$A = \begin{bmatrix} 1 & 2 \\ 1+10^{-4} & 2 \end{bmatrix}$ $\quad \det A = -2\cdot 10^{-4}$

~~$1\cdot 10^{-4}$ / $2\cdot 10^{-4}$~~

~~$\begin{bmatrix} 2 & -(1+10^{-4}) \\ -2 & 1 \end{bmatrix}$~~

$A^T = \begin{bmatrix} 1 & 1+10^{-4} \\ 2 & 2 \end{bmatrix}$

↙ transponiraš

/ matrika kofaktorjev transponirane matrike

$A^{-1} = \frac{1}{-2\cdot 10^{-4}} \cdot \begin{bmatrix} 2 & -2 \\ -(1+10^{-4}) & 1 \end{bmatrix}$ $\quad \|A^{-1}\|_\infty = 0.5$

$A^{-1} = \begin{bmatrix} -1\cdot 10^{+4} & 1\cdot 10^{4} \\ \frac{(1+10^{-4})}{2\cdot 10^{-4}} & -\frac{1}{2}\cdot 10^{4} \end{bmatrix}$

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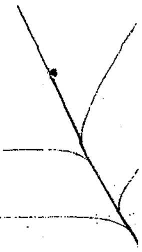


$$\int_0^1 \frac{f(x)}{\sqrt{1-x}}\,dx = \sum_{i=0}^{2} w_i f(x_i), \quad x_i = \frac{i}{2}$$

$$= w_1 f(0) + w_2 f(\tfrac{1}{2}) + w_3 f(1)$$

$1, x, x^2$

1) $f(x) = 1;$ $\quad \int_0^1 \frac{1}{\sqrt{1-x}}\,dx = w_1 + w_2 + w_3$

$1 - x = u \qquad -x = u - 1 \qquad x^2 = (1-u)^2$

$\qquad\qquad\qquad x = 1 - u$

$-dx = du$

$$-\int_1^0 \frac{1}{\sqrt{u}}\,du = \int_0^1 \frac{du}{\sqrt{u}}$$

$$\int_0^1 \frac{1\,du}{\sqrt{u}} = w_1 + w_2 + w_3 = \frac{2u^{1/2}}{1} = 2\sqrt{u}\Big|_0^1 = 2$$

$\swarrow u^{-1/2}$

② $f(x) = x$ $\qquad 1 - \frac{1}{2} = u^{1/2}$

$$\int_0^1 \frac{1-u}{\sqrt{u}} = \int_0^1 \frac{1}{\sqrt{u}} - \int_0^1 \frac{u}{\sqrt{u}}$$

$$= \underbrace{2\sqrt{u}}_{2} - \frac{2u^{3/2}}{3} = 2 - 2/3 = \frac{4}{3}$$

$$\int_0^1 \frac{1-u}{\sqrt{u}} = \frac{4}{3} = w_1 \cdot 0 + w_2 \cdot \frac{1}{2} + w_3 \cdot 1$$

$1^2 - 2u + u^2$

③ $\int_0^1 \frac{(1-u)^2}{\sqrt{u}}\,du \qquad 2 - 1/2 = 3/2$

$$= \int_0^1 \frac{1}{\sqrt{u}} - 2\int_0^1 \frac{u}{\sqrt{u}} + \int_0^1 \frac{u^2}{\sqrt{u}} = \qquad 2 - 4/3 + 2/5 =$$

$$= 2 - 2 \cdot 2/3 + \underbrace{\frac{2u^{5/2}}{5}}_{\downarrow\ 2/5} \qquad \frac{30 - 20 + 6}{15} = \frac{16}{15}$$

$$w_1 + w_2 + w_3 = 2$$

$$\frac{2}{4}\frac{w_2}{2} + w_3 = \frac{4}{3}$$

$$\frac{w_2}{4} + w_3 = \frac{16}{15}$$

$$\frac{w_2}{4} = \frac{4}{3} - \frac{16}{15} \qquad \frac{w_2}{4} = \frac{4}{5}$$

$$\frac{20-16}{15} = \frac{4}{5} \qquad w_2 = \frac{16}{5}$$

$$w_3 = \frac{4}{3} - \frac{16}{5 \cdot 2}$$

$$= \frac{4}{3} - \frac{8}{5} = \frac{20-24}{15} = -\frac{4}{15} \quad (w_3)$$

$$w_1 = 2 - w_3 - w_2 = 2 + \frac{4}{15} - \frac{16}{5} = \frac{30+4-16}{15} = \frac{18}{15} \; \frac{6}{5}$$

$$\int_0^1 \frac{f(x)}{\sqrt{1-x}}\,dx = \frac{6}{5}f(0) + \frac{16}{5}f\left(\frac{1}{2}\right) - \frac{4}{15}f(1)$$

Izpit iz numeričnih metod
19. januarja 2001

1. Določite interval, na katerem konvergira iteracija

$$x_{n+1} = \sqrt{\frac{e^{x_n}}{3}}$$

in določite negibno točko na dve decimalni mesti natančno.

2. Razcepite matriko

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

matrika $\begin{bmatrix} a & \emptyset \\ c & b \end{bmatrix}$

po Choleskem.

$A = L \cdot L^T$

3. Izračunajte integral

$$\int_{-1}^{2} x^3 e^x \, dx$$

s pomočjo Simpsonove formule ($n = 4$).

19. januar 2001

1. $x_{n+1} = \sqrt{\frac{e^{x_n}}{3}}$ $g(x) = \sqrt{\frac{e^x}{3}}$

$$g'(x) = \frac{1}{2}\left(\frac{e^x}{3}\right)^{-1/2} \cdot e^x$$

$$= \frac{1}{2} \frac{e^x}{\sqrt{\frac{e^x}{3}}}$$

$$= \frac{1}{2} \frac{e^{\frac{x}{2}}}{\sqrt{3}}$$

$= \frac{x}{2}$

$-\frac{x}{2} + x = \frac{x}{2}$

Da to konvergira, more bit

$$|g'(x)| < 1$$

$$\frac{1}{2} \frac{e^{\frac{x}{2}}}{\sqrt{3}} < 1$$

$$e^{\frac{x}{2}} < 2\sqrt{3}$$

$$\frac{x}{2} < \ln(2\sqrt{3})$$

$$x < 2 \cdot \ln(2\sqrt{3})$$

$$x < 2.4849$$

$x = g(x)$ Negibna točka

začetni približek:

$x_0 = 0$ Rešitev: $x = 0.9100$

60

2. Razcepi matriko po Choleskem

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$A = L \cdot L^T$$

$$\begin{bmatrix} a & \emptyset \\ c & b \end{bmatrix} \begin{bmatrix} a & c \\ \emptyset & b \end{bmatrix} = \begin{bmatrix} a^2 ; & ac \\ ac ; & c^2+b^2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$a^2 = 2$

$a = \pm\sqrt{2}$

$a \cdot c = 1$

$c = \frac{1}{a}$

$c = \frac{1}{\pm\sqrt{2}}$

$c^2 + b^2 = 3$

$b^2 = 3 - \frac{1}{2} = \frac{5}{2}$

$b = \pm\sqrt{\frac{5}{2}}$

$$L = \begin{bmatrix} \sqrt{2}, & \emptyset \\ \frac{1}{\sqrt{2}}, & \sqrt{\frac{5}{2}} \end{bmatrix} = \begin{bmatrix} 1.4142; & \emptyset \\ 0.7071; & 1.5811 \end{bmatrix}$$

Rešitev je samo L!

