OSNOVE ELEKTROTEHNIKE I
zapiski predavanj

Šolsko leto	2007 / 2008
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Sodelavci	

UREJANJE DOKUMENTA

VERZIJA	01	**REVIZIJA**	01
DATUM	15. 8. 2009		
ZADNJI POPRAVLJAL		Blaž Potočnik	
PREGLEDAL			

OPOMBE

Zapiski so uporabni za sledenje pri predavanjih in ponovitev pred izpitom. Za razumevanje obravnavane tematike bo potrebno prebrati še kaj več.

POPRAVKI

LORENTZOVA ELEKTROMAGNETNA SILA (1895)

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3.10.2007

Hertz 1888 - oscilator.

To sila je ena pomembnejših za življenje
V elektromagnetiki so pomembni 3 poskusi, vsi govorijo o sili na naboj)

Coulombova električna sila (sila med naboji)
sila med tokovi (gibajočimi se delci)
Faradayeva inducirana sila

Sila bo tista, okoli katere se bo vse vrtelo. Je osnova za vse mehanizme. Lorentz je izpeljal tudi transformatorske enačbe. Pri sledeči enačbi ni imel nič, skupaj je združil 2 sili (električno in magnetno)

$$\delta\vec{F}_L = \delta Q\,(\vec{E} + \vec{w}\times\vec{B})$$

← nekateri tej enačbi pravijo tudi def. elektromagnetnega polja

delta, tudi δ, majhna količina to predstavlja; sila; naboj; poljska jakost $E(\vec{r},t)$ v V/m; hitrost delca; magnetno polje, dejansko funkcija časa in jakosti $\vec{B}(\vec{r},t)$

Za majhne delce vzamemo čim manjše količine, zato enačbo uporabljamo le pri majhnih nabitih delcih. Za pomikajoče delce pa δ (delta) zamenjamo z diferencial

$$d\vec{F}_L = dQ\,(\vec{E} + \vec{w}\times\vec{B})$$

$\vec{E}$ - jakost el. polja (V/m)
$\vec{w}$ - hitrost delca z nabojem (m/s)
$\vec{F}_L$ - Lor. sila (N)
Q - naboj (C)
$\vec{B}$ - mag. polje (jakost v enotah T) v elektromotorjih od 1-1,5 T

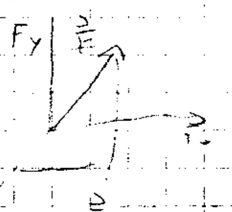
gostota v MRI je 2 T je precej težja stabilizacija, bi se super prelomi magnet

Poljska jakost po žici je okoli 10 mV/m, napon v katerih so jakosti je polovica na 0, pri mag. polju od 1,5 T navzdol.

Lorentzova sila nastane iz 2 enačb

$$\delta\vec{F}_L = \underbrace{\delta Q\vec{E}}_{\vec{F}_e} + \underbrace{\delta Q\,\vec{w}\times\vec{B}}_{\vec{F}_m}$$

vektorski produkt



Mikrovalovka izkorišča polarnost vode in s hitrim obračanjem polja (2,1 - 2,5 GHz) začno molekule zadevati ena ob drugo zaradi navora (F_e) in s tem se ustvari toplota (F_m)

$\vec{F}(F_x, F_y, F_z)$

Vsaka ploskev ima svoj vektor, ki ima svoje komponente

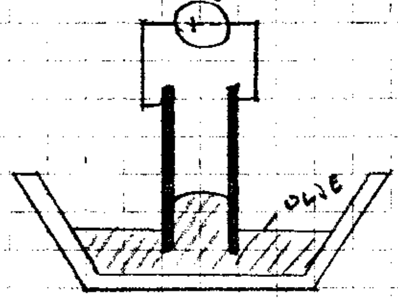
$|\vec{S}| = \sqrt{\ldots}$

Značilnosti električne in magnetne sile

učinki – električna sila učinki:
- sila na delec (elektroforeza – [illegible])
- sila na dipol
- sila na objekt
- navor na dipol
- kot razlaga odziva snovi na polje

magnetna sila
- sila na delec
- navor na tokovno zanko
- Hallova sonda (merilnik, ki pokaže napetost v mag. poljem)
- kot razlaga odziva snovi (magnetika) na polje

Drugače se da silo na objekt zapisati tudi s pomočjo energije (gibalni moment) in ne samo z [illegible] formulo. Tako lahko govorimo tudi o učinkih na [illegible] objekte kot je npr. olje (v kond. se kapljice [illegible])



[illegible] je [illegible] element. [illegible] tekstom [illegible] - [illegible] modeli, konstrukt.

Porazdelitev funkcije el. naboja (elektrine)

VENE

Pri [illegible] hitrosti [illegible], masa pa se spreminja

Elektrina je drugo ime za el. naboj. Uporablja se predvsem v elektrotehniki. Enako velja za kapaciteto kondenzatorja, ki ji pravimo kapacitivnost.

[illegible] porazdelitve:

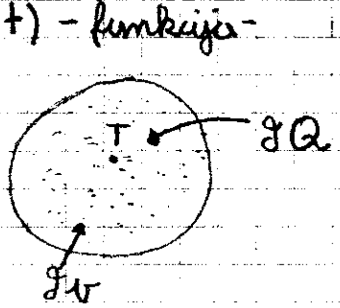
prostorska: imamo oblak nabitih delcev – o prostorninski gostoti govorimo ko so naboji v prostoru. Ta gostota se spreminja od točke do točke, zato mora biti [illegible] zelo majhen. [illegible] z majhno količino [illegible] tudi volumenska gostota

$$\rho = \lim_{\text{"}\delta v \to 0\text{"}} \frac{\delta Q \vee \delta v \text{ od } T}{\delta v}$$

$\rho(T,t)$ – funkcija

gostota vode je krajevno, časovna funkcija

v – volumen
T – kraj



Zgled:
Vzamimo baker (Cu), ki v periodnem sistemu leži na 29 mestu, kar pomeni, da ima en prost elektron. Število elektronov je $n \simeq 8,4 \cdot 10^{-8}$ el/m³, elektron pa ima naboj $Q_e = -1,602 \cdot 10^{-19}$ C. Kakšna je gostota prostih elektronov?

$n \simeq 8,4 \cdot 10^{-8}$ el/m³

$e = -1,602 \cdot 10^{-19}$ C

$\rho = -e \cdot n$

$\rho = -1,34 \cdot 10^{10}$ C/m³

samo 1. vzamemo, ker je samo 1 prost, če bi bila 2, bi to množili z 2.

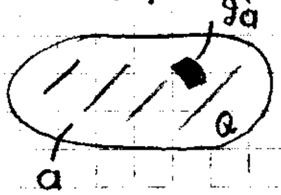
Ko v nevihtnem oblaku je ρ dosti manjša in strela drugače nima veliko W

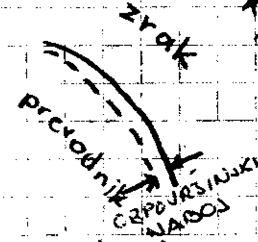
ploskovna porazdelitev: govorimo o naboju na telesu, npr. krompirju. Govorimo, da so naboji na površini. Dejansko se naboj nahaja v globinskem pasu amstrem ~ $Å = 10^{-10}$ m. To je - bistveni obpovršinski naboj, samo kakšen atom je dejansko na površini.

$$\sigma = \lim_{\text{„}\partial a \to 0\text{"}} \frac{\partial Q \text{ na } \partial a \text{ v } t}{\partial a}$$

sigma

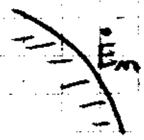
$\sigma(T,t)$ - površinska gostota





Na kondenzatorju niha harmonično z frekvenco 50 Hz. Naboj se znotraj plošč ves čas ne spreminja - je enak. Presežek naboja (+ ali -) pa se pojavlja na površini in od tega je odvisno, kateri naboj bo v prebitku.

Zgled:
Poljska jakost ob površini je $E_n \simeq 1$ MV/m



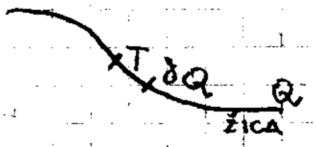
$\sigma = \varepsilon_0 \cdot E_n$

$\sigma = \frac{10^{-9}}{36\pi} \frac{As}{Vm} \cdot 1$ MV/m

$\sigma = 8,85$ μC/m²

POLJSKA JAKOST V PREVODNIKIH JE 0 RAZEN ČE JE TEČE TOK, VENDAR JE TAKRAT NAJ ZANEMARLJIVA

linijska porazdelitev: spomne nas na žico, porazdelitev naboja v neki črti.

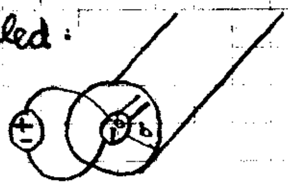


$$q = \lim_{\text{„}\partial l \to 0\text{"}} \frac{\partial Q \text{ na } \partial l \text{ ob } t}{\partial l}$$

$q(T,t)$

Zgled:

U - 100K



$c = \frac{2\pi\varepsilon_0}{\ln b/a}$

$q = cU$
$(Q(t) = CU)$

Najbolj efektiven je ta kabel, ko je $b/a \simeq e = 2,718$

$q = \frac{2\pi\varepsilon_0}{\ln e} \, 100$ V

$q = 0,055 \cdot 10^{-7}$ C/m

$q = 5,5$ nC/m

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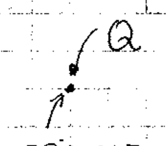
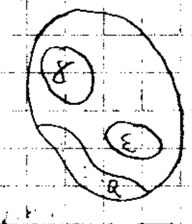
točkast naboj: drugače točka brez mase.
Ko so deli glede na ostalo tako majhni, jih jemljemo kot točka. Najbolj se točkastemu naboju približa elektron.



Točkaste naboje ne smemo dobesedno jemati kot točko, ker nam lahko nekatere fizikalne količine podivjajo

Prostorska porazdelitev je najbolj pogosta in največkrat jo obravnavamo. Če je stisnjena je ploskovni naboj. Če to ploskev stisnem dobim linijsko, ... itd. Naravna porazdelitev je vedno prostorska.

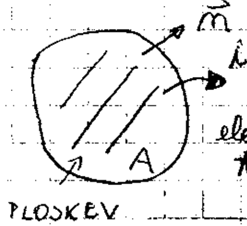


$$Q = \int \rho \, dV$$

$\rho \, dV$ ← dQ

ELEKTRIČNI TOK (PRETOK NABOJA OZ. ELEKTRINE)

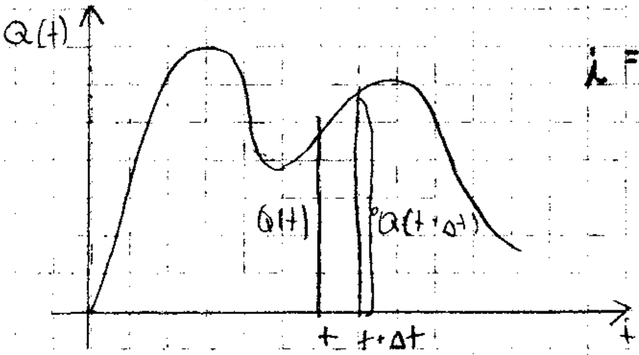
Naboji se tudi gibljejo in zato uporabljamo novo količino: električni tok. Količina tok se pojavlja večkrat (masni, vodni, energijski, denarni)
Vsi toki so definirani na enak način: imamo ploskev, skozi katero nas zanima, koliko gre nekakšnih elementov. Toliko in toliko skozi kaj. (orientirane ploskve).



$$i = \lim_{\Delta t \to 0} \frac{\Delta Q \text{, ki gre skozi A v času } \Delta t}{\Delta t}$$

električni tok

Bodi ali naj bo $Q(t)$ množina naboja, ki je ploskev v izbrani smeri prestopila od časa $t=0$ do časa t. ← funkcija, ki govori o pretoku naboja skozi orient. ploskev.



$$i = \lim_{\Delta t \to 0} \frac{Q(t+\Delta t) - Q(t)}{\Delta t} = \frac{dQ}{dt} = Q'$$

Zgled: Tok je harmonična funkcija:

$i = I_m \cos(\omega t + \alpha)$

amplituda temenska vrednost

krožna hitrost, krožna frekvenca

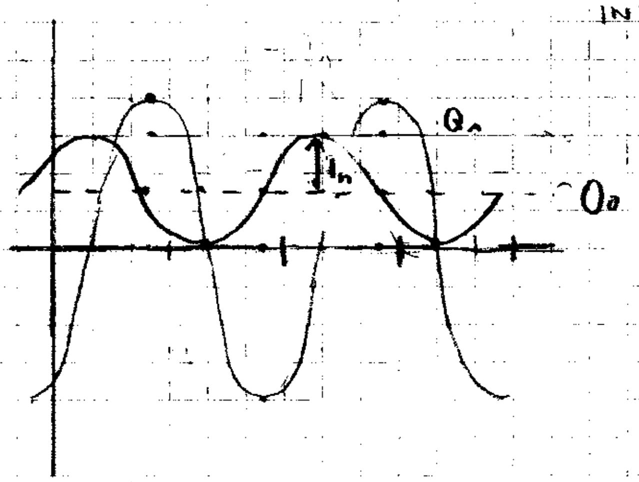
začetni kot

$\sin x = \cos(x - \frac{\pi}{2})$

konstanta

$$i = \frac{dQ}{dt} \Rightarrow Q_t = \underbrace{-\frac{I_m \sin(\omega t + \alpha)}{\omega}}_{\text{INTEGRAL } I_m \cos(\omega t + \alpha)} + Q_0$$

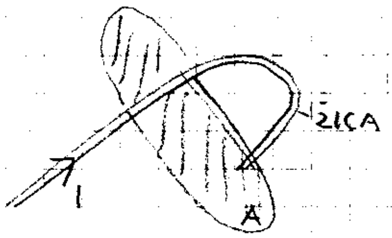
$\omega = 2\pi \nu$



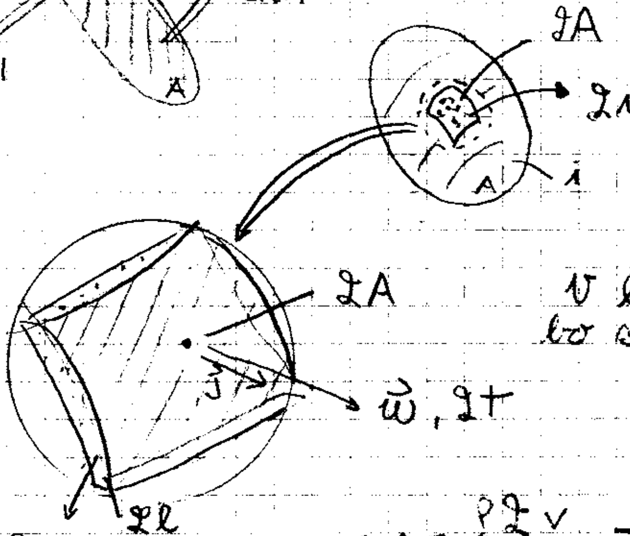
$I_m = 6A \quad f = 50 Hz \quad \omega = 100\pi s^{-1}$

$$Q_m = \frac{I_m}{\omega} = \frac{6A}{100\pi s^{-1}} = 0.018 As = 18 mC$$

GOSTOTA EL TOKA



Tok skozi ploskev je 0. Vendar to ne pomeni, da se naboj ne pretaka.



V listku dolim namišljen volumen, ki bo odaj odaj prečkal ploskvico

$dQ = \rho\, dV$

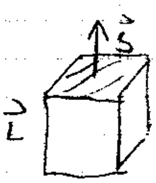
$$di = \frac{\rho\, dV}{dt} = \frac{\rho\, d\vec{a} \cdot d\vec{l}}{dt} = \rho\, \vec{w} \cdot d\vec{a}$$

ploščina

ploščina

hitrost

volumen



$vol = \vec{S} \cdot \vec{L} = S \cdot L_s$

$\vec{J} = \rho\, \vec{w}$

gostota ~~...~~ hitrost naboja

Zgled: Baker in gostota prostih elektronov v njem:

$$\rho_{Cu} \cong 1{,}34\cdot 10^{10}\ C/m^2$$

$$|\vec{J}| \sim 4\ A/mm^2 = 4\ MA/m^2$$

so še znosne meje gostote

10 A/m² je zadnja meja (še oljno hlajenje), če ne bi se vodnik pregreval

Znosne gostote

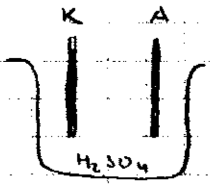
$$|\vec J| = J = |\rho||\vec w| = |\rho|\cdot|\vec w|$$

$$\langle w\rangle = \frac{J}{|\rho|} = \frac{4\cdot 10^6\ MA/m^2}{1{,}34\cdot 10^{10}} = 300\ \mu m/s$$

Električno in magnetno polje pa potuje s svetlobno hitrostjo in onedva nosita energijo.

Drugače imamo več nosilcev naboja. Če imamo več nosilcev naboja:

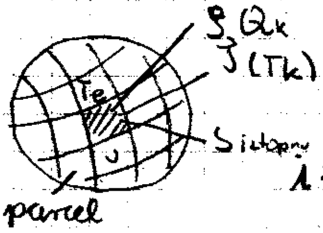
$$\vec J = \rho_1 \vec w_1 + \rho_2 \vec w_2$$



$$\vec J = \rho_+ \vec w_+ + \rho_- \vec w_-$$

J in w sta si vzporedna

$$\rho_- = -\rho_+$$
$$w_- = w_+$$



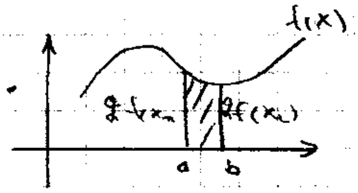
$$\delta i = \vec J(T_k)\cdot \delta \vec a_k$$

ploščina

$$i = \sum_{k=1}^{n} \delta i_k = \sum_{k=1}^{n} \vec J(T_k)\cdot \delta \vec a_k$$

$$\vec J = \rho \cdot \vec w$$ vektor gostote električnega polja

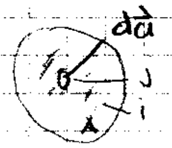
gostota, hitrost



$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\delta x_k = \int_a^b f(x)\,dx$$

$$i = \lim_{n\to\infty} \sum_{k=1}^{n} \vec J(T_k)\,\delta a_k = \int_A \overbrace{\vec J\, d\vec a}^{di}$$

ploščina



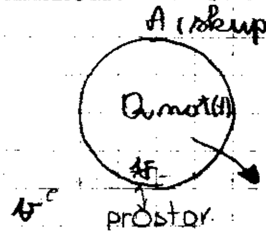
$$i = \int \vec J\, d\vec a$$ – ploščina manjšega dela

ZAKON O OHRANITVI NABOJA ALI ELEKTRINE IN KONTINUITETNA ENAČBA

Naboj ne doživi defekta. Množina nabojev je tudi po separaciji enaka.

velja

A (skupna ploskev)



Naboji se lahko selijo noter ven v tem prostoru (zgolj dogovor da je prostor)

Naboj ne nastaja ali izginja

ob nekem trenutku t: $Q_{notr.} + Q_{zunanji} = kons.$ (ali 0)

$t + \Delta t$: $Q_{notr.} + Q_{zunanji} = C$ (kons.)

pomeni odvod

$$\lim_{\Delta t \to 0} \frac{Q_{notr.}(t+\Delta t) - Q_{notr.}(t) + (Q_{zun.}(t+\Delta t) - Q_{zun}(t))}{\Delta t} = 0$$

velja za vsako sklenjeno ploskev

$$\frac{dQ_{notr.}}{dt} + \underbrace{\frac{dQ_{zun}}{dt}}_{i} = 0$$

$$\text{ekstemi tok } -i = -\frac{dQ_{notranji}}{dt} = \frac{dQ_{zunanji}}{dt} = i$$

Hitrost upadanja je enaka hitrosti ??hajanja zunanja toku

Zgled: Imamo kondenzator (ob toku in u času teče naboj iz ene plošče na drugo) Imamo polnilne tokove (ki polnijo oz. praznijo kond.

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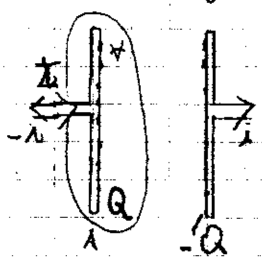
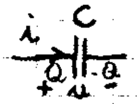


$$-i = -\frac{dQ}{dt}$$

$$i = \frac{dQ}{dt} \qquad i = Q'$$

Tok je enak hitrosti naraščanja naboja.



simbol za kondenzator

enačba kondenzatorja izhaja iz zakona o ohranitvi naboja, ta pa iz ?? PROSTORA ELEKT.

$Q = Cu$ napetost

$Q(t_2) = C\,u(t_2)$

$Q(t_1) = C\,u(t_1)$

$$i = Q' = Cu'$$

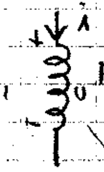
$$i = Cu'$$

$$\int_{t_1}^{t_2} du = \int_{t_1}^{t_2} \frac{i}{C}\, dt$$

V kondenzatorju skoncentriramo električno polje, zunaj pa je malo polja

$$u(t_2) - u(t_1) = \frac{1}{C} \int_{t_1}^{t_2} \boxed{i\, dt}\quad Q$$

$$u(t_2) - u(t_1) = \frac{1}{C}(Q(t_2) - Q(t_1))$$



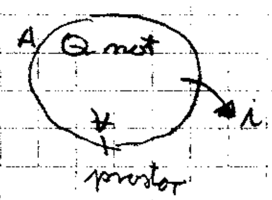
Faradayev zakon

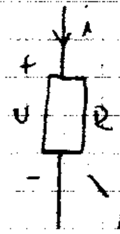
$u = L i'$

skoncentrirano magnetno polje

RAZREDI ELEKTROMAGNETNIH POLJ

$i = -\frac{dQ_{not.}}{dt}$





Ohmov zakon

$\oint_A \vec{J} \cdot d\vec{a} = -\frac{d}{dt} \int_V \rho \cdot dv$ - volumen

ploskev (zaključena) · skoncentrirano tokovno polje · tokovna gostota · nabojna gostota

$\rho(T,t)$ krajevno-časovna funkcija

$u = R i$

$= -\int \frac{\partial \rho}{\partial t} dv$ - časovna hitrost spreminjanja naboja

$\frac{df}{dx} = f'(x) = Df$

$g(x,y,z,t)$ funkcija več odvodov

$= \frac{\partial g}{\partial x} = g_x$ - parcialni ali delni odvod

A:) elektrostatično - 1 preučevano polje, polje mirujočih nabojev

$\vec{J} = 0 \Rightarrow \vec{J} = 0$, česar ga mirovanja v listu ni, kond. se prazni, ker je izolant tud prevoden - vendar slabo

To je drugače idealizacija, vendar za kratke čase to velja.

$\Rightarrow \rho(T)$ - zgolj krajevna funkcija

$0 = -\int \underbrace{\frac{\partial \rho}{\partial t} dv}_{0 = \text{konstantna}}$

To obdeluje elektrostatika: el. polje, C-vezja

B) $\vec{J}(T)$ - naj bo neodvisno to od časa (enosmerna vezja)

$\oint_A \vec{J} \cdot d\vec{a}$ = je neka vrednost = konstanta

$-\frac{d}{dt}\int \rho \, dv$ - mora biti konstanta - linearna funkcija

enosmerni tok - tokovno polje, R-vezja

$\oint_A \vec{J} \cdot d\vec{a} = K = -\frac{d}{dt} Q_{not.}$

$Q_{not} = -Kt + K_1$ - linearna funkcija

$\Rightarrow K = 0 \Rightarrow \oint_A \vec{J} \cdot d\vec{a} = 0$ pretok je vedno 0

$i_1 + i_2 - i_3 = 0$

$\sum_k (\pm) i_k = 0$

Kirchhofov zakon o ohranitvi toka - posledica zakona o ohranitvi Q

$\vec{J}(T) \rightarrow$ magnetostatika (magnetno polje okoli vodnikov), 1 "magnetna vezja"

c) $\vec{J}(T,t)$ - dinamika
dopustimo, da se spreminjajo

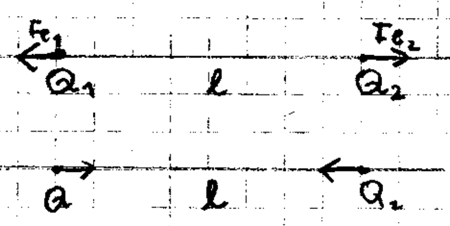
$\vec{J}(T,t) = \varrho(r,t)$

zu bo: inducirano električno polje, RLC vezja

COLOMBOV ZAKON (1785)

("COLOMB")

Izhaja iz elektrostatike. Je preučevanje električnih sil med naelektrenimi delci. Izumil je torzijsko tehtnico. Meril je silo med 2 krogličama, ki so majhna, da jih lahko obravnavamo kot točkasto telo



Sili sta si:
privlačni ali odbojni, ravna
absolutno enaki
→ odvisno od vrednosti Q
nasprotni smeri

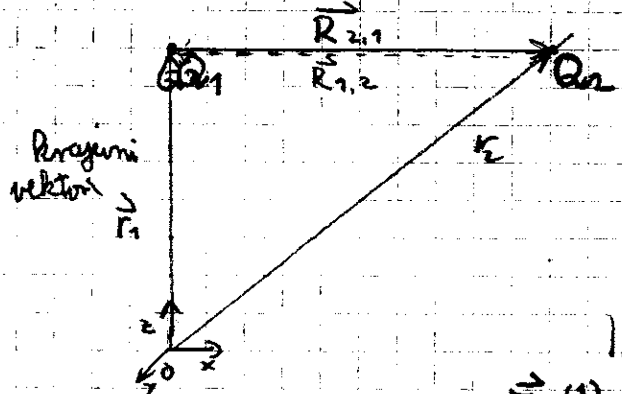
$|\vec{F}_{e1}| = |\vec{F}_{e2}|$

$\vec{F}_{e1} = -\vec{F}_{e2}$

$F_{e2} = k \cdot \frac{Q_1 Q_2}{l^2}$

$|\vec{F}_{e1}| = F_{e1} = F_{e2} \propto \frac{Q_1 \cdot Q_2}{l^2}$ - mi nimamo samo 2 nabojev ampak mnogo
Zadeva spominja na gravitacijski zakon

$\vec{F} \propto \frac{m_1 m_2}{r^2}$



$\vec{R}_{21} = \vec{r}_2 - \vec{r}_1 = -\vec{R}_{12}$

$\vec{r}_1 = (x_1, y_1, z_1) \quad \vec{r}_2 = (x_2, y_2, z_2)$

$\vec{R}_{2,1} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

$|\vec{R}_{2,1}| = |\vec{R}_{1,2}| = R_{21} = R_{12} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$\vec{F}_{e2}^{(1)} = k \frac{Q_1 \cdot Q_2}{R_{21}^2} \frac{\vec{R}_{21}}{R_{21}}$ $\frac{R_{21}}{R_{21}}$ - to dodam za smer in vrednost (+ ali -)

nujno morata biti 2 (najmanj) za Colombov zakon

$F_{e1}^{(2)} = -k \frac{Q_1 \cdot Q_2}{R_{21}^2} \frac{\vec{R}_{21}}{R_{21}}$ } - enotski vektor

$= k \frac{Q_1 \cdot Q_2}{R_{12}^2} \frac{\vec{R}_{12}}{R_{12}}$ $\qquad \vec{e}_u = \frac{\vec{a}}{|\vec{a}|}$

V CGS sistemu (centimeter, gram, sekunda) je bil $k = 1$. Vsak meritev
Sedaj pa imamo merski sistem SI (tu je zdaj amper), sistem je legalen

$k = \frac{1}{4\pi\varepsilon_0}$ $\qquad \varepsilon_0$ - influenčna konstanta vakuma ~~...~~ C v CGS so ... ali dielektričnost vakuma

SI { $\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$
c_0 - hitrost svetlobe $= 2{,}99792459 \frac{m}{s}$

$$\varepsilon_0 \mu_0 c_0^2 = 1$$

$$\varepsilon_0 = \frac{1}{\mu_0 c_0^2} \simeq 8{,}8542 \cdot 10^{-12} \frac{As}{Vm}$$

$$c_0 \simeq 3 \cdot 10^8 \frac{m}{s} \Rightarrow \varepsilon_0 \simeq \frac{10^{-9}}{36\pi} \frac{As}{Vm}$$

$$\frac{1}{k} = 9 \cdot 10^9 \ldots$$

$$Q_1 \xrightarrow{\vec{R}_{21}} Q_2$$

$$\vec{F}_{e_1}^{(2)} = \frac{R_{12} Q_1 Q_2}{4\pi \varepsilon_0 R_{12}^3}$$

$$F_{e_2}^{(1)} =$$

Sila na 2

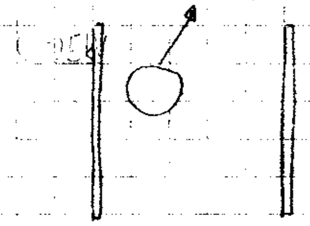
$$\vec{F}_{e_2}^{(1)} = \frac{Q_1 Q_2 \vec{R}_{21}}{4\pi \varepsilon_0 R_{21}^3}$$

Elektrotehnike se ne moremo iti samo z 2 nabojema.

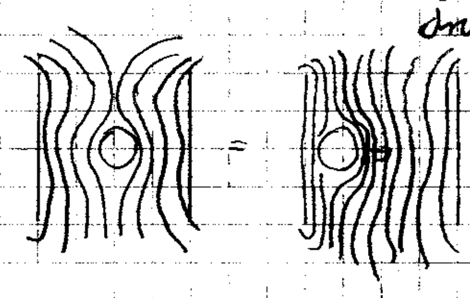
OE I
16. 10. 2007

Demonstracija Colombove sile

pinkponk žogica pobarvana s prevodno barvo



Ko se kroglica dotakne, prevzame del te elektrine. Sila zato odrine kroglico, ker ima elektrina enak predznak. Ko se dotakne druge, ki jo privlači, se situacija ponovi, samo na drugi plošči.



Elektrine imajo tudi neko potencialno energijo (posledica elektrine)

Sila narašča s ^{2} jakosti polja (skoraj)

Če je polje dosti velika lahko pride do preboja. Če pa večamo razdaljo je ravno obratno (kroglica se upočasnuje ker je sila tudi se premajhna (kroglica obmiruje)

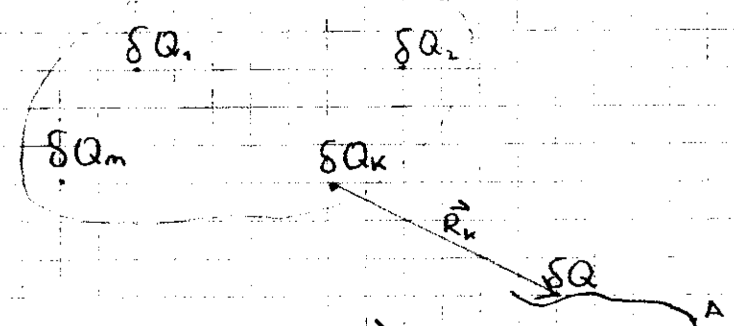
Če imamo prevodno kroglico, ki je polna, je manj vztrajna. Če se dotaknemo, šele takrat se začne gibati.

Če izolimo kroglico namočimo v slano vodo se ta giba toliko časa dokler se ne osuši.

Do visoke napetosti pridemo s pomočjo izolatorja, vmes ima dober dielektrik.
V ~~diodo~~ kaskadi je veliko kondenzatorjev, ki se polnijo (praznijo počasi).

kaskada!
dioda=!

SILA GRUČE NABOJEV NA IZBRAN NABOJ



Princip superpozicije – seštevanja delnih sil

$$\delta \vec{F}_e^{(k)} = \frac{\delta Q \cdot \delta Q_K \vec{R}_K}{4\pi\varepsilon_0 \; R_K^3}$$

$$\Downarrow$$

$$\delta \vec{F}_e = \sum_{k=1}^{m} \delta \vec{F}_e^{(k)}$$

$$= \sum_{k=1}^{m} \frac{\delta Q \cdot \delta Q_K \vec{P}_K}{4\pi\varepsilon_0 \; P_K^3} = \frac{\delta Q}{4\pi\varepsilon_0} \sum_{k=1}^{m} \frac{\delta Q_K \cdot \vec{R}_K}{R_K}$$

Dobimo množico rešitev, ki se dreguje od kraja do kraja. Sila je tudi pod močnim vplivom subjekta.

Če silo vzamemo podobno kot enačbo $F = m \cdot a$. Silo delimo s subjektom

ELEKTRIČNA POLJSKA JAKOST

$$\frac{\delta F \text{ na } \delta Q \text{ v } T}{\delta Q} = \vec{E}(T)$$

Dobimo bolj univerzalno enoto / količino. Normirano silo delimo z nabojem.

enote: $[V/m]$ v zraku je:

$$\Rightarrow E(T) = \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{m} \frac{\delta Q_n \vec{R}_K}{R_K^3}$$

$$\delta \vec{F}_e = \delta Q \cdot \vec{E}(T)$$

pri Lorentzovi sili $\delta F_L = \delta Q \, (E \cdot \vec{w} \, B)$ je polje v bistvu normirana Lorentzova sila na mirujoč naboj

$$\vec{E} = \frac{\delta F_L}{\delta Q} \quad w = 0$$

Sila nas prisili, da obravnavamo še ostale stvari. Sila je vzrok za vse ostale stvari (tok). Zelo je pomembna dinamika, ne samo statika pri opisovanju sil.

Zakaj polje? Predlagal Faraday (prej smatrali kot interakcijo na daljavo). Polje je prostor v stanju, stanje v vektorju. Je prostor, kjer se objekt nahaja.

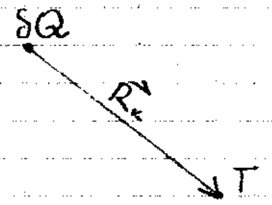
 Podobno velja tudi za magnetno polje.

Polje je tudi pokazatelj energije, ki je v prostoru

$$\vec{E}(T) = \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{n} \frac{\delta Q_k \cdot \vec{R}_k}{R_k^3} = \sum_{k=1}^{n} \frac{\delta Q_k \cdot \vec{R}_k}{R_k^3\, 4\pi\varepsilon_0} =$$

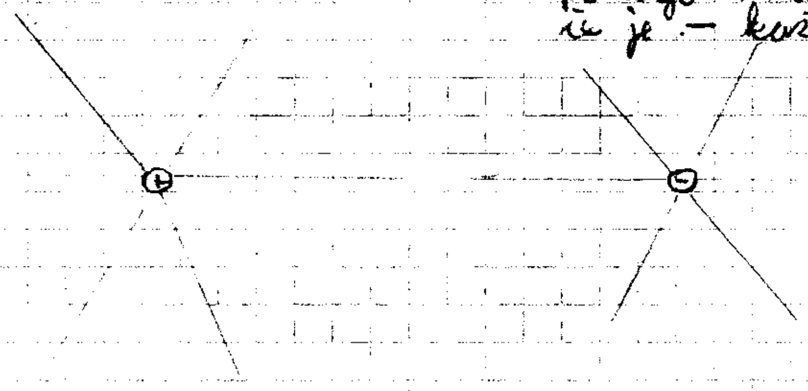
$$= \sum_{k=1}^{n} \frac{\delta \vec{F}_e^{(k)}}{\delta Q} = \sum_{m=1}^{k} \delta \vec{E}^{(k)}(T)$$

Polja lahko seštevam



$$\delta \vec{E}^{(k)}(T) = \frac{\vec{R}_k\, \delta Q_k}{4\pi\varepsilon_0 R_k^3}$$

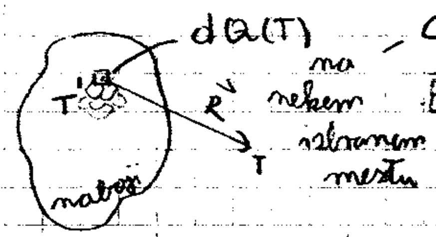
Če je naboj + kaže puščica vektorja stran od naboja, če je − kaže k naboju



Absolutna vrednost polja upada s kvadratom oddaljenosti

$$|\delta \vec{E}^{(k)}(T)| = \frac{|\delta Q_k| \cdot |\vec{R}_k|}{4\pi\varepsilon_0 R_k^3} = \frac{|\delta Q_k|}{4\pi\varepsilon_0 R_k^2}$$

Če gremo počasi k vedno manjšim delcem nabitih delcev, s pridemo v diferenciale



$$d\vec{E}(T) = \frac{dQ(T')\vec{R}}{4\pi\varepsilon_0 R^3}$$

na nekem izbranem mestu

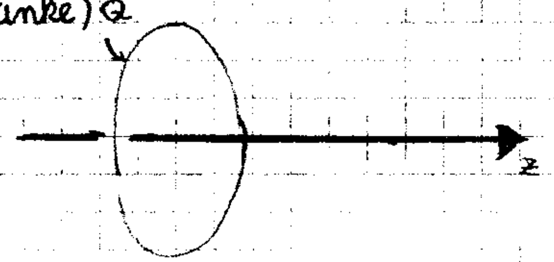
$$\vec{E}(T) = \int d\vec{E}(T) = \int_{(Q\ vseh)} \frac{dQ\,\vec{R}}{4\pi\varepsilon_0 R^3}$$

$$\vec{E}(T) = \frac{1}{4\pi\varepsilon_0} \int_{(Q\ vseh)} \frac{\vec{R}\, dQ(T')}{R^3}$$

tudi Coulombovo polje (mirujoči naboji)

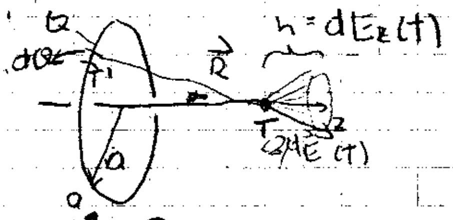
Zgled: $\vec{E}$ v osi naelektrenega krožnega obroča (prostora, obroča, zanke) Q

Izven osi, se z zgornjo enačbo ne problem.



