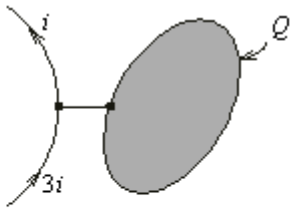
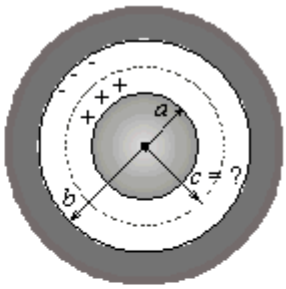
Določite električni tok i, če se množina električnega naboja Q na telesu spreminja po zakonitosti: $Q(t)/\mathrm{C} = 10^{-3}\mathrm{e}^{-10^3 t}$!



Rešitev:

$$\frac{\mathrm{d}Q(t)}{\mathrm{d}t} = 3i - i = 2i \;\Rightarrow\; i = \frac{1}{2}\frac{\mathrm{d}Q(t)}{\mathrm{d}t} = \frac{1}{2}\cdot 10^{-3}\left(-10^3\right)\mathrm{e}^{-10^3 t}\,\frac{\mathrm{C}}{s} \;\Rightarrow\; i = -0.5\,\mathrm{e}^{-10^3 t}\,\mathrm{A}$$

V naelektrenem sferičnem kondenzatorju z radijema $a = 10\,\text{cm}$ in $b = 20\,\text{cm}$ poiščite radij c ekvipotencialke, da bo $U_{ac} = U_{cb}$!



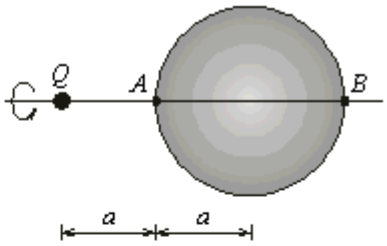
Rešitev:

$$E_r(r) = \frac{Q}{4\pi\varepsilon_0 r^2}\,,\ a < r < b$$

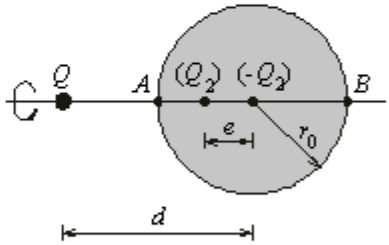
$$U_{ac} = \int_a^c E_r(r)\,\mathrm{d}r = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{c}\right)\ ,\ U_{cb} = \int_c^b E_r(r)\,\mathrm{d}r = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{c} - \frac{1}{b}\right)$$

$$U_{ac} = U_{cb} \ \Rightarrow\ \frac{1}{a} - \frac{1}{c} = \frac{1}{c} - \frac{1}{b} \ \Rightarrow\ \frac{2}{c} = \frac{1}{a} + \frac{1}{b} \ \Rightarrow\ c = \frac{2ab}{a+b} \ \Rightarrow\ c \cong 13.33\,\text{cm}$$

Majhno kovinsko kroglico z nabojem Q približamo k električno nevtralni prevodni krogli. Določite razmerje $\sigma(A)/\sigma(B)$!



Rešitev:



$d = 2a \quad , \quad r_0 = a$

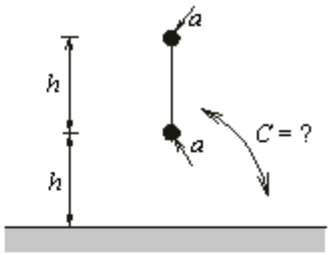
$$ed = r_0^2 \;\Rightarrow\; e = \frac{a}{2} \;,\; Qr_0 = -Q_2 d \;\Rightarrow\; Q_2 = -\frac{Q}{2}$$

$$\sigma(A) = \varepsilon_0 E_n(A_+) = \frac{-Q}{4\pi(d-r_0)^2} + \frac{Q_2}{4\pi(r_0-e)^2} + \frac{-Q_2}{4\pi r_0^2}$$