③ Izračunajte integral $\int_{-1}^{2} x^3 e^x \, dx$ s pomočjo simpsonove formule. ($n=4$)

→ interval razdelimo na 4 dele

$h = \frac{2-(-1)}{4}$

$= \frac{3}{4}$

$-1 \quad -1/4 \quad 1/2 \quad 5/4 \quad 2$

$-1 + 3/4 = -1/4$

$-1/4 + 3/4 = 2/4 = 1/2$

$1/2 + 3/4 = 5/4$

$5/4 + 3/4 = 8/4 = 2$

	x	y
y_0	-1	-0.367879441
y_1	$-1/4$	-0.012168762
y_2	$1/2$	0.206090158
y_3	$5/4$	6.817076089
y_4	2	59.11244879

Simpsonova formula

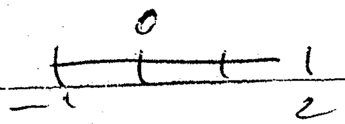
$P = \frac{h}{3}\left[(y_0 + y_{2n}) + 4(y_1 + y_3 + \dots + y_{2n-1}) + 2(y_2 + y_4 + \dots + y_{2n-2})\right]$

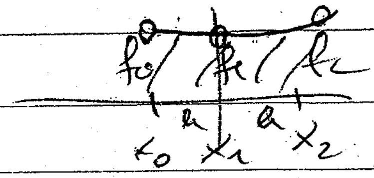
$P = \frac{3}{4} \cdot \frac{1}{3}\left[(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2)\right]$

$=$

$= 21.59409474$

19.1.07





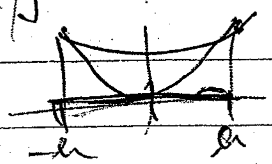
$\int$

$h = \frac{3}{4} = \frac{2-(-1)}{4}$

$$P = \frac{h}{3}\left(y_0 + 4y_1 + y_2\right)$$

$x_0 = -1, -\frac{1}{4}, \frac{2}{4}, \frac{5}{4}, \frac{8}{4}$

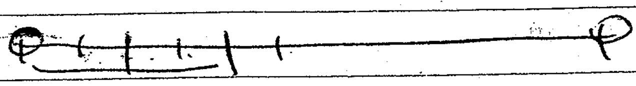
$\int \quad y = x^2$

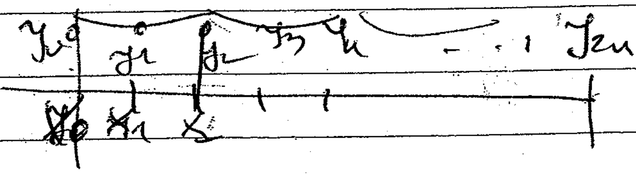


x	y
-1	y_0
$-\frac{1}{4}$	y_1
$\frac{1}{2}$	y_2
$\frac{5}{4}$	y_3
2	y_4

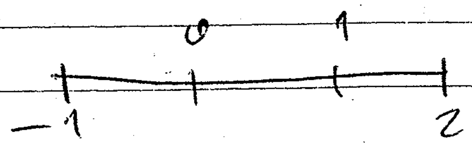
$$\int_{-h}^{h} x^2\,dx = 2\int_0^h x^2\,dx = 2\frac{h^3}{3}$$

$$P = \frac{h}{3}\left(h^2 + h^2\right) = \frac{2h^3}{3}$$





$P = \frac{3}{4 \cdot 3}\left[(y_0 + \right.$



$$P = \frac{h}{3}\left[(y_0 + 4y_1 + y_2) + (y_2 + 4y_3 + y_4) + \ldots + (y_{2n-2} + 4y_{2n-1} + y_{2n})\right]$$

$h = \frac{3}{4}$

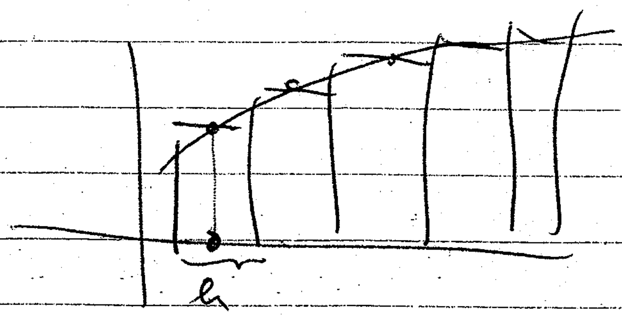
$h = \frac{b-a}{2n}$

$$= \frac{h}{3}\left[(y_0 + y_{2n}) + 4(y_1 + y_3 + \ldots + y_{2n-1}) + 2(y_2 + y_4 + \ldots + y_{2n-2})\right]$$

x	
-1	y_0
$-\frac{1}{4}$	y_1
$\frac{2}{4}$	y_2
$\frac{5}{4}$	y_3
2	y_4

$$P = \frac{3}{4} \cdot \frac{1}{3}\left[(y_0 + y_4) + 4(y_1 + y_3) + 2y_2\right]$$

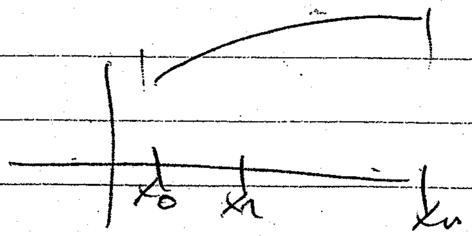
PRAVOKOTNIŠKA

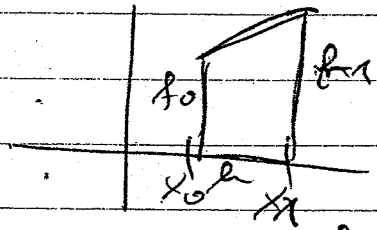


$$h = \frac{b-a}{n}$$

$$P = h\left[f_{1/2} + f_{3/2} \dots\right]$$

TRAPEZNA





$$P = h\,\frac{f_0+f_1}{2}$$

$$P = \frac{h}{2}\left[(f_0+f_1)+(f_1+f_2)+\dots+(f_{n-1}+f_n)\right]$$

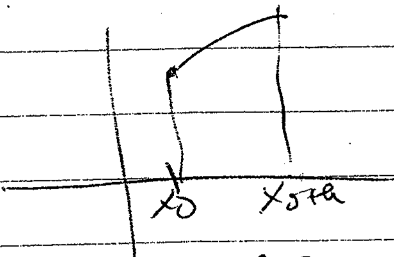
$$= \frac{h}{2}\left[(f_0+f_n)+2(f_1+f_2+\dots+f_{n-1})\right]$$

$$h\left[\frac{f_0+f_n}{2}+(f_1+f_2+\dots+f_{n-1})\right]$$

$$y(x_0+h) = y(x_0) + h\,y'(x_0) + \frac{h^2}{2!}y''(x_0) + \frac{h^3}{3!}y'''(x_0) + \dots$$

$$y' = f(x,y) \quad ; \quad y(x_0) = y_0$$

$$f(x,y) = y - \sin x + 1$$



$$y'(x) = f(x, y(x))$$

$$y'(x_0) = f(x_0, y_0)$$

$$y''(x) = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\,y'$$

$$y' = y - \sin x + 1$$

$y(x_0) = y_0$ ... podano

$$y'(x_0) = y_0 - \sin x_0 + 1$$

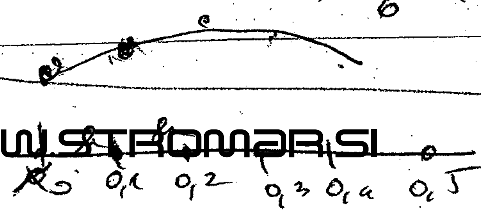
$$y'' = y' - \cos x = y - \sin x + 1 - \cos x$$

$$y''(x_0) = y_0 - \sin x_0 - \cos x_0 + 1$$

$$y''' = y' - \cos x + \sin x = y - \sin x + 1 - \cos x + \sin x$$

$$y'''(x_0) = y_0 - \cos x_0 + 1$$

$$y(x_0+h) = y_0 + h(y_0 - \sin x_0 + 1) + \frac{h^2}{2}(y_0 - \sin x_0 - \cos x_0 + 1) + \frac{h^3}{6}(y_0 - \cos x_0 + 1)$$



x	Δy
x_0	
x_0+h	
x_0+2h	

Ime, priimek ....