- $\vec{E}$ v osi naelektrene krožne zanke

Žica je tanka (da ne bo površinske porazdelitve naboja)



$$q = \frac{Q}{2\pi a}$$

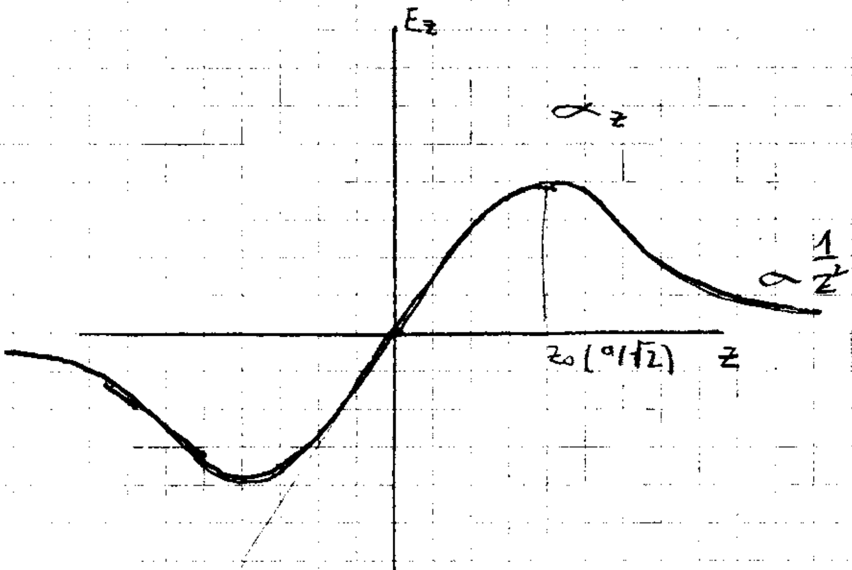
$$|d\vec{E}(T)| = \left|\frac{\vec{R}\, dQ}{4\pi\varepsilon_0 R^3}\right| = \text{konst.}$$

$$d\vec{E}_z(T) = |d\vec{E}(T)| \cdot \cos\alpha$$
$$= \frac{dQ}{4\pi\varepsilon_0 R^2} \cdot \frac{z}{R}$$

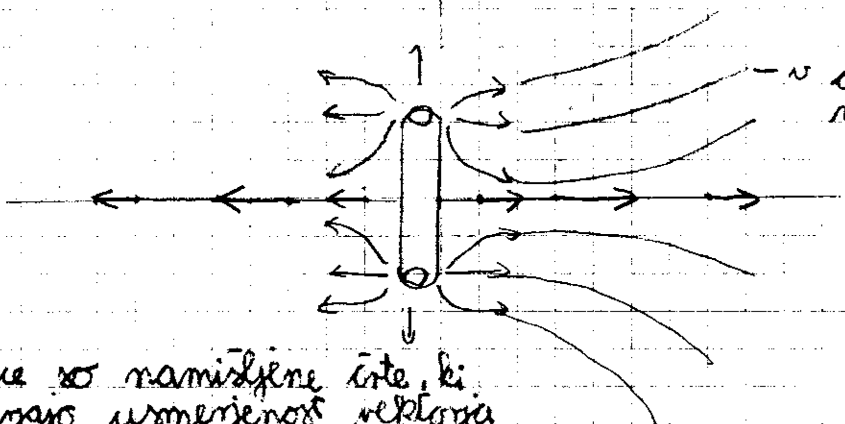
$$\vec{E}(T) = (0, 0, E_z)$$

$$E_z(T) = \int dE_z(T) = \int\limits_{\substack{\text{po vseh}\\ \text{nabojih}(Q)}} \frac{dQ\, z}{4\pi\varepsilon_0 R^3} = \frac{Q z}{4\pi\varepsilon_0 R^3}$$

$$E_z(T) = \frac{Q}{4\pi\varepsilon_0} \frac{z}{(a^2+z^2)^{3/2}} = E_z(z)$$



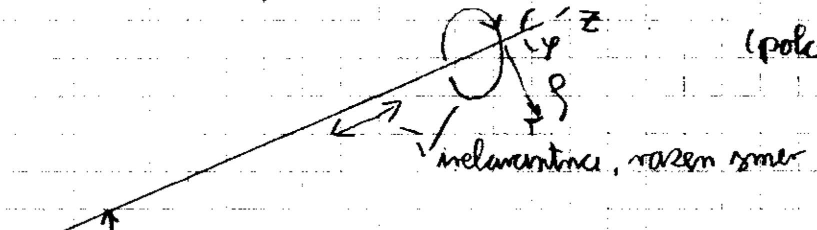
$V \tilde{=} E$



– v daljavi kot točkast naboj

Silnice so namišljene črte, ki ponazarjajo usmerjenost vektorja v točkah prostora.

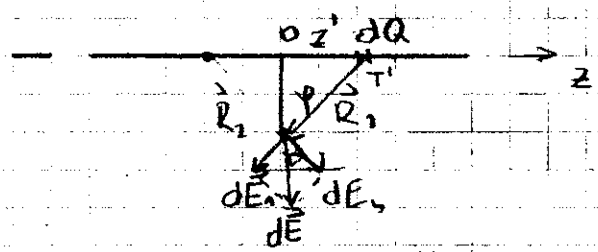
Zgled: Dolga ravna žica (veliko ravnih struktur v elektrotehniki) in E ob njej. (naelektrena premica)



(polarni sistem)
$T(\rho, \varphi, z)$
oddaljenost

$q = \text{konst}$

$q > 0$



smer je to in je to radialna smer

$d\vec{E}_1 = d\vec{E}_2$

$d\vec{E} = (dE_\rho, 0, 0)$

$\vec{E} = (E_\rho, 0, 0)$

$$dE_r = 2 \cdot \underbrace{\frac{dQ}{4\pi\varepsilon_0 R_1^2}}_{|dE_{1,2}|} \cdot \cos\beta \quad r/R_1$$

$$dE_r = \frac{dQ \; q\,dz}{R\, 2\pi\varepsilon_0 R_1^2} \cdot \cos\beta = \frac{q\,dz}{2\pi\varepsilon_0} \frac{r}{\cos^2\beta} \frac{\cos^2\beta}{\rho^2} \cdot \cos^2\beta$$

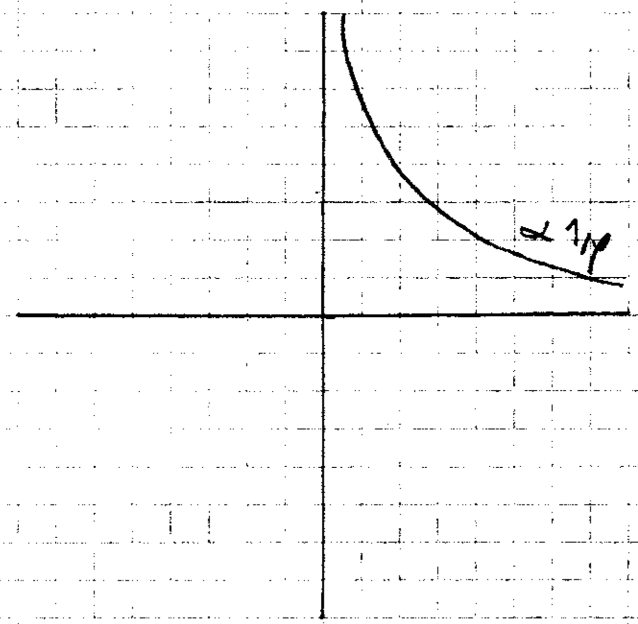
$z' = R_1 \cdot \sin\beta \;, \quad dz' = dz'/d\beta \; d\beta = //$

$z = r \cdot \operatorname{tg}\beta \quad dz' = \frac{dz'}{d\beta} d\beta = \frac{r}{\cos^2\beta} d\beta$

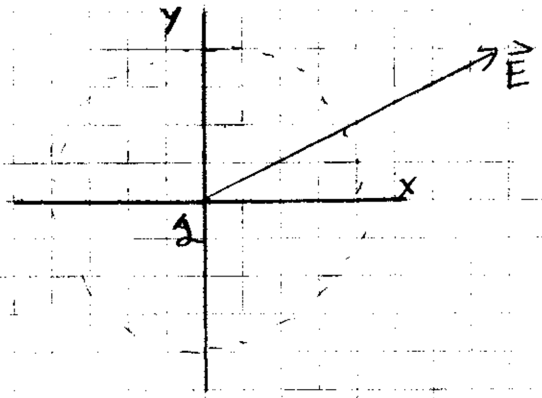
$R_1 = r/\cos\beta$

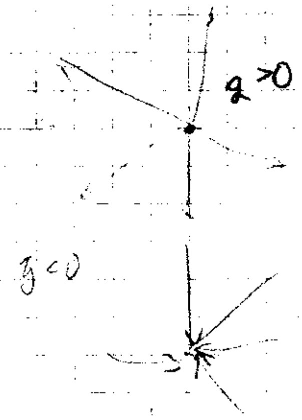
$$dE_r = \frac{q}{2\pi\varepsilon_0} \cos\beta \, d\beta$$

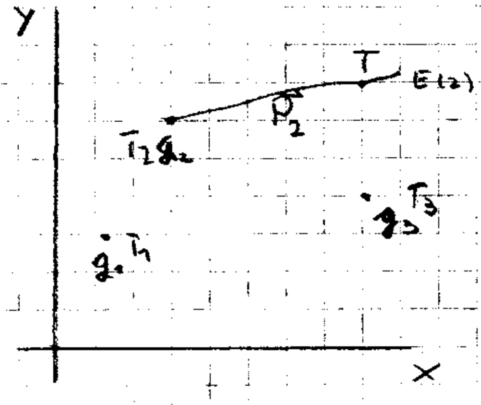
$$E_r = \int dE_r = \int_0^{\pi/2} \frac{q}{2\pi\varepsilon_0 r} \cos\beta \, d\beta = \frac{q}{2\pi\varepsilon_0 r}$$



Če imamo točkast naboj pa je $E = \frac{Q}{4\pi\varepsilon_0 r^2}$







$P_2 = \overrightarrow{T_2 T}$

$\vec{E}^{(2)}(T) = \frac{q_2}{2\pi\varepsilon_0 P_2} \cdot \frac{\vec{P}_2}{P_2}$

$\vec{E}^{(1)}(T)$

$\vec{E}^{(3)}(T)$

$E(T) = \sum_{k=1}^{n} \vec{E}^{(k)}(T)$

$\vec{E}^{(k)}(T) = \frac{Q\,\vec{P}_k}{4\pi\varepsilon_0 R_k^3}$

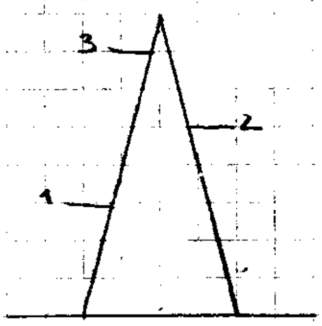
$E^{(k)}(T) = \frac{q_k\,\vec{P}_k}{2\pi\varepsilon_0 P_k^2}$

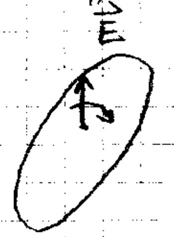
Daljnovod

$q_k = q_{k0}\cos(\overset{2\pi\nu}{\omega} t + \varphi_k) = q_k(t)$

$\vec{E}^2(T,t) = \frac{q_k(t)\,\vec{P}_k}{2\pi\varepsilon_0 P_k^2}$

vzemimo primer trifaznega sistema



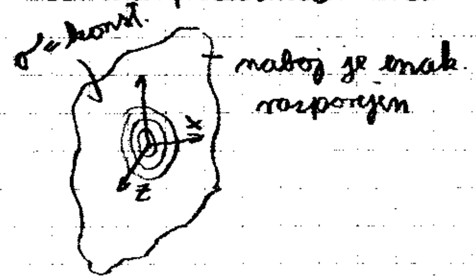


$E_{ef} = \frac{\sqrt{E^2_{max} + E^2_{min}}}{\sqrt{2}}$

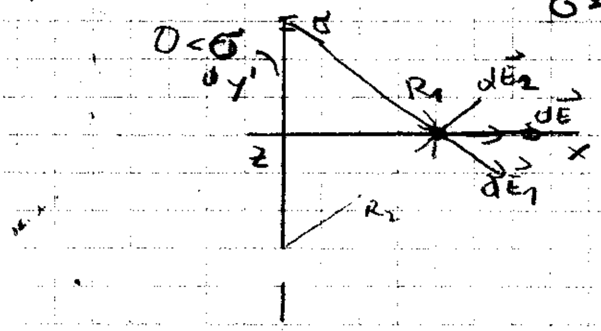
Naboj na naelektreni razsežni (razsežni) plošči. V bistvu temu pravimo tudi naelektrena ravnina.

σ = konst.

+ naboj je enak razporejen



Lahko jo razumemo kot složeno iz premic (trakov) ali obročev



$\sigma \cdot dy' \cdot l \quad d\vec{E}_x = \frac{2\sigma\, dy'}{2\pi\varepsilon_0 P_1} \cdot \cos\gamma \quad \frac{x}{P_1} = \frac{\sigma dy'}{\pi\varepsilon_0} \frac{x}{P_1^2}$

$$E_x = \int_0^{\infty} \frac{\sigma dy'}{\pi\varepsilon_0} \frac{x}{x^2+y^2}$$

$$E_x = \frac{\sigma}{\pi\varepsilon_0} \int_0^{\infty} \frac{x\, dy'}{x^2+y^2} = \frac{\sigma}{\pi\varepsilon_0} \int_0^{\infty} \frac{\overbrace{\frac{dy'}{x}}^{\mathrm{arctg}\, x}}{1+\frac{y^2}{x^2}}$$

$\underline{E_x = \frac{\sigma}{2\varepsilon_0}} \quad x>0 \qquad \underline{E_x = -\frac{\sigma}{2\varepsilon_0}} \quad x<0$

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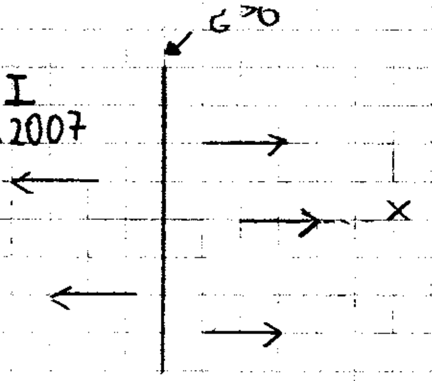
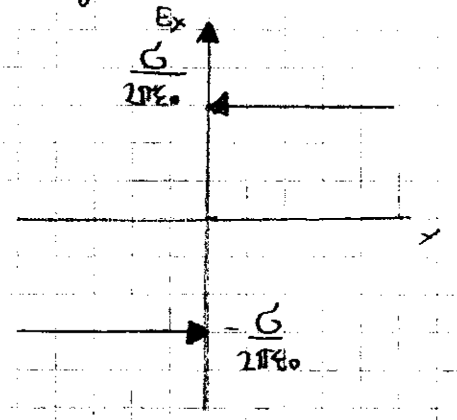


Pri plošči je homogeno polje. Ravnina je taka struktura. Homogeno je ko ima povsod enako smer in jakost!

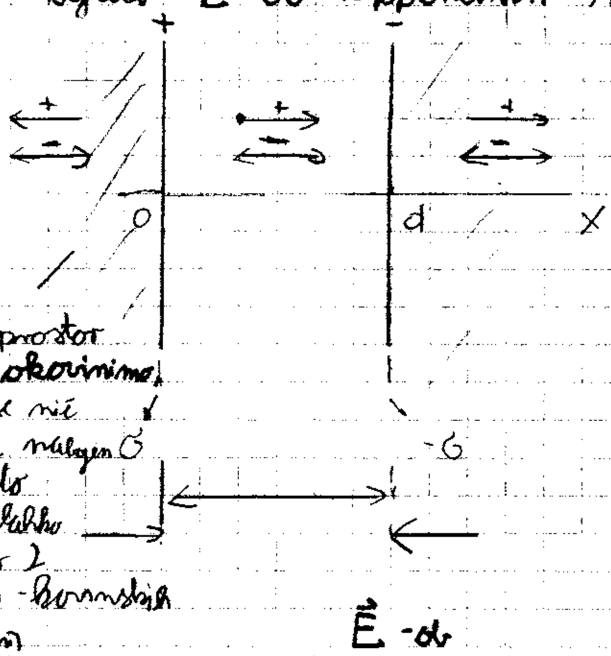
Polje v / znotraj ravnine nič, ne obstaja.

$$E_x = \begin{cases} -\frac{\sigma}{2\varepsilon_0}, & x<0 \\ 0, & x=0 \\ \frac{\sigma}{2\varepsilon_0}, & x>0 \end{cases}$$



To je naša bodoča plošča kondenzatorja

Zgled: E ob vzporednih ravninah s +, - naelektritvama



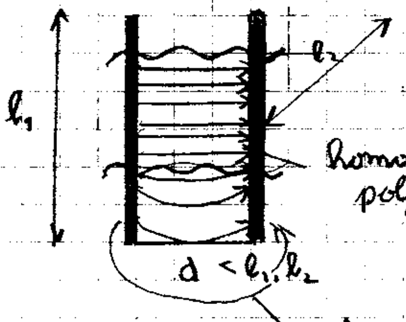
ta prostor lahko okrnimo, ne bo se nič zgodilo z nabojem σ tu. Zato računamo lahko govorimo o 2 ploščah - končnih ravninah

$\vec{E}$ -ob

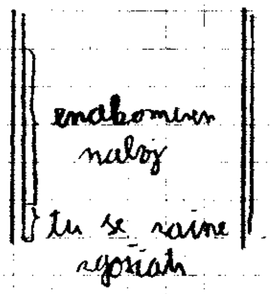
V mejah preraspored naboje, jih ne ustvarja

$$E_x = \begin{cases} 0, & x<0 \\ \frac{\sigma}{2\varepsilon_0} & x=0 \\ \frac{\sigma}{\varepsilon_0} & x<d \\ \frac{\sigma}{2\varepsilon_0} & x=d \\ 0; & x>d \end{cases}$$

E - ob vzporednih ploščah s +, - nabojema



stresano polje, ki pa je lahko tudi 1000x manjše od homogenega polja

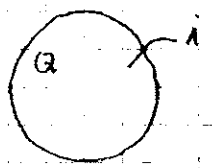


V homogenem polju govorimo o coni, ko smo v sredini in je to enako d - razmaku - i vejetno od roba plošč.

IZVORNOST POLJA $\vec{E}$

Pri točkastem naboju izgleda kot da bi nekaj izviralo, pri pozitivnem delcu. Pri negativnem pa kot da bi nekaj ponikalo. (Ponikanje)

Tudi za izvornost rabimo eno matematično formo. So bomo dobili z eno od Maxwellovih enačb.

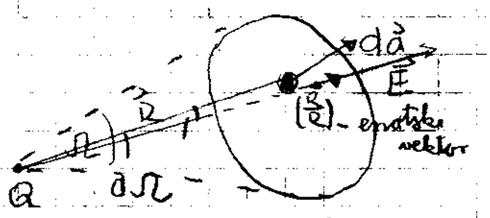


$i = \oint_A J \cdot d\vec{a} = -\frac{dQ\,not}{dt}$ je izvorno

$\oint_A \vec{E} \cdot d\vec{a} = ?$ | $\vec{E}(T) = \frac{1}{4\pi\varepsilon_0} \int \frac{\vec{R} \cdot dQ(T)}{R^3}$ ali je to Coulombovo polje izvorno.

A - ssklenjene ploskev

polje točkastega naboja



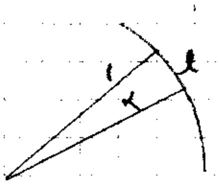
$\vec{E} = \frac{Q\vec{R}}{4\pi\varepsilon_0 R^3}$

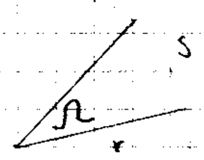
$d\phi_E = \vec{E} \cdot d\vec{a}$

$= \frac{Q}{4\pi\varepsilon_0} \frac{\vec{R} \cdot d\vec{a}}{R^3}$

$= \frac{Q}{4\pi\varepsilon_0} \underbrace{\frac{\left(\frac{\vec{R}}{R}\right) \cdot d\vec{a}}{R^2}}$ - dobim projekcijo ploskve na R^3

prostorski kot pod katerim vidimo ploskvico pod kotom Q $d\Omega$



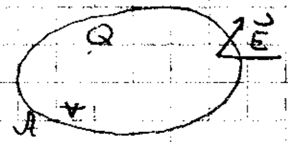


$$\int_A \vec{E}\cdot d\vec{a} = \Phi_E = \int d\Phi_E = \int \frac{Q}{4\pi\varepsilon_0}\, d\Omega = \frac{Q}{4\pi\varepsilon_0}\int d\Omega = \frac{Q}{4\pi\varepsilon_0}\,\Omega$$

vidimo celo polavinko

$$\int_A \vec{E}\cdot d\vec{a} = \frac{Q}{4\pi\varepsilon_0}\,\Omega$$

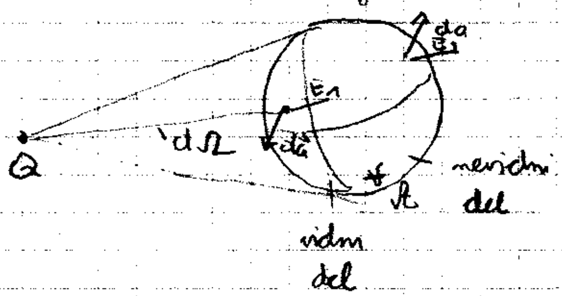
Sklenjena ploskev (2 pisma, oznake jo def. ...) Gre za nekakšen balon



$$\oint_A \vec{E}\cdot d\vec{a} = \frac{Q}{4\pi\varepsilon_0}\,4\pi = \frac{Q}{\varepsilon_0}$$

sklenjena ploskev

Lahko pa naboj ostane zunaj te vreče, balona

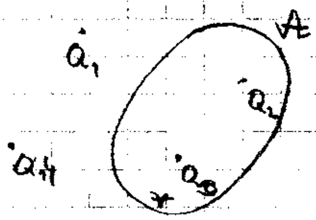


Koliko gre pretoka noter, ga gre tudi ven. Takih parov je neskončno

$$\vec{E}_1 d\vec{a}_1 = -\vec{E}_2 d\vec{a}_2$$

$$\vec{E}_1 d\vec{a} + E_2 d\vec{a}_2 = 0$$

$$\oint_A \vec{E}\cdot d\vec{a} = 0$$



Če imamo več akterjev

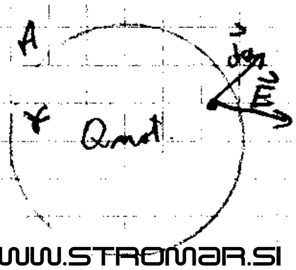
$$\oint_A \vec{E}\cdot d\vec{a} = \oint_A (\vec{E}_1+\vec{E}_2+\vec{E}_3+\vec{E}_4)\cdot da$$

$$= \underbrace{\oint_A \vec{E}_1 d\vec{a}}_{\emptyset} + \underbrace{\oint_A \vec{E}_2 d\vec{a}}_{Q/\varepsilon_0} + \underbrace{\oint_A \vec{E}_3 d\vec{a}}_{Q/\varepsilon_0} + \underbrace{\oint_A \vec{E}_4 d\vec{a}}_{\emptyset}$$

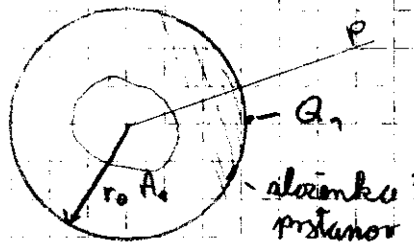
$$= \frac{Q_2+Q_3}{\varepsilon_0} = \frac{Q_{not}}{\varepsilon_0}$$

$$\oint_A \vec{E}\cdot d\vec{a} = \frac{Q_{not.}}{\varepsilon_0}$$ – 1. Maxwellova enačba

Električno polje je izvorno



Zgled: Električno polje naboja na krogelni lupini



$$\sigma = \frac{Q}{4\pi R_0^2} = \text{konst.}$$

$$\Rightarrow \vec{E} = (E_r, 0, 0)$$

notranjost $r < r_0$
krogla polmer r, koncentrično k naelektreni sferi

$$\oint \vec{E}\cdot d\vec{a} = E_r \cdot 4\pi r^2 = 0 \Rightarrow E_r(r<r_0) = 0$$

zunanjost $r > r_0$
krogla polmera r, koncentrična k naelektreni sferi

$$\oint \vec{E}\cdot d\vec{a} \Rightarrow E_r \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E_r(r>r_0) = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E_r = \begin{cases} 0 & r < r_0 \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & r > r_0 \end{cases}$$

Znotraj krogle prostor lahko okovimo. Notranji naboj je enak zunanjemu

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

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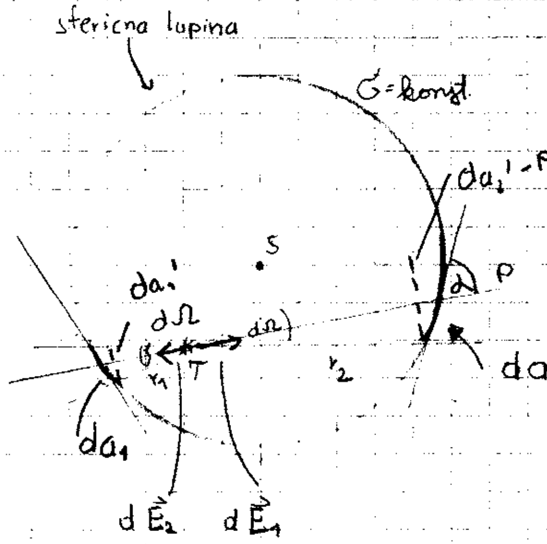
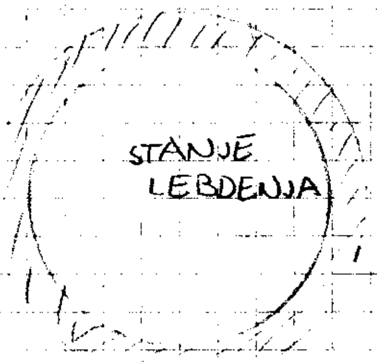


Količina naboja:
$\sigma\, da_1$
$\sigma\, da_2$

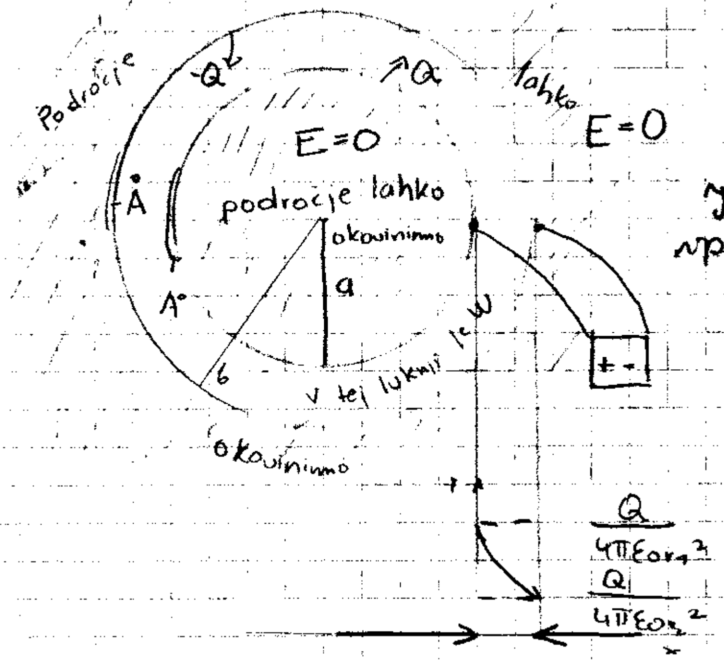
$$|d\vec{E}_1| = dE_1 = \frac{\sigma\, da_1}{4\pi\varepsilon_0 r_1^2}\,\frac{\sin\alpha}{\sin\alpha}\, d\Omega$$

$$|d\vec{E}_2| = dE_2 = \frac{\sigma\, da_2}{4\pi\varepsilon_0 r_2^2}\,\frac{\sin\alpha}{\sin\alpha}\, d\Omega$$

$$\Rightarrow d\vec{E}_1 + d\vec{E}_2 = \vec{0}$$



Zgled: Primer sferičnega kondenzatorja

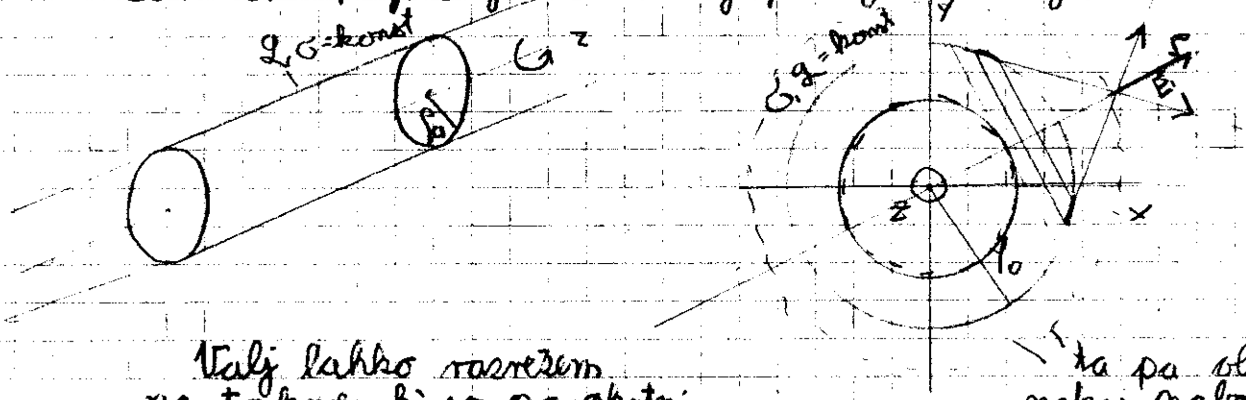


$$E_r \begin{cases} 0 \ ; \ r<a \\ \frac{Q}{4\pi\varepsilon_0 r^2}, \ a \le r < b \\ 0 \ , \ r > b \end{cases}$$

Imamo snovi, ki ne čutijo vpliva električnega polja

Energija se shranjuje v solenoidih (glavnina le-te)

Električna poljska jakost naboja, ki je na valju



Valj lahko razrežem na trakove, ki so pravokotni na premico

ta pa objame nekaj naboja

$$\Rightarrow \vec{E} = (E_r, 0, 0)$$

notranjost:
$\rho < \rho_0$

A_1: plašč valja dolžine l, vzdolž osi z, ki je soosen k naelektrenemu valju

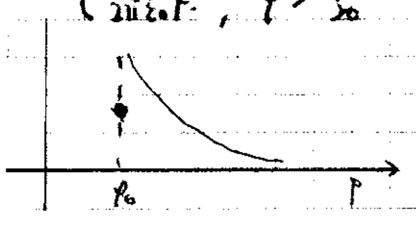
$$\oint_{A_1} \vec{E}\cdot d\vec{a} = \underset{(\pi\rho^2)\,0}{\int_{S_1} \underbrace{\vec{E}_\perp d\vec{a}}} + \underset{(\pi\rho^2)\,0}{\int_{S_2} \underbrace{\vec{E}\cdot d\vec{a}}_{\perp}} + \underset{\underbrace{(2\pi r l)}_{E_r\, 2\pi r l}}{\int_{S_3} \vec{E}_\parallel d\vec{a}} = 0 / \varepsilon_0 \Rightarrow E_r(r<\rho_0)=0$$

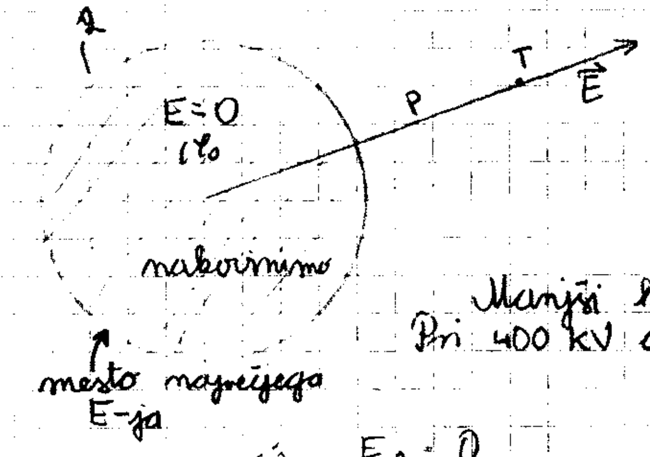
zunanjost: $\rho > \rho_0$
A_2 - plašč polmera ρ, dolžine l vzdolž osi z, ki je soosen k naelektrenemu valju.

$$E_r = \begin{cases} 0, \ \rho < \rho_0 \\ \frac{q}{2\pi\varepsilon_0 \rho}, \ \rho > \rho_0 \end{cases}$$

$$\oint E\cdot d\vec{a} = E_r\, 2\pi r l = \frac{q l}{\varepsilon_0}$$

$$E_r(\rho>\rho_0) = \frac{q}{2\pi\varepsilon_0 r}$$



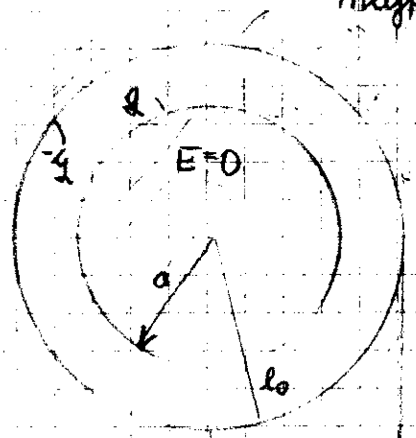


$$E_{(T)} = \frac{q\,\vec{p}}{2\pi\varepsilon_0 p^2}$$

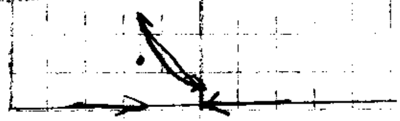
Manjši ko je radij žice, večje je polje okoli. Pri 400 kV daljnovodih so tudi 1 MV/m².

$E_1 = 0$
$E_2 = \sigma/2\varepsilon_0$
$E_3 = \sigma/\varepsilon_0$

Zgled: Koksialni kabel (koaksialc)
- najpogostejša povezava med aparaturami

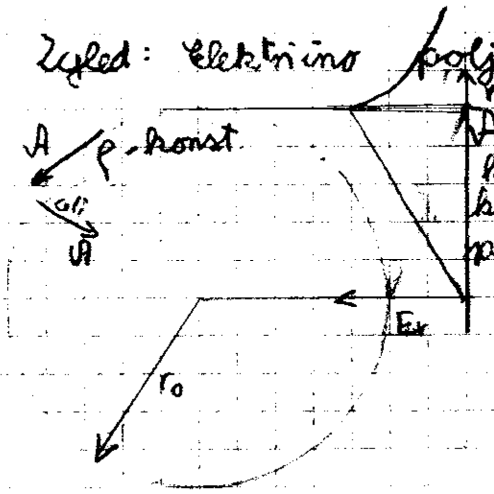


$$E_\rho = \begin{cases} 0 & ; \rho < a \\ \dfrac{q}{4\pi\varepsilon_0 \rho} & ; a < \rho < l_0 \\ 0 & ; \rho > l_0 \end{cases}$$



Antensko telo samo usmerja, sprejme pa seva - Dodajamo jim sevalne diagrame. (uklonska slika - interferenca valovanj, ki izhajajo iz točkastih izvirov)

Zgled: Električno polje krogelnega oblaka



A: naj je krogla polmera r, ki je koncentrična k oblaku. Lahko si mislimo kot zloženko lupino. Za vsako lupino je polje radialno ⇒ tudi za oblak

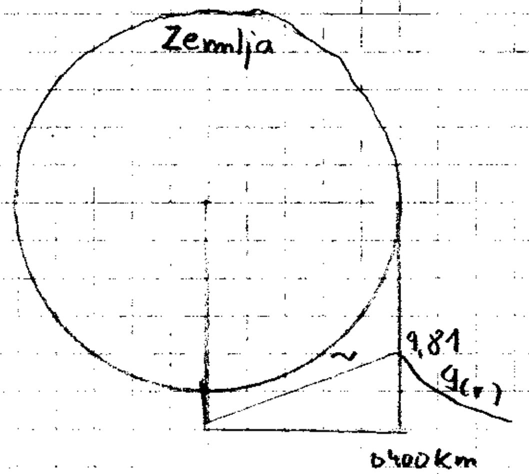
$$\vec{E} = (E_r, 0, 0)$$

$$\oint_A \vec{E} \cdot d\vec{a} = \int_A \overset{\text{konst}}{E_r} \cdot da_r = E_r \cdot 4\pi r^2 = \begin{cases} \dfrac{\rho \cdot \frac{4}{3}\pi r^3}{\varepsilon_0} & , r \le r_0 \\ \dfrac{\rho \cdot \frac{4}{3}\pi r_0^3}{\varepsilon_0} & r > r_0 \end{cases}$$

Vsaka sklenjena ploskev je Gaussova

$$E_r = \begin{cases} \dfrac{\rho r}{3\varepsilon_0} & r \le r_0 \\ \dfrac{\rho r_0^3}{3\varepsilon_0 r^2} & r > r_0 \end{cases}$$

Ob tem si lahko mislimo to kroglo kot planet, npr. zemljo.