$$\sigma(B) = \varepsilon_0 E_n(B_+) = \frac{Q}{4\pi(d+r_0)^2} + \frac{Q_2}{4\pi(r_0+e)^2} + \frac{-Q_2}{4\pi r_0^2}$$

$$\sigma(A) = \frac{-Q}{4\pi a^2}\left(1 + \frac{1/2}{(1/2)^2} - \frac{1}{2}\right) \;,\; \sigma(B) = \frac{-Q}{4\pi a^2}\left(\frac{-1}{(3)^2} + \frac{1/2}{(3/2)^2} - \frac{1}{2}\right) \;,\; \frac{\sigma(A)}{\sigma(B)} \cong -\frac{45}{7}$$

Dve daljnovodni vrvi, ki sta obešeni ena vrh druge, galvansko povežemo. Določite kapacitivnost sistema: dvojček-zemlja!



Rešitev:

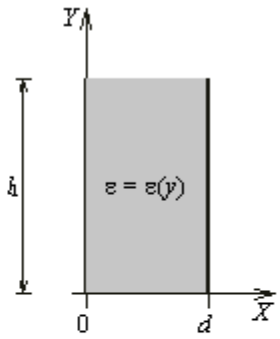
Linijski naboj zgornje vrvi označimo z q_1 ter spodnje z q_2. Upoštevamo zrcalna nadomestna linijska naboja $(-q_2)$ in $(-q_1)$ na globini h oziroma $2h$ v zemlji. Potencial dvojčka je:

$$V = \frac{q_1}{2\pi\varepsilon_0}\ln\frac{4h}{a} + \frac{q_2}{2\pi\varepsilon_0}\ln\frac{3h}{h} = \frac{q_1}{2\pi\varepsilon_0}\ln\frac{3h}{h} + \frac{q_2}{2\pi\varepsilon_0}\ln\frac{2h}{a} \;\Rightarrow\; q_2\ln\frac{2h}{3a} = q_1\ln\frac{4h}{3a}$$

$$q_2 = q_1\frac{\ln\dfrac{4h}{3a}}{\ln\dfrac{2h}{3a}} \quad , \quad q_1 + q_2 = q_1\left(1 + \frac{\ln\dfrac{4h}{3a}}{\ln\dfrac{2h}{3a}}\right)$$

$$V = \frac{q_1}{2\pi\varepsilon_0}\left(\ln\frac{4h}{a} + \frac{\ln 3\ln\dfrac{4h}{3a}}{\ln\dfrac{2h}{3a}}\right) \quad , \quad C = \frac{q_1+q_2}{V} = \frac{2\pi\varepsilon_0\left(\ln\dfrac{2h}{3a}+\ln\dfrac{4h}{3a}\right)}{\ln\dfrac{2h}{3a}\ln\dfrac{4h}{a} + \ln 3\ln\dfrac{4h}{3a}}$$

Med ploščama kondenzatorja je izolant, ki ima zaradi vertikalnega temperaturnega gradienta funkcijsko odvisno dielektričnost $\varepsilon = \varepsilon(y) = Ay + B$; A in B sta znani konstanti. Določite izraz za kapacitivnost takšnega kondenzatorja, če je l_2 dolžina kondenzatorja, privzamemo pa, da je razmak $d << l_1, l_2$!



Rešitev:

$$E(y) = \text{konst.} = \frac{U}{d} = \frac{D(y)}{\varepsilon(y)} = \frac{\sigma(y)}{\varepsilon(y)} \Rightarrow \sigma(y) = \varepsilon(y)\frac{U}{d}\ ,\ Q = \int_0^h \sigma(y) l_2\, \mathrm{d}y = \frac{U l_2}{d}\left(A\frac{l_1^2}{2} + B l_1\right)$$

$$C = \frac{Q}{U} = \frac{l_1 l_2}{d}\left(A\frac{l_1}{2} + B\right)$$