Naloga	točke
1.	
2.	
3.	
Skupaj	

NUMERIČNE METODE

19. junij 2001

1. Z uporabo Newtonove metode poiščite iteracijsko formulo za izračun $\sqrt[3]{2}$. (Rešite enačbo $x^3 - 2 = 0$). Ali iteracija vedno konvergira?

2. Določite a_0, a_1, k tako, da bo formula

UBISTVU JE TO IZPELJAVA

$$\int_{x_0}^{x_1} f(x)dx = a_0 f(x_0) + a_1 f(x_1) + k f''(\xi)$$

točna za $f(x) = 1, x$. Konstanto k določite s funkcijo $f(x) = x^2$.

3. S pomočjo Taylorjeve vrste poiščite formulo tretjega reda za reševanje enačbe

$$y' = y - \sin x + 1$$

s korakom h !

19. junij 2001

Ali je to že to?

1. $$x_{n+1} = \frac{x_n}{1} - \frac{x_n^3-2}{3x_n^2} = \frac{3x_n^3 - x_n^3+2}{3x_n^2} = \frac{2x_n ...}{3x ...}$$

$$x_{n+1} = g(x_n)$$

$$g(x) = \frac{2x^3+2}{3x^2} = \frac{2}{3}x + \frac{2}{3x^2}$$

$$g'(x) = \frac{2}{3} + \frac{2}{3}(-2)\frac{1}{x^3} = \frac{2}{3} - \frac{4}{3}\frac{1}{x^3}$$

$$|g'(x)| < 1$$

$$\left|\frac{2}{3} - \frac{4}{3}\frac{1}{x^3}\right| < 1$$

$$-1 < \left(\frac{2}{3} - \frac{4}{3x^3}\right) < 1$$

$$-1 - 2/3 < -\frac{4}{3x^3} < 1 - 2/3$$

$$-5/3 < -\frac{4}{3}x^3 < 1/3$$

$$-5 < \frac{4}{x^3} < 1$$

$$5 > \frac{4}{x^3} > -1$$

$$\frac{5}{4} > \frac{1}{x^3} > -\frac{1}{4}$$

$$\frac{4}{5} < x^3 < -4$$ za take x-e konvergira

$$\sqrt[3]{\frac{4}{5}} < x < \sqrt[3]{-4}$$

~~pogoj konstanta na [a,a]~~

Iščemo samo za > ∅

ker je $\sqrt[3]{2}$ vedno več kot ∅

1.) $\int_{x_0}^{x_1} f(x)\,dx = a_0 f(x_0) + a_1 f(x_1) + k f''(\xi)$

točka za $f(x) = 1, x$ $k \Rightarrow$ $f(x) = x^2$ $f'(x) = 2x$ $f''(x) = 2$

$= a_0 f(x_0) + a_1 f(x_1) + k \cdot 2$

$\int_{x_0}^{x_1} x^2\,dx = a_0 f(x_0) + a_1 f(x_1) + 2k$

2.) $\int_{x_0}^{x_1} f(x)\,dx = a_0 f(x_0) + a_1 f(x_1) + k f''(\xi)$

$f(x) = 1;$

drugi odvod od 1 je $\emptyset$

$\int_{x_0}^{x_1} dx = a_0 + a_1 + \emptyset = (x_1 - x_0) = h$

$f(x) = x;$

$\int_{x_0}^{x_1} x\,dx = a_0 x_0 + a_1 x_1 = \frac{x^2}{2}\Big|_{x_0}^{x_1} = \frac{x_1^2 - x_0^2}{2}$

$\int_{x_0}^{x_1} x^2\,dx = a_0 x_0^2 + a_1 x_1^2 + k \cdot 2 = \frac{x^3}{3}\Big|_{x_0}^{x_1} = \frac{x_1^3 - x_0^3}{3}$

$a_0 + a_1 = (x_1 - x_0)$

$a_0 x_0 + a_1 x_1 = \frac{x_1^2 - x_0^2}{2}$

$2a_0 x_0 + 2a_1 x_1 = x_1^2 - x_0^2$

$a_0 + a_1 = x_1 - x_0$

$y' = y - \sin x + 1$ s korakom h

Poišči formulo tretjega reda za reševanje enačbe

$$y(x_0+h) = y(x_0) + hy'(x_0) + \frac{h^2}{2!}y''(x_0) + \frac{h^3}{3!}y'''(x_0)$$

(ostale člene naprej odrežemo, ker rabimo samo 3. reda)

$y(x_0) = y_0$ (podano)

$y'(x_0) = y_0 - \sin x_0 + 1$

$y''(x_0) = y' - \cos x_0 = y_0 - \sin x_0 - \cos x_0 + 1$

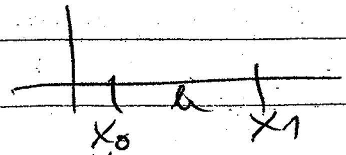
$y'''(x_0) = y'' + \sin x = y_0 - \sin x_0 - \cos x_0 + \sin x_0 + 1$

$= y_0 - \cos x_0 + 1$

$$y(x_0+h) = y_0 + h(y_0 - \sin x_0 + 1) + \frac{h^2}{2}(y_0 - \sin x_0 - \cos x_0 + 1) + \frac{h^3}{6}(y_0 - \cos x_0 + 1)$$

19.6.2001

2.) $$\int_{x_0}^{x_1} f(x)\,dx = a_0 f(x_0) + a_1 f(x_1) + k f''(\xi)$$



$x_1 - x_0 = h$

$f(x) = 1$; $\int_{x_0}^{x_1} 1\,dx = a_0 + a_1 = (x_1 - x_0) = h$

$f(x) = x$: $\int_{x_0}^{x_1} x\,dx = a_0 x_0 + a_1 x_1 = \frac{x^2}{2}\Big|_{x_0}^{x_1} = \frac{x_1^2 - x_0^3}{2}$

$$a_0 + a_1 = h \quad / \cdot x_1$$

$$a_0 x_0 + a_1 (\cancel{x_0 + h}\; x_1) = \frac{h(x_1 + x_0)}{2}$$

$$a_0 x_1 + a_1 x_1 = h x_1$$

$$a_0 x_0 + a_1 x_1 = \frac{h(x_1 + x_0)}{2}$$

$$a_0 h = h x_1 - \frac{h(x_1 + x_0)}{2}$$

$$a_0 = x_1 - \frac{x_1 + x_0}{2} = \frac{x_1 - x_0}{2} = \frac{h}{2} = a_1$$

$$\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_1)\right] + k f''(\xi)$$

$$\int_{x_0}^{x_1} x^2\,dx = \frac{h}{2}\left[x_0^2 + x_1^2\right] + k \cdot 2 = \frac{x^3}{3}\Big|_{x_0}^{x_1} = \frac{x_1^3 - x_0^3}{3} =$$