Drugače je to velik akumulator energije.

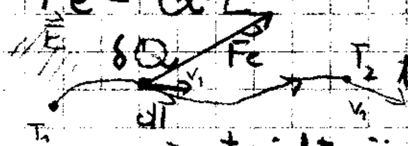
Pomembno za elektrotehniko sta 2 žici in s tem znanjem o tem še ne moremo govoriti.

O Ø

Delo el. sil, potencialna energija, potencial, napetost, nevrtinčnost, II. KIRCHOFFOV ZAKON

Električna sila je naboj × električno polje.

$$\vec{F_e} = Q \cdot \vec{E}$$



ta delec bo šel po neki poti.
$\vec{E}$ - je kot pospešek, ima podoben učinek.
trajektorij, sled

$$d\vec{A_e} = \vec{F_e} \cdot d\vec{l} = Q \cdot \vec{E} \cdot d\vec{l}$$

$$A_{e_{12}} = Q \int_{T_1 \mathcal{L} T_2} \vec{E} \cdot d\vec{l}$$

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Vektor hitrosti je tangenten

$$\delta m \cdot \vec{a} = \delta Q \vec{E} = \delta \vec{F_e}$$

lahko povemo kot III. Newtnov zakon:

$$-\delta m \cdot \vec{a} + \delta Q \vec{E} = \delta \vec{F_e}$$

vztrajnostna sila — skupna gib. moč

Tu smo veliko izpustili, npr. magnetno silo, upora ter druge zunanje sile.

$$(-\delta m \vec{a} + \delta Q \vec{E} + \delta Q \vec{v} \cdot \vec{B} + \delta \vec{F_u} + \delta \vec{F_z} = \vec{0})$$

gibalna enačba — lahko tudi neelektrična

$$dA_e = \delta \vec{F}_e \cdot d\vec{l} = \delta Q \cdot \vec{E} \cdot d\vec{l}$$ za delo

$$\vec{A}_e (T_1 \mathcal{L} T_2) = \delta Q \int_{T_1 \mathcal{L} T_2} \vec{E} \cdot d\vec{l} = \delta m \int_{T_1 \mathcal{L} T_2} \vec{a} \cdot d\vec{l}$$

po poti

$$\int_{T_1 \mathcal{L} T_2} \vec{a} \cdot d\vec{l} = \int_{T_1 \mathcal{L} T_2} \frac{d\vec{v}}{dt} \cdot \vec{v} \cdot dt = \int_{T_1 \mathcal{L} T_2} \vec{v} \cdot d\vec{v} = \int_{T_1 \mathcal{L} T_2} (v_x \cdot dv_x + v_y \cdot dv_y + v_z \cdot dv_z)$$

$$f\,df = f(x)\,df(x)$$
$$(f \cdot f')dx =$$
$$= \frac{1}{2} f^2 dx = \frac{1}{2}\frac{d(f^2)}{dx} dx = \frac{1}{2} d(f^2)$$

$$= \int_{T_1 \mathcal{L} T_2} d(v_x^2 + v_y^2 + v_z^2)$$

$$= \int_{T_1 \mathcal{L} T_2} d(v^2) = v^2(T_2) - v^2(T_1)$$

$$\delta A_e = \frac{\delta m v_2^2}{2} - \frac{\delta m v_1^2}{2} = \delta W_{k_2} - \delta W_{k_1}$$

razlika kinetičnih energij

1. Delo električne sile $\delta \vec{F}_e$, dajale oziroma odvzemale bodo te sile delcu kinetično energijo

$$\delta A_e (T_1 \mathcal{L} T_2) = \delta Q \int_{T_1 \mathcal{L} T_2} \vec{E} \cdot d\vec{l}$$

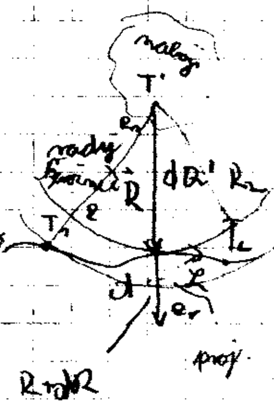
$$= \delta Q \int \left(\frac{1}{4\pi\varepsilon_0} \int \frac{dQ(T')\vec{R}}{R^3} \right) \cdot dl$$

$$= \frac{\delta Q}{4\pi\varepsilon_0} \int dQ(T) \int_{T_1 \mathcal{L} T_2} \frac{\vec{R} \cdot d\vec{l}}{R^3} \Rightarrow \int_{T_1 \mathcal{L} T_2} \frac{\vec{R} \cdot d\vec{l}}{R^3}$$

$$= \int_{T_1 \mathcal{L} T_2} \frac{\left(\frac{\vec{R}}{R}\right) dl}{R^2}$$ – dolžina projekcije

$$= \int_{T_1 \mathcal{L} T_2} \frac{dR}{R^2} = \left(-\frac{1}{R}\right)\Big|_{T_1}^{T_2}$$

$$= \frac{1}{R_1} - \frac{1}{R_2}$$

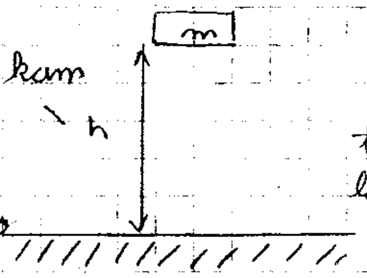


oddaljenost – samo razdalja začetka in konca

$$A_e (T_1 \mathcal{L} T_2) = \delta A_e (T_1, T_2)$$
$$= \delta Q \int_{T_1}^{T_2} \vec{E} \cdot d\vec{l} = \frac{\delta Q}{4\pi\varepsilon_0} \int \left(\frac{1}{R_1} - \frac{1}{R_2}\right) dQ(T')$$

Delo je neodvisno od poti, enaka lastnost kot pri gravitacijski sili.

2. Električna potencialna energija

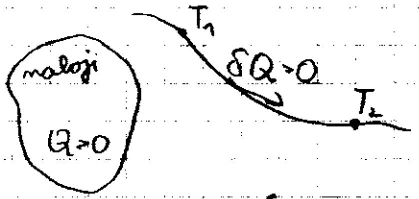


do kam lahko gre

*zmenit se moramo za referenco

$W_p = m g h$

pridobljena W_k do tja kamor lahko gre (lahko pridobi toliko W_k)



naboj se lahko giblje do neskončnosti

$$\delta W_{ep}\ \delta Q \text{ v } T_1 = \delta A_e(T_1, T_\infty)$$

številsko enaki

$$= \delta Q \int_{T_1}^{T_\infty} \vec{E}\cdot d\vec{l}$$

$$= \frac{\delta Q}{4\pi\varepsilon_0} \int \frac{dQ(T')}{R_1}$$

$$\delta W_{ep}\ \delta Q \text{ v } T = \delta Q \int_{T}^{T_\infty} \vec{E}\cdot d\vec{l} = \frac{\delta Q}{4\pi\varepsilon_0} \int \frac{dQ(T')}{R}$$

3. Električni potencial (V)

Wep se nanaša na naboj, pri potencialu pa bi se radi znebili Q-ja.

$$V(T) = \frac{\delta W_{ep}\ \delta Q \text{ v } T}{\delta Q}$$

točka

$$V(T) = \int_{T}^{T_\infty} \vec{E}\cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0} \int \frac{dQ(T')}{R}$$

Označevanje potenciala

T	A	B	C
V(T) ali	V(A) ali	5V ali	-7V

Govorili bomo tudi o negativnih potencialih

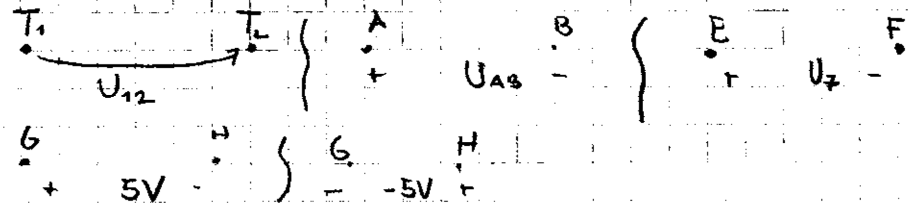
Ekvipotencialne ploskve (imajo enak potencial)

4. Električna napetost → je normirano delo.

$$U_{12} = \frac{\delta A_e(T_1, T_2)}{\delta Q} = \frac{\delta W_{ep}(T_1) - \delta W_{ep}(T_2)}{\delta Q}$$

$$= V(T_1) - V(T_2) = \int_{T_1}^{T_2} \vec{E}\cdot d\vec{l}$$

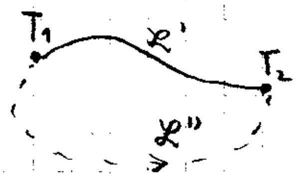
označevanje napetosti : vedno stvar 2 točk



$$U_{A\infty} = V(A) - \underbrace{V(\infty)}_{0}$$

V bistvu bi s tem pojem potenciala lahko ukinili.

5. Nevrtinčnost



Ae je enako $\int_{T_1 \mathcal{L}' T_2} \vec{E} \cdot d\vec{l} = \int_{T_1 \mathcal{L}'' T_2} \vec{E} \cdot d\vec{l}$

$$\int_{T_1 \mathcal{L}' T_2} \vec{E} \cdot d\vec{l} - \int_{T_1 \mathcal{L}'' T_2} \vec{E} \cdot d\vec{l} = 0$$

$$\Rightarrow \int_{T_1 \mathcal{L} T_2} \vec{E} \cdot d\vec{l} + \int_{T_2 \mathcal{L}'' T_1} \vec{E} \cdot d\vec{l} = \vec{0}$$

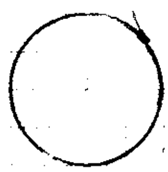
$\langle E_t \rangle = 0$

$$\Rightarrow \oint_{\mathcal{L}} \vec{E} \cdot d\vec{l} = 0 \quad \text{– nevrtinčnost}$$

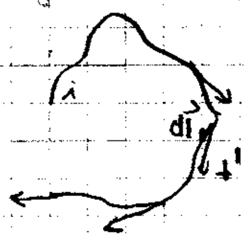
sklenjena pot, po kakršnikoli poti

za inducirano poljsko jakost to ne velja

Vrtinčnost in izvornost - s tem opišemo izviranje in ponikanje, s tem ovrednotimo določene pojave $\vec{E}$. Pokažemo njegove značilnosti. Dobimo občutek vrtinca pri sliki mag. polja.



Če na krivuljo rišem vektorje imajo tangentne vektorje

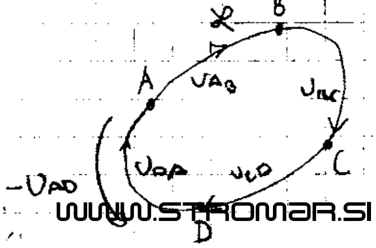


$$\vec{E} \cdot d\vec{l} = \vec{E} \cdot \vec{t}\ dl = E_t$$

$\oint E_t \, dl = 0$ povprečje funkcije je potem 0 $\Rightarrow \langle E_t \rangle = 0$

ne izkazuje nobene komp. Ae.

6. II. Kirchoffov zakon. Počiva na formuli nevrtinčnosti.

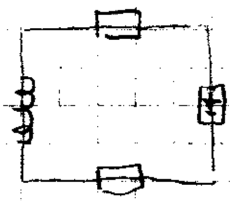


$$\oint_{\mathcal{L}} \vec{E} \cdot d\vec{l} = \underbrace{\int_A^B \vec{E} \cdot d\vec{l}}_{U_{AB}} + \underbrace{\int_B^C \vec{E} \cdot d\vec{l}}_{U_{BC}} + \underbrace{\int_C^D \vec{E} \cdot d\vec{l}}_{U_{CD}} + \underbrace{\int_D^A \vec{E} \cdot d\vec{l}}_{U_{DA}}$$

$$\Rightarrow U_{AB} + U_{BC} + U_{CD} + U_{DA} = 0 \rightarrow \sum_m (\pm) U_m = 0$$

1. Kirch: $\sum_k (\pm) I_k = 0$ ↙ časovno nespremenljive oblike pišemo z velikimi, nečasovne pa z malo (tihi dogovor)

Primer: zančna enačba

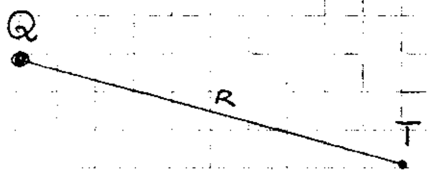


OET I
o.11.2007

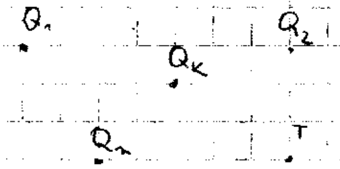
ZGLEDI

1. Potencial okoli točkastega naboja

$$\vec{E}(T) = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ(T')\vec{R}}{R^3}$$

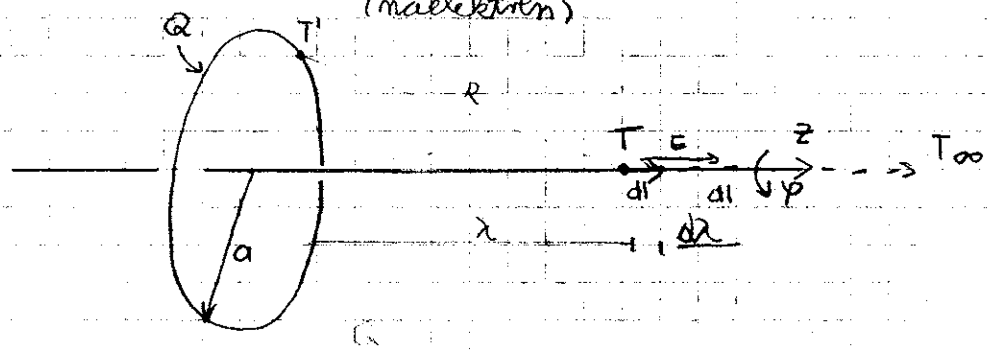


$$V(T) = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ(T')}{R} = \frac{Q}{4\pi\varepsilon_0 R}$$

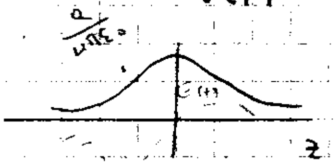


$$V(T) = \sum_{k=1}^{n} V^k(T) = \sum_{k=1} \frac{Q_k}{4\pi\varepsilon_0 R_k} = \frac{1}{4\pi\varepsilon_0}\sum_k^n \frac{Q_k}{R_k}$$

2. Potencial okoli prstana/navoja v osi.
(naelektren)



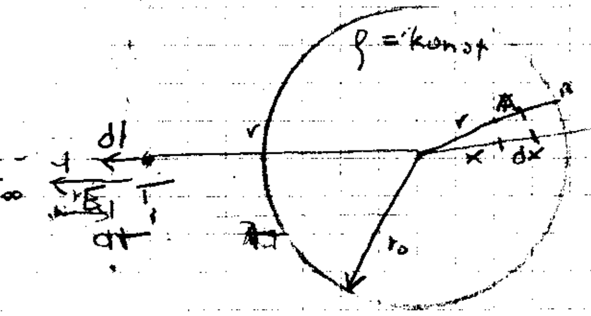
a) $V(T) = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ(T')}{R} = \frac{Q}{4\pi\varepsilon_0 R} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{\sqrt{a^2+z^2}} = V(z)$



b) $V(T) = \int_T^{T_\infty} \vec{E}\cdot d\vec{l} = \int_z^\infty E_z(\lambda)\,d\lambda = \int_z^\infty \frac{Q}{4\pi\varepsilon_0}\frac{\lambda}{(a^2+\lambda^2)^{3/2}}\,d\lambda$

$$= \frac{Q}{4\pi\varepsilon_0}\left(-\frac{1}{\sqrt{a^2+\lambda^2}}\right)\Bigg|_z^\infty = \frac{Q}{4\pi\varepsilon_0}\frac{1}{\sqrt{a^2+z^2}} = V(z)$$

3. V in ob krogelnem oblaku nabojev



$$E_r = \begin{cases} \dfrac{\rho r}{3\varepsilon_0} & r < r_0 \\ \dfrac{\rho r_0^3}{3\varepsilon_0 r^2} & r > r_0 \end{cases}$$

zunanjost, $r > r_0$

$$V(T) = \int_{T}^{T_\infty} \vec{E}\cdot d\vec{l} =$$

$$= \int_r^\infty E_r(t)\,dt = \int_r^\infty \frac{\rho_0 r_0^3}{3\varepsilon_0}\frac{dt}{t^2} = \frac{\rho_0 r_0^3}{3\varepsilon_0}\left(-\frac{1}{t}\right)\Big|_r^\infty = \frac{\rho r_0^3}{3\varepsilon_0 r} = V(r)$$

$r > r_0$

na površini $(r = r_0)$

$$V(r = r_0) = \frac{\rho r_0^2}{3\varepsilon_0} = V(B)$$

v notranjosti $(r < r_0)$

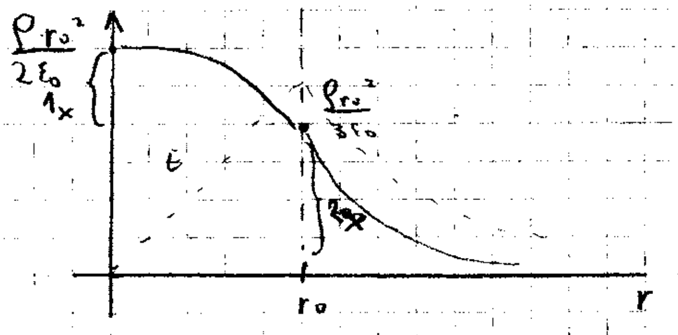
$$V(A) - V(B) = \int_A^B \vec{E}\cdot d\vec{l} = \int_r^{r_0} \frac{\rho x}{3\varepsilon_0}\,dx$$

$$= \frac{\rho}{3\varepsilon_0}\frac{x^2}{2}\Big|_r^{r_0} = \frac{\rho_0}{6\varepsilon_0}(r_0^2 - r^2)$$

$$V(r) - V(r_0) = \frac{\rho}{6\varepsilon_0}(r_0^2 - r^2)$$

$$V(r) = \frac{\rho r_0^2}{3\varepsilon_0} + \frac{\rho}{6\varepsilon_0}(r_0^2 - r^2)$$

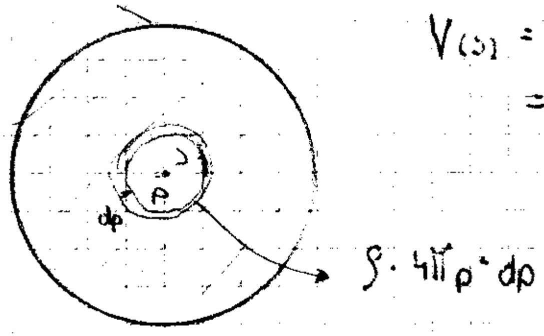
$$= \frac{\rho r_0^2}{2\varepsilon_0} - \frac{\rho r^2}{6\varepsilon_0} = \frac{\rho r_0^2}{2\varepsilon_0}\left(1 - \frac{1}{3}\left(\frac{r}{r_0}\right)^2\right)$$



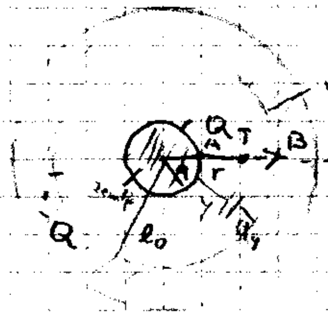
4. Potencial v centru oblaka

$$V(0) = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ(T)}{R}$$

$$= \frac{1}{4\pi\varepsilon_0}\int_0^{r_0} \frac{\rho\, 4\pi\, \rho^2\, d\rho}{\rho} = \frac{\rho r_0^2}{2\varepsilon_0}$$



5. Potencial v sferičnem kondenzatorju



$$E_r = \begin{cases} 0, & r < a \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}, & a < r < b \\ 0, & r > b \end{cases}$$

zunanjost $r > b$

$$V(r \geq b) = 0$$

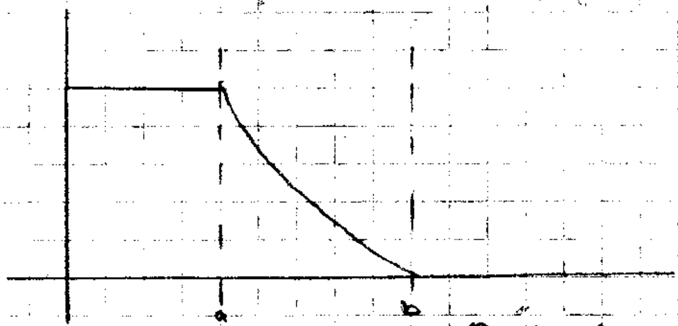
vmes:

$$V(T) - \underbrace{V(B)}_{0} = \int_T^B \vec{E}_i \, d\vec{l} = \int_r^b \frac{Q}{4\pi\varepsilon_0 y^2}\, dy = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{b}\right) = V(r)$$

$$V(A) = V(r=a) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q(b-a)}{4\pi\varepsilon_0 ab}$$

notranjost $(r < a)$

$$V(r \leq a) = V(A) = \frac{Q(b-a)}{4\pi\varepsilon_0 ab}$$



napetost: $U_{AB} = V(a) - V(b) = \dfrac{Q(b-a)}{4\pi\varepsilon_0 ab}$

Kapacitivnost je merilo za sposobnost prejemanja, se opremiti z nabojem.

$$C = \frac{Q}{U_{AB}} = \frac{4\pi\varepsilon_0 ab}{b-a}$$

Šum je motnja, ki se pojavlja v sferičnem kond.
$\nu_{min} = 8\,Hz$ ↓
ker je prazen prostor

primer: zemlja, ionosfera
$a \cong 6400\,km$ $\quad b \cong 6550\,km$

uporabimo lahko tudi pri osamljeni krogli. Te motnje domujejo le tu

$b \to \infty$

$C = 4\pi\varepsilon_0 a$

primer $b \cong a$

$\Rightarrow b - a = d$

$$\Rightarrow C \cong \frac{4\pi\varepsilon_0 a^2}{d} = \frac{\varepsilon_0 S}{d}$$

kapacitivnost ploščnega kondenzatorja

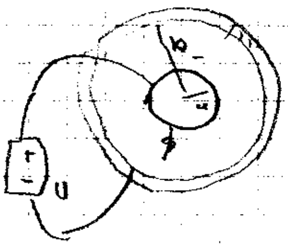
Optimizacija geometrije sf. kond.

$$E_r(r=a) = \frac{Q}{4\pi\varepsilon_0 a^2} = \frac{abU}{(b-a)a^2} = \frac{bU}{a(b-a)}$$

spremen.

$$4\pi\varepsilon_0 = \frac{abU}{b-a}$$

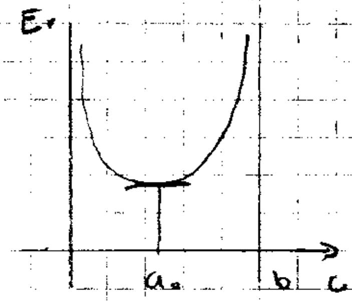
To je izbira rešitve optimalnega v danem primeru.



rešimo: U, b sta izbrana

iščem primeren radij za največjo E, da se ne bodo presegali prebojno trdnosti zraka

a je sedaj spremen.



$$g(a) = a(b-a)$$
$$g'(a) = b-2a$$
$$g'(a_0) = b - 2a_0 = 0$$
$$a_0 = \frac{b}{2}$$

6. Primer: potencial pri ploščnem kondenzatorju

OE I
7. 11. 2007

$$E_x = \begin{cases} 0, & x<0 \\ \frac{\sigma}{\varepsilon_0}, & 0<x<d \\ 0, & x>d \end{cases}$$

Slika je zrcalna preko simetrale

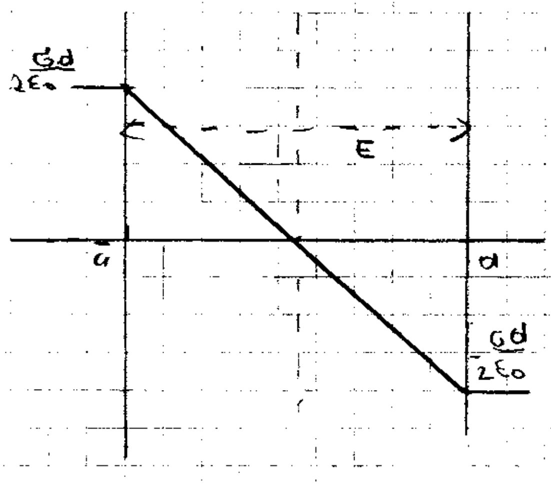
$$V(T) = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ(T')}{R}$$

poz. V | neg V

$$V(x = d/2) = 0$$
$$V>0 \text{ za } x<d/2$$
$$V<0 \text{ in } x>d/2$$

$$V(A) - V(B) = 0 - V(x) = \int_A^B \vec{E}\cdot d\vec{l} = \int_{d/2}^{x} \frac{\sigma}{\varepsilon_0}\,dt = \frac{\sigma}{\varepsilon_0}\left(x - \frac{d}{2}\right)$$

$$V(x) = \frac{\sigma}{\varepsilon_0}\left(\frac{d}{2} - x\right) \quad 0<x<d$$

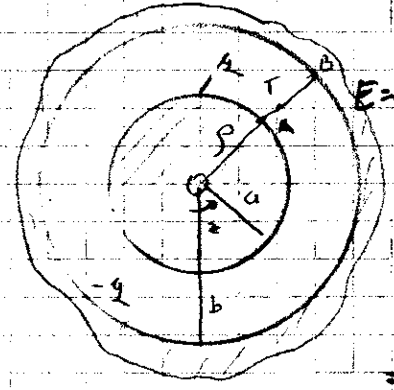


$$U_{1,2} = V(x=0) - V(x=d) = \frac{\sigma}{\varepsilon_0}d = E_x d \qquad E_x = \frac{U_{12}}{d}$$

$$U_{12} = \frac{\sigma}{\varepsilon_0}d = \frac{Q/S}{\varepsilon_0}d$$

$$\Rightarrow C = \frac{Q}{U_{12}} = \varepsilon_0\frac{S}{d}$$

7. Potencial v koaksialnem kablu



$$E_\rho = \begin{cases} 0 & \rho < a \\ \frac{q}{2\pi\varepsilon_0\rho}, & a < \rho < b \\ 0 & \rho > b \end{cases}$$ – polje enakega pot.

$$V(\rho > b) = 0$$

$$V(T) - \underbrace{V(B)}_{0} = V(\rho) - 0 = \int_T^B \vec{E}\cdot d\vec{l}$$

$$= \int_\rho^b \frac{q}{2\pi\varepsilon_0 s}\, ds = \frac{q}{2\pi\varepsilon_0} (\ln s)\Big|_\rho^b$$

$$V(\rho) = \frac{q}{2\pi\varepsilon_0} \ln \frac{b}{\rho}$$

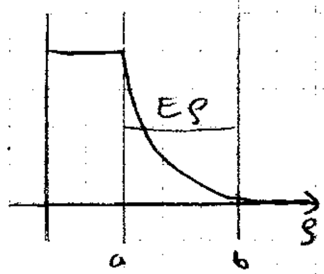
$$V(A) = V(a) = \frac{q}{2\pi\varepsilon_0} \ln \frac{b}{a} \qquad V(\rho < a) = V(a)$$

$$U_{AB} = V(A) - V(B) = \frac{\cancel{q\,l}\ Q}{2\pi\varepsilon_0 l} \ln \frac{b}{a}$$

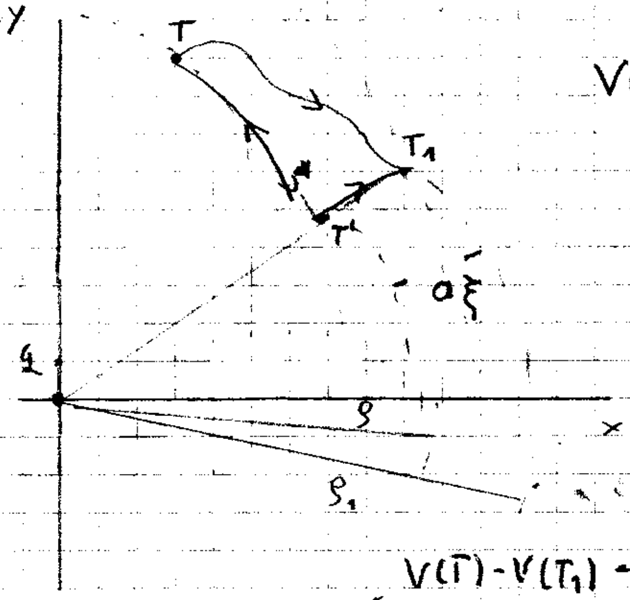
$$\frac{Q}{U_{AB}} = C = \frac{2\pi\varepsilon_0 l}{\ln b/a} = \frac{q}{U_{AB}} = \frac{2\pi\varepsilon_0}{\ln b/a} \qquad b/a = e$$

$= c$ kapacitivnost na enoto dolžine

$$c = \frac{2\pi\cdot 10^{-9}}{36\pi} = \frac{10^{-9}}{18} = 0{,}055\cdot 10^{-9} = 55\cdot 10^{-12} = 55\ pF/m$$



8. Potencial v okolici linijskega naboja (naelektrena premica)



$$V(T) - V(T_1) = \int_T^{T_1} \vec{E}\cdot d\vec{l}$$

$$= \underbrace{\int_T^{T'} \vec{E}\cdot dl}_{0} + \int_{T'}^{T_1} E\cdot dl$$

$$= \int_\rho^{\rho_1} \frac{q}{2\pi\varepsilon_0 \xi}\, d\xi$$

$$= \frac{q}{2\pi\varepsilon_0} \ln \frac{\rho_1}{\rho}$$

$$\Downarrow$$

$$V(T) - V(T_1) = V(\rho) - V(\rho_1) = \frac{q}{2\pi\varepsilon_0}\ln \rho_1 - \frac{q}{2\pi\varepsilon_0}\rho$$

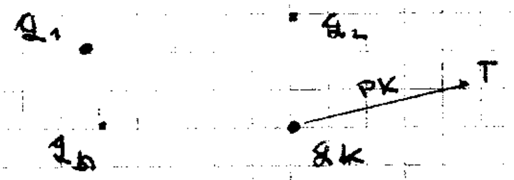
$$V(\rho) = -\frac{q}{2\pi\varepsilon_0} \ln \rho + C$$ – potencial ob linijskem naboju

$$V(\rho_1) = -\frac{q}{2\pi\varepsilon_0} \ln \rho_1 + C$$

$$V(T) = -\frac{q}{2\pi\varepsilon_0}\ln\rho + C$$ ← določen je do konstante natančno, 2 problem je logaritem vrednosti

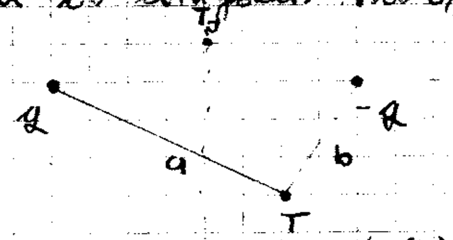
$$= \frac{q}{2\pi\varepsilon_0}\ln\frac{1}{\rho} + C$$

Kakšen pa je potencial večih linijskih nabojev



$$V(T) = \sum_{k=1}^{n} V^{(k)}(T) = \left(\sum_{k=1}^{n}\frac{q_k}{2\pi\varepsilon_0}\ln\frac{1}{\rho_k}\right) + C$$

8. Potencial od linijskih nabojih $\pm q$.



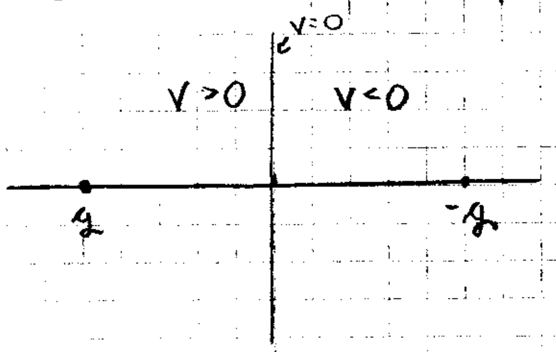
$$V(T) = \frac{q}{2\pi\varepsilon_0}\ln\frac{1}{a} + C_1 + \frac{(-q)}{2\pi\varepsilon_0}\ln\frac{1}{b} + C_2$$

$$V(T) = \frac{q}{2\pi\varepsilon_0}\left(\ln\frac{b}{a} + C\right)$$

↘ ko gre točka proti neskončnosti je koef 1 in V_∞ ln je potem 0

$V(T_\infty) = 0 \Rightarrow C = 0$

$V(T) = \frac{q}{2\pi\varepsilon_0}\ln\frac{b}{a}$ ⇒ $T_0 = 0$, če je na simetrali



Sedaj poznam tole $V(T) = \int_T^{T_0}\vec{E}\cdot d\vec{l}$. Sedaj se sprašujem ali poznam pot iz V k E-ju, ali obstoja.

KAKO OD V-JA K E-JU

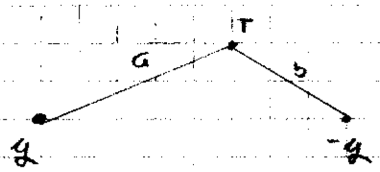
$\vec{E}$ je vektor s komponentami (E_x, E_y, E_z). Nekje bomo kot kaže imeli 3 odvode.

Ekvipotencial(ke) – množica točk na katerih je potencial enake vrednosti. Ekvipotencialke oblikuje družina koncentričnih krogel.

$\frac{Q}{4\pi\varepsilon_0 R}$

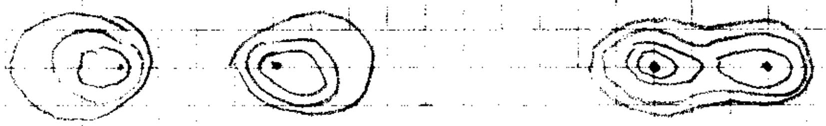
Potencialke pri linijskem naboju so koncentrični valji - tak primer je pri koaksialcu.

$$\frac{q}{2\pi\varepsilon_0} \ln \frac{A}{B}$$



$$\frac{q}{2\pi\varepsilon_0} \ln \frac{b}{a} \qquad b/a \text{ konst.}$$

a/b = krog (graf)
$a \cdot b$ = Cassinijeve krogle



Imejmo eno ekvipotencialko

$$V(T) - V(T_1) = \int_T^{T_1} \vec{E} \cdot \vec{dl} = V(\vec{r}) - V(\vec{r_1}) \qquad r_1 = r + \vec{\delta l}$$

$$\int_a^{a+\delta a} f(x)\,dx \cong f\left(a + \frac{\delta a}{2}\right) \cdot \delta a$$

$$\vec{E} \cdot \delta \vec{l}$$

$$V(\vec{r} + \delta \vec{l}) - V(\vec{r}) \simeq -\vec{E} \cdot \delta \vec{l}$$

$$\delta V \cong -\vec{E}\, \delta \vec{l} \quad \text{- če je dovolj majhen}$$

$$dV = -\vec{E}\, d\vec{l}$$

a) bodi $\delta \vec{l} = \delta \vec{l}_t$
(premikam se po ekvipotencialki)
$\Rightarrow \delta V = 0$
$\Rightarrow \vec{E} \cdot \delta \vec{l}_t = 0 \quad \Rightarrow \vec{E} \perp \delta \vec{l}_t$

vektor leži na premici, ki je $\perp$ na potencialko

$$\Rightarrow \vec{E} = \vec{n} \cdot E_n$$

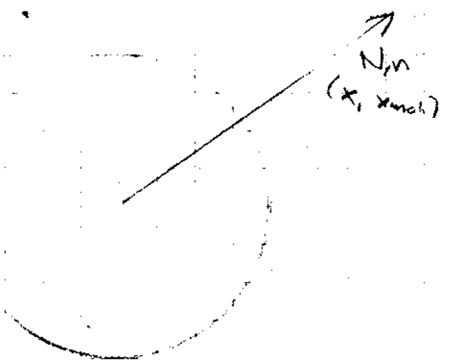
normalni vektor

$\delta V \cong -E\,\delta l$

1. $\delta l = \delta l_T$, $\delta V = 0$, $E \cdot \delta \vec{l}_T = 0$

$\Rightarrow E \perp \delta \vec{l}_T \Rightarrow \vec{E} = n \cdot E_n$

2. $\delta \vec{l} = \delta \vec{l}_n = \vec{n} \cdot \delta n$



$\delta V = -(\vec{n} \cdot \vec{E}_n) \cdot (\vec{n}\,\delta n) = -E_n\,\delta n$

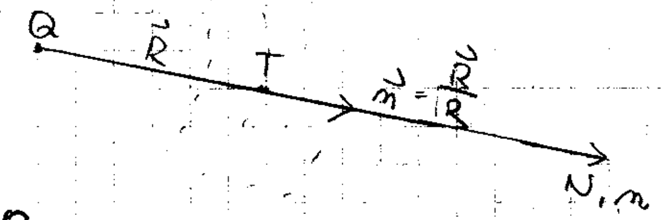
$E_n \simeq -\frac{\delta V}{\delta n} \Rightarrow E_n = -\lim_{\delta n \to 0} \frac{\delta V}{\delta n}$

$E_n = -\frac{\partial V}{\partial n}$ – parcialni / delni odvod (hitrost spreminj. funkcije v določeni smeri)

$\vec{E} = -\vec{n} \frac{\partial V}{\partial n}$

ZGLEDI

1. točkast naboj

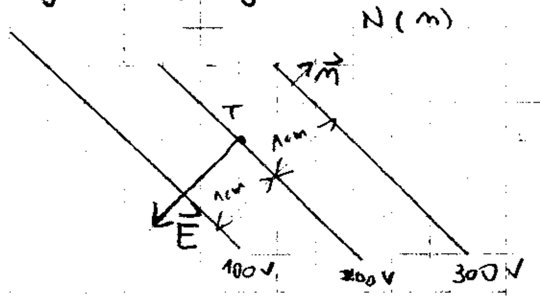


$V(T) = \frac{Q}{4\pi\varepsilon_0 R}$

$\frac{\partial V}{\partial n} = \frac{\partial V}{\partial R} = -\frac{Q}{4\pi\varepsilon_0 R^2}$

$E = -\vec{n} \frac{\partial V}{\partial n}$
$= -\frac{\vec{R}}{R}\left(-\frac{Q}{4\pi\varepsilon_0 R^2}\right)$
$= \frac{\vec{R} Q}{4\pi\varepsilon_0 R^3}$

2. homogeno polje



1. $\frac{\partial V}{\partial n} = \frac{300\,V - 100\,V}{2 \cdot 10^{-2}\,m} = 10^4\,V/m$

$\vec{E} = -n \cdot 10^4\,V/m$

2. normala v drugo smer

$\frac{\partial V}{\partial n} = \frac{100\,V - 300\,V}{2 \cdot 10^{-2}} = -10^4\,V/m$

$\vec{E} = -\vec{n} \cdot 10^4\,V/m = n \cdot 10^4\,V/m$

E kaže v smer potenciala, ki pada. Kaže od večjega potenciala k manjšemu.

3. $\delta \vec{l}_1 = (\delta x, 0, 0)$

$$\vec{E} = (E_x, E_y, E_z)$$

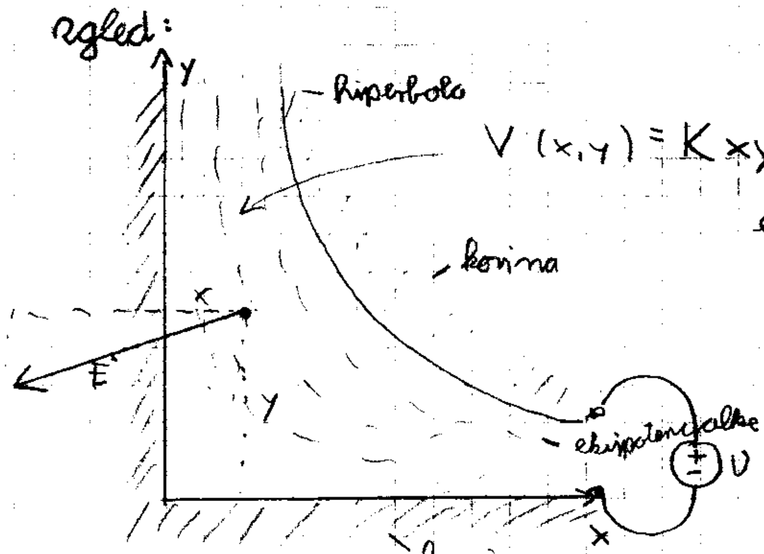
$$\delta V = -\vec{E}\cdot\delta\vec{l}_1 = -E_x \delta x$$

$$E_x = -\lim_{\delta x \to 0} \frac{\delta V}{\delta x} = -\frac{\partial V}{\partial x}$$ enako velja za y in z.

$$E = \left(-\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z}\right)$$

x komponenta E_y E_z

zgled:



ekvipotencialka $V_1 = Kxy \Rightarrow y = \frac{V_1}{K}\frac{1}{x}$

kovina (tu potencial vedno nič)

y in z je konst

$$E_x = -\frac{\partial V}{\partial x} = -Ky$$

$$E_y = -\frac{\partial V}{\partial y} = -Kx$$

E je pravokoten na potencialke in kaže padanje potenciala

4. $\delta \vec{l} = (\delta \rho, 0, 0)$

$$E = (E_\rho, E_\varphi, E_z)$$

$$\delta V = -E_\rho \delta \rho$$

$$E_\rho = -\lim_{\delta \rho \to 0} \frac{\delta V}{\delta \rho} = -\frac{\partial V}{\partial \rho}$$

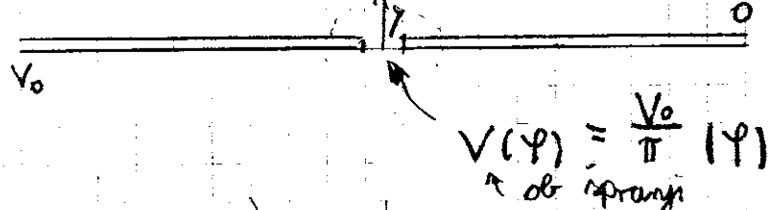
$\delta l_2 = (0, \rho \delta \varphi, 0)$

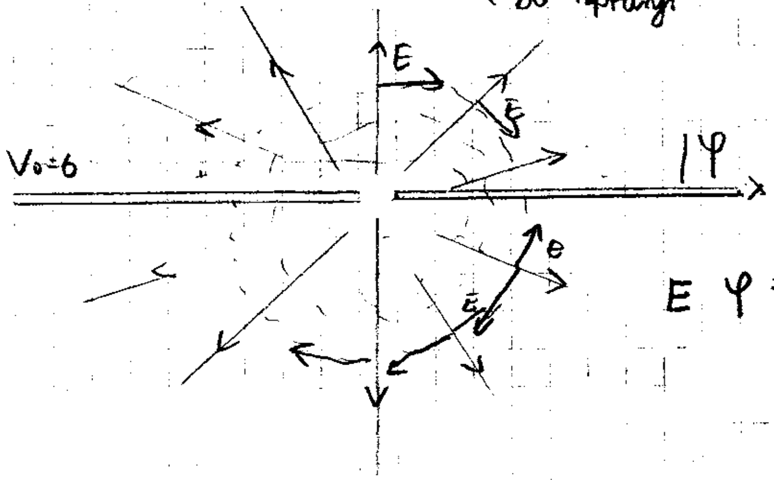
$$\delta V = -E_\varphi \rho \delta \varphi$$

$$E_\varphi = -\frac{1}{\rho}\frac{\partial V}{\partial \varphi}$$

valjne struk.: $E = \left(-\frac{\partial V}{\partial \rho}, -\frac{1}{\rho}\frac{\partial V}{\partial \varphi}, -\frac{\partial V}{\partial z}\right)$

nazled





$$E\varphi = -\frac{1}{\rho}\frac{\partial V}{\partial \varphi} = -\frac{V_0}{\pi\rho}$$

5 $\delta \ell_1 = (\delta r, 0, 0)$

$E = (E_r, E_\vartheta, E_\varphi)$

$\delta V \simeq -E_r \delta r$

$E_r = -\frac{\partial V}{\partial r}$

$\delta \ell_2 = (0, E\vartheta, 0)$

$E_\vartheta = -\frac{1}{r}\frac{\partial V}{\partial \vartheta}$

$\delta \ell_3 = (0, r\cdot \sin\vartheta, 0)$

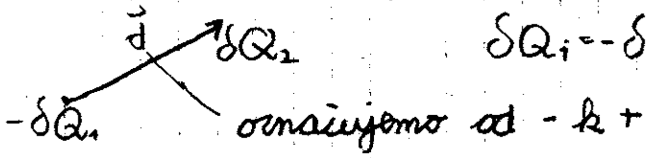
$E_\varphi = -\frac{1}{r\sin\vartheta}\frac{\partial V}{\partial \varphi}$

$$E = \left(-\frac{\partial V}{\partial r}, -\frac{1}{r}\frac{\partial V}{\partial \vartheta}, -\frac{1}{r\sin\vartheta}\frac{\partial V}{\partial \varphi}\right)$$

ELEKTRIČNI DIPOL

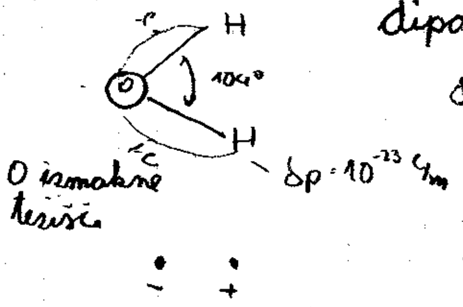
Lahko bi rekli tudi dvopol, samo za elektrotehnika to pomeni 2 sponki v škatli. Lahko je tudi n pol (čip).

... sta mišljena 2 naboja.



$\delta Q_1 = -\delta Q_2$

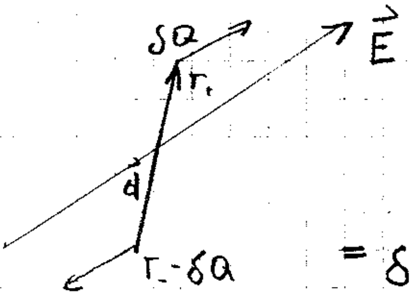
V kemiji polarne in nepolarne molekule. Polarna je npr. voda



dipolski moment

$\delta\vec{p} = \delta Q \vec{d}$

1. Navor na dipol

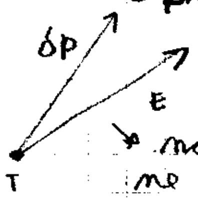


$$\delta \vec{M}_e = \left(\frac{\vec{d}}{2}\right) \times \delta Q E(T_+) + \left(-\frac{\vec{d}}{2}\right) \times (-\delta Q) \vec{E}(T_-)$$

$$= \delta Q \vec{d} \times \underbrace{\frac{1}{2}\left[\vec{E}(T_+) + \vec{E}(T_-)\right]}_{\sim E(T)}$$

$$\simeq \delta \vec{p} \times \vec{E}$$

$\delta \vec{M}_e = \delta \vec{p} \times \vec{E}$, če gre za točkast dipol



navor bo deloval toliko časa, dokler ne bosta vektorja poravnana

2. Potencialna energija dipola:

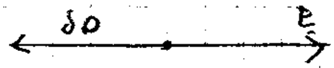
$$\delta W_p = \delta Q V(T_+) + (-\delta Q) V(T_-)$$

$$= \delta Q \underbrace{\left[V(T_+) - V(T_-)\right]}_{\int_{T_+}^{T_-} (E \cdot dl) \ \sim \vec{E} \cdot d}$$

$$= -\delta Q \cdot d \cdot \vec{E} = -\delta p \cdot \vec{E}$$

$$\delta W_p = -\delta \vec{p} \cdot \vec{E}$$

labilna lega



$$\delta \vec{M}_e = 0$$

$$\delta W_p = |\delta \vec{p}| \, |\vec{E}| > 0$$

stabilna

$$\delta \vec{M}_e = 0$$

$$\delta W_p = -|\delta \vec{p}| \cdot |\vec{E}| < 0$$

3. Sila na dipol

$$\delta \vec{F}_e = \delta Q E(T_+) + (-\delta Q) E(T_-)$$

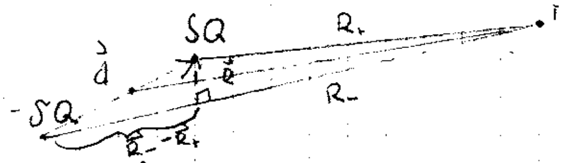
$$= \delta Q \left[E(T_+) - E(T_-)\right]$$

temu se reče dielektroforeza

$$\delta \vec{F} = \left(\delta p_x \frac{\partial \vec{E}}{\partial x} + \delta p_y \frac{\partial \vec{E}}{\partial y} + \delta p_z \frac{\partial \vec{E}}{\partial z}\right)$$

če bo nehomogeno polje

Električno polje dipola:



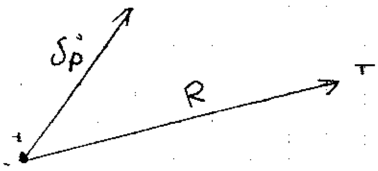
$$|\vec d| <<< |\vec R|$$

$$\delta V(T) = \frac{\delta Q}{4\pi\varepsilon_0 R_+} + \frac{-\delta Q}{4\pi\varepsilon_0 R_-} = \frac{\delta Q}{4\pi\varepsilon_0}\boxed{\frac{R_- - R_+}{R_+ R_-}} =$$

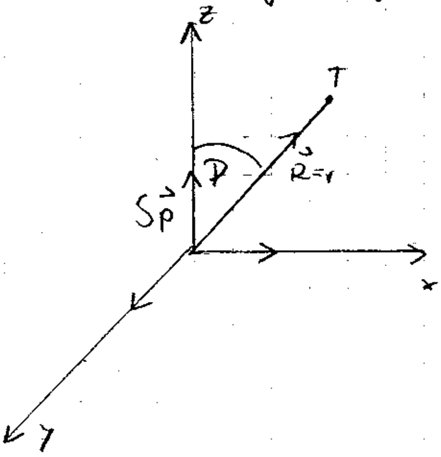
$$= \frac{\delta Q}{4\pi\varepsilon_0}\frac{d\cdot(\vec R/R)}{R^2}$$ — pravokotna projekcija

$$= \frac{\delta Q\,\vec d}{4\pi\varepsilon_0}\frac{\vec R}{R^3}$$

$$\delta V(T) = \frac{\delta p\cdot\vec R}{4\pi\varepsilon_0 R^3}$$



Vektor poljske jakosti



$\vec R = \vec r$, $\delta\vec p = (0, 0, \delta p_z)$

$$\delta V(T) = \frac{\delta p_z \cdot r\cdot\cos\vartheta}{4\pi\varepsilon_0 r^3} = \frac{\delta p}{4\pi\varepsilon_0}\frac{\cos\vartheta}{r^2}$$

$$= \delta V(T, \vartheta)$$

sferični kondenzator

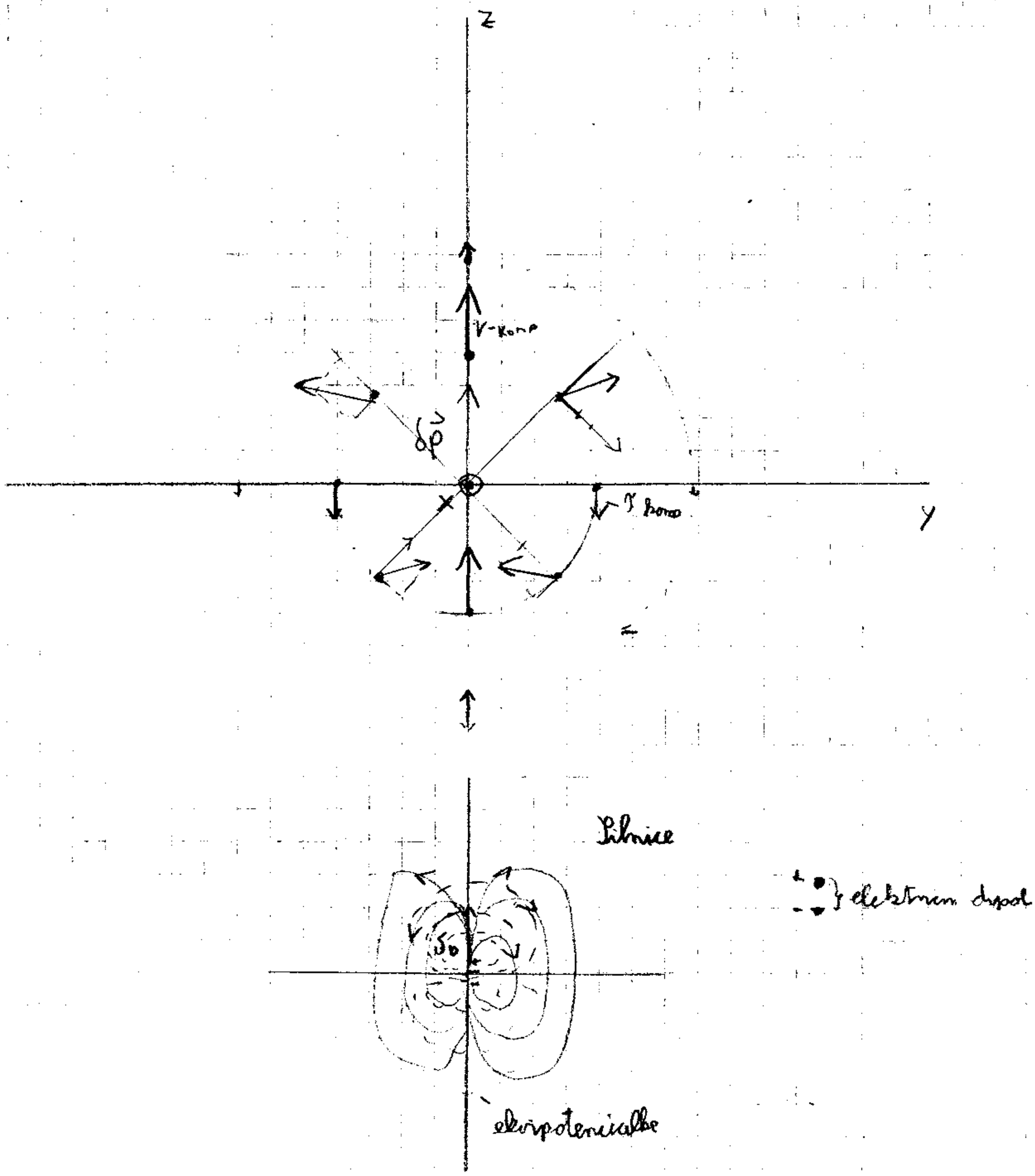
POLJE V SFERIČNEM SISTEMU

$$\vec E = \Big(\underbrace{-\frac{\partial V}{\partial r}}_{E_r};\ \underbrace{\frac{1}{r}\frac{\partial V}{\partial\vartheta}}_{E_\varphi},\ -\frac{1}{r\cdot\sin\vartheta}\frac{\partial V}{\partial\varphi}\Big).$$

$$\delta E_r = -\frac{\partial\,\delta V}{\partial r} = \frac{\delta p}{4\pi\varepsilon_0}\frac{2\cos\vartheta}{r^3}$$

$$\delta E_\vartheta = -\frac{1}{r}\frac{\delta V}{\delta\vartheta} = \frac{\delta p}{4\pi\varepsilon_0}\frac{\sin\vartheta}{r^3}$$

$$\delta E_\varphi = 0$$



Rekapitulacija

* $\delta F_L = \delta Q(\vec E + w \times \vec B)$
* $\rho\ (\sigma, q)$; $J = \vec w \cdot \rho$
* $i = \lim \frac{\delta Q}{\delta t}$ $\quad i = -\frac{dQ_{not.}}{dt}$

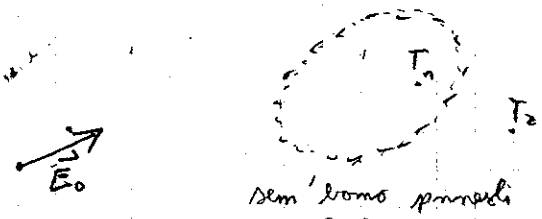
viri električ polja so naboji in tudi tisti, ki se gibljejo

- $E(T) = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ(T')}{R^3}\vec R$
- $\oint_A \vec E \cdot d\vec A = \frac{Q_{not}}{\varepsilon_0}$
- $\vec E = \vec n \frac{\partial V}{\partial n}$
- $V(A) - V(B) = \int_A^B \vec E \cdot d\vec l = U_{AB}$
- $V(A) = \int_A^{r_0} \vec E \cdot d\vec l = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ(T')}{R}$

PREVODNIK v ELEKT POLJE

INFLUENCA

PRED



sem bomo prinesli objekt

PO

na nabite delce deluje neka sila
Na proton deluje zelo majhna sila.

presežek elektronov

Ko se razdelitev začne električna influenca

$$\vec{E} = \vec{E}_0 + \vec{E}_1$$

novo polje

staro

reakcijsko polje

ker naboji funkcionirajo kot dipol

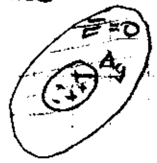
$\vec{E}(T_n) = ? = 0$
pokažemo s protislovjem

$* E(T_n) \neq \vec{0}$ - v majhnem območju ne more biti neskončno nabojev
gibali bi se delci, tekel bi tok in prevodnik bi se segreval, kar pa po naših skušnjah ni mogoče.

$\Rightarrow$ ostane nam le $E(T_n) = 0$
velja za elektrostatično polje

Posledice:

1.



$$\underbrace{\oint_A \vec{E} \cdot d\vec{a}}_{0} = \frac{Q_{not}}{\varepsilon_0}$$

$\Rightarrow Q_{not.} = 0$ imam znotraj toliko + kot -.

$\Rightarrow \rho(T_n) = 0$

2. Lokalni presežek nabojev se pojavi le na robu prevodnika. Influenciran naboj se pojavi ob površini prevodnika z gostoto σ.

3. Kreatorji reakcijskega polja E_1 so naboji s površine prevodnika.

$$E_1(T) = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma(T') da'}{R^3}$$

$$V_1(T) = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma(T') da'}{R}$$

R - oddaljenost

4. $E(T_n) = E_0(T_n) + E_1(T_n) = 0$

$$\Rightarrow \vec{E}_1(T_n) = -\vec{E}_0(T_n)$$

5.) Imamo točki A, B v prevodniku potem sledi:

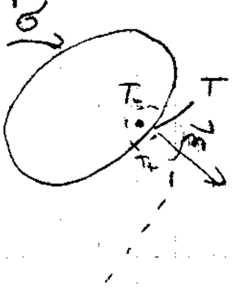
$$\int_A^B \vec{E} \cdot dl = V(A) - V(B) = U_{AB} = 0$$

$\Rightarrow$ prevodnik je ekvipotencialno telo

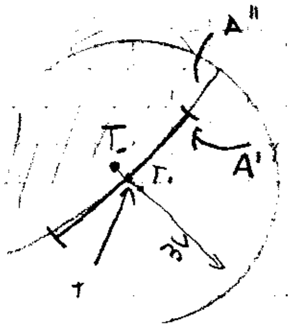
6.) Površina prevodnika je ekvipotencialka, kjer V = konst.

$$\vec{E}(T_+) = \vec{n} \cdot E_n(T_+)$$
$$= -\vec{n} \, \frac{\partial V}{\partial n}(T_+)$$

7. E ob površini prevodnika je nezvezna.



Polje preskoči iz 0 k vrednosti
T - točka kjer se nahaja naboj (površini)



neposredno površino ob točkah, ki se nam zdi ravna označimo z A', ostalo z A''.

$$E = E' + E''$$

polje nabojev na A' — polje nabojev na A'' in izven telesa

$$\vec{E}(T_+) = \vec{E}'(T_+) + \vec{E}''(T_+) = \vec{n} \, \frac{\sigma(T_+)}{\varepsilon_0}$$

$$\vec{E}(T) = \vec{E}'(T) + \vec{E}''(T) = \vec{n} \, \frac{\sigma(T)}{2\varepsilon_0}$$

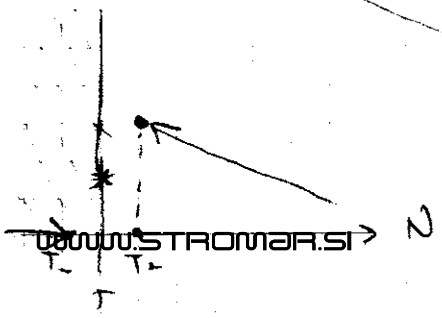
$$\vec{E}(T_-) = \vec{E}'(T_-) + \vec{E}''(T_-) = \vec{0}$$

$$E' = \begin{cases} \vec{n} \, \dfrac{\sigma(T)}{2\varepsilon_0}, & v\ T_+ \\ 0, & v\ T \\ -\vec{n} \, \dfrac{\sigma(T)}{2\varepsilon_0} & v\ T_- \end{cases}$$

E ravnine

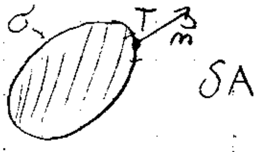
$$E''(T_+) = E''(T) = E''(T_-) = \vec{n} \, \frac{\sigma(T)}{2\varepsilon_0}$$

relativno daleč stran so naboji tega polja



8. Ploskovne sile

VET I
20. 11. 2007



$$\delta Q = \sigma(T)\,\delta a$$

$$\delta \vec{F}_e = \delta Q\, E(T) = \delta a \cdot \sigma(T)\, \vec{n}\, \frac{\sigma(T)}{2\varepsilon_0}$$

$$\frac{\delta \vec{F}_e}{\delta a} = \vec{n}\, \frac{\sigma(T)}{2\varepsilon_0} = \vec{f}_e$$

$$f_{en} > 0$$

$$f_{en} = \frac{\sigma(T)}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0 \Big[\underbrace{\frac{\sigma(T)}{\varepsilon_0}}_{E_0(T_+)}\Big]^2 = \underbrace{\frac{1}{2}\varepsilon_0 E^2}_{\text{gostota el energije}}$$

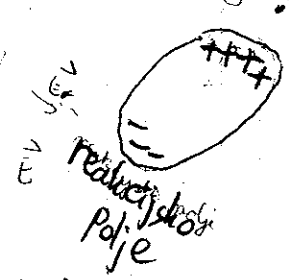
Primer: $E = 1\,MV/m$

$$f_{en} = \frac{1}{2}\, 8{,}85 \cdot 10^{-12} \cdot 10^{12} = 4{,}4\ Pa$$

tlak je v bistvu ploskovna sila

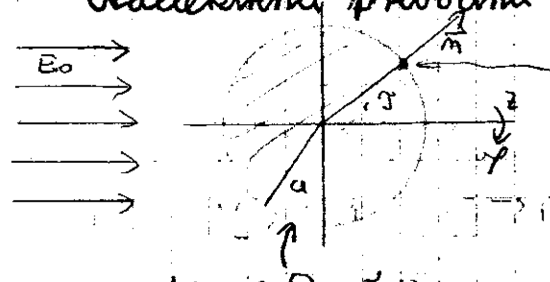
V 2 SEMESTRU $\frac{1}{2}\frac{B^2}{\mu_0}$ gostota mag energije

$\sigma = ?$



Zgled 1:

Naelektrena prevodna krogla v homogenem polju (18.3)



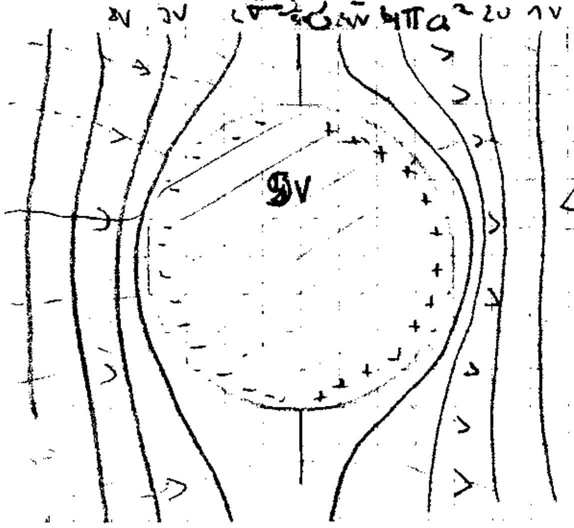
$$E_n(T) = \frac{\sigma(T)}{2\varepsilon_0}$$

$$\sigma(T) = \frac{Q}{4\pi a^2} + 3\varepsilon_0 E_0 \cos\vartheta$$

dan je Q, $\sigma = ?$

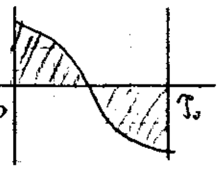
1. primer, da poenostavimo: $E_0 = 0$

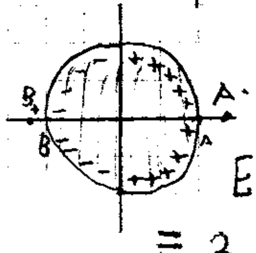
$$\sigma = \frac{Q}{4\pi a^2}$$



2 primer, $Q = 0$

$$\Rightarrow \sigma(T) = 3\varepsilon_0 E_0 \cos\vartheta_0$$

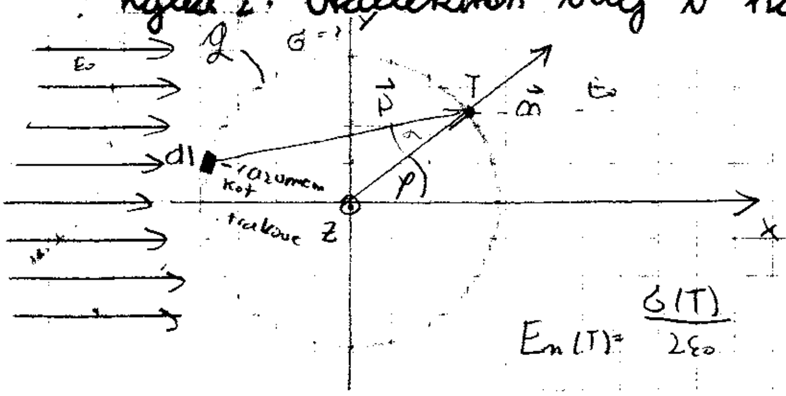




$$E_n(A_+) = \frac{\sigma(A)}{\varepsilon_0} = 3E_0$$

motnja, ki jo vnesemo v polje poveča polje, ga spremeni

Zgled 2: Naelektren valj v homogenem polju.



$$\vec{E} = \vec{E_0} + \vec{E_a}$$

$$E_n(T) = E_{0n}(T) + E_{an}(T)$$

$$\vec{E_{on}} = E_0 \cos \varphi$$

$$E_n(T) = \frac{\sigma(T)}{2\varepsilon_0}$$

$$\vec{E_{1n}}(T) = \vec{n}\,\vec{E_1}(T)$$

$$= \vec{n} \cdot \int \frac{\sigma(T')\, dl'}{2\pi\varepsilon_0 \rho} \frac{\vec{\rho}}{\rho} =$$

$$\frac{\vec{n}\cdot\vec{\rho}}{\rho^2} = \frac{\rho \cdot \cos\alpha}{\rho^2} = \frac{1}{\rho/\cos\alpha} = \frac{1}{2a}$$

$$= \frac{1}{2\pi\varepsilon_0} \int \underbrace{\left(\frac{\vec{n}\cdot\vec{\rho}}{\rho^2}\right)}_{1/2a} \sigma(T')\, dl'$$

Za valj je vsa slika kot pri krogli.

$$= \frac{1}{4\pi\varepsilon_0 a} \overbrace{\int \sigma(T')\, dl'}^{Q}$$

$$\vec{E_n}(T) = \frac{\sigma(T)}{2\varepsilon_0} = E_0 \cdot \cos\varphi + \frac{Q}{4\pi\varepsilon_0 a}$$

$$\sigma(T) = \frac{Q}{2\pi a} + 2\,\varepsilon_0 E_0 \cos\varphi = \sigma(\varphi)$$

$$* \quad V = V_0 + V_1$$

$$V_0 = -E_0 x + C \quad \Rightarrow V_0(x=0) = C$$

naše ekvipotencialke

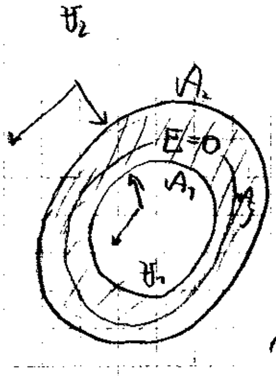
V_1 pri $x=0$ $=0$

$V(x=0) = C$

Če je to žica, prevzame tak potencial, kot ga je imela simetralna ravnina.

(KNJIGA 25.2)

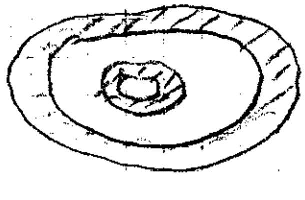
PREVODNA SRAJČKA, ELEKTRIČNO ZAKLJUČENI SISTEMI IN NJIHOVA AVTONOMNOST



$$\oint_{A_3} \vec{E} \cdot d\vec{l} = \frac{Q_{not.}}{\varepsilon_0} \Rightarrow Q_{not.} = 0$$

$\Rightarrow$ vsota nabojev v V_1 in na A_1 je 0. To imenujem električno zaključen sistem. Ker pa je celoten prostor tudi tak, da je vsota nabojev enaka 0 (V_c) $\Rightarrow$ vsota nabojev V_2 in na A_2 je enaka 0

Če bi meli srajčko v srajčki



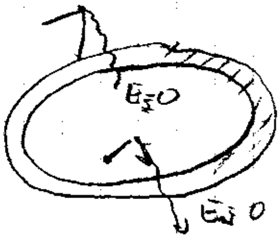
Avtonomnost = samostojnost

Predpostavimo da je polje $\vec{E}$ zunanjih nabojev v notranjosti različno od 0.

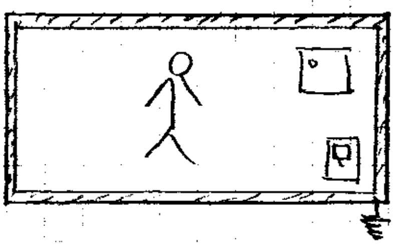
8v - Gaussova ploskev A_g

$$\oint \underbrace{\underbrace{\vec{E}\, d\vec{a}}_{>0}}_{>0} = \underbrace{\frac{Q_{not}}{\varepsilon_0}}_{\neq 0} \Rightarrow$$ predpostavka je napačna

$\Rightarrow E$ zunanjih nabojev znotraj ovojka je 0.