$$= \frac{(x_1 - x_0)(x_1^2 + x_1 x_0 + x_0^2)}{3}$$

$$2k = \frac{h}{3}(x_1^2 + x_1 x_0 + x_0^2) - \frac{h}{2}(x_0^2 + x_1^2) =$$

$$= \frac{h}{6}\left[2x_1^2 + 2x_1 x_0 + 2x_0^2 - 3x_0^2 - 3x_1^2\right] =$$

$$= \frac{h}{6}\left[-x_1^2 - x_0^2 + 2x_1 x_0\right] = -\frac{h}{6}\left[x_1^2 - 2x_1 x_0 + x_0^2\right] =$$

$$= -\frac{h}{6}(x_1 - x_0)^2 = -\frac{h^3}{6} \Rightarrow \boxed{k = -\frac{h^3}{12}}$$

$$\boxed{-\frac{h^3}{12} f''(\xi)}$$

12. 9. 2003

	x	y	$f[\,,\,]$	$f[\,,\,,\,]$
x_0	8,0	16,6355		
			9,8	
	8,1	17,6155		−33,51
			−0,2530	
	8,3	17,5649		

$$9{,}8 = \frac{17{,}6155 - 16{,}6355}{8{,}1 - 8{,}0}$$

$$f[\,,\,,\,] = \frac{-0{,}2530 - 9{,}8}{8{,}3 - 8{,}0} = -33{,}51$$

$$f(x) = +16{,}6355 + 9{,}8\underbrace{(x-8{,}0)}_{(x-x_0)} - 33{,}51(x-8{,}0)(x-8{,}1)$$

(3.) $$I = \int_1^{\infty} \frac{\sin\left(\frac{1}{x}\right)}{x^{3/2}}\,dx = -\int_1^{0} \frac{\sin u}{\frac{1}{u^{3/2}}}\,\frac{du}{u^2} = \int_0^1 \frac{\sin u}{\sqrt{u}}\,du$$

$\frac{1}{x} = u$; $x = \frac{1}{u}$; $dx = -\frac{du}{u^2}$

$$\int_0^1 \frac{\sin u}{\sqrt{u}}\,du = \sim\sim$$

4.)

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7.6.2001

1.) ~~$(x-1)^2$~~ $f(x) = 1 - 6x + 32x^3$

$f'(x) = -6 + 32 \cdot 3x^2 = 0$

$32 \cdot 3x^2 = 6$

$32 \cdot x^2 = 2$

$x = 0{,}25$

$x^2 = \frac{1}{16}$; $x_{1,2} = \pm \frac{1}{4}$

3) $f(x,d) = x^2 e^{dx}$; $\ln f = \ln x^2 + dx =$

~~$= 2x + dx$~~

$2 \ln x + dx$

x_i	y_i	$\ln y_i$	
1	0,35		
2	0,54		
3	0,45		
4	0,30		
5	0,17		

$$\sum_{i=1}^{N} (2\ln x_i + d x_i - \ln y_i)^2 = E$$

$$\frac{\partial E}{\partial d} = 0$$

$$\sum_{i=1}^{N} 2 (2\ln x_i + d x_i - \ln y_i) x_i = 0$$

$$\sum_{i=1}^{N} (2\ln x_i \cdot x_i - x_i \ln y_i) + d \sum_{i=1}^{n} x_i^2 = 0$$

$$d = \frac{\sum_{i=1}^{N} (x_i \ln y_i - 2 x_i \ln x_i)}{\sum_{i=1}^{N} x_i^2} =$$

$$= \frac{\sum_{i=1}^{n} x_i \ln \frac{y_i}{x_i^2}}{\sum_{i=1}^{N} x_i^2}$$

$y = x^2 e^{dx}$

$\ln y = \ln x^2 + dx$

$\ln \frac{y}{x^2} = dx = Y$

~~aaaa~~, ~~bbbbb~~ ...

0, 00 000 0 x

$$y = 1 - e^{\lambda x}$$

$$e^{+\lambda x} = 1 - y$$

$$\lambda x = \ln(1-y) = Y$$

$$\sum_{i=1}^{N} y_i x_i = k \sum_{i=1}^{N} x_i^2 + m \sum_{i=1}^{N} x_i$$

$$\sum_{i=1}^{N} y_i = k \sum_{i=1}^{N} x_i + m \sum_{i=1}^{N} 1$$

$$\begin{array}{|l|} 57{,}24 = k \cdot 385 + m \cdot 55 \\ 81 = k \cdot 55 + m \cdot 10 \end{array} \quad : 55$$

$$1{,}040727273 = k7 + m \quad | \cdot 10$$

$$81 = k55 + 10m$$

$$10{,}40727273 = 70k + 10m$$

$$81 = 55k + 10m$$

$$70{,}59272727 = -15k$$

$$\boxed{k = -4{,}706181818}$$

$$\boxed{m = 33{,}9675}$$

Numerične metode
1. september 2000

1. Pokažite, da leži vsaj en koren enačbe

$$x^3 + 4x^2 - 10 = 0$$

na intervalu $[1, 2]$. Preverite, da za vsak začetni približek s tega intervala, iteracija

$$x_{n+1} = \sqrt{\frac{10}{4 + x_n}},$$

konvergira k edinemu korenu enačbe na tem intervalu. Izberite začetni približek $x_0 = 1.5$ in izračunajte koren na dve decimalni mesti natančno.

2. Matriko

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

zapišite kot produkt spodnjetrikotne matrike L z enkami na diagonali in zgornje trikotne matrike U.

3. Izberite metodo, izračunajte integral

$$\int_0^1 e^{\sqrt{x}}\, dx$$

in ocenite napako.

77

1.9. 2000

Izberite metodo, izračunajte integral

$$\int_0^1 e^{\sqrt{x}}\,dx = 2\int_0^1 e^{u}\,du$$

$\sqrt{x} = u \qquad u^2 = x$

$dx = 2du$

$$= 2\cdot e^{u}\Big|_0^1 = 2\cdot e^1 - 2\cdot e^0 =$$

$$= 2e^1 - 2$$

$$= 3.436563657$$

rešitev

Pokažite, da leži vsaj en koren enačbe $x^3 + 4x^2 - 10 = 0$ na intervalu $[1,2]$. Preverite, da za vsak začetni približek s tega intervala, iteracija konvergira. $x_0 = 1{,}5$ in izračunaj na 2 mesti.

$$f(1) = 1^3 + 4 \cdot 1 - 10 = 1 + 4 - 10 = -5$$

$$f(2) = 2^3 + 4 \cdot 4 - 10 = 8 + 16 - 10 = 14$$

Vsaj eden leži na tem intervalu

$$x_{n+1} = \sqrt{\frac{10}{4+x_n}}$$

$$g'(x) = \frac{1}{2}\left(\frac{10}{4+x_n}\right)^{-1/2} \cdot \frac{(-1)\,10}{(4+x)^2}$$

$$= -5\,\frac{1}{\sqrt{\frac{10}{4+x_n}}} \cdot \frac{1}{(4+x)^2}$$

$$= -5\,\frac{\sqrt{4+x_n}}{\sqrt{10}} \cdot \frac{1}{(4+x)^2}$$

$$= \frac{-5}{\sqrt{10}}\,\frac{1}{(4+x)^{3/2}}$$

$$|g'(x)| = \frac{5}{\sqrt{10}}\,\frac{1}{(4+x)^{3/2}} < 1 \quad \text{konvergira}$$