To je princip Faradayeve kletke

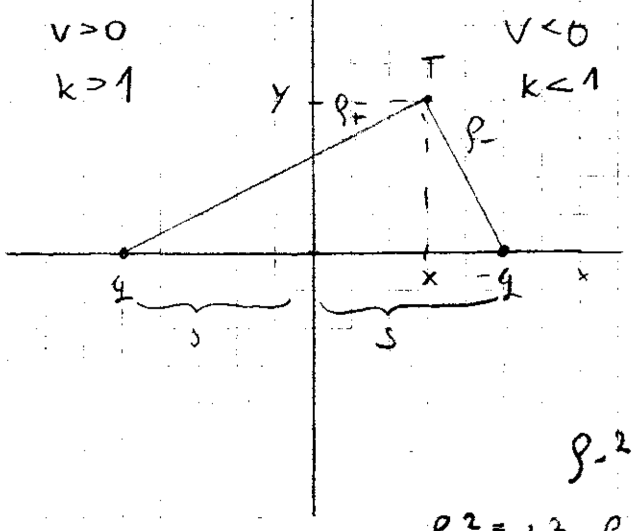


Če je stena prevodnika, lahko nekaj EM valovanja vstopi

„Okovinjanje" ekvipotencialk

OE I
21. 11. 2007

Ekvipotencialke nabojev $\pm q$



$$V(T) = \frac{q}{2\pi\varepsilon_0} \ln \frac{\rho_-}{\rho_+}$$

$$\frac{\rho_-}{\rho_+} = k$$ (značka ekvipotencialke)

$$k = e^{\frac{2\pi\varepsilon_0 V}{q}}$$

$$\rho_-^2 = (s-x)^2 + y^2 \qquad \rho_+^2 = (s+x)^2 + y^2$$

$$\rho_-^2 = k^2 \rho_+^2$$

$$(s-x)^2 + y^2 = k^2((s+x)^2 + k^2 y^2$$

$$s^2 - 2sx + x^2 + y^2 = k^2 s^2 + 2sk^2 x + k^2 x^2 + k^2 y^2$$

$$(1-k^2)x^2 - 2s(1+k^2)x + (1-k^2)y^2 + (1-k^2)s^2 = 0 \;/ : (1-k^2)$$

$$x^2 - 2s\frac{1+k^2}{1-k^2}x + y^2 + s^2 = 0$$

$$(x-p)^2 + (y-q)^2 - r^2 = 0$$

$$x^2 - \underbrace{2p}_{s\frac{1+k^2}{1-k^2}}x + y^2 + \underbrace{\boxed{p^2 - r^2}}_{s^2} = 0$$

$$p = \frac{1+k^2}{1-k^2} \cdot s$$

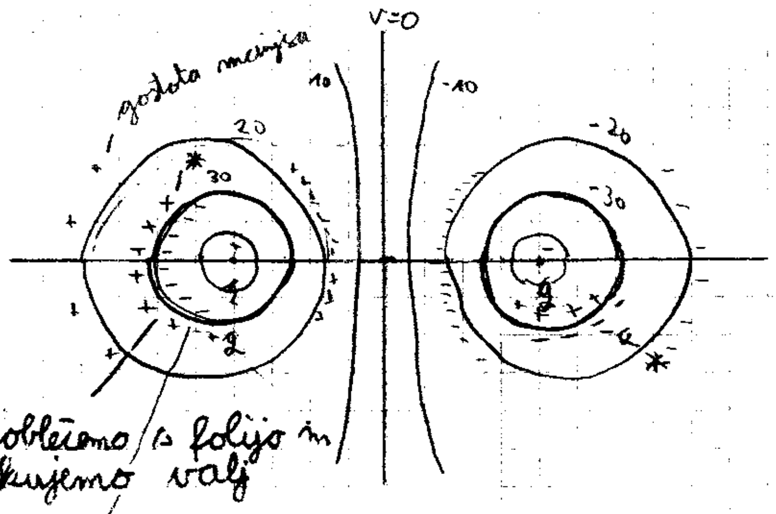
$$r^2 = p^2 - s^2 = \left[\frac{(1+k^2)^2}{(1-k^2)^2} - 1\right] s^2 = \frac{2k}{|1-k^2|} s$$

$$= \frac{\cancel{1} + 2k^2 + \cancel{k^4} - \cancel{1} + 2k^2 - \cancel{k^4}}{(1-k^2)} s^2$$

$$= \frac{4k^2}{(1-k)^2} s^2$$

$V \rightarrow -V \Rightarrow k \rightarrow 1/k$

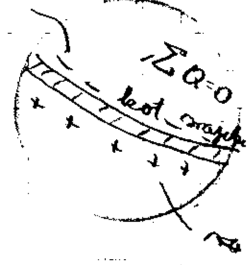
$p \rightarrow -p \Rightarrow r \rightarrow r$



oblečemo s folijo in oblikujemo valj

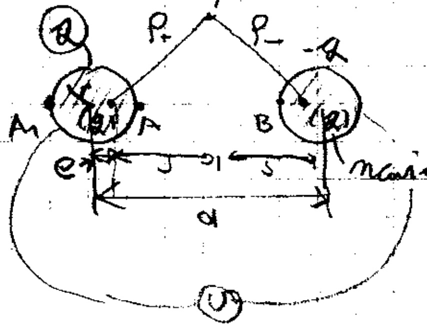
silnice so krogi, ki ležijo na ~~y~~ osi

* Ta naboja sta odgovorna za vmesni prostor



To je popolna 100% influenca. S primarnim nabojem lahko delam kar hočem in okolica ne bo nič zaznala

Simetričen dvovod



Pri dani napetosti in geometriji določiti ekscentričnost in množino naboja

Poznamo U, d, r:

$$V(T) = \frac{q}{2\pi\varepsilon_0} \ln \frac{\rho_-}{\rho_+}$$

$$V(A) = \frac{q}{2\pi\varepsilon_0} \ln \frac{d-r-e}{r-e}, \quad V(A_1) = \frac{q}{2\pi\varepsilon_0} \ln \frac{d+r-e}{r+e}$$

$$V(A) = V(A_1) \qquad \frac{d-r-e}{r-e} = \frac{d+r-e}{r+e}$$

$$dr + de - r^2 - \cancel{re} - \cancel{er} - e^2 = \cancel{rd} + r^2 - re - de - \cancel{er} + e^2$$

$$e^2 - de + r^2 = 0$$

$$e_{1,2} = \frac{d \pm \sqrt{d^2-4r^2}}{2} \qquad \underbrace{e_1 = e = \frac{d-\sqrt{d^2-4r^2}}{2}}$$

samo e

$$V(B) = \frac{q}{2\pi\varepsilon_0}\ln\frac{r-e}{d-r-e} = -V(A) \qquad U = V(A) - V(B)$$

$$U = \frac{q}{\pi\varepsilon_0}\ln\frac{d-r-e}{r-e}$$

$$q = \frac{\pi\varepsilon_0 U}{\ln\frac{d-r-e}{r-e}}$$

Kapacitivnost:

$$C = \frac{q\,l}{U} = \frac{\pi\varepsilon_0 l}{\ln\frac{d-r-e}{r-e}} = \ldots\ldots = \frac{\pi\varepsilon_0 l}{\ln\frac{d+\sqrt{d^2-4r^2}}{2r}} - c\ [F/m]$$

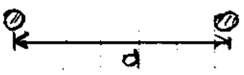
Če je $r \ll d$ potem:

$$e = \frac{d-\sqrt{d^2-4r^2}}{2} = \frac{d}{2}\left[1-\sqrt{1-\left(\frac{2r}{d}\right)^2}\right] \cong \frac{d}{2}\left[1-1+\frac{2r^2}{d^2}\right] = \frac{r^2}{d}$$

$$\sqrt{1-x} = 1 - \frac{0{,}01}{2} = 0{,}995$$

Če je $r \ll d$ je ekscentričnost dosti manjša od r oziroma zanemarljiva. Geometrična in ekscentrična os sovpadata

$$C \cong \frac{\pi\varepsilon_0}{\ln d/r}$$



$$\sigma(A) = \varepsilon_0 E_n(A_+) = \varepsilon_0\left[\frac{q}{2\pi\varepsilon_0}\cdot\frac{1}{r-e} + \frac{q}{2\pi\varepsilon_0}\frac{1}{d-r-e}\right]$$

$$\sigma(A_1) = \varepsilon_0 E_n(A_{1+}) = \varepsilon_0\left[\frac{q}{2\pi\varepsilon_0}\frac{1}{r+e} - \frac{q}{2\pi\varepsilon_0}\frac{1}{d+r-e}\right]$$

$$\frac{\sigma(A)}{\sigma(A_1)} = \frac{\frac{1}{r-e}+\frac{1}{d-r-e}}{\frac{1}{r+e}-\frac{1}{d+r-e}}$$

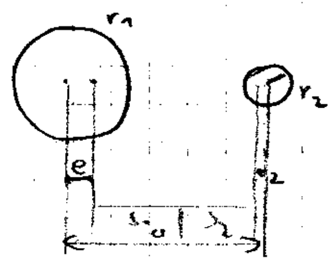
Primer: $d = 10\,cm$, $r = 3\,cm$

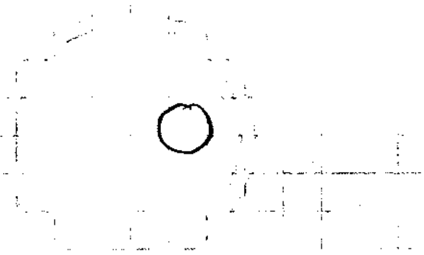
$$e = \frac{10-\sqrt{100-36}}{2} = \frac{10-8}{2} = \frac{2}{2} = 1\,cm$$

$$\frac{\sigma(A)}{\sigma(A)_1} = ?$$

$$\frac{\sigma(A)}{\sigma(A_1)} = \frac{\frac{1}{2}+\frac{1}{6}}{\frac{1}{4}-\frac{1}{12}} = \frac{\frac{4}{6}}{\frac{2}{12}} = 4$$ večja kot na drugem koncu

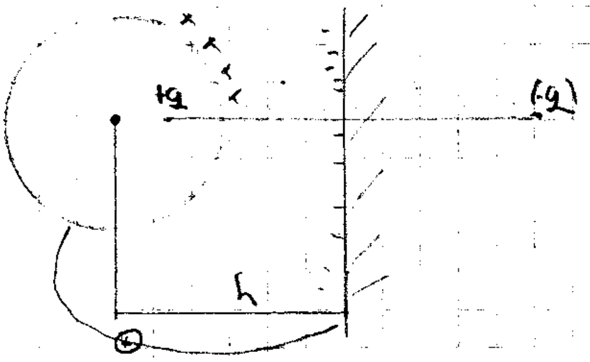
Primer debele in tanke žice





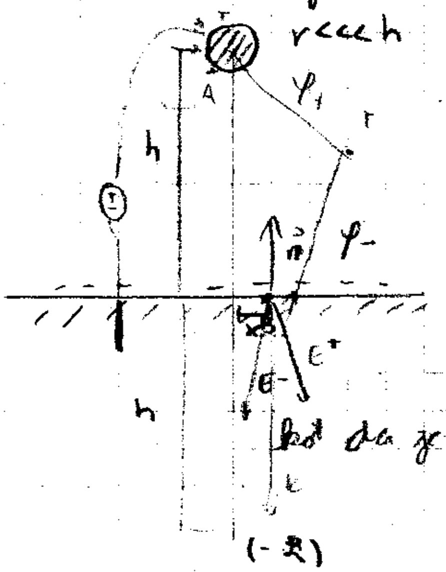
ekscentrični kabel

3 Primer: Vodnik in ozemljena stena



vodnik pred prevodno steno

Vodnik nad zemljo



$r \ll h$

$e \doteq 0$

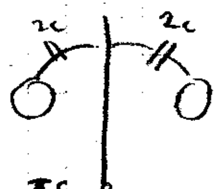
$$V(T) = \frac{q}{2\pi\varepsilon_0} \ln \frac{\rho_-}{\rho_+}$$

$$V(A) \approx \frac{q}{2\pi\varepsilon_0} \ln \frac{2h}{r} \; ; \quad V(B) = 0$$

$$U = V(A) - V(B) = \frac{q}{2\pi\varepsilon_0} \ln \frac{2h}{r}$$

$$C = \frac{q}{U} = \frac{2\pi\varepsilon_0}{\ln \frac{2h}{r}}$$

kot da je v globini

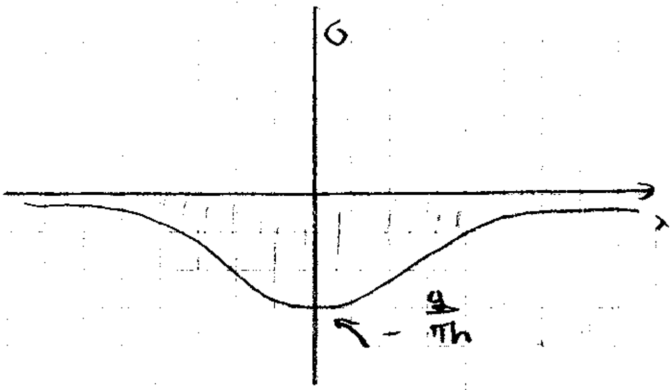


$$C = \frac{\pi\varepsilon_0 l}{\ln \frac{2h}{r}}$$

Gostota naboja na zemlji

$$\sigma(B) = \varepsilon_0 E_n(B_r) = \varepsilon_0 \left[-2\cdot \frac{q}{2\pi\varepsilon_0 \sqrt{h^2+x^2}} \frac{h}{\sqrt{h^2+x^2}}\right]$$

$$= -\frac{q h}{\pi(h^2+x^2)}$$

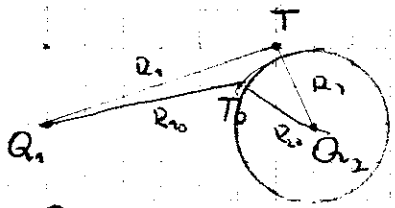


Izračunajmo

$$\int_{-\infty}^{\infty} \sigma(x)\,dx = -\frac{q}{\pi}\, 2\int_0^{\infty} \frac{h\,dx}{h^2+x^2} = -\frac{2q}{\pi}\underbrace{\int_0^{\infty} \frac{d(\frac{x}{h})}{1+(\frac{x}{h})^2}}_{\pi/2}^{\text{arctg } x} = -q$$

POLJE 2. TOČKASTIH NABOJEV

OET I
27.11.2007



$Q_1, Q_2 < 0 \;:\; |Q_1| \geq |Q_2|$

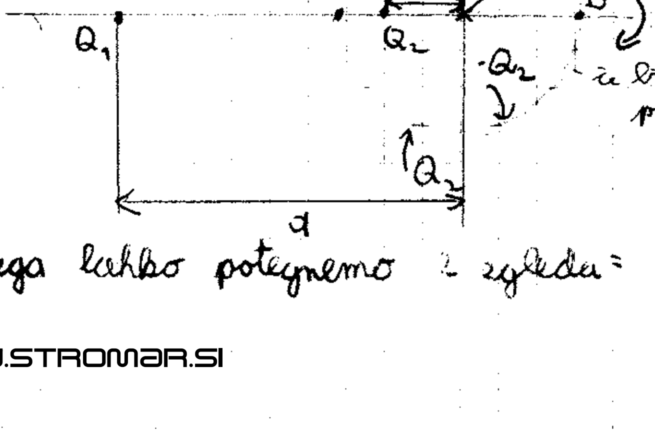
v tem prostoru verjetno obstajajo točke kjer je $V=0$, predstavnik T_0

$$V(T) = \frac{Q_1}{4\pi\varepsilon_0 R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{R_1} + \frac{Q_2}{R_2}\right)$$

$$V(T_0) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{R_{10}} + \frac{Q_2}{R_{20}}\right) \Rightarrow \frac{R_{10}}{R_{20}} = -\frac{Q_1}{Q_2} = k > 1$$

to je krog, ker pa je rotacijska je to krogla

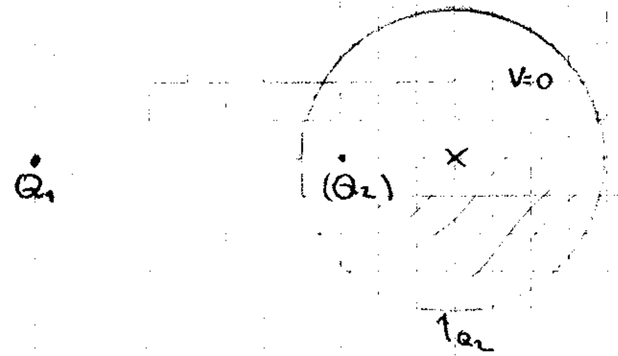
Naboj se nahaja ekscentrično glede na ekvipotencialno kroglo.



če bi okovinili bi delovalo kot prevodna srajčka

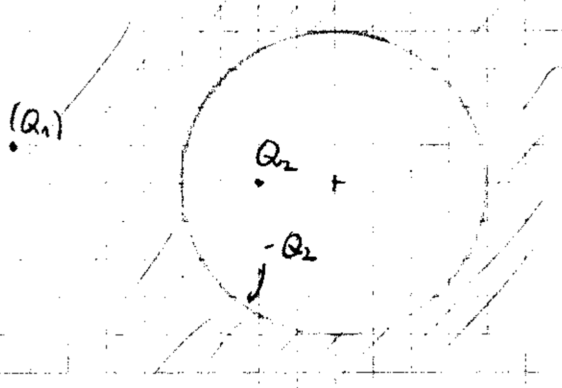
Iz tega lahko potegnemo 2 zgleda:

a)



v listu zemlja je to prevodno telo

b)



kot kaverna v zemlji. Zunaj je $\vec{E}=0$

Iz prejšnje strani

zrcalni točki glede na kroglo

$$V(A) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{d-r_0} + \frac{Q_2}{r_0-e}\right) = 0$$

$$V(B) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{d+r_0} + \frac{Q_2}{r_0+e}\right) = 0$$

1. V_A / V_B

$$= \frac{d+r_0}{d-r_0} = \frac{r_0+e}{r_0-e}$$

$$\Rightarrow \cancel{dr_0} - de + r_0^2 - \cancel{er_0} = \cancel{dr_0} + de - r_0^2 - r_0 e$$

$$de = r_0^2$$

Če je slučajno $Q_1 = -Q_2$

Q_1 | $V=0$ $-Q_1$

simetralna ravnina je potem $V=0$

Q_1 | $\cdot -Q_2$

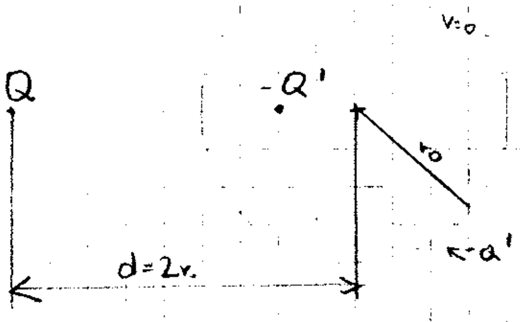
$-Q$

Sedaj moramo uskladiti še naboje

$$Q_2 = -Q_1\left(\frac{r_0+e}{d+r_0}\right) = -\frac{r_0+\frac{r_0^2}{d}}{d+r_0}Q_1 = -\frac{r_0(d+r_0)}{(d+r_0)\,d}Q_1 = -\frac{r_0}{d}Q_1$$

$$Q_2 = -\frac{r_0}{d}Q_1 = -\frac{e}{r_0}Q_1$$

Zgled: Naboj pred prevodno kroglo s potencialom 0.



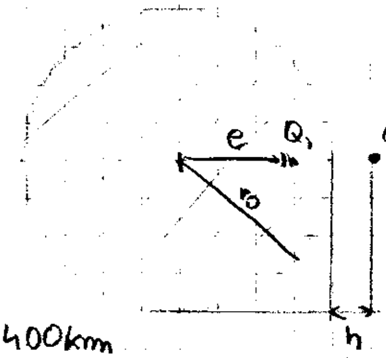
Resnični naboj ni na krogli vendar navidezni naboj ima enakega

$$e = \frac{r_0^2}{d} = \frac{r_0^2}{2r_0} = \frac{r_0}{2}$$

$$Q' = \frac{r_0}{d}Q = \frac{r_0}{2r_0}Q = \frac{Q}{2}$$

$$|\vec{F}_e| = \frac{Q\cdot Q'}{4\pi\varepsilon_0(d-e)^2} = \frac{Q^2/2}{4\pi\varepsilon_0\left(2r_0-\frac{r_0}{2}\right)^2} = \frac{Q^2}{18\pi\varepsilon_0 r_0^2}$$

Naboj nad /pod/ ob Zemljo.



$r_0 = 6400km$

h = nekaj metrov nad tlemi

Če gre $d \Rightarrow r_0$ gre tudi $e \Rightarrow r_0$

$Q' = ?$

$e = ?$

$$e = \frac{r_0^2}{d} = \frac{r_0^2}{h+r_0} = \frac{1}{1+\frac{h}{r_0}}\,r_0 \simeq \left(1-\frac{h}{r_0}\right)r_0 = r_0-h$$

$$= \frac{1}{1+x}\ \Big|\simeq 1-x \quad |x| \ll 1$$

$$* \ \frac{1}{1+0{,}1} \doteq 0{,}99$$

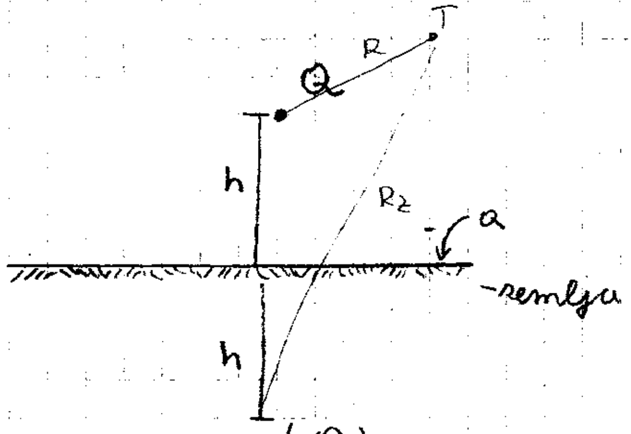
$$\frac{1}{1+0{,}001} = 0{,}999$$

$$(1-x)(1+x) \simeq 1-x^2$$

$$Q' = \frac{r_0}{r_0+h}Q = \frac{1}{1+\frac{h}{r_0}}Q \simeq \left(1-\frac{h}{r_0}\right)Q \simeq Q$$

$Q' \simeq Q$

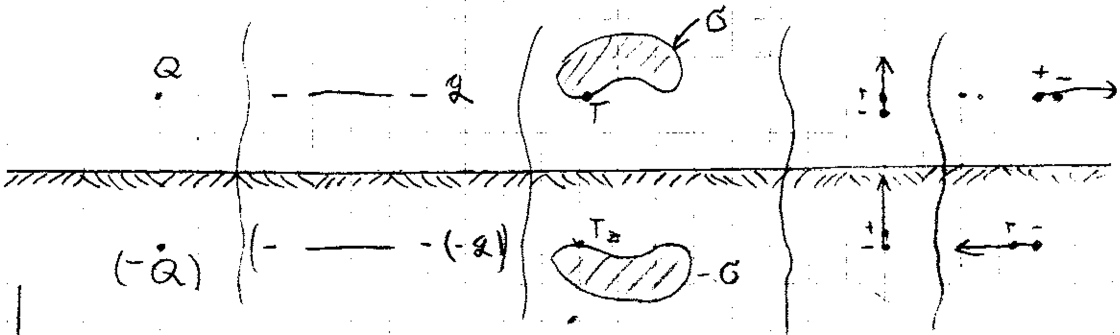
$e \simeq r_0 - h$

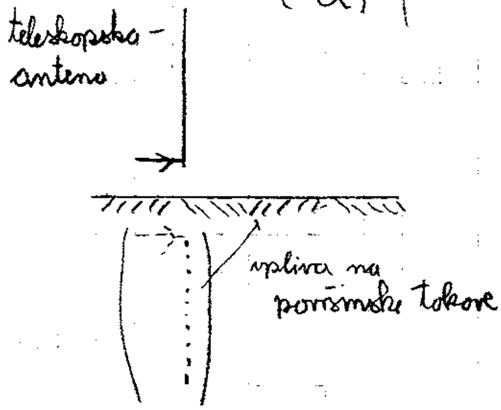


$$V(T) = \frac{Q}{4\pi\varepsilon_0 R} + \frac{(-Q)}{4\pi\varepsilon_0 R_z} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_z}\right)$$

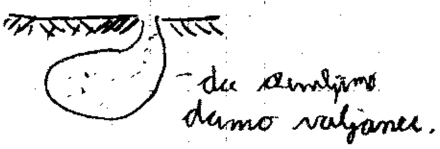
po starem po novem

ZRCALJENJE NA RAVNI PREVODNI PODLAGI





Zemlja postaja sestavina naših razmišljanj.



IZOLANT / DIELEKTRIK IN ELEKTRIČNO POLJE

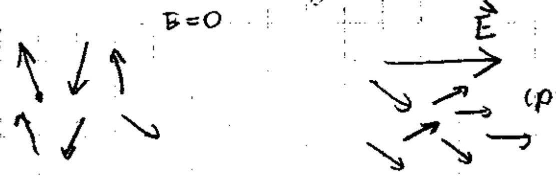
(ni to isto enako)

Vsakemu izolantu lahko rečem dielektrik, vsakemu dielektriku pa ne izolant, npr. voda.

Težko je postaviti mejo med dielektrikom/izolantom in prevodnikom.

Načini polariziranja molekulskih dielektričnih snovi
- poljska
- elektronska
- ionska

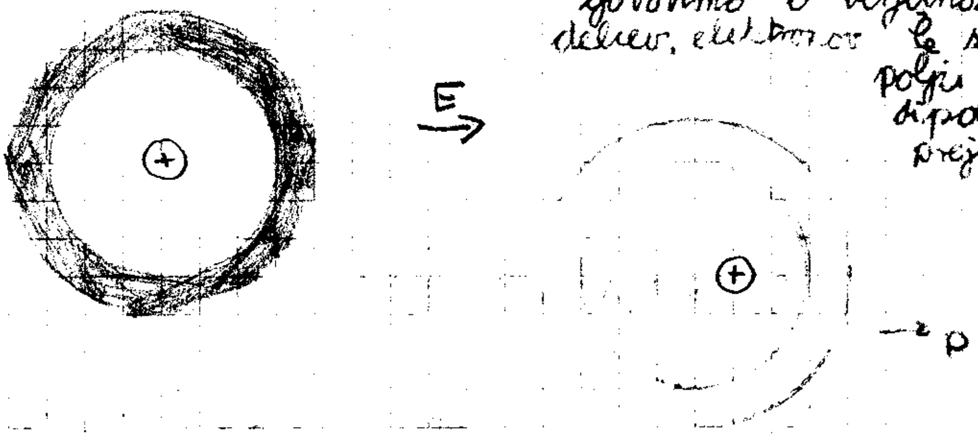
1.) Značilna za vodo, alkohole (polarne molekule). Pokažemo jih lahko kot dipole. So kot vektorji ki pa se vseskozi premikajo. Bolj kot je vroče bolj mižgajo



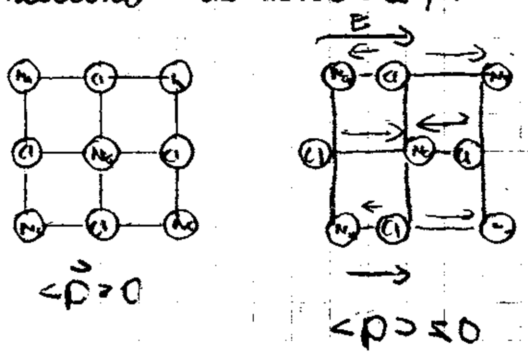
Izkazujejo nek povprečen dipolski moment

$\langle \vec{p} \rangle \neq 0$ Taka snov ni dipolsko polarizira

2. Značilna za trdne snovi (plastika, guma). Mehanizem: govorimo o verjetnosti nahajanja delcev, elektronov če se znajde v polju se naredi dipolski moment, prej jih ni bilo.



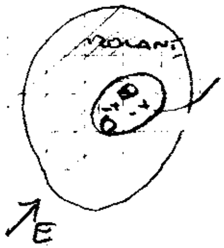
3. Značilno za kristale, ki imajo jonske vezi (npr: NaCl) Pod vplivom polja se pozitivni premaknejo v eno stran negativni v drugo. Navzven se dipolski moment ne izkazuje pri brezpoljskem prostoru



Piezoelektrični efekt z neko gnanjo ali mehansko silo ustvarimo

Vsem 3 je skupno da $\vec{E}$ ustvari neke povprečne dipolske momente ali pa jih naredi: zato mora količina

vektor polarizacije: $\vec{E} \to \langle \vec{p} \rangle$



$\lim_{\Delta V \to 0} \frac{\sum \delta \vec{p}}{\delta V} = \vec{P}$ ali prostorska

enote $[C/m^2]$

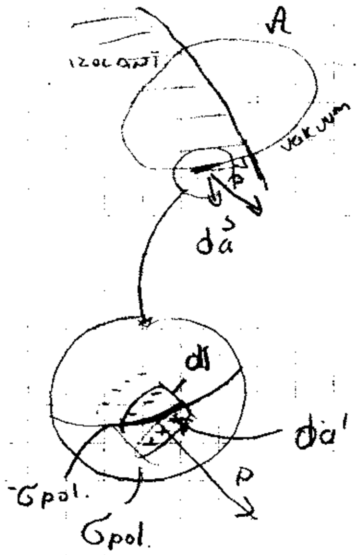
Izpeljani $\oint_A \vec{P} \cdot d\vec{a} = -Q_{notr.}$ polarizaciskih nabojev

vezani naboji, ki so znotraj vezeti te strukture gibanje je omejeno, vzhaja podobno prostoru

Izvornost vektorja $\vec{P}$

$$\oint_A \vec{P}\cdot d\vec{a} = ?$$

$$\vec{P} \neq \vec{0}$$



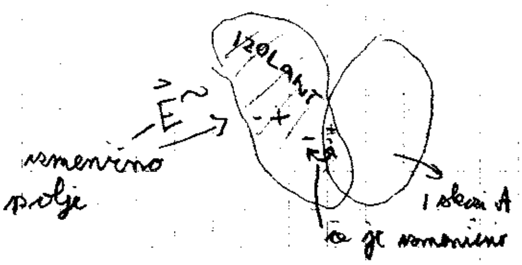
$$\vec{P}\cdot d\vec{a} = \frac{\overset{\text{naboj}}{\sigma_{pol}\cdot da'}\cdot dl}{dV}\cdot d\vec{a} = dQ_{pol.} \text{ zunaj } A$$

$$= -dQ_{pol} \text{ enotraj } A$$

$$\oint_A \vec{P}\cdot d\vec{a} = \int -dQ_{not.\,pol.} = -Q_{not.\,pol.}$$

$$\oint_A \vec{P}\cdot d\vec{a} = -Q_{not.\,pol}$$

POLARIZACIJSKI TOK



$$\frac{d}{dt}\int \vec{P}\cdot d\vec{a} = -\frac{dQ_{not.\,pol}}{dt}$$

1 skozi A => 1 polarizacijski

$$\int_A \frac{\partial \vec{P}}{\partial t}\cdot d\vec{a} = \oint_A J_{Pol}\cdot d\vec{a}$$

$$\vec{J}_{POL.} = \frac{\partial \vec{P}}{\partial t}$$

VEKTOR GOSTOTE EL. PRETOKA

$$\oint_A \vec{E}\cdot d\vec{a} = \frac{Q_{not.}}{\varepsilon_0} \quad / \varepsilon_0 \qquad \Rightarrow \oint_A \varepsilon_0 E\cdot da = Q_{not}$$

$$+$$

velja tudi $\quad \oint_A \vec{P}\cdot d\vec{a} = -Q_{not.\,pol.} \qquad \oint_A P\cdot d\vec{a} = -Q_{not.\,pol.}$

$$= \oint_A (\varepsilon_0 \vec{E} + \vec{P})\cdot d\vec{a} = Q_{not.} - Q_{not.\,pol.}$$

$$= Q_{not\ prosti}$$

$$\varepsilon_0\vec{E} + \vec{P} = \vec{D} \qquad \oint_A \vec{D}\cdot d\vec{a} = Q_{notr.\,prosti}$$

$D = C/m$

DIELEKTRIČNOST

Rečemo tudi influenca v vakumu. Je podatek, ki nam pove, koliko se zmore neka snov polarizirati.

Večinsko je $\vec{P} \parallel \vec{E}$ in $\vec{P} \propto \vec{E}$ - to velja za večino snovi. So pa tudi izjeme - pri kristalih.

Če velja $\vec{P} \parallel \vec{E}$ pravimo temu izotropnost, če je $\vec{P} \nparallel \vec{E}$ je pa snov anizotropna. (pozor tudi pri magnetiki, mišicah, antenah)
$P \propto E$ je linearnost

Za linearne in izotropne medije je vektor $\vec{P}$ možno zapisati v obliki:

$$\vec{P} = \chi_e \varepsilon_0 \vec{E}$$

električna susceptibilnost (možnost sprejemanja $\vec{E}$)

Vpemo v enačbo

$$D = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E} + \chi_e \varepsilon_0 \vec{E} = \underbrace{\underbrace{(1+\chi_e)}_{\varepsilon_r} \varepsilon_0}_{\varepsilon} \vec{E} = \varepsilon_r \varepsilon_0 \vec{E} = \varepsilon \vec{E}$$

$$\vec{P} = \chi_e \varepsilon_0 \vec{E} = (\varepsilon_r - 1) \varepsilon_0 \vec{E}$$

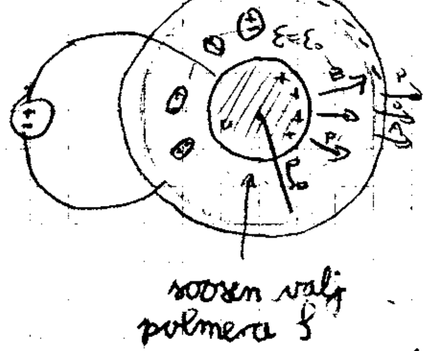
Vektorji $\vec{E}, \vec{P}$ in $\vec{D}$ so kolinearni

Poglej v knjigi snov in njen ε_r.

ε_r - postaja snovna lastnost, neka konstanta snovi.

Pri visokih V zveznost med D in E ni več linearna, ampak kot elipsa - merilo za dielektrične izgube (toplota)

Zgled: koaksialni kabel z izolantom.



Vektorji $\vec{E}$, P, D so ~~neko~~ radialni

$$\oint \vec{D} \cdot d\vec{a} = D_\rho 2\pi \rho l = q l \Rightarrow D_\rho = \frac{q}{2\pi\rho}$$

$$E_\rho = \frac{D_\rho}{\varepsilon} = \frac{q}{2\pi\varepsilon\rho}$$

$$U = \int_a^b E_\rho \cdot d\rho = \frac{q}{2\pi\varepsilon} \int_a^b \frac{d\rho}{\rho} = \frac{q}{2\pi\varepsilon} \ln\frac{b}{a}$$

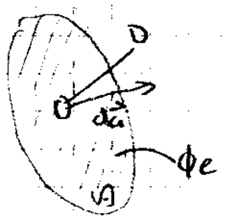
$$\frac{q l}{U} = \frac{2\pi\varepsilon l}{\ln b/a} = C$$

Električni pretok

$i = \int_A J \cdot da$

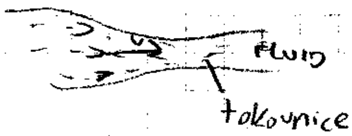
$\Phi_e = \int_A \vec{D} \cdot d\vec{a}$ Tu se nič več ne premika.

Enota $\Phi_e = [C]$

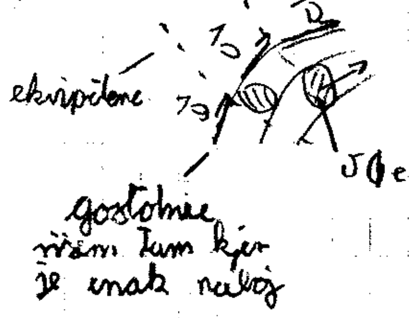


Gostotnice in pretočne cevke

Predstavljajo neke vrste tokovnice



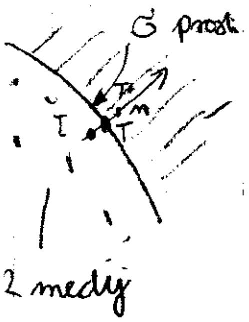
- Nekaj podobnega lahko naredimo z električnim pretokom. Črte podolne silnicam, kjer je D tangenten. Pri linearnih medijih so tudi silnice



Električni pretok predstavim s pretočnimi cevkami

Mejni pogoji

Kaj lahko rečem o $\vec{E}$ na mejnih ploskvah naših medijev.

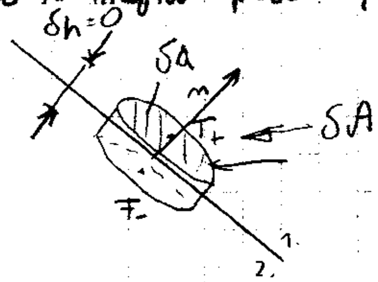


$\oint_A \vec{D} \cdot d\vec{a} = Q$ notr. to mora splošno veljati.

in tudi tole

$\oint_A \vec{E} \cdot du = 0$

Zamislimo si mejno ploskev, ki bo malo tu malo tam



$$\oint_{\delta A} \vec{D}\cdot d\vec{a} \simeq \underbrace{\vec{D}(T_+)\cdot n\cdot \delta a}_{D_n(T_+)} + \underbrace{D(T_-)\cdot(-n)\cdot\delta a}_{-(D_n(T_-))} + (\sim 0)$$

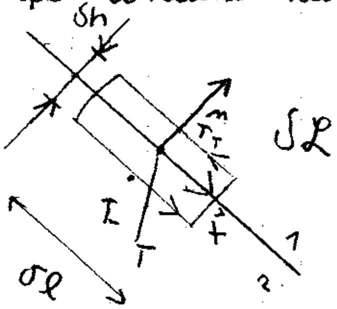
poljubno kratek interval, poljubno malo naboja

$$\cong \sigma_{prosti}(T)\delta a$$

$$D_n(T_+) - D_n(T_-) = \sigma_{prosti}(T)$$

$$n = [\vec{D}(T_+) - D(T_-)] = \sigma_{prosti}(T)$$

2. spet lokalna scena



$$\oint_{\delta L} \vec{E}\cdot d\vec{l} \cong \underbrace{E(T_-)\cdot + \delta\ell}_{} + (\sim 0) + \overbrace{\vec{E}(T_+)}^{\cdot E_T(T_+)}\cdot(-+)\delta\ell + (\sim 0) = 0$$

Tangenta vrednost $\vec{E}$ na obeh straneh je enaka

$$\vec{E}_+(T_+) - E(T_-) = 0$$

$$E_{T2}(T_T) - E_{T1}(T_-) - 0$$

$$\vec{n}\cdot[\vec{E}(T_T) - E(T_-)] = \vec{0}$$

Mejne pogoje dobimo iz

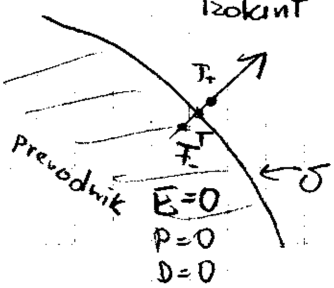
$\oint D\cdot da$

– teh lastnosti

$\oint E\cdot da$

Zgled:

Meja izolant prevodnik

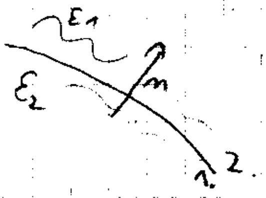


$$\Rightarrow D_n(T_-) = 0 \Rightarrow \underbrace{D_n(T_+)}_{\varepsilon_0\cdot E_n(T)} = \sigma_{prosti}(T)$$

$$\Rightarrow E_n(T_+) = \frac{\sigma_{prosti}(T)}{\varepsilon}$$

$E_T(T_-) = 0$
$E_T(T_+) = 0$ $\Rightarrow E(T_+) \parallel \vec{n}$

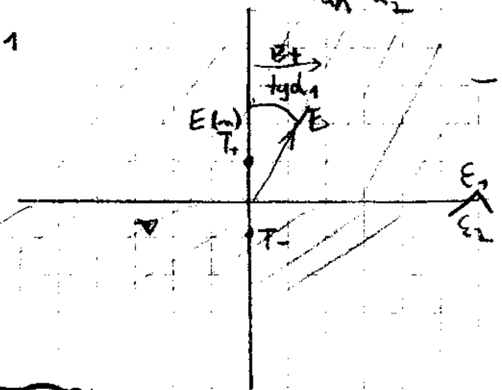
Meja izolant–izolant ($\sigma_{prosti} = 0$)



$E_T(T_+) = E_T(T_-)$
in
$D_n(T_+) = D_n(T_-)$

$\varepsilon_1 E_n(T_+) = \varepsilon_2 E_n(T_-)$

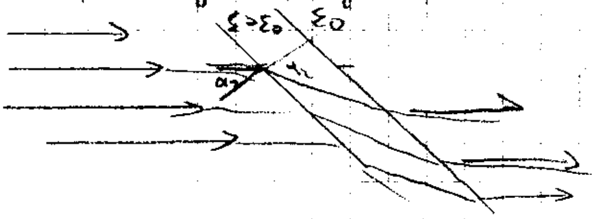
$$\underbrace{\frac{E_T(T_+)}{\varepsilon_1 E_n(T_+)}}_{\tan\alpha_1} = \underbrace{\frac{E_T(T_-)}{\varepsilon_2 E_n(T_-)}}_{\tan\alpha_2}$$



— polje predstavim s silnicami

$$\boxed{\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\varepsilon_1}{\varepsilon_2}}$$ pri $\sigma_{prosti} = 0$

Imamo homogeno polje



gostotnice ali silnice

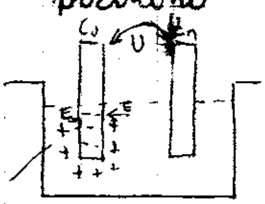
$$\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\varepsilon_0}{\varepsilon} = \frac{1}{\varepsilon_r}$$

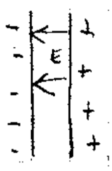
OET I
4. 12. 2007

IDEALNI NAPETOSTNI VIR

Pri galvanskih členih prihaja do separacije naboja. Pri termočlenu sta 2 kovini z različnim potencialom in naboj prehaja iz ene na drugo.