(če damo noter mejni vrednosti vidimo, da sta obe manjše k ena)

$$x_{n+1} = \sqrt{\frac{10}{4+x_n}} \qquad x_0 = 1{,}5$$

rešitev je $x = 1{,}365$ oz $\approx 1{,}37$

80

$x^{1-1/2}$

$2^{-1/2} =$

$3^{-1/2} = 2^{1/2}$

$4 - 1/2 = 3^{1/2} =$

25. july → 31. july

Matriko

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

zapiši kot produkt sp. trikotne matrike L z enkami na diagonali in zg. trikotne matrike U.

$$A = L \cdot U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_2 & 1 & 0 & 0 \\ 0 & l_3 & 1 & 0 \\ 0 & 0 & l_4 & 1 \end{bmatrix} \begin{bmatrix} a_1 & b_1 & 0 & 0 \\ 0 & a_2 & b_2 & 0 \\ 0 & 0 & a_3 & b_3 \\ 0 & 0 & 0 & a_4 \end{bmatrix}$$

sp. trikotna — zg. trikotna

$$= \begin{bmatrix} a_1, & b_1, & 0, & 0 \\ l_2 a_1, & l_2 b_1 + a_2, & b_2, & 0 \\ 0, & l_3 a_2, & l_3 b_2 + a_3, & b_3 \\ 0, & 0, & l_4 a_3, & l_4 b_3 + a_4 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$a_1 = 2$ $\quad b_1 = -1$

$l_2 a_1 = -1$

$l_2 \cdot 2 = -1$

$\underline{\underline{l_2 = -1/2}}$

$l_2 b_1 + a_2 =$

$= -1/2 \cdot (-1) + a_2 = 2$

$= \frac{1}{2} + a_2 = 2$

$\underline{\underline{a_2 = \frac{3}{2}}}$

$\underline{\underline{b_2 = -1}}$

$l_3 \cdot a_2 = -1$

$l_3 \cdot \frac{3}{2} = -1$

$l_3 = \frac{-1 \cdot 2}{3}$

$\underline{\underline{l_3 = -2/3}}$

$l_3 \cdot b_2 + a_3 = 2$

$-\frac{2}{3} \cdot (-1) + a_3 = 2$

$\frac{2}{3} + a_3 = 2$

$a_3 = 2 - 2/3$

$\underline{\underline{a_3 = 4/3}}$

$\underline{\underline{b_3 = -1}}$

$l_4 \cdot a_3 = -1$

$l_4 \cdot \frac{4}{3} = -1$

$l_4 = \frac{-1 \cdot 3}{4}$

$\underline{\underline{l_4 = -3/4}}$

$l_4 \cdot b_3 + a_4 = 2$

$-\frac{3}{4} \cdot (-1) + a_4 = 2$

$\frac{3}{4} + a_4 = 2$

$a_4 = 2 - 3/4$

$a_4 = \frac{5}{\ }$

$$K = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 0 & -2/3 & 1 & 0 \\ 0 & 0 & -3/4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & 0 & 5/4 \end{bmatrix}$$

Ime, priimek ....

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NUMERIČNE METODE

20. januar 2000

1. Pokažite, da leži vsaj en koren enačbe

$$x^3 + 4x^2 - 10 = 0$$

na intervalu $[1, 2]$. Koren določite z iteracijo

$$x = \sqrt{\frac{10}{4+x}}$$

, če ta iteracija konvergira.

2. Matriko

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

zapišite kot produkt spodnje trikotne matrike L in zgornje trikotne matrike U.

3. Kako bi numerično izračunali integral

$$\int_0^1 \frac{e^x}{\sqrt{x}}\, dx$$

?

20.1. 2000

1.

$x^3 + 4x^2 - 10 = 0$

$f(1) = 1^3 + 4 \cdot 1 - 10 = 1 + 4 - 10 = -5$

$f(2) = 2^3 + 4 \cdot 2^2 - 10 = 8 + 16 - 10 = 14$

$$\begin{array}{cc} 1 & 2 \\ <0 & >0 \end{array}$$

(vsaj en koren leži na tem intervalu)

$$x = \sqrt{\frac{10}{4+x}}$$

$$x_{n+1} = x_n - \frac{x_n^3 + 4x_n^2 - 10}{3x_n^2 + 4 \cdot 2 x_n}$$

$$= x_n - \frac{x_n^3 + 4x_n^2 - 10}{3x_n^2 + 8x_n}$$

približek je $x_0 = 0.5$

2.36842

1.6494

1.3657

1.36523 rešitev

$$g(x) = \sqrt{\frac{10}{4+x}} \; ; \; g'(x) = \frac{10}{2\sqrt{\frac{10}{4+x}}} \frac{(-1)}{(4+x)^2} =$$

$$= \frac{-5}{\sqrt{10}} \frac{1}{(4+x)^{3/2}}$$

$$|g'(x)| = + \frac{5}{\sqrt{10}} \frac{1}{(4+x)^{3/2}} \quad [0{,}44442,$$

$$|g'(x)| < 1 \qquad x_n = 0{,}5$$

$$x_{n+1} = \sqrt{\frac{10}{4+x_n}}$$

Pride ista rešitev k gornji

(2) 20.1.2000

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

izberemo si tako, da ima diagonala enke!

$$L \cdot U = \begin{bmatrix} d_1 & 0 & 0 & 0 \\ l_2 & d_2 & 0 & 0 \\ 0 & l_3 & d_3 & 0 \\ 0 & 0 & l_4 & d_4 \end{bmatrix} \begin{bmatrix} a_1 & b_1 & 0 & 0 \\ 0 & a_2 & b_2 & 0 \\ 0 & 0 & a_3 & b_3 \\ 0 & 0 & 0 & a_4 \end{bmatrix}$$

$$= \begin{bmatrix} d_1a_1 & d_1b_1 & 0 & 0 \\ l_2a_1 & l_2b_1+d_2a_2 & d_2b_2 & 0 \\ 0 & l_3a_2 & l_3b_2+d_3a_3 & d_3b_3 \\ 0 & 0 & l_4a_3 & l_4b_3+d_4a_4 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$d_1a_1 = 2$ $\quad d_1b_1 = -1$ $\quad l_2a_1 = -1$

$(-1)(-\frac{1}{2})$

$l_2b_1 + d_2a_2 = 2$ (1,5) $\quad d_2b_2 = -1$

$l_3a_2 = -1$ $\quad l_3b_2 + d_3a_3 = 2$ $\quad d_3b_3 = -1$

$-1\cdot(-\frac{1}{2})+$ $\quad l_4a_3 = -1$ $\quad l_4b_3 + d_4a_4 = 2$

$d_2a_2 = 2$

$d_1 = \frac{2}{a_1}$ $\quad \frac{2}{a_1}b_1 = -1$

$2 - \frac{1}{2} = d_2a_2$ $\quad \frac{b_1}{a_1} = -\frac{1}{2}$ $\quad l_2 = -\frac{1}{a_1}$

$a_1d_1 = 2$

$\frac{3}{2} = d_2a_2$

$a_1 = \frac{2}{d_1}$ $\quad \frac{2}{d_1}\cdot l_2 = -1$ $\quad \boxed{l_2 = -1}$

$d_1 \cdot 1 = 2$ $\quad a_1 \cdot \frac{-1}{a_1} = -1$

$\boxed{d_1 = 2}$ $\quad 2\cdot b_1 = -1$ $\quad \boxed{a_1 = 1}$

$\boxed{b_1 = -\frac{1}{2}}$

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$\frac{3}{2} = d_2 a_2$

$\frac{3}{2a_2} = d_2$

$\frac{3}{a_2} = 2d_2$

$\frac{1}{a_2} = \frac{2d_2}{3}$

$a_2 = \frac{3}{2d_2}$

$a_2 = \frac{3 \cdot b_2}{2 \cdot -1}$

$a_2 = -\frac{3b_2}{2}$

$\frac{3}{2a_2} \cdot b_2 = -1$

$\frac{3b_2}{2a_2} = -1$

$\frac{3b_2 \cdot 2d_2}{2 \cdot 3} = -1$

$d_2 = -\frac{1}{b_2}$

$a_2 = -\frac{1}{l_3}$

$d_2 = -\frac{1}{b_2}$

$1{,}5 \cdot b_2 = -1$

$b_2 = \frac{-1}{1{,}5}$

$b_2 = -\frac{2}{3}$

$\frac{1}{2} + d_2 a_2 = 2$

$d_2 a_2 = 2 - \frac{1}{2}$

$d_2 a_2 = \frac{3}{2}$

$d_2 = 1{,}5$

$d_2 = \frac{3}{2a_2}$

$-\frac{1}{a_2} \cdot a_2 = -1$

$d_2 a_2 = 1{,}5$

$-\frac{a_2}{b_2} = 1{,}5$

$\frac{b_2}{a_2} = -1{,}5$

$l_3 a_2 = -1$

$a_2 = 1$

$l_3 = \frac{-1}{a_2}$

$l_3 = -1$

$^2/_3$

$d_3 b_3 = -1$ $\quad -1\cdot(-\frac{2}{3}) + d_3 a_3 = 2$

$d_3 = \frac{-1}{b_3}$ $\quad d_3 a_3 = 2 - ^2/_3$

$d_3 a_3 = \frac{4}{3}$

$-\frac{a_3}{b_3} = \frac{4}{3}$

$l_4 \cdot a_3 = -1$ $\quad l_4 = -\frac{1}{a_3}$ $\quad \frac{a_3}{b_3} = -\frac{4}{3}$

$a_3 = -\frac{1}{l_4}$

$\frac{b_3}{a_3} = -\frac{3}{4}$

$-\frac{1}{a_3} \cdot a_3 = -1$

$a_3 = 1$

$l_4 = -1$

$4 b_3 = -3 a_3$

$4 b_3 = -3$

$b_3 = -^3/_4$

$d_3 = \frac{-1 \cdot 4}{-3}$

$= \frac{4}{3}$

$l_4 b_3 + d_4 a_4 = 2$

$-1\cdot(-^3/_4) + d_4 a_4 = 2$

$\frac{3}{4} + d_4 a_4 = 2$

$d_4 a_4 = 2 - ^3/_4$

$d_4 a_4 = \frac{5}{4}$ $\quad a_4 = 1$

$d_4 = ^5/_4$

$$= \begin{bmatrix} 2 & 0 & 0 & 0 \\ -1 & 3/2 & 0 & 0 \\ 0 & -1 & 4/3 & 0 \\ 0 & 0 & -1 & 5/4 \end{bmatrix} \begin{bmatrix} 1 & -1/2 & 0 & 0 \\ 0 & 1 & -2/3 & 0 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ime, priimek ....

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1.	
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NUMERIČNE METODE

3. september 1998

1. Pokažite, da leži vsaj en koren enačbe

$$x^3 - x - 1 = 0$$

na intervalu $[1, 2]$. Koren želimo določiti z metodo bisekcije tako, da bo napaka manjša od 10^{-3}. Koliko korakov iteracije je potrebnih?

2. Izpeljite formule za LU razcep tridiagonalne matrike reda $n \times n$

teorija

$$A = \begin{bmatrix} b_1 & c_1 & 0 & \cdots & 0 \\ a_2 & b_2 & c_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & b_n \end{bmatrix}$$

3. Določite uteži w_i kvadraturne formule

$$\int_0^1 \frac{f(x)}{\sqrt{x}}\, dx = \sum_{i=0}^{2} w_i f(x_i), \quad x_i = i\frac{1}{2}$$

tako, da bo točna za polinome druge stopnje.

$\frac{a+b}{2}$

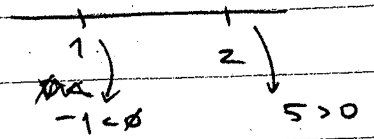
a	f(a)	b	f(b)	c	f(c)
1	−1	2	5	1.5	0.875
1	−1	1,5	0,875	1,25	−0,296875
1,25	−0,296875	1,5	0,875	1,375	0,224609375
1,25	−0,296875	1,375	0,224609375	1.3125	−0,051513671
1.3125	−0,051513671	1.375	0,224609375	1.34375	0,082611083
1.3125	−0,05151367	1.34375	0,082611083	1.328125	0,014575958
1.3125	−0,05151367	1.328125	0.014575958	1.3203125	−0,018710613
1.3203125	−0,018710613	1.328125	0,014575958	1.32421875	$-2,12794543 \cdot 10^{-3}$
1.32421875	$-2,12794543 \times 10^{-3}$	1.328125	0,014575958	1.326171875	$6.20882958 \cdot 10^{-3}$
1.32421875	−ıı−	1.326171875	$6.20882958 \times 10^{-3}$	1.3251953125	$2.03665279 \cdot 10^{-3}$
1.32421875	−ıı−	1.3251953125	$2.03665279 \times 10^{-3}$	1.324707032	$-4.659169 \cdot 10^{-5}$

1.324

3. 9. 1998

$x^3 - x - 1 = 0$ $[1,2]$

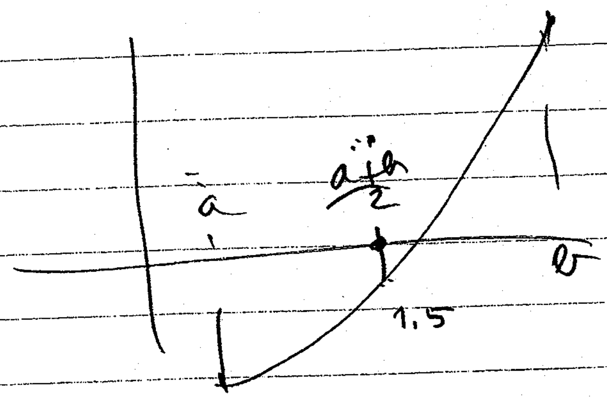
koren nekje med 1 in 2



$f(1)$ $1 - 1 - 1 = -1$

$f(2)$ $2^3 - 2 - 1 = 8 - 3 = 5$

Bisekcija



$a - b$

$\frac{a-b}{2}$

$\frac{a-b}{2^n}$

$\frac{1}{2^n} < \frac{1}{10^3}$

$2^n > 10^3$

$n \cdot \ln 2 > 3 \ln 10$

a	b

$c = \frac{a+b}{2}$

a	f(a)	b	f(b)	c	f(c)

$n > \frac{3 \ln 10}{\ln 2}$

$n \geq 10$

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$$\int_0^1 \frac{f(x)}{\sqrt{x}}\,dx = \sum_{i=\emptyset}^{2} w_i f(x_i); \quad x_i = i\frac{1}{2}$$

$i=\emptyset \quad x_1=\emptyset$

$i=1 \quad x_i = 1/2$

$i=2 \quad x_1 = 1$

$$= w_1 f(\emptyset) + w_2 f(\tfrac{1}{2}) + w_3 f(1)$$

$1, x, x^2$

$f(x) = 1 \qquad \int_0^1 \frac{1}{\sqrt{x}}\,dx = w_1 + w_2 + w_3 = 2\sqrt{x}\Big|_0^1 = \underline{\underline{2}}$