Steklaste in smolnate snovi. Ene se bolj neg. naelektrijo, druge bolj pozitivno



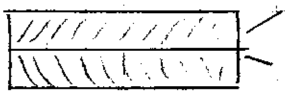


kot kond.

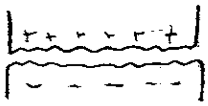
naboji funkcionirajo kot kondenzator

~~Temu~~ Temu rečem elektrodna napetost

Termočlen: če ga segrevamo izvemo migracije in posledično višjo napetost, pri visokih temp. je linearno

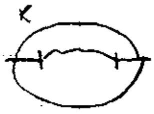


Tornočlen: z trenjem omogočimo prenašanje naboja (steklasta / smolnata snov)

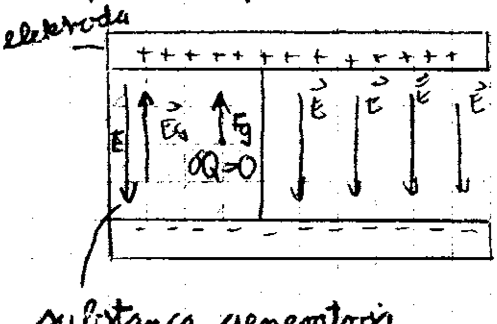


Fotodioda

To so vsi neki elementarni enosmerni viri.



Splošna predstava o viru:



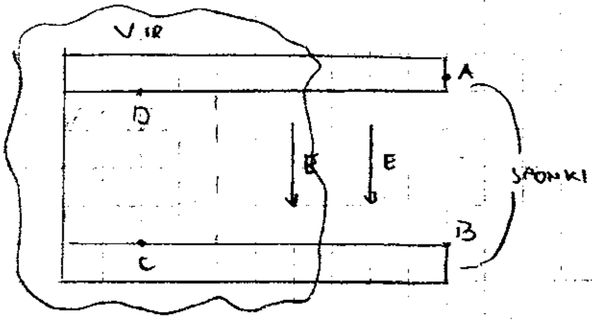
$\delta \vec{F}_g$ - neelektrična sila, ki razdvaja naboj.

$\frac{\delta F_g}{\delta Q} = \vec{E}_g$ (V/m) - normirana neelektrična sila (ne poljska jakost) !!!

Proces je zaključen, ko se F_e in F_g izenačita

$\delta Q \cdot E + \delta \vec{F}_g = 0$ ravnovesje

$$\delta Q (\vec{E} + \vec{E}_g) = 0 \Rightarrow \vec{E} + \vec{E}_g = 0$$



$$\oint E \cdot dl = \underbrace{\int_A^B E \cdot dl}_{U_{AB}} + \underbrace{\int_B^C E \cdot dl}_{0} + \underbrace{\underbrace{\int_C^D E \cdot dl}_{-\int_C^D E_g \cdot dl}} + \underbrace{\int_D^A E \cdot dl}_{0} = 0$$

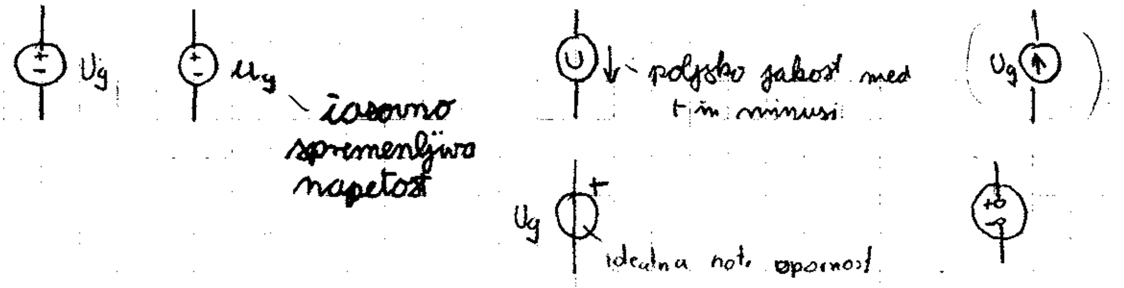
U_g - notranja ali generatorska napetost

$$U - U_g = 0 \Rightarrow U = U_g$$

$$\delta A_g = \int_C^D \delta F_g \cdot dl = \delta Q \int_C^D E_g \cdot dl = \delta Q \cdot U_g = \delta A_g$$

SIMBOL NAP.

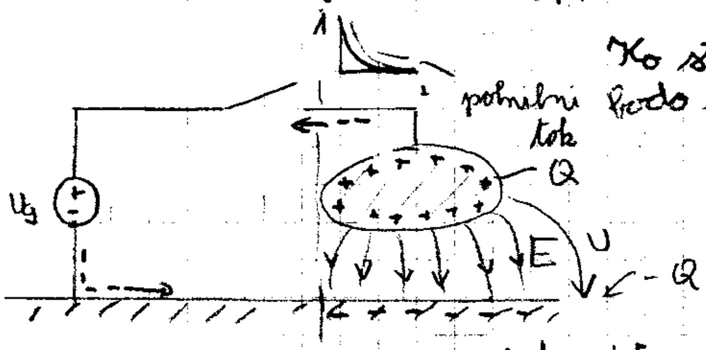
SIMBOL NAP VIRA KOT STRNJENEGA ELEMENTA ELEKTR. VEZIJ



Simbol mora sporočati jasno polariteto

NAELEKTRITEV SISTEMA

Ko staknem stikalo pohitijo krenili elektroni

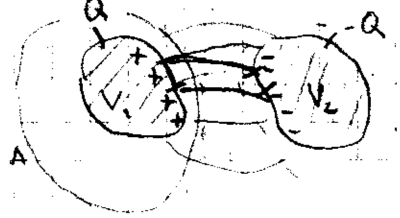


ta sistem lahko menujem naelektritev

$-U_g = U$

To oblikuje kondenzator

KAPACITIVNOST IN KONDENZATOR KOT STRNJEN ELEMENT ELEKTRIČNIH VEZIJ



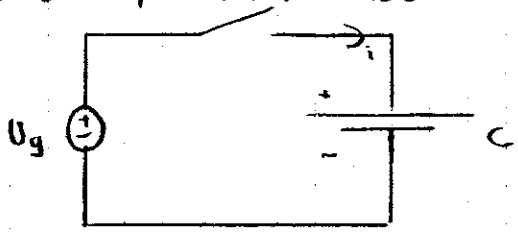
To je naš posplošen kondenzator

$$\oint D \cdot da = Q_{not} = \Phi_{e12}$$

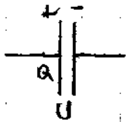
+ se navežejo številsko na − in to je enako

$$C = \frac{\Phi_{e12}}{V_1 - V_2} \quad \left(= Q/U\right)$$

Obravnavali smo še ploščati kond., koaksialni, sferični kondenzator, dvovod, vodnik nad ozemljeno steno, zemljo, ekscentrični kabel.

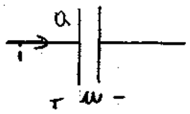


Simbol kondenzatorja



$Q = CU$

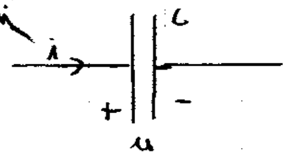
KONDENZATOR V "ŽIVIH" STRUKTURAH



$$(-i) = -\frac{dQ}{dt}$$

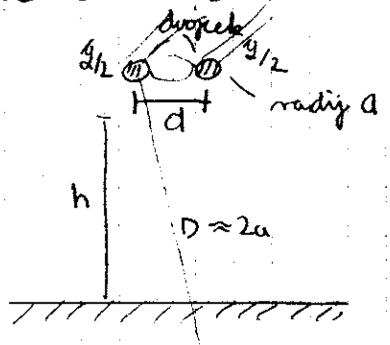
$$i = \frac{dQ}{dt} = \frac{d(Cu)}{dt} = C\frac{du}{dt}$$

polnilni tok



$$i = C\frac{du}{dt}$$

Zgled: polnilni tok „dvojčka" med Ljubljano in Postojno ki je harmonično vzbujan in ustrezno modelno vezje.



$h \gg d \gg a$

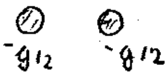
$$V_{dvojčka} = \frac{q}{2\pi\varepsilon_0}\ln\frac{2h}{a} + \underbrace{\frac{q/2}{2\pi\varepsilon_0}\ln\frac{D}{d}}_{\text{sredina žice prej smo jo postavili tja}}$$

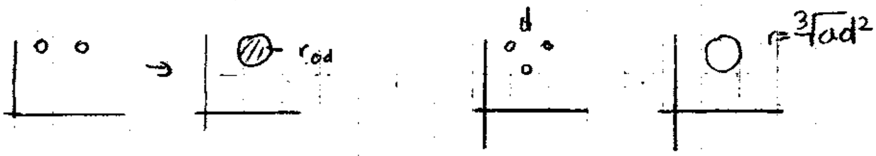
$$= \frac{q/2}{2\,2\pi\varepsilon_0}\ln\frac{(2h)^2}{(\sqrt{ad})^2} =$$

$$= \frac{Q}{2\pi\varepsilon_0}\ln\frac{2h}{\sqrt{ad}}$$

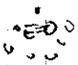
$$c = \frac{q}{U} = \frac{2\pi\varepsilon_0}{\ln\frac{2h}{\sqrt{ad}}}$$

funkcionira koren kot nek ekvivalentni radij nadomestne žice radij





če je četverček



$$r = \sqrt[4]{a\sqrt{2}d^3}$$

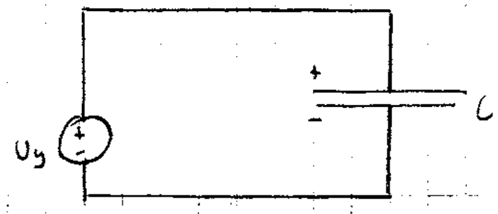
ker diagonala

Med žicami smo tako polje odrinili, ~~polje~~ tako zmanjšamo ionizacijo zraka, ker zunaj žice ne teče velik polnilni tok

Primer: $r = 2\,cm$ $d = 50\,cm$ $h = 10\,m$, $\ell = 50\,km$

$$C = c\cdot\ell = \frac{\cancel{2\pi}\ \frac{10^{-9}}{\cancel{2\pi 18}}\cdot 50\cdot 10^3}{\ln\frac{2\cdot 20}{\sqrt{0{,}02\cdot 0{,}5}}} = \frac{50}{108} = 0{,}46\cdot 10^{-6}\,\mu F$$

10^{2}



medfazna napetost je 400kV,

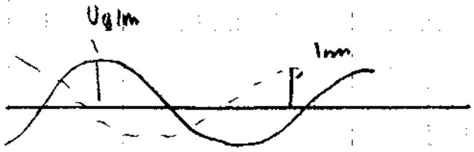
fazna (zemlja vodnik) $\frac{400kV}{\sqrt{3}}$, efektivna pa $\frac{400kV}{\sqrt{3}} \cdot \sqrt{2}$

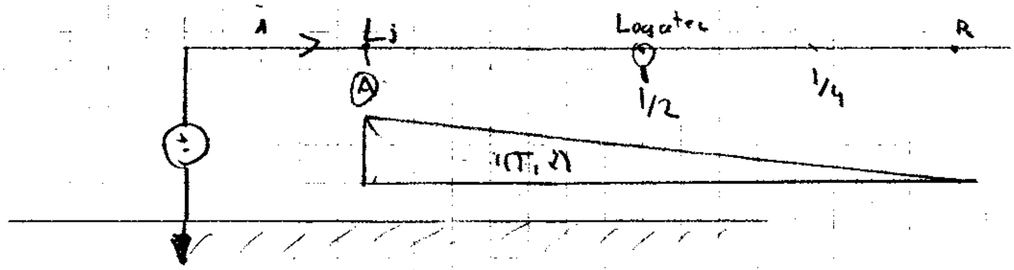
$$U_g = U_{gm} \sin(\underbrace{\omega}_{2\pi f} t + \alpha)$$

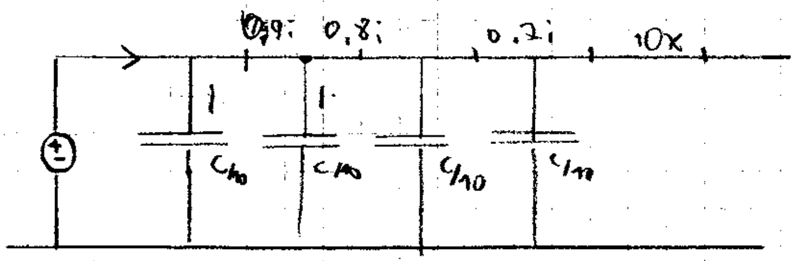
$$-U_g + u = 0 \qquad u = U_g$$

$$i = C\frac{du}{dt} = \underbrace{\omega C U_{gm}}_{I_m} \cos(\omega t + \alpha)$$

$$I_m = \omega \cdot C \cdot U_{gm} = 47A \simeq 50A$$



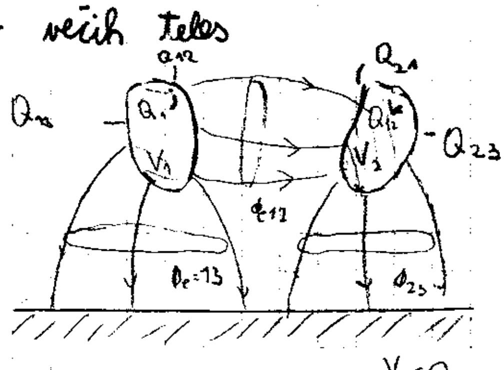




OET I

DELNE KAPACITIVNOSTI

Sistem večih teles



$$Q_1 + Q_2 + Q_3 = 0$$

$$V_1 > V_2 > V_3 = 0$$

$$\phi_{12} = Q_{12} = +Q_{21}$$

$$\phi_{13} = Q_{13} \; ; \; \phi_{23} = Q_{23}$$

$V_3 = 0$

$$C_{jk} = \frac{\phi_{ejk}}{V_j - V_k} = \frac{Q_{jk}}{U_{jk}}$$

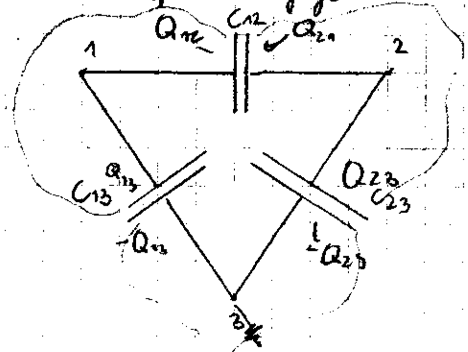
$$Q_1 = Q_{12} + Q_{13} = C_{12}(V_1 - V_2) + C_{13}(V_1 - \overset{=0}{V_3}) = (C_{12}+C_{13})V_1 - C_{12}V_2$$

$$C_{kj} = \frac{Q_{kj}}{V_k - V_j} = \frac{-Q_{jk}}{-V_j + V_k}$$

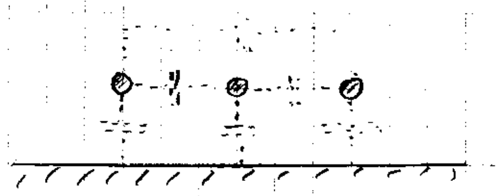
$$Q_2 = Q_{21} + Q_{23} = C_{21}(V_2 - V_1) + C_{23}(V_2 - \overset{=0}{V_3}) = (C_{12}+C_{23})V_2 - C_{12}V_1$$

C_{jk}

Površine predstavljajo kond.

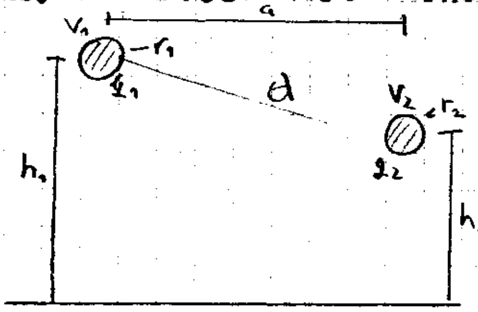


Sistem večih teles lahko predstavim z modelom kondenzatorskega vezja. Modelno kond. vezje



$$n\text{-teles} : \frac{n(n-1)}{2} \text{ delnih kapac.}$$

Zgled: dvovod nad zemljo



$$V_1 = \frac{q_1}{2\pi\varepsilon_0}\ln\frac{2h_1}{r_1} + \frac{q_2}{2\pi\varepsilon_0}\ln\frac{D}{d} \qquad p_{11}q_1 + p_{12}q_2$$

$$V_2 = \frac{q_1}{2\pi\varepsilon_0}\ln\frac{D}{d} + \frac{q_2}{2\pi\varepsilon_0}\ln\frac{2h_2}{r_2} \qquad p_{12}q_1 + p_{22}q_2$$

$$l\,q_1 = l\,\frac{p_{22}V_1 - p_{12}V_2}{p_{11}p_{22} - p_{12}^2} \qquad l\,q_2 = \frac{-p_{12}V_1 + p_{11}V_2}{p_{11}p_{22} - p_{12}^2}\,l$$

$$Q_1 = \frac{p_{22}\,l\cdot V_1 - p_{12}\,l\,V_2}{p_{11}p_{22} - p_{12}^2}$$

$$Q_2 = \frac{p_{12}\,lV_1 + p_{11}\,lV_2}{p_{11}p_{22} - p_{12}^2}$$

$(-q_2)$

(q_1)

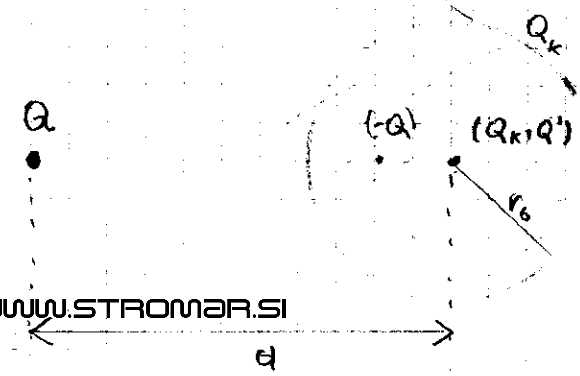
$$\Longrightarrow \quad C_{12} = \frac{p_{12}\,l}{p_{11}p_{22} - p_{12}^2}$$

$$C_{13} = \frac{(p_{22} - p_{12})\,l}{p_{11}\cdot p_{22} - p_{12}^2}$$

$$C_{23} = \frac{(p_{11} - p_{12})\,l}{p_{11}p_{22} - p_{12}^2}$$

ZRCALJENJE

Zgled: naboj pred naelektreno kroglo:



← to mora biti potencialko n 0 poten.

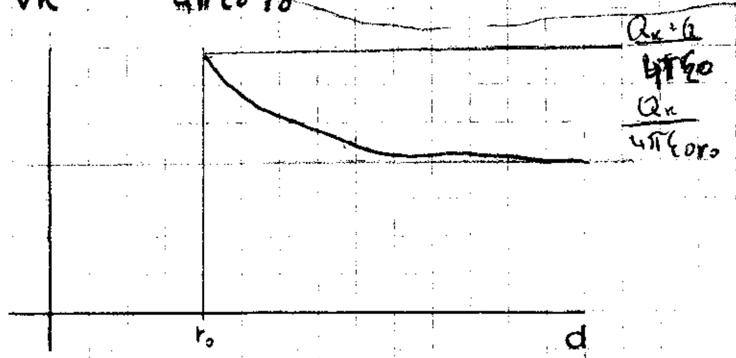
$$Q' = \frac{r_0}{d}Q, \quad e = \frac{r_0^2}{d}$$

zato v sredisce dodam $(Q_k \cdot Q')$

Določimo potencial krogle

$$V_K = V_{Q_K-Q'} + V_{Q_K+Q'} = \frac{Q_K+Q'}{4\pi\varepsilon_0 r_0} \Rightarrow$$

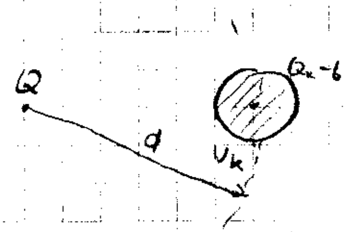
$$V_K = \frac{Q_K + \frac{r_0}{d}Q}{4\pi\varepsilon_0 r_0}$$



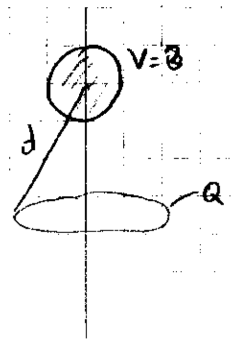
lahko prihajajo še drugi proces elektrenja

$Q_K=0$

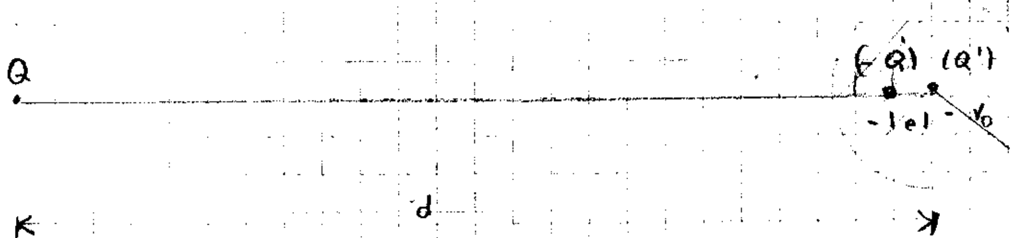
$V_K = \frac{Q}{4\pi\varepsilon_0 d}$



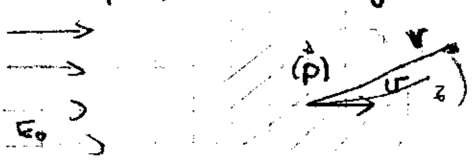
Tak potencial kot ga ima središče krogle



Nov primer $d \gg r_0$



Ta zgodba gre v primer homogenega polja in krogla



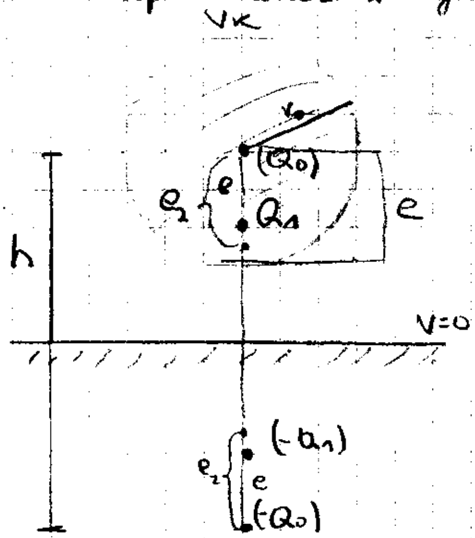
$$p = Q'e = \frac{r_0}{d}Q\,\frac{r_0^2}{d} = \frac{r_0^3 Q}{d^2} \Rightarrow \frac{Q}{d^2} = \frac{p}{r_0^3}$$

$$E_0 = \frac{Q}{4\pi\varepsilon_0 d^2} = \frac{p}{4\pi\varepsilon_0 r_0^3} \Rightarrow p = 4\pi\varepsilon_0 r_0^3 E_0$$

$$\vec{E} = \vec{E}_0 + \vec{E}_p$$

dipolno polje

Zgled: Kapacitivnost krogle nad prevodno podlago



$Q_0 = 4\pi \varepsilon_0 r_0 V_K$

$Q_1 = \frac{r_0}{2h} Q_0 \; ; \quad e = \frac{r_0^2}{2h}$

$Q_2 = \frac{r_0}{2h-e_1} Q_1 \; , \quad e_2 = \frac{r_0^2}{2h-e_1}$

$Q_{n+1} = \frac{r_0}{2h-e_n} Q_n \quad e_{n+1} = \frac{r_0^2}{2h-e_n}$

$e = \lim_{n\to\infty} e_n$

$e = \frac{r_0^2}{2h-e} \Rightarrow -2he + e^2 + r_0^2 = 0$

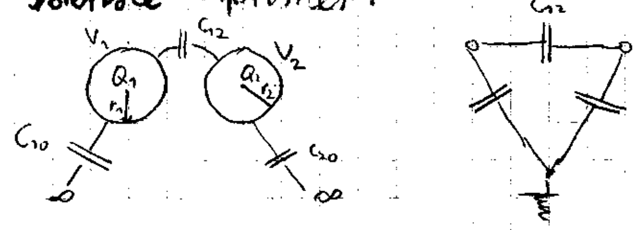
$e = \frac{2h \pm \sqrt{(2h)^2 - 4r_0^2}}{2}$ da dobimo e ne vzdolž od V=0

$e = h - \sqrt{h^2 - r_0^2}$

$Q_{n+1} = \frac{r_0}{2h-e_n} Q_n < \frac{r_0}{2h-e} Q_n$

$Q_K = Q_0 + Q_1 + Q_2 + \dots = \underbrace{C_0 \left[1 + \frac{r_0}{2h} + \frac{r_0^2}{2h(2h-e_1)} + \frac{r_0^3}{2h(2h-e_1)(2h-e_2)} + \dots \right]}_{\text{kapacitivnost sistema krogla podlaga}} V_K$

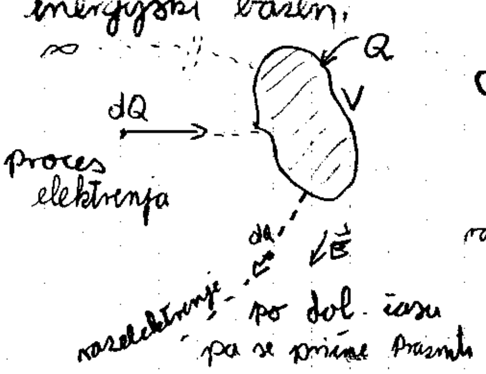
Iskrišče - primer:



Modelno vezje ne izraža okolice

ELEKTRIČNA ENERGIJA

Če delec opravi neko delo, potem moram imeti v ozadju nek energijski bazen.



$dA_z = V\,dQ = \frac{Q}{C}\,dQ$ (vzmemgi)

$A_z = \int dA_z = \int_0^{Q} \frac{Q}{C}\,dQ = \frac{Q_{kon}^2}{2C} = \frac{1}{2} C V_{kon}^2 = \frac{1}{2} Q_{kon} V_{kon}$

razelektr.

$dA_e = dQ \cdot \frac{Q_{kon} - Q}{C}$

$A_e = \int dA_e = \int_0^{Q_{kon}} \frac{Q_{kon} - Q}{C}\,dQ$

$= \frac{Q_{kon}^2}{C} - \frac{Q_{kon}^2}{2C} = \frac{Q_{kon}^2}{2C}$

$W_e = A_z = A_e$

$W_e(t) = \frac{Q^2(t)}{2C}$

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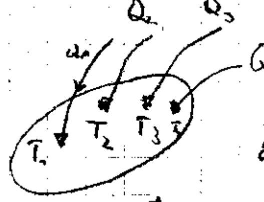


$A_{a1} = 0$

$A_{a2} = Q_2 V^{(1)}(T_2)$

je že poz. 1. $A_{a3} = Q_3 [V^{(1)}(T_3) + V^{(2)} T_3$

$A_{a4} = Q_4 [V^{(1)}(T_4) + V^{(2)}(T_4) + V^{(3)} T_4]$

Če premikam polje potem je delo, če ne ni. -

$A_z = A_{z1} + A_{z2} + A_{z3} + A_{z4}$

Takšno je delo da odide $A_{e1} = Q_1 [V^{(2)} T_1 + V^{(3)} T_1 + V^{(4)}(T_1)]$

$A_{e2} = Q_2 [V^{(3)} T_2 + V^{(4)} T_2]$

$A_{e3} = Q_3 [V^{(4)} T_3]$

$A_{e4} = 0$

$A_e = A_{e1} + A_{e2} + A_{e3} + A_{e4}$

$2 W_e = A_z + A_e$

To je seštevek produktov nabojev s potenciali ostalih nabojev na njegovem mestu.

Sedaj gremo od 4 k n nabojem

$\delta Q_1 + \delta Q_2, \ldots\ldots \delta Q_n$

$$2 W_e = \sum_{k=1}^{n} \delta Q_k \sum_{j \neq k}^{n} \delta V^{(j)}(T_k)$$

če gre $n \to \infty$, gre $\delta Q \to 0$
, postaja diferencial

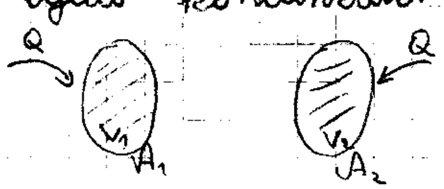
$$W_e = \frac{1}{2} \lim \Big[\sum_{k=1}^{n} \delta Q_k \underbrace{\lim \sum_{\substack{j=1 \\ j \neq k}}^{n} \delta V^{(k)}(T_k)}_{V(T_k)}\Big] = \frac{1}{2} \lim \sum_{k=1}^{n} \delta Q_k V(T_k)$$

$$W_e = \frac{1}{2} \int V dQ$$

$$\frac{\rho \cdot \delta r_k}{2 \varepsilon_0} = \delta V^{(k)}(T_k)$$

$n \to \infty$; $\delta r_k \to 0 \Rightarrow \delta V^{(k)}(T_k) \to 0$

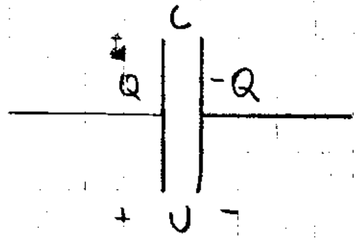
Zgled: kondenzator



$$W_e = \frac{1}{2} \int_{A_1} V_1 dQ + \frac{1}{2} \int_{A_2} V_2 dQ$$ – ker sta konst. – KOVINA!!!

$$= \frac{1}{2} V_1 Q + \frac{1}{2} V_2 (-Q) = \frac{1}{2} Q \overbrace{(V_1 - V_2)}^{U} = \frac{1}{2} QU$$

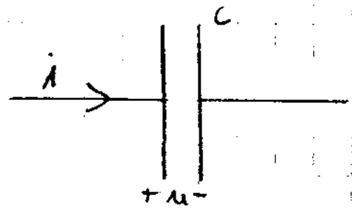
$$= \frac{1}{2} CU^2 = \frac{Q^2}{2C}$$



$$Q = CU$$

$$W_e = \frac{1}{2} C U^2$$

Kondenzator v "živih" vezjih



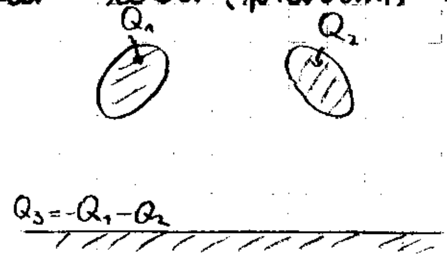
$$i = C\frac{du}{dt}$$

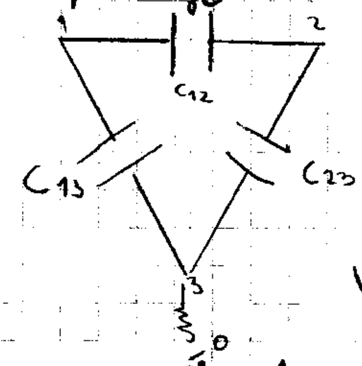
$$W_e = \frac{1}{2} C u^2$$

$$p_e = \frac{dW_e}{dt} = \tfrac{1}{2} C\, 2u \frac{du}{dt} = u \cdot i$$

moč elektrenja, kako hitro se polni, prazni - moč polnenja

Zgled: telesi (prevodni) nad podlago





$$W_e = \frac{1}{2}\int V\,dQ$$

$$= \frac{1}{2} V_1 Q_1 + \frac{1}{2} V_2 Q_2 + \frac{1}{2} \overset{0}{V_3} (Q_3)$$

n-teles nad podlago

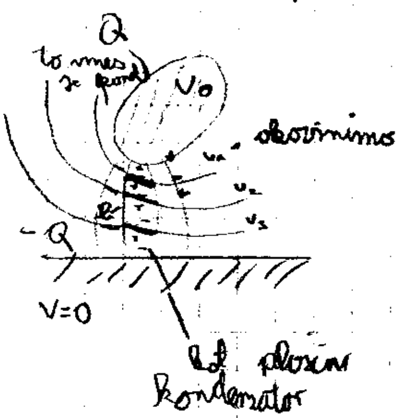
$$W_e = \frac{1}{2} \sum_{k=1}^{n} V_k Q_k$$

$$W_e = \frac{1}{2} V_1 \left[C_{12} (V_1 - V_2) + C_{13} [V_1 - \overset{0}{V_3}] \right] + \frac{1}{2} V_2 \left[C_{12} (V_2 - V_1) + C_{23} (V_2 - V_3) \right]$$

$$= \frac{1}{2} C_{12} \underbrace{(V_1 - V_2)^2}_{U_{12}} + \frac{1}{2} C_{13} \underbrace{V_1^2}_{U_{13}} + \frac{1}{2} C_{23} \underbrace{V_2^2}_{U_{23}}$$

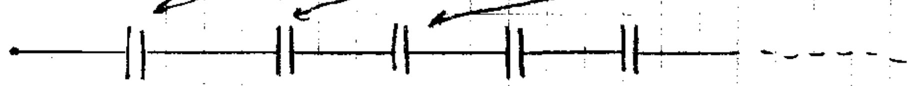
$$W_e = \frac{1}{2} C_{12} U_{12}^2 + C_{13} U_{13}^2 + \frac{1}{2} C_{23} U_{23}^2$$

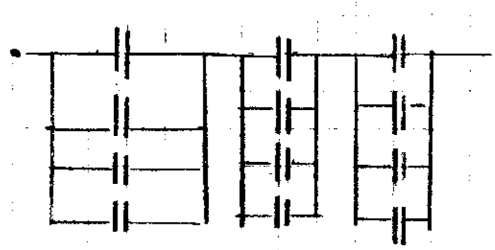
GOSTOTA ENERGIJE



$$W_e = \tfrac{1}{2} Q V_0$$

$$W_e = \frac{1}{2} Q (V_0 - V_1) + \frac{1}{2} Q (V_1 - V_2) + \frac{1}{2} Q (V_2 - V_3) \ldots$$





bodi n - ekvipotencialk in m - pretočnih cevk

$m, n \to \infty$

$$W_e = \frac{1}{2} Q V_0 = \frac{1}{2} \overset{\Phi_e}{Q} \sum_{k=1}^{n} \delta U_k = \frac{1}{2} \sum_{j=1}^{m} \delta \Phi_{e_j} \sum_{k=1}^{n} \delta U_k$$

$$= \frac{1}{2} \sum_{j=1}^{m} \sum_{k=1}^{n} (\vec{D} \cdot \delta \vec{a}_j)(\vec{E} \cdot \delta \vec{l}_k) = \frac{1}{2} \sum_{j=1}^{m} \sum_{k=1}^{n} (\vec{D} \cdot \vec{E}) \cdot \underbrace{(\delta \vec{a}_j \cdot \vec{l}_k)}_{\delta V_{jk}}$$

$(\vec{a} \cdot \vec{b})(\vec{c} \cdot \vec{d})$

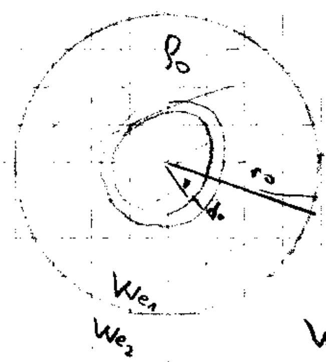
$(\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$

| če greda n in m proti neskončnosti $n \to \infty$

$$W_e = \frac{1}{2} \int_V \vec{D} \cdot \vec{E} \cdot dv = \int_V \underbrace{\left(\frac{1}{2} \vec{D} \cdot \vec{E}\right)}_{w_e} \cdot dv$$

$$w_e = \frac{\vec{D} \cdot \vec{E}}{2} = \frac{\varepsilon E^2}{2} = \frac{D^2}{2\varepsilon} \quad \text{če velja } \vec{D} = \varepsilon \vec{E}$$

Zgled: Krogelni oblak nabojev



$$W_e = \frac{1}{2} \int V dQ$$

$$= \frac{1}{2} \int_0^{r_0} \frac{\rho r_0}{2} \left[1 - \frac{1}{3}\left(\frac{r}{r_0}\right)^2\right] \cdot \rho \pi 4 r^2 dr =$$

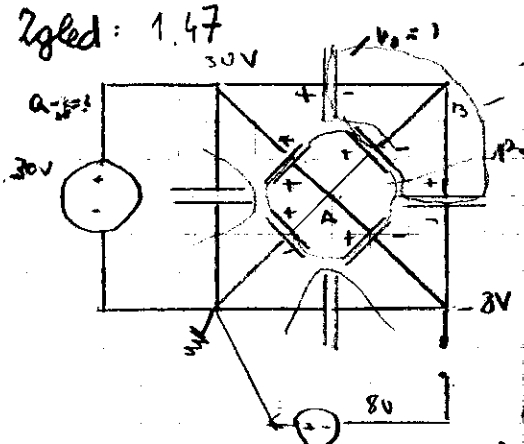
$=$ ......