$x^{-1/2+1} = \frac{2x^{\frac{1}{2}}}{1} = 2\sqrt{x}$

$f(x) = x \qquad \int_0^1 \frac{x}{\sqrt{x}}\,dx = w_1\cdot\emptyset + w_2\cdot\frac{1}{2} + w_3\cdot 1 = \frac{2x^{3/2}}{3}\Big|_0^1 = \underline{\underline{\frac{2}{3}}}$

$x^{1+1/2}$

$\underline{\underline{x^{1-1/2}}} \quad x^{1/2}$

$f(x) = x^2 \qquad \int_0^1 \frac{x^2}{\sqrt{x}}\,dx = w_1\cdot\emptyset + \frac{w_2}{4} + w_3 = \frac{2x^{5/2}}{5} = \frac{2}{5}$

$2-1/2 = 1\frac{1}{2} = 3/2$

$w_1 + w_2 + w_3 = 2$

$\frac{2}{4}\; w_2\frac{1}{2} + w_3 = \frac{2}{3}$

$w_2\frac{1}{4} + w_3 = \frac{2}{5}\Big] -$

$\frac{w_2}{4} = \frac{2}{3} - \frac{2}{5} = \frac{10-6}{15}$

$\frac{w_2}{4} = \frac{4}{15}$

$w_2 = \frac{4\cdot 4}{15} = \underline{\underline{\frac{16}{15}}}$

$\frac{w_2}{4} + w_3 = \frac{2}{5}$

$w_3 = \frac{2}{5} - \frac{16}{15\cdot 4} = \frac{2}{5} - \frac{4}{15} = \frac{6-4}{15} = \underline{\underline{\frac{2}{15}}}$

$w_1 = 2 - \frac{16}{15} - \frac{2}{15} = \frac{12}{15} = \underline{\underline{\frac{4}{5}}}$

$$= \int_0^1 \frac{f(x)}{\sqrt{x}} = \frac{4}{5}f(\emptyset) + \frac{16}{15}f(\tfrac{1}{2}) + \frac{2}{15}f(1)$$

11.6.1998

3) Poiščite interpolacijski polinom, ki aproksimira funkcijo $f(x) = \frac{1}{x}$ v vozelnih točkah $x_0 = 2.0$, $x_1 = 2.5$, $x_2 = 4.0$

Npr. Lagrangeov polinom $(x-x_0), (x-x_1), (x-x_2)$

$$P(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f_2 \quad \text{(Formula)}$$

$x_0 = 2.0$

$x_1 = 2.5$

$x_2 = 4.0$

$$P(x) = \frac{(x-2.5)(x-4)}{(2-2.5)(2-4)} f_0 + \frac{(x-2)(x-4)}{(2.5-2)(2.5-4)} f_1 + \frac{(x-2)(x-2.5)}{(4-2)(4-2.5)} f_2$$

$$P(x) = (x-2.5)(x-4) f_0 + \frac{4(x-2)(x-4)}{-0.75 \; -3} f_1 + \frac{(x-2)(x-2.5)}{3} f_2$$

$$P(x) = (x-2.5)(x-4)\frac{1}{2} - \frac{4(x-2)(x-4)}{3}\frac{1}{2.5} + \frac{(x-2)(x-2.5)}{3}\frac{1}{4}$$

$f = \frac{1}{x}$

$f_0 = \frac{1}{x_0} = \frac{1}{2}$

$f_1 = \frac{1}{x_1} = \frac{1}{2.5}$

$f_2 = \frac{1}{x_2} = \frac{1}{4}$

$$= \frac{1}{2}(x^2 - 4x - 2.5x + 10) - \frac{4}{7.5}\left[x^2 - 2x - 4x + 8\right) + \frac{1}{12}\left(x^2 - 2x - 2.5x + 5\right)$$

Ime, priimek ....MATJAŽ TERČELJ....

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NUMERIČNE METODE

11. junij 1998

1. Koliko računskih operacij je potrebnih za izračun izraza

$$\sum_{i=1}^{n}\sum_{j=1}^{i} a_i b_j$$

?

2. Pokažite, da navadna iteracija $x = g(x)$, kjer je

$$g(x) = \frac{1}{3}(x^2 - 1)$$

konvergira k rešitvi, ki leži na intervalu $[-1, 1]$.

3. Poiščite interpolacijski polinom, ki aproksimira funkcijo $f(x) = \frac{1}{x}$ v vozelnih točkah $x_0 = 2.0$, $x_1 = 2.5$, $x_2 = 4.0$.

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$$\frac{1}{2}(x^2-6.5x+10) - \frac{4}{7.5}(x^2-6x+8) + \frac{1}{12}(x^2-4.5x+5)$$

$$= \frac{x^2}{2} - \frac{6.5}{2}x + 5 - \frac{4}{7.5}x^2 + \frac{4\cdot 6}{7.5}x - \frac{4\cdot 8}{7.5}$$

$$+ \frac{x^2}{12} - \frac{4.5}{12}x + \frac{5}{12}$$

$$= \frac{x^2}{2} - \frac{13}{4}x + 5 - \frac{8}{15}x^2 + \frac{24\cdot 2}{15}x - \frac{32\cdot 2}{15}$$

$$+ \frac{x^2}{12} - \frac{9}{2\cdot 12}x + \frac{5}{12}$$

$$= \frac{90x^2 - 96x^2 + 15x^2}{180} + \frac{-1170x + 1152x - 135x}{360}$$

$$+ \frac{900 - 768 + 75}{180}$$

$$= \frac{9x^2}{180} + \frac{-153x}{360} + \frac{207}{180}$$

$$= \frac{x^2}{20} - \frac{17x}{40} + \frac{23}{20}$$

$$= \frac{1}{20}\left[x^2 - \frac{17}{2}x + 23\right]$$

$$=$$

$$\frac{1}{20}\cdot\frac{x^2}{1} - \frac{17}{20}\cdot\frac{1}{20}$$

(2) $g(x) = \frac{1}{3}(x^2 - 1)$

$g'(x) = \left(\frac{1}{3}x^2 - \frac{1}{3}\right)' = \frac{2}{3}x$

$\left|\frac{2}{3}x\right| < \frac{2}{3} < 1$ ker je $x\,[-1,1]$

zato povsod konvergira

zato je x lahko največ 1

Ime, priimek ....JOŽE.....ANDOLŠEK..........

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NUMERIČNE METODE

22. junij 1998

1. Z uporabo Newtonove metode poiščite iteracijsko formulo za izračun $\sqrt{2}$. (Rešite enačbo $x^2 - 2 = 0$). Ali iteracija vedno konvergira?

2. Določite a_0, a_1, k tako, da bo formula

(izpeljava) rešeno že na enem drugem izpitu

$$\int_{x_0}^{x_1} f(x)dx = a_0 f(x_0) + a_1 f(x_1) + k f''(\xi)$$

točna za $f(x) = 1, x$. Konstanto k določite s funkcijo $f(x) = x^2$.

3. S pomočjo Taylorjeve vrste poiščite formulo tretjega reda za reševanje enačbe

$$y' = y - x^2 + 1$$

s korakom h !