$$W_{e1} = \int_0^{r_0} \frac{1}{2} \varepsilon_0 \left(\frac{\rho r}{3\varepsilon_0}\right)^2 4\pi r^2 \cdot dr = \dots \quad 1/6 \ 100\%$$

$$W_{e2} = \int \frac{1}{2} \varepsilon_0 \left(\frac{\rho r_0^3}{3\varepsilon_0 r^2}\right) 4\pi r^2 dr = \dots \quad 5/6 \ \%$$

KONDENZATORSKA VEZJA

Zgled: 1.47



$V_A, V_B = ?$

A: $C(V_A - 30V) + C(V_A - V_B) + C(V_A - (-8)V) + C(V_A - 0) = 0$

ker ko sonje nevtralno, ko je bilo pri

B: $-C(30V - V_B) - C(V_A - V_B) + C(V_B - (-8V)) = 0$

$V_A - V_B = 22V$

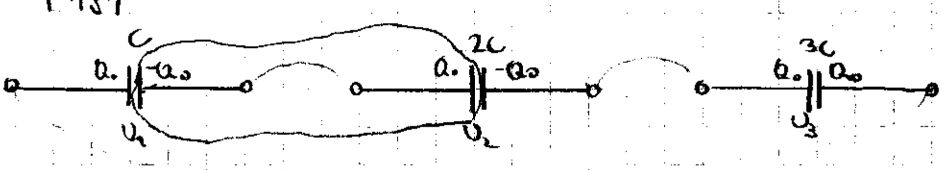
$-V_A + 3V_B = 22V$

$V_A = 8V \qquad V_B = 10V$

$$W_e = \frac{1}{2} C\,(30V - V_B)^2$$

$$Q_{30} = C\cdot(30_V - 0) + C\,(30 - V_A) + C\,(30 - V_B)$$

Zgled 1.131



Stalne napetosti $\frac{Q_0}{C}$, $\frac{Q_0}{2C}$, $\frac{Q_0}{3C}$ Nove napetosti: U_1, U_2, U_3

$$U_1 + U_2 + U_3 = 0 \quad \oint E\, dl = 0$$

novi naboji

$$-CU_1 + 2CU_2 = -Q_0 + Q_0 = 0$$

$$CU_1 + 3CU_3 - Q_0 + Q_0 = 2Q_0$$

$$-U_1 + 2U_2 = 0 \quad / 4$$

$$4U_1 + 3U_2 = \frac{2Q_0}{C}$$

$$U_2 = \frac{2}{11}\frac{Q_0}{C} \qquad U_1 = \frac{4}{11}\frac{Q_0}{C} \qquad U_3 = \frac{6}{11}\frac{Q_0}{C}$$

VODNIK ... MEJNI POGOJI

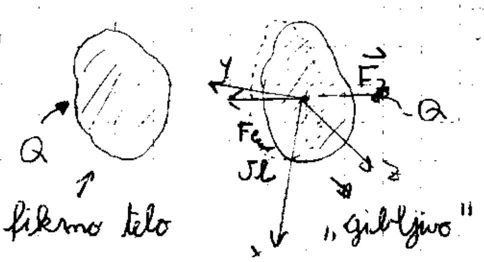
—| |— KONDENZATOR

GIBALNI PROCESI

OET I
12.12.2007

a) SISTEM BREZ VIROV - avtonomen sistem
b) Z VIRI

a)



$\delta A = \vec{F_e}\,\delta l$ Delo gre na račun energije, ki se bo zmanjšala

$$\delta A + \delta W_e = 0$$

$$\vec{F_e} \cdot \delta l + \delta W_e = 0$$

$$F_e \cdot dl + dW_e = 0$$

$$\delta l = (\delta x, 0, 0)$$

$$F_{ex}\, \delta x + \delta W_e = 0$$

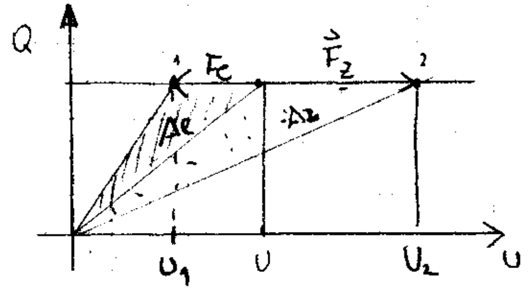
$\delta x \to 0$

$$F_{ex} = -\frac{\partial W_e}{\partial x} \qquad \left\{ E_x = -\frac{\partial V}{\partial x} \right\}$$

$$\vec{F_e} = \left(-\frac{\partial W_e}{\partial x}, -\frac{\partial W_e}{\partial y}, -\frac{\partial W_e}{\partial z} \right)$$

Približevanje gre na zmanjševanje energije z nabojev, oblakov

$\vec{F_e}$ -zunanja, če vlečem narazen bomo pa povečevali električno energijo

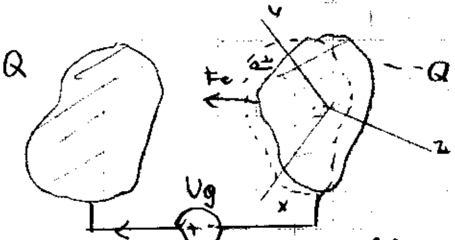


Razlika energij je opravljeno delo

$$A_e = \frac{QU}{2} - \frac{QU_1}{2}$$

$$A_2 = \frac{QU_2}{2} - \frac{QU}{2}$$

B.



se oblinala δQ

$\delta A_e = F_e \cdot \delta l$ opravljeno delo el. sile

$\underset{>0}{\delta W_e}, \underset{>0}{\delta A_g}$ vir opravi delo in se energija se spremeni (se poveča

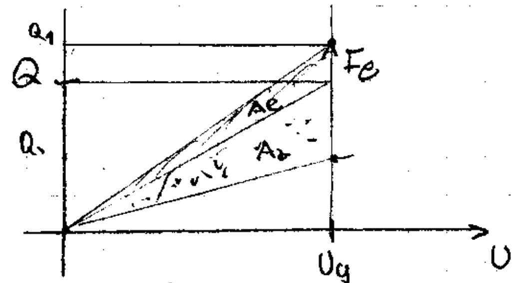
$$\delta A_g = \delta A_e + \delta W_e$$

$$\delta Q \cdot U_g = F_e \cdot \delta l + \underbrace{\tfrac{1}{2}(Q + \delta Q)U_g - \tfrac{1}{2} Q \cdot U_g}_{1/2\, \delta Q\, U_g}$$

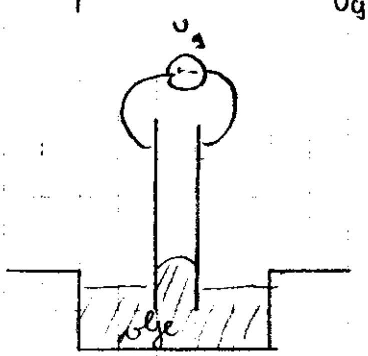
$$\Longrightarrow \frac{1}{2}\delta Q U_g = \vec{F}_e \cdot \delta l = \delta W_e$$

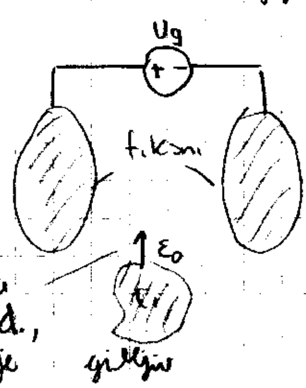
$$\vec{F}_e \cdot \delta l - \delta W_e = 0$$

$$\Longrightarrow \vec{F}_e = \left(\frac{\partial W_e}{\partial x}, \frac{\partial W_e}{\partial y}, \frac{\partial W_e}{\partial z}\right)$$



Lahko pa imam tudi zunanjo silo, ki dajejo kond. narazen pa teče naboj v drugo smer. Zakrat se vir polni (zmanjšuje se akumulacijo We), vir dobi 2We energije (1 smo dali mi z vlečenjem narazen, 1 pa energija)



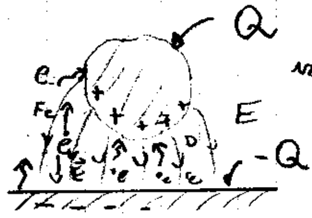


KNJIGA!!!

poveča se C kond., bolj ko je bližje kond. večja je C

Težnja električne sile je, da premakne dielektrik, dipol v električno polje z večjo jakostjo

TOKOVNO POLJE

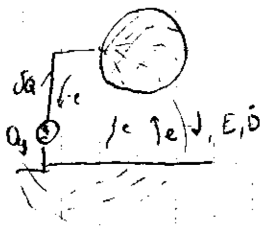


izolant — ima primanjkljaj prostih nabojev $C_0 = 10^{-8}$, pri izolantu $10^{8,5}$ e/m³

To migracijo e imenujemo razelektritev

$Q(t_2 > t_1) < Q(t_1)$ Liste elektrostatike ni

Kaj storiti da bo $Q(t_2) = Q(t_1)$. Priključimo na vir



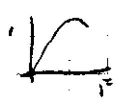
Ta tok prehajanja ~~e~~ imenujem izolacijski tok
Takemu prevajanju pravimo konduktivno prevajanje
ali konduktivni tok

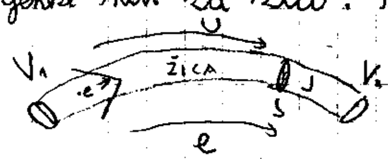
Konvektivni tok je gibanje delcev v neki snovi (plini), skozi neko usmerjeno ...

KONDUKTIVNO PREVAJANJE

1. OHMOV ZAKON , ki ni zakon
2. JOULOV ZAKON

1 Ta zakon ima toliko izjem, da ne more biti zakon. Ne velja za halogenke niti za žico. Paralela v fiziki je HOOKOV ZAKON





$I \propto U$ res za majhne toke

$I \propto U = V_1 - U_2$

$J \cdot S \propto E \cdot l \Rightarrow J \propto E$

ρw, F_e; $w \propto E$, $w \propto F_e$

v fiziki je $\sum F_i = 0$

$F_e - F_3 = 0$

to je sila upora

$\vec{F_e} + \vec{F_U} = \vec{0}$

$\Rightarrow F_U \propto w$ veljaven linearni zakon upora

LINEARNI ZAKON UPORA

Model trkov (v knjigi). Je gibalna enačba:

$$\delta m \cdot \dot{\vec{v}} = -e \cdot E + \vec{F_o}$$
$$= -e \cdot E - k \cdot \vec{v}$$

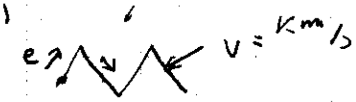
zaključek enačbe:

$$J = \underbrace{\frac{n e^2 T}{m}}_{\gamma} \cdot E$$

to konstanto imenujem specifična električna prevodnost $[A/Vm] = [S/m]$ SIEMENSI $S = \frac{1}{\Omega}$ $\Omega\, [\frac{V}{A}]$

$$\gamma = \frac{n \cdot e^2 T_r}{m} \cdot E$$

n - št. prostih el.
e^2 - ² nabojo osnovnega
T - čas svobodnega preleta povprečni čas
m - masa delca (el.)
f - $1/T$ - frekvenca trkov



$J = \gamma \cdot E$ podobno kot $D = \varepsilon E$
to izmerimo

v = μm/s

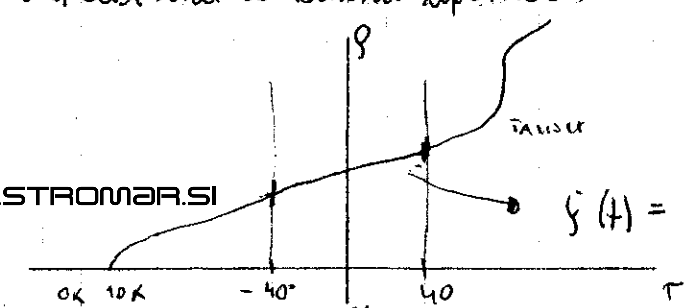
Če bo čas daljši bo prevodnost večja in če bo masa večja bo specifična prevodnost manjša.

Temperatura se skriva v času preleto Večja kot je manjši je čas

SNOV	SP. EL PREVODNOST [S/m]	α (temp. koeficient $[K^{-1}]$)
supraprevodnik	∞	
baker	$56 \cdot 10^6$	0,0039
aluminij	$35 \cdot 10^6$	
železo in jeklo	$10 \cdot 10^6$	
oglje, grafit	10^4	<0
morska voda	4	
sladka voda	$\sim 10^{-2}$	
zemlja	$\sim 10^{-3}$	
izolanti	$\sim 10^{-10}$	
idealni izolant	0	

~~TEMPERATURNA ODVISNOST~~ $\boxed{\text{TEMP ODVISNOST}}$

$\frac{1}{\gamma} = \rho$ (specifična električna upornost)



$$\rho(T) = \rho(20)\,[1 + \alpha(T - 20°C)]$$

$$\left.\frac{\frac{\rho(T)-\rho(T_0)}{\rho(T_0)}}{T-T_0}\right\} = \alpha$$ relativno povečanje

α - temperaturni koeficient ali količnik

pri bakru je $\alpha \simeq \frac{0{,}4\%}{K}$

$\rho(80°C)$ za Cu $\simeq 1{,}24\ \rho(20°C)$

OET I 18. 12. 2007

Razširjen Ohmov zakon

$J = \gamma \cdot E \; | \; J = \gamma(\vec{E} + \vec{w}\times\vec{B} + \vec{E}_g)$

Nekdo potiska e, da dovoljajo upor skozi toka trke.

$$\vec{J} = \gamma(\vec{E} + \frac{\vec{J}}{\rho_w}\times B + \vec{E}_g)$$

$$\frac{J}{\gamma} - \frac{J}{\rho_w}\times B = \vec{E} + \vec{E}_g \quad \Rightarrow \vec{J} = \gamma(\vec{E} + \vec{E}_g)$$

Primer: vzamemo baker

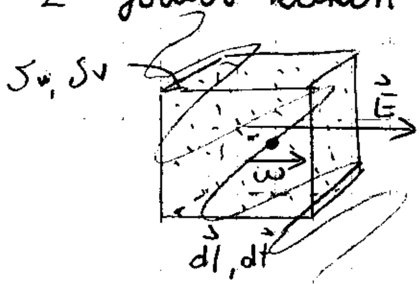
$\gamma = 56 \cdot 10^6\ S/m$

$\rho = -1{,}34 \cdot 10^{?}\ C/m^3$

$B = mT$

$$\Rightarrow \frac{1}{\gamma} \gg \left|\frac{\vec{B}}{\rho_w}\right|$$

2 Joulov zakon



$$dA_e = (\underbrace{\rho_w \cdot \delta v \vec{E}}_{\vec{F}}) \cdot (\underbrace{\vec{w} \cdot dt}_{dl})$$

$$= (J \cdot \vec{E})\,\delta v\, dt$$

$$\delta P_e = \frac{dA_e}{dt} = (\vec{J}\cdot\vec{E})\,\delta v$$ — volumen

$$\frac{\delta P_e}{\delta v} = \vec{J}\cdot\vec{E} = p_e = p_t$$ — gostota moči sproščanja toplote

Pri modelu trkov sta 2 sili nasprotno enaki:

$$\delta\vec{F}_e + \delta\vec{F}_u = 0$$ (F_u — upora)

če je / kjer je $\vec{E}_g = 0 \Rightarrow \vec{J}\cdot\vec{E} = \frac{J^2}{\gamma} = \gamma \cdot E^2$

Primer:

$J = 4\,A/mm^2$ če so večje je treba že hladiti, vsaj zračno

$\gamma_{Cu} = 56 \cdot 10^6\ S/m$

$$p_e = \frac{16 \cdot 10^{12}}{56 \cdot 10^6} \simeq 300\ kW/m^3$$

WWW.STROMAR.SI

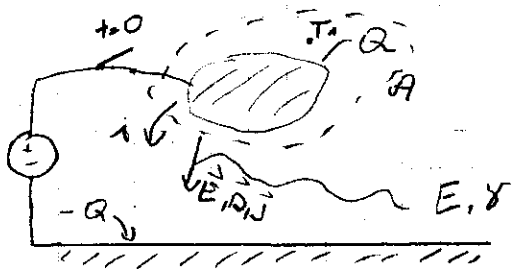
OHMOV IN JOULOV ZAKON V ČASOVNEM SPREMENLJIVEM PROSTORU

Toki sinhrono sledijo električnemu polju

$$\underline{J(T,t) = \gamma \vec{E}(T,t)} \Rightarrow p_T(T,t) = \vec{J}(T,t)\cdot\vec{E}(T,t)$$

Frekvence tokov so $10^5 - 10^6$ višje kot so frekvence naših virov

PRAZNJENJE KONDENZATORJA



Če hočemo imeti stalno množino naboja, rabimo vir

$t>0$

$$i = \oint_A \vec{J}\cdot d\vec{a} \qquad J=\gamma E,\quad D=\varepsilon \vec{E}$$

$$= \frac{\gamma}{\varepsilon}\underbrace{\oint_A D\cdot d\vec{a}}_{Q} = \frac{\gamma}{\varepsilon} Q = -\frac{dQ}{dt}$$

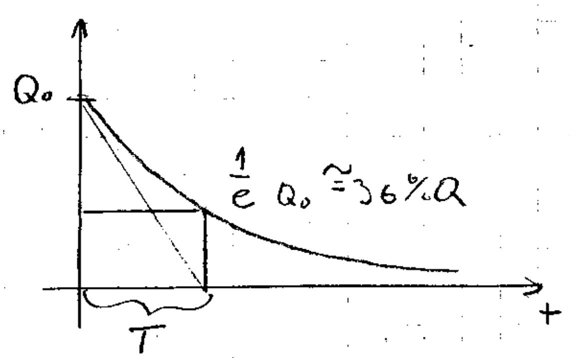
$$\frac{dQ}{dt} + \frac{\gamma}{\varepsilon} Q = 0;\quad Q(t) = A\cdot e^{\lambda t};$$

$$\frac{dQ}{dt} = \lambda A\cdot e^{\lambda t}$$

$$\Rightarrow (\lambda + \frac{\gamma}{\varepsilon})A e^{\lambda t} = 0 \rightarrow \lambda = -\gamma/\varepsilon$$

$$\frac{\varepsilon}{\gamma} = T \quad ;\ e^{\lambda t} = {}^{-t}/T$$

$$\left.\begin{array}{l} Q(t=0) = Q_0 \\ Q_0 = A\cdot e^{\lambda 0} = A \end{array}\right\} \underline{Q(t) = Q_0 e^{-t/T}}$$



Primer: $\varepsilon = 10^{-10}$
$\gamma = 10^{-13}$

$$T = \varepsilon/\gamma \sim 10^3 = 1000\,s$$

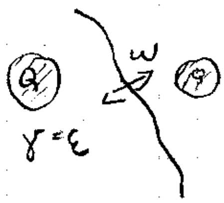
$\underline{J(T_1,t) = J(T_1,0)\,e^{-t/T}}$ vse zadeve se sesedajo, tudi energija

$$p_T(T_1,t) = \frac{J^2(T_1,0)}{\gamma}\cdot e^{-2t/T}$$

$$w_{TITF}\int_0^\infty p_t\,dt = \frac{J^2(T_1,0)}{\gamma}\underbrace{\int_0^\infty e^{-2t/\tau}\,dt}_{-\frac{\tau}{2}\cdot e^{-2t/\tau}\Big|_0^\infty = \tau/2} \overset{\varepsilon/\gamma}{=} \frac{\tau}{2}\,\frac{J^2(T_1,0)}{\gamma^2} =$$

$$= \frac{\varepsilon}{2}\,E^2(T_1,0) = w_e(T_1,0)$$

spremeni svojo naravo, in prvotna električna energija se spreminja v toploto



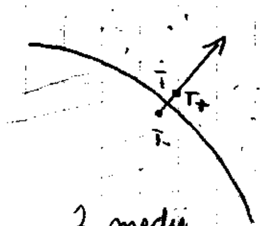
MEJNI POGOJI ČASOVNO STALNEGA TOKOVNEGA POLJA

$\oint_A \vec{J}\cdot d\vec{a} = 0$ časovno stalne razmere

$\oint_L \vec{E}\cdot d\vec{l} = 0 \;\Rightarrow\; E_t(T_+) - E_t(T_-) = 0$ $\quad \vec{D} = \varepsilon\vec{E}$, $\quad \vec{J} = \gamma(\vec{E} + \vec{E}_g)$

$\oint D\cdot da = Q_{\text{not. prosti}}$ $\quad - D_n(T_+) - D(T_-) = \sigma_{\text{prosti}}(T)$

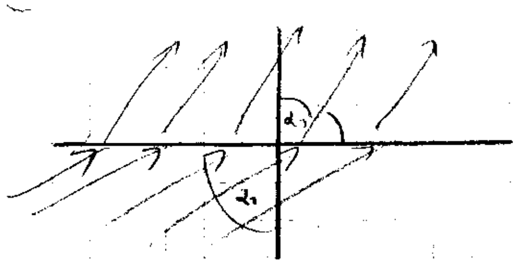
$\oint J\cdot d\vec{a} = 0 \qquad J_N(T_+) - J_n(T_-) = 0$



$E_T(T_+) - E_T(T_-)$

$\gamma\,E_n(T_+) = \gamma_2\,E_n(T_-)$

$$\frac{E_T(T_+)}{\gamma_1 E_n(T_+)} = \frac{E_t(T_-)}{\gamma_2(E_n T_-)} \Rightarrow$$



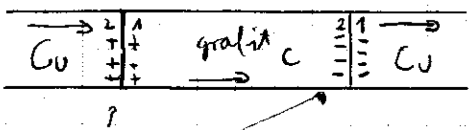
$$\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\gamma_1}{\gamma_2}$$

$$\frac{\varepsilon_1}{\gamma_1}J_n(T_+) - \frac{\varepsilon_2}{\gamma_2}J_n(T_-) = \sigma_{\text{prosti}}(T)$$

$$\left(\frac{\varepsilon_1}{\gamma_1} - \frac{\varepsilon_2}{\gamma_2}\right)J_n(T=) = \sigma_{\text{prosti}}(T)$$

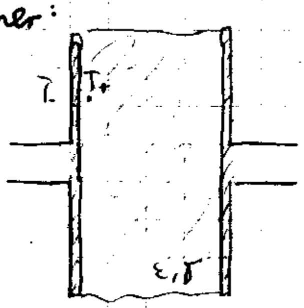
Primer: 1.

Vektor polarizacije je 0



$\left(\frac{\varepsilon_1}{\gamma_1} - \frac{\varepsilon_2}{\gamma_2}\right)$

Primer:



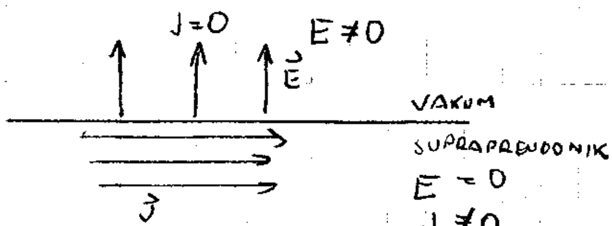
$\frac{\varepsilon_{iz}}{\gamma_{iz}}$ $J_n(T_+) \cong \sigma_{prosti}$

$E_n(T_+) \simeq \frac{\sigma_{prosti}}{\varepsilon}$

DVE MEJI

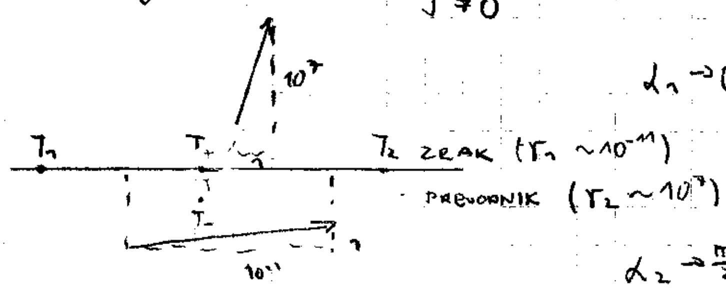
1) idealen izolant - idealen prevodnik
2) meja realen izolant - realen prevodnik

1.)



v enem prostoru imamo zgolj tokovno drugje zgolj električno polje

2.)



$\alpha_1 \to 0$

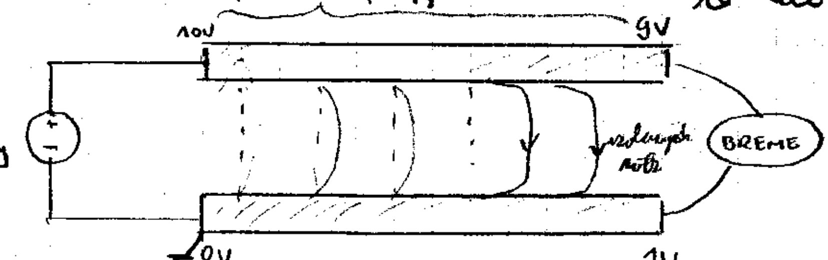
$\alpha_2 \to \frac{\pi}{2}$

$E_n(T_+) \sim 10^6\ V/m$, $J_n(T_+) = 10^{-5}\ A/m^2$ $\quad$ $J_n(T_-) \sim 10^{-5}\ A/m^2$

$J_t(T_-) \sim 10^6\ A/m^2$, $E_t(T_-) \sim 0{,}1\ V/m$ $\quad$ $E_t(T_+) \sim 10^{-1}\ V/m$

$U_{12} = \int_{T_1}^{T_2} \vec{E}\cdot d\vec{l} = \int_{T_1}^{T_2} E_T \cdot dl$

Prevodnik ni več ekvipotencialka, ko teče skozi njega.

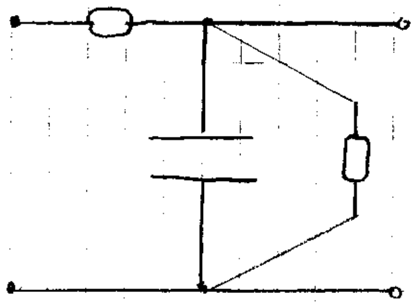


Breme dobi samo 8V

$\frac{U_g - U}{U_g} \cdot 100\% \sim 2\%$

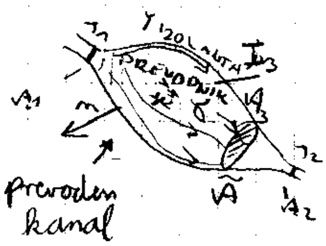
k temu stremimo

Če toka ni modeliramo s kondenzatorji, sedaj pa moramo dodati še upor, celo 2.



ELEKTRIČNA PREVODNOST IN UPORNOST

OET I 19.12.200



$A_1 \cup A_2 \cup A_3 = A$ γ izolanta <<< γ prevodnika

I_3 - izolacijski tok

$$\oint \vec{J}\cdot d\vec{a} = 0 \Rightarrow \underbrace{\overbrace{\int_{A_1} J\cdot da}^{<0}}_{-I_{vstopni}} + \underbrace{\overbrace{\int_{A_2} J\cdot da}^{>0}}_{I_{izstopni}} + \underbrace{\int_{A_3} J\cdot da}_{\text{izolacijski tok}} = 0$$

$-I_1 + I_2 + I_3 = 0$, ako je γ izolanta <<< γ $\Rightarrow I_3 <<< I_1, I_2$

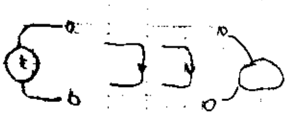
sledi: $I_2 \simeq I_1 \Rightarrow I_1 = I_2 = I$ (tok konduktivnega kanala)

Imamo pa tudi električno polje:

$\int_{T_1}^{T_2} \vec{E}\cdot d\vec{l} = V_1 - V_2 = U$ (napetost med koncema kanala)

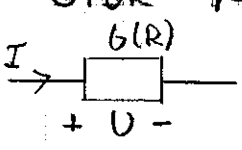
$I = \int_{\tilde{A}} J\cdot d\vec{a}$; če v kanalu vlada relacija $\vec{J} = \gamma\cdot\vec{E}$ potem velja med I in U sorazmerna zveza. (velja linearna upornost) $I \propto U$

$\frac{I}{U} = G$ (električna prevodnost); $1/G = R$ (električna upornost); $R = \frac{U}{I}$



če dam v slano vodo ne morem povedati upornosti

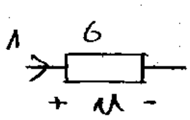
UPOR KOT ELEMENT EL. VEZIJ



$I = GU$ podobno pri kondenzatorju $Q = CU$

upor v "živih" vezjih

$\vec{J}(T,t) = \gamma\cdot\vec{E}(T,t)$



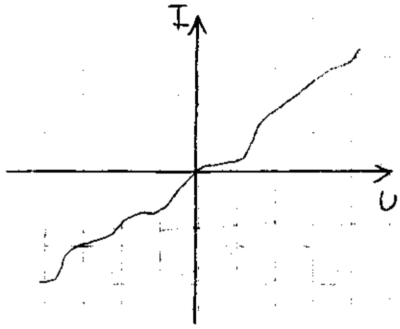
$i = Gu$ kondenzator $i = C\frac{du}{dt}$

$\Rightarrow i = Gu$

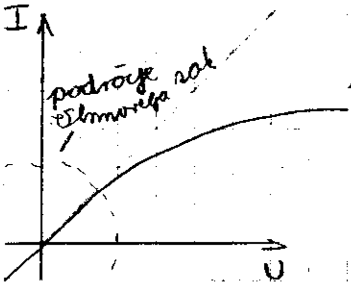
NELINEAREN UPOR

Tok ni sorazmeren napetosti $I \not\propto U$. Pišemo, da je tok funkcija napetosti

$I = f(U)$ to funkcijo poznamemo tabelarično.

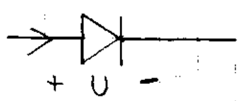


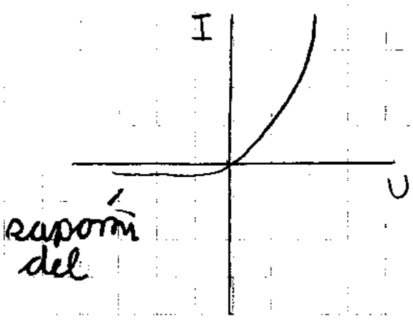
Primer: žica



s segrevanjem žice se specifična upornost poveča

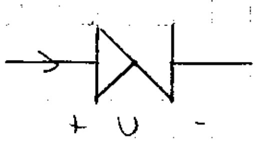
Primer: dioda

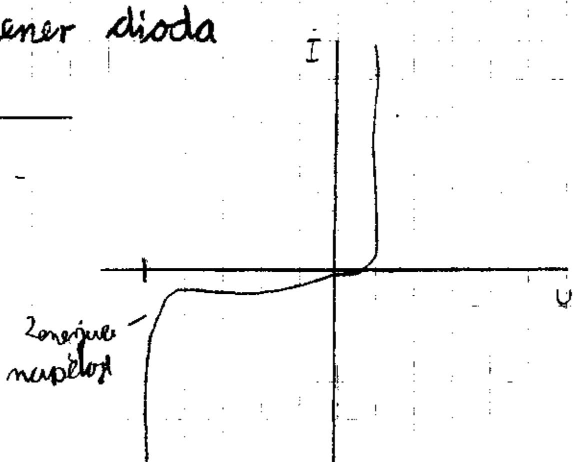




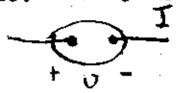
dostikrat se riše kar takole

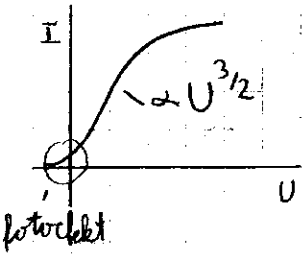
Primer: Zener dioda





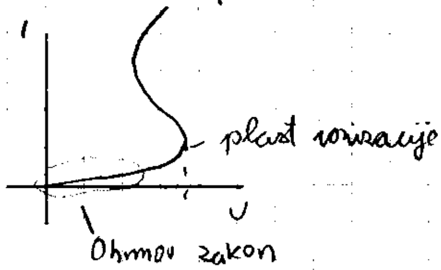
Primer: vakumska dioda



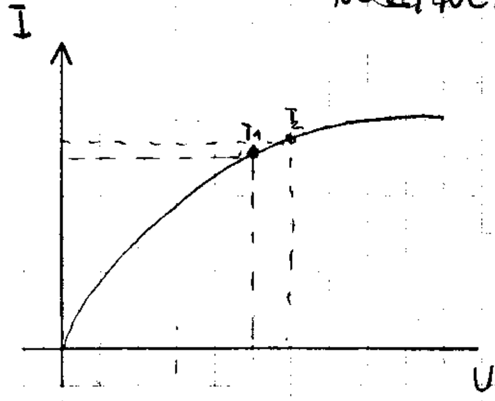


do zasičenja pride zaradi velikosti katode

Primer: tlivka (floresentna cev)



STATIČNA IN DINAMIČNA PREVODNOST / UPORNOST NELINEARNEGA UPORA



statična ~~upor~~ prevodnost ustreza eni točki in je od točke do točke različna

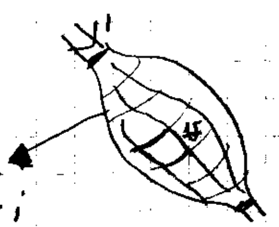
$G_{(s)}(T_1) = I_1/U_1 = 1/R(T_1) = f(U_1)/U_1$

Dinamična

$$G_D = \lim_{U_2 \to U_1} \frac{I_2 - I_1}{U_2 - U_1} = \frac{dI}{dU} = f'(T_1)$$

$\boxed{\text{Moč}}$

Moč konduktivnega kanala



$\vec{J} = \gamma \cdot \vec{E}$;

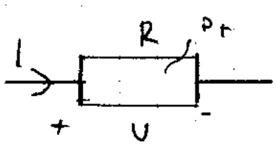
$p_T = J \cdot \vec{E}$

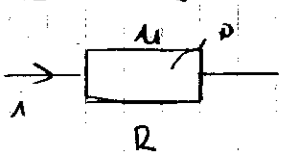
$$P_T = \int p_T \, dv = \int (\vec{J} \cdot \vec{E}) \, d\vec{a} \cdot d\vec{l} = \int (\vec{J} \cdot d\vec{a})(\vec{E} \cdot d\vec{l})$$

$$= \int di \underbrace{\int \vec{E} \cdot d\vec{l}}_{U} = U \int dI = U \cdot I$$

če je kanal linearen, da velja $I = GU$ potem je:

$$P_T = U \cdot I = GU^2 = RI^2$$





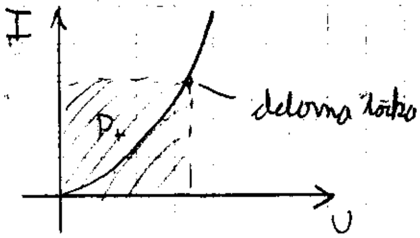
$u = Ri$

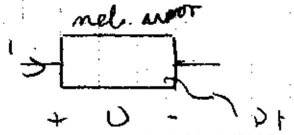
$p_T = Ri^2$

$U = R \cdot I$

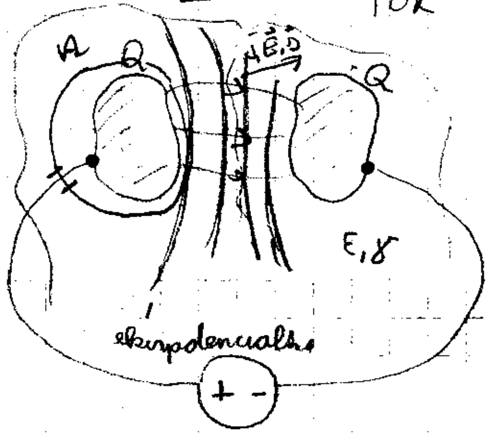
$P_T = RI$

Če je kanal nelinearen





$\boxed{\text{DUALNOST}}$ (SVOJNOST) ELEKT IN TOK POLJA



$$\vec{J} = \gamma \cdot \vec{E}$$
$$\vec{D} = \varepsilon \vec{E}$$

$$\oint_A \overbrace{\vec{J} \cdot d\vec{a}}^{I} = \frac{\gamma}{\varepsilon} \oint_A \overbrace{\vec{D} \cdot d\vec{a}}^{Q}$$

izgubni izolacijski tok

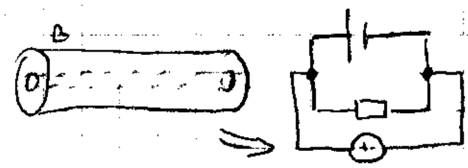
$$\Rightarrow I = \frac{\gamma}{\varepsilon} Q$$

izolacijski pr. GU   C·U

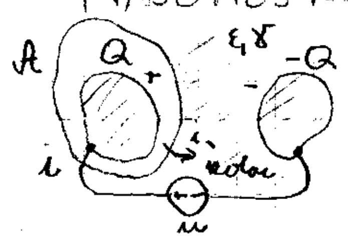
$$GU = \frac{\gamma}{\varepsilon} CU$$
$$\frac{G}{C} = \frac{\gamma}{\varepsilon}$$

Zgled: koaksialni kabel

$$C = \frac{2\pi \varepsilon l}{\ln b/a}$$
$$\Rightarrow G = \frac{2\pi \gamma l}{\ln b/a}$$