Pregled W

3. S pomočjo T.V. poišči formulo 3. reda za reševanje enačbe s korakom h

$$\boxed{y' = y - x^2 + 1} \qquad y(x_0) = y_0$$

$$y(x_0+h) = y(x_0) + h y'(x_0) + \frac{h^2}{2!} y''(x_0) + \frac{h^3}{3!} y'''(x_0)$$

$$y(x_0+h) = y_0 + h(y_0 - x_0^2 + 1) + \frac{h^2}{2}(y_0 - x^2 + 1 - 2x) + \frac{h^3}{6}(y - x^2 - 2x - 1)$$

$$y'' = y' - 2x = y - x^2 + 1 - 2x$$

~~$y''' = y'' - 2 = y - x^2 + 1 - 2x - 2$~~

~~$= y - x^2 - 2x - 1$~~

$$y''' = y'' - 2 = y - x^2 + 1 - 2x - 2$$

Num. met. 22.9.2004

	x	f(x)
0	1.8	3.12014
1	2.0	4.42569
2	2.2	6.04241
3	2.4	8.03014
4	2.6	10.46675

$h = 0{,}2$

$$\int_{1.8}^{2.6} f(x)\,dx$$

Simpsonova formula

$$P = \frac{0.2}{3}\Big[(3.12014 + 10.46675) + 4(4.42569 + 8.0301 + 2\cdot 6.04241\Big]$$

$$= \underline{\underline{5.033002}}$$

Num. met 17.9.2001

$\det[A - \lambda I]$

3.) Ali je matrika pozitivno definitna?

$$\begin{bmatrix} 2 & -1 & \emptyset \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

matrika je pozitivno definitna, če so vse lastne vrednosti $> \emptyset$ in realne!

šahovnica

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix} \qquad \begin{vmatrix} 2-\lambda & -1 & \emptyset \\ -1 & 2-\lambda & -1 \\ \emptyset & -1 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)\begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ 0 & 2-\lambda \end{vmatrix} + \emptyset \cdot *$$

$$= (2-\lambda)\big[(2-\lambda^2) - 1\big] + \underset{(-2+\lambda)}{-(2-\lambda)}$$

$$= (2-\lambda)\big[(2-\lambda^2) - 1 - 1\big]$$

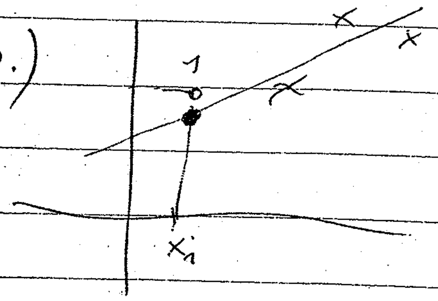
$$= (2-\lambda)(\lambda^2 - 4\lambda + 2)$$

$\underline{\underline{\lambda_1 = 2}}$ Pozitivno def.

$\lambda_{2,3} = 2 \pm \sqrt{2}$

3.8.2001

3.)



$y = kx + n$

~~$y_i = kx_i + n$~~

$$\sum_{i=1}^{N} (y_i - kx_i - n)^2 = m$$

Poiščite premico, ki se najbolje prilega naslednjim merilnim rezultatom

$y = kx + n$

$(y - kx - n)$

x	1	2	3	4	5	6	7	8	9	10
y	1.3	3.5	4.2	5.0	7.0	8.8	10.1	12.5	13.0	15.6

$$\frac{\partial E}{\partial k} = \sum_{i=1}^{N} 2(y_i - kx_i - n)(-x_i) = \emptyset$$

$$\frac{\partial E}{\partial n} = \sum_{i=1}^{N} 2(y_i - kx_i + n)(1) = \emptyset$$

$$-2\sum_{i=1}^{N} y_i x_i - 2\sum_{i=1}^{N} kx_i^2 - 2\sum_{i=1}^{N} x_i n = \emptyset$$

$$\sum_{i=1}^{N} y_i x_i = k\sum_{i=1}^{N} x_i^2 + n\sum_{i=1}^{N} x_i$$

$$\sum_{i=1}^{N} y_i = k\sum_{i=1}^{N} x_i + \sum_{i=1}^{N} n$$

$y = 1.48073k - 0.44$

$57.24 = k \cdot 385 + n \cdot 55$

$81 = k \cdot 55 + n10 \quad / 55$

$4455 = 55^2 k + n55$

$4397.76 = 2970k$

$\frac{4397.76}{2970} = k$

$n = -0.44$

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$k = 1.480727273$

30.1.2004

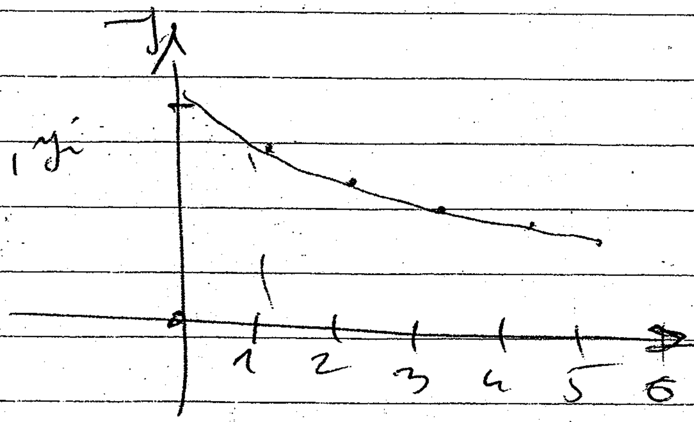
(3) $\left(\frac{1}{n^2}\right) < 10^{-2} = n^2 > 100$

1 - - $\varepsilon < 10^{-2}$ $n > 10$

(2)

x	y
0	1,14
1	0,89
2	0,71
3	0,67
4	0,64
5	0,48

x_i, y_i



$$y = ae^{bx}$$

$$y = ae^{bx_i} \quad ; \quad x_i, y_i$$

$$E = \sum_{i=1}^{n} \left(y_i - a e^{bx_i}\right)^2 = \min$$

x_i	$\ln y_i$ (Y_i)
0	0,13102
1	-0,11653
2	-0,34249
3	-0,40047
4	-0,44628
5	-0,73397

~~$\ln y = a + b$~~

$$\ln y = \ln a + bx$$

$$Y = A + bx$$

$Y \to \ln y$, $A \to \ln a$

~~$\sum_{i=1} (Y_i - A$~~ $E = \sum_{i=1}^{N} (Y_i - A - bx_i)^2 = \min$

$$\frac{\partial E}{\partial A} = 0 = \sum_{i=0}^{N} 2(Y_i - A - bx_i)(-1)$$

$$\frac{\partial E}{\partial b} = 0 = \sum_{i=0}^{N} 2(Y_i - A - bx_i)(-x_i)$$

$$\sum_{i=0}^{N} (Y_i - A - b x_i) = 0$$

$$\sum_{i=0}^{N} (Y_i - A - b x_i) x_i = 0$$

$$\sum_{i=0}^{N} Y_i = A \sum_{i=0}^{N} 1 + b \sum_{i=0}^{N} x_i$$

$$\sum_{i=0}^{N} Y_i x_i = A \sum_{i=0}^{N} x_i + b \sum_{i=0}^{N} x_i^2$$

$$-1,90873 = 6A + b\,15$$

$$-7,45793 = 15A + b\,55$$

$$-2,983172 = 6A + 22b$$

$$7b = -1,074442$$

$$\boxed{b = -0,153492}$$

$$\boxed{A = 0,0656087} = \ln a$$

$$\boxed{a = 1,06781} \leftarrow e^A = a$$

$$\boxed{y = 1,06781 \cdot e^{-0,153492x}}$$

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