NADOMESTNO VEZJE, VEZJE KONDENZATORJA

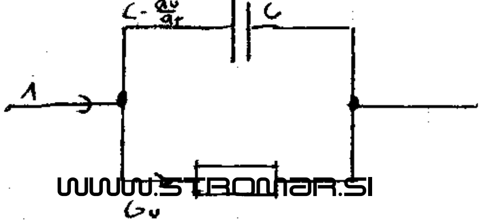


$$A: \ -i + i_{\text{izol.}} = -\frac{dQ}{dt}$$
$$-i + Gu = -\frac{d}{dt} C \cdot u = -C \frac{du}{dt}$$

$$i = Gu + C\frac{du}{dt} \text{ - polnilni}$$

izolacijski   enačba kondenzatorja

To enačbo realiziramo z nadomestnim vezjem



Zgled: izgubni polnilni tok

Imamo kondenzator s C in G. vzbujamo ga z napetostjo $u = U_m \sin \omega t$

$$i = Gu + C\frac{du}{dt} = \underbrace{G U_m}_{\text{amplituda izgubnega toka}} \sin \omega t + \underbrace{\omega C U_m}_{\text{amplituda polnilnega toka}} \cos \omega t$$

$$= I_m \sin(\omega t + \alpha)$$

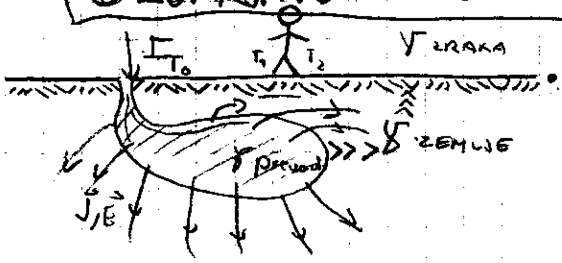
Ocena: kakšno je razmerje

$$\frac{G U_m}{\omega C U_m} = \frac{G}{\omega C} = \frac{\gamma}{\omega \varepsilon}$$ ta formula nam pove kako dober je ta prevodnik

če $\frac{\gamma}{\omega \varepsilon} \ll 1$ dober izolant

$\frac{\gamma}{\omega \varepsilon} \gg 1$ dober prevodnik

OZEMLJITVENA (PONIKALNA) UPORNOST

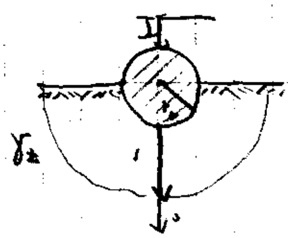


$$\int_{T_0}^{T_\infty} \vec E \cdot d\vec l = V(T_0)$$ - potencial ozemljilnega telesa

$$R_{oz} = \frac{V(T_0)}{I}$$ - potencial ozemljila

$$U_{12} = \int_{T_1}^{T_2} \vec E \cdot d\vec l$$ (napetost koraka) Ozemljitvena upornost mora biti majhna

Zgled: polkroglno ozemljilo



$$J_r = \frac{I}{2\pi r^2} \quad ; \quad E_r = \frac{J_r}{\gamma_z} = \frac{I}{2\pi \gamma_z r^2}$$

$$V(T_0) = \int_{r_0}^{\infty} \frac{1}{2\pi \gamma_z r^2} = \frac{1}{2\pi \gamma_z r_0}$$

$$R_{oz} = \frac{V(T_0)}{I} = \frac{1}{2\pi \gamma_z r_0}$$

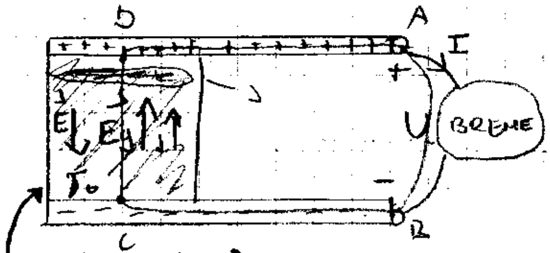
Primer

$r_0 = 1m$

$\gamma_z = 10^{-2}$

$R_{oz} \cong 16\ \Omega$

Realen napetostni vir



$J = \gamma_G (\vec{E}+\vec{E}_g)$

Razlika žene tok
generatorski substanci

Za zanko velja:

$$\oint_L \vec{E}\cdot d\vec{l} = 0 \Rightarrow \underbrace{\int_A^B E\cdot dl}_{U} + \underbrace{\int_B^C E\cdot dl}_{R_{BC}\cdot I} + \underbrace{\int_C^D E\cdot dl}_{\int_C^D \frac{J}{\gamma_G} dl - \int_C^D E_g\, dl} + \underbrace{\int_D^A E\cdot dl}_{R_{AD}\cdot I} = 0$$

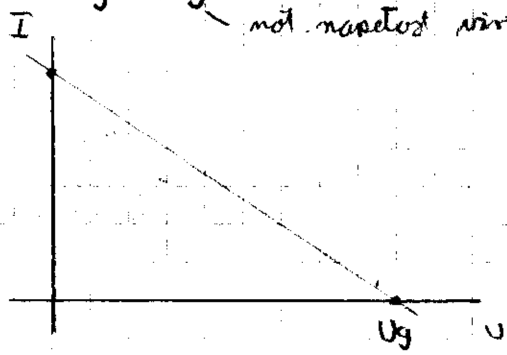
$$\int_C^D \frac{J}{\gamma_G} dl \Rightarrow I \int \frac{dl}{S_g \gamma_g} \Rightarrow R_{CD}, \qquad \int_C^D E_g\, dl = U_g$$

$$\underbrace{R_{CD} I - U_g}$$

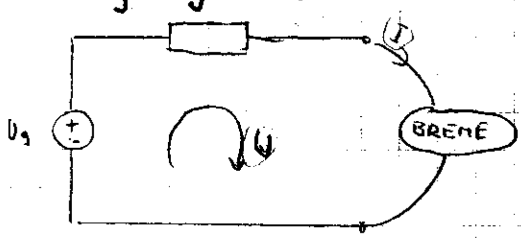
$$U + \overbrace{(R_{BC}+R_{AD}+R_{CD})}^{R_g} I - U_g = 0$$

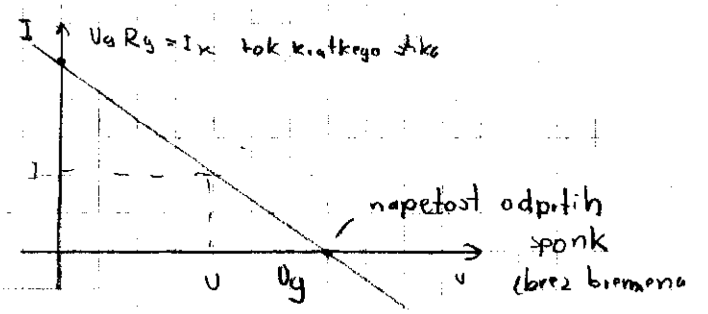
$$U = U_g - R_g\cdot I$$

not. napetost vira



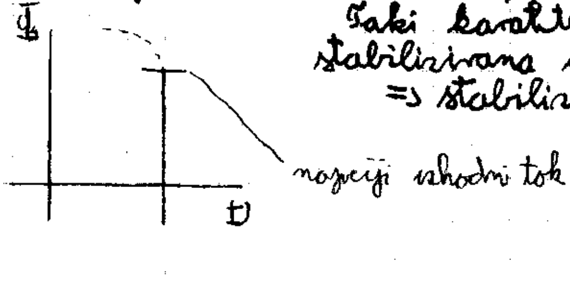
$$-U_g + R_g I + U = 0$$





Idealen napetostni vir

Ko bo premica vertikalna, bo vir idealen.
Taki karakteristiki se približajo
stabilizirana usmerna vezja
=> stabiliziran usmernik

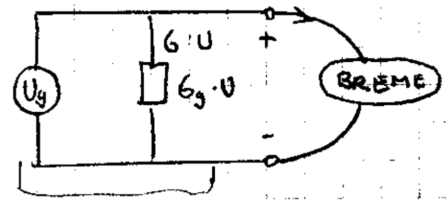


REALEN TOKOVNI VIR

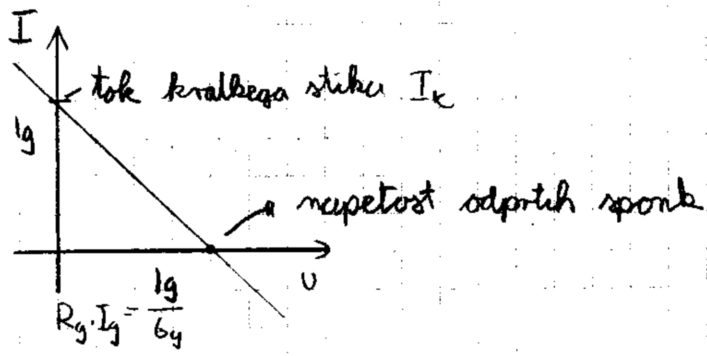
$$-U_g + R_g I + U = 0$$

$$I = \frac{U_g}{R_g} - \frac{U}{R_g} = I_g - G_g U$$

$$I = I_g - G_g U$$ – asociira me na Kirchhoffov tokovni zakon

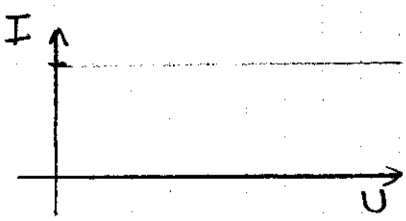


realen tokovni vir



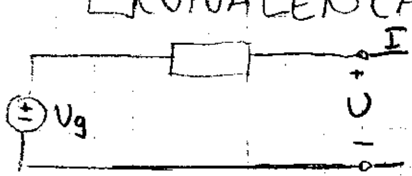
Idealen tokovni vir

IDEALNI TOKOVNI VIR



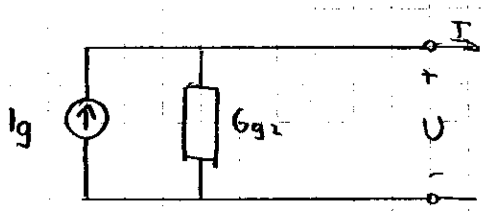
$R_g = \infty$

EKUIVALENCA VIR



$$-U_{g1} + R_{g1} I + U = 0$$

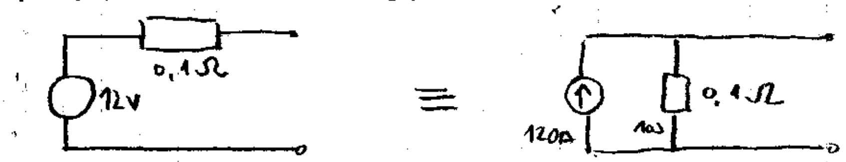
$$I = \frac{U_{g1}}{R_{g1}} - \frac{U}{R_{g1}} = G_{g1} U_{g1} - G_{g1} U$$



$\rightarrow * \quad I = I_{g2} - G_{g2} U$

$I_{g2} = G_{g1} U_{g1} \qquad G_{g1} = G_{g2}$

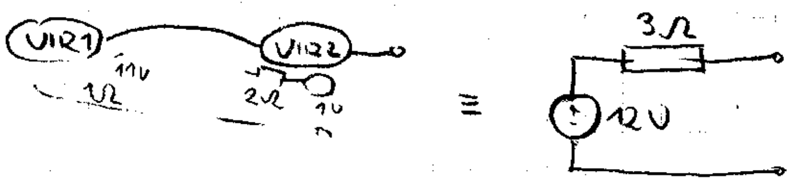
Primer: akumulator

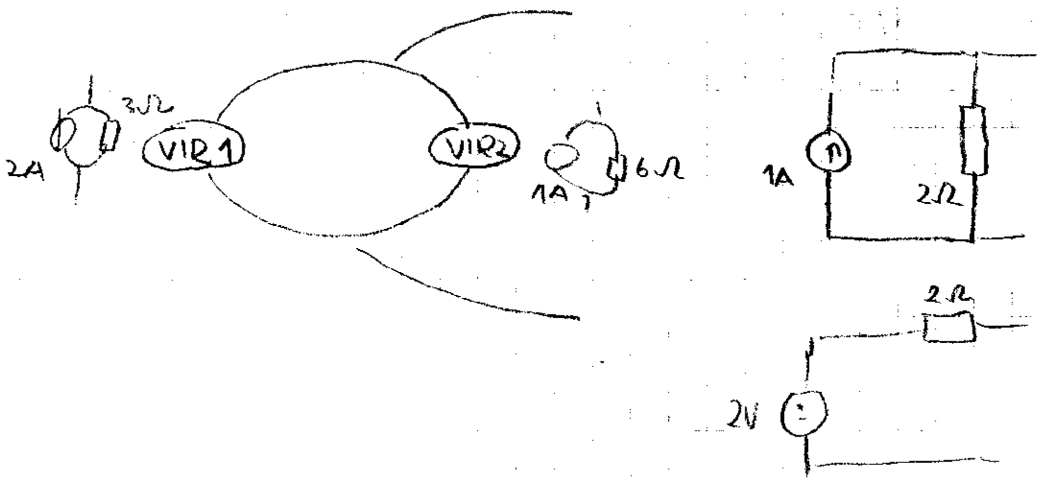


Če želimo imeti večje napetosti, vežemo vse.

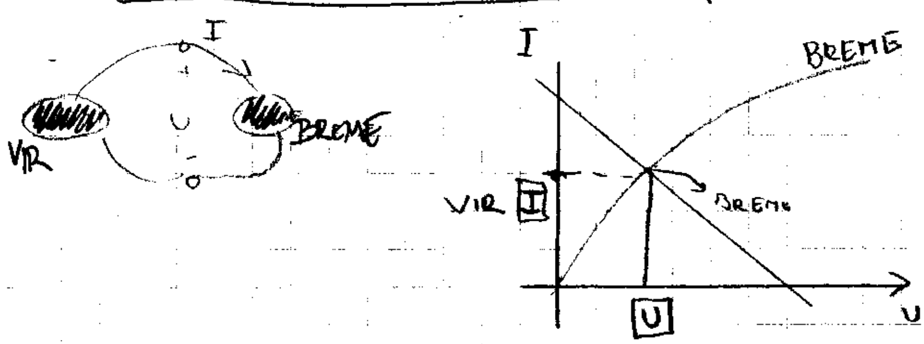
VEZAVE VIROV

Če hočemo imeti večjo napetost, jih vežemo , če pa višje tokove, pa jih vežemo zaporedno



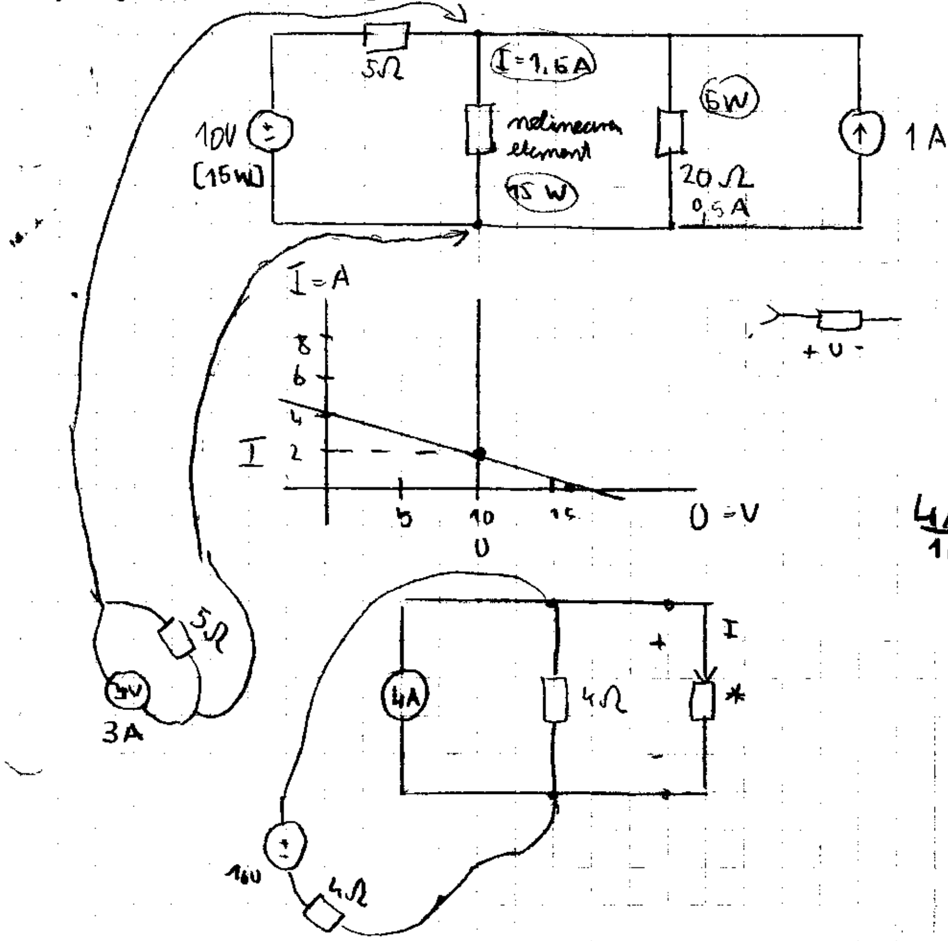


ENOSTAVEN TOKOKROG



$\boxed{I}$ in $\boxed{U}$ - delovni količini tega vezja, kar sta na sponkah.

Primer 3.13



[10 W]

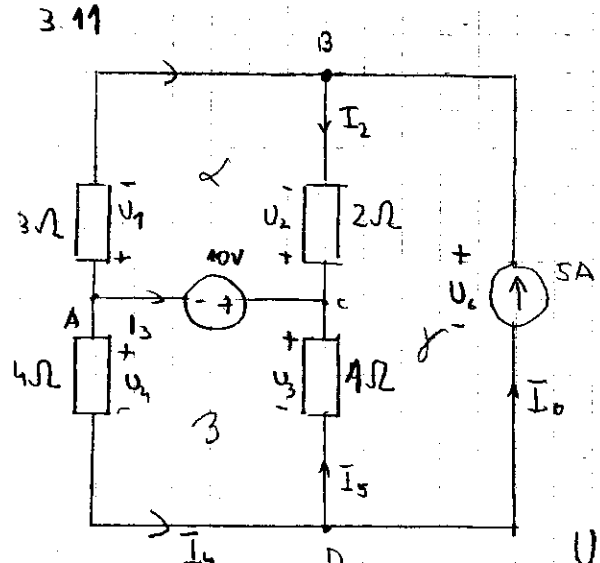
$$\frac{4A}{16} = \frac{I}{6}$$

$I = 1,5\,A \qquad U = 10\,V$

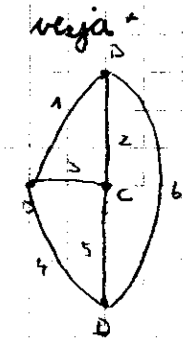
METODE ANALIZE ENOSMERNIH LINEARNIH EL. VEZIJ

1. direktna metoda
2. metoda spojiščnih potencialov
3. metoda zančnih tokov

Primer 3.11



graf vej:



neznanke so:
- U_1 ali I_1
- U_2 ali I_2
- U_3 I_3
- U_4 ali I_4
- U_5 ali I_5
- U_6

$U_1 = 3\,\Omega \cdot I_1$

$U_2 = -2\,\Omega \cdot I_2$

$U_4 = 4\,\Omega \cdot I_4$

$U_5 = -4\,\Omega \cdot I_5$

WWW.STROMAR.SI

I. Kirchhoffov zakon

A: $\boxed{I_1} + I_3 + \boxed{I_4} = 0$
B: $-\boxed{I_1} + I_2 - 5A = 0$
C: $-I_2 - I_3 - I_5 = 0$
D: $\cancel{-I_4 + I_5} + 5A = 0$

A+B+C : $I_4 - 5A + I_5 = 0 = D$

2. Kirchhof: $U = 0$

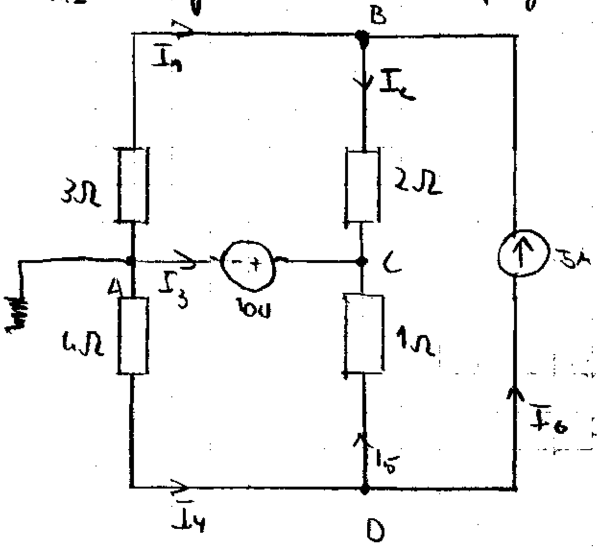
zberem si kaj so neznanke iz prejšnje strani.

α: $U_1 - \boxed{U_2} + 10V = 0$

β: $-U_4 - 10 + \boxed{U_5} = 0$

γ: $-\boxed{U_5} + \boxed{U_2} + U_4 = 0$

a12 Reši z metodo spojiščnih potencialov



$V_A = 0$, $V_B = ?$, $V_C = 10V$
$V_D = ?$

B: $-I_1 + I_2 - 5A = 0$
D: $-I_4 + I_5 + 5A = 0$

$-\frac{0V - V_B}{3\Omega} + \frac{V_B - 10V}{2\Omega} - 5A = 0 \quad / 6\Omega$

$-\frac{0V - V_D}{4\Omega} + \frac{V_D - 10V}{1\Omega} + 5A = 0 \quad / 4\Omega$

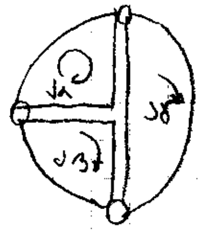
$5V_B - 30V - 30V = 0$
$V_B = \frac{60}{5} = 12V$

$5V_D - 40V + 20V = 0 \Rightarrow V_D = 4V$

$I_1 = -4A$, $U_1 = -12V$, $P_1 = U_1 I_1 = 48W$
$I_2 = 1A$, $U_2 = -2V$, $P_2 = -U_2 I_2 = 2W$
$I_3 = 5A$, $U_3 = 10V$, $P_3 = 10V \cdot 5 = 50W$
$I_4 = -1A$, $U_4 = -4V$, $P_4 = U_4 \cdot I_4 = 4W$
$I_5 = -6A$, $U_5 = 6V$, $P_5 = 36W$
$I_6 = 5A$, $U_6 = 8V$, $P_6 = 5A \cdot U_6 = 40W$

$P_3 = 50W$
$P_3 = 50W$

a13



$I_1 = J\alpha$
$I_2 = J\alpha + J\gamma = -5A$
$I_3 = J\alpha - J\beta$
$I_4 = -J\beta - 5A$
$I_5 = J\beta - J\gamma$
$I_6 = -J\gamma = +5A$

Zapiši neodvisno enačbo v tisti zanki, v kateri vsiljen tok

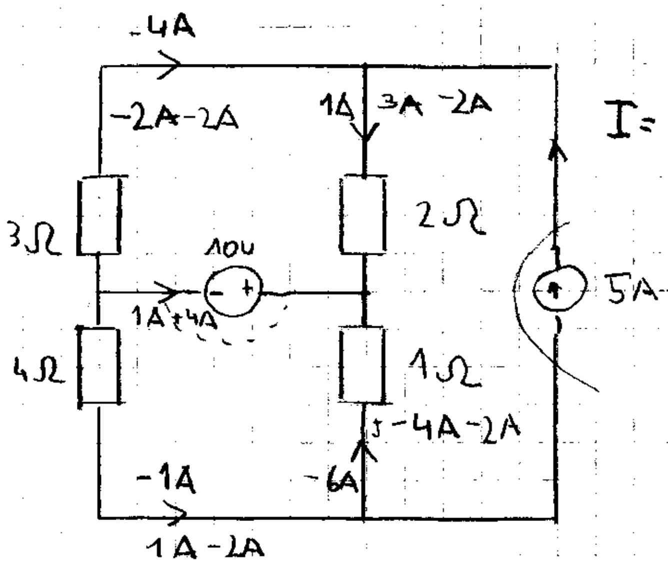
α: $3\Omega\, I_\alpha + 2\Omega\,(I_\alpha - (-5A)) + 10V = 0$
β: $+4\Omega\,(-I_\alpha - I_\beta)) - 10V - 1\Omega\,(-5A - I_\beta) = 0$

$5 I_\alpha = -20A \qquad I_\alpha = -4A$
$3 I_\beta = +5A \qquad I_\beta = -1A$

TEOREMI

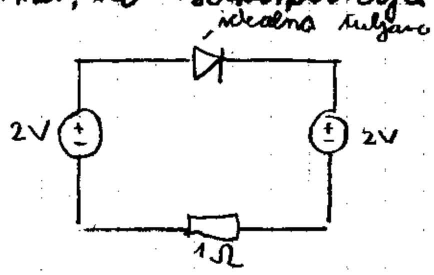
1. superpozicija
nadomestitev
Tellegen stavek
Thevenin, Nortonov stavek
maksimum moči
recipročnost

1. Nalaganje, superponiranje, seštevanje prispevkov. Mislјeno za vsi viri v vezju. Velja le za linearna vezja. Deaktiviran vir - napetost nič, ali pri tokovnem viru, tok nič, kar pomeni da je to kot odprto stikalo. (sklenjeno stikalo)



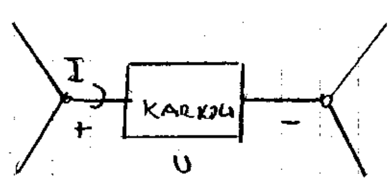
$I = I_1' + I_1''$
od tok. vira — od nap. vira

primer, ko superpozicija ne velja

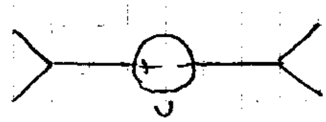


$I = I_1 + I_2$

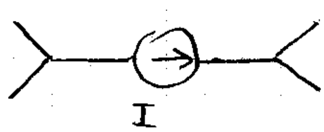
$0 = 2A + 0$

2.


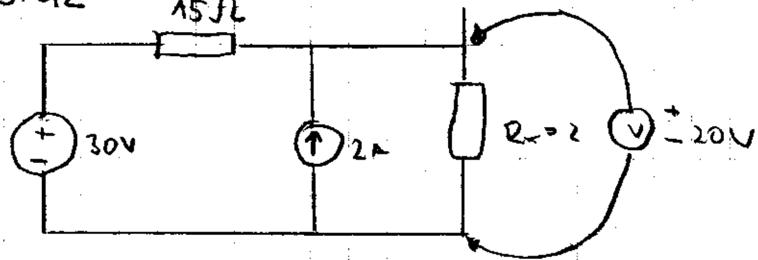
to lahko zamenjamo z

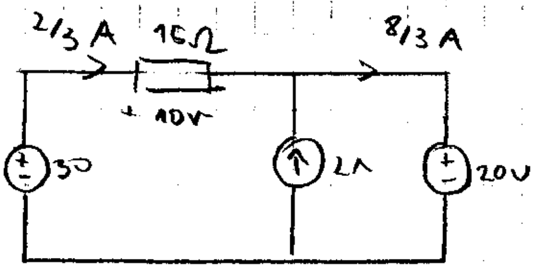


ali



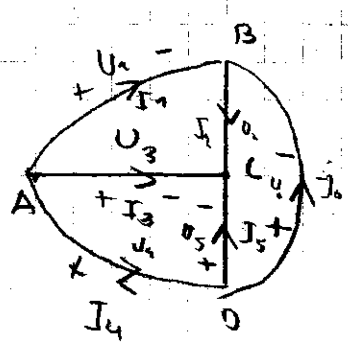
Zgled 3.42





$$R_x = \frac{20V}{8/3A} = 7{,}5\,\Omega$$

3. Če tokove in napetosti v nekem vezju označimo istosmiselno



Lahko uporabim

bremenski $\xrightarrow{\ \ }$ + U −

ali

generatorski $\xrightarrow{I}$ − U +

$$\sum_{k=1}^{n} U_k I_k = 0$$

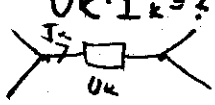
$$\sum_{k=1}^{6} U_k I_k = \overset{U_1}{(V_A - V_B)} I_1 + \overset{U_2}{(V_B - V_C)} I_2 + \overset{U_3}{(V_A - V_C)} I_3 +$$

$$\overset{U_4}{(V_A - V_D)} I_4 + \overset{U_5}{(V_D - V_C)} I_5 + \overset{U_6}{(V_D - V_B)} I_6$$

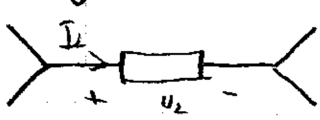
$$= V_A \overbrace{(I_1 + I_2 + I_3)}^{0} + V_B \overbrace{(-I_1 + I_2 - I_6)} +$$

$$V_C \underbrace{(-I_2 - I_3 - I_5)}_{0} + V_D \underbrace{(-I_4 + I_5 + I_6)}_{0}$$

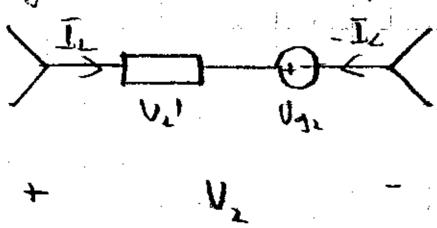
$U_k \cdot I_k = ?$ − moč



če je to pasiven element

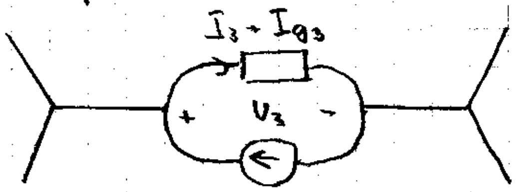


Če je to realen napetostni vir



$$I_2 U_2 = I_2 (U_2' + U_{g2}) = \underbrace{I_2 U_2'}_{P_{b2}} - \underbrace{(-I_2 \cdot U_{g2})}_{P_{g2}}$$

Če pa imamo tokovni vir:



$$U_3 I_3 = U_3 (I_3 + I_{g3} - I_{g3}) = U_3 \underbrace{(I_3 + I_{g3})}_{P_{b3}} - \underbrace{U_3 I_{g3}}_{P_{g3}}$$

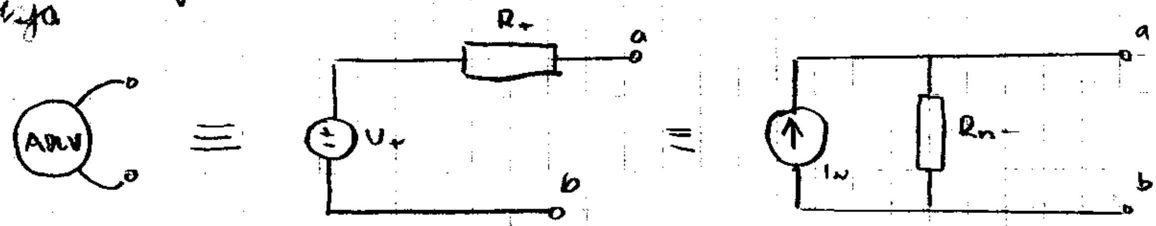
$$U_k I_k = P_{bk} - P_{gk}$$

Če gremo s tem dalje

$$\sum_{k=1}^{n} U_k I_k = \sum_{k=1}^{n} (P_{bk} - P_{gk}) = 0$$

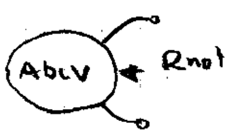
$$\sum_{k=1}^{n} P_{bk} = \sum_{k=1}^{n} P_{gk}$$

4. Naslanjata se ta dva sponka na aktivna dvopolna linearna vezja

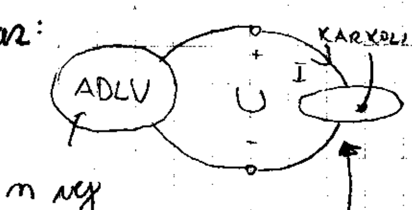


Theveninovo vezje   Nortonovo vezje

U_T - napetost odprtih sponk ADLV
I_N - tok kratkega stika ADLV
$R_T = R_N = U_T / I_N = R_{not}$ - notranja upornost originalnega vezja pri deaktiviranih virih



dokaz:



neznanke:

$\begin{pmatrix} I_1 \\ \text{ali} \\ U_1 \end{pmatrix} \cdots \begin{pmatrix} I_n \\ \text{ali} \\ U_n \end{pmatrix}$ $n+2$ neznank

$I = f(U)$ — $n+2$ enačb

zapišemo po Kirchoffu: $n+1$ enačb za $(n+2)$ neznanki
- to so linearne enačbe

Z eliminacijo notranjih neznank v ADLV dobimo eno enačbo z neznankama U in I.

$$AU + BI + C = 0 \rightarrow I = f(U)$$

kakoli naj bo:

1 odprto stikalo; $I = 0$, $U = U_g$

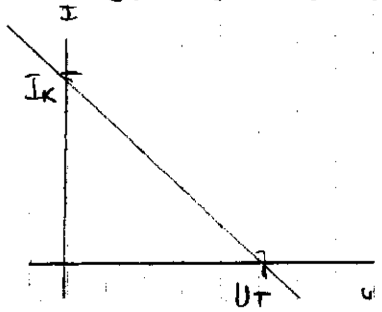
$AU_T + B + C = 0$ $\quad A = C/-U_T$

2 kratek stik, $U = 0$; $I = I_k$

$0 + BI + C$ $\quad B = -\frac{C}{I_K}$

$-C\frac{U}{U_T} - C\frac{I}{I_K} + C = 0$

$\frac{U}{U_T} + \frac{I}{I_K} = 1$ odsekovna enačba



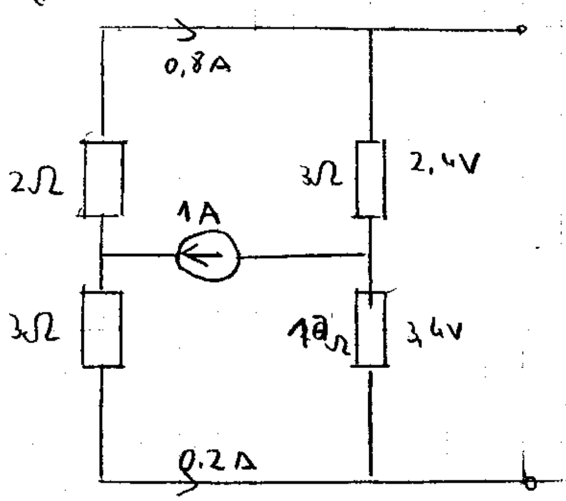
in to lahko predstavimo z vezji stra

Dokaz da je R notranja upornost

ko je aktiven vir je

$R_{not} = \frac{U_T}{I_K}$

Zgled: 3-30

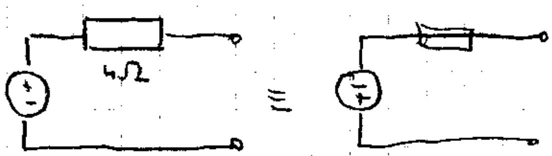


V odprtih sponkah

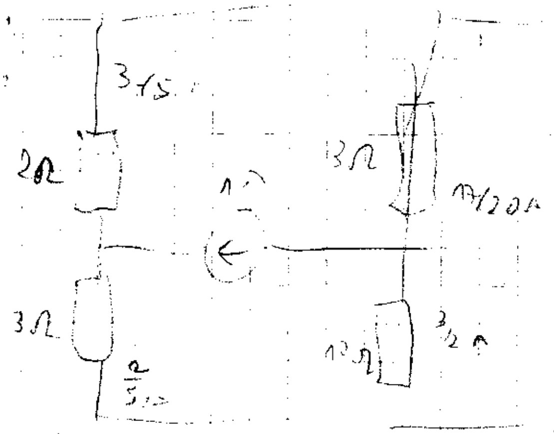
$U_T = 2{,}4 - 3{,}4 = 1V$

vzporedno

$R_{not} = (20 \parallel 5)\Omega = 4\Omega$

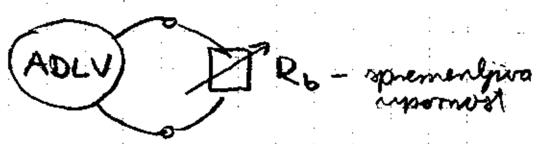


Če pa bi hoteli izračunati tok kratkega stika

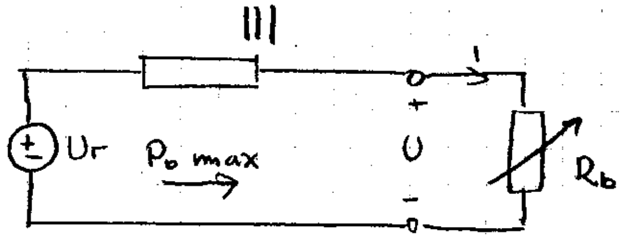


$$I_K = \left(\frac{3}{5} - \frac{17}{20}\right)A = -\frac{1}{4}A$$

5. maksimum moči



Kakšna mora biti upornost, da bo moč vezja največja



$$P_b = R_b \cdot I^2 = \frac{R_b U_T^2}{(R_b+R_T)^2} = P_b(R_b)$$

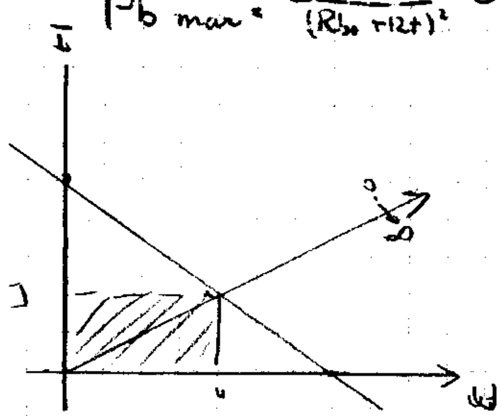
$$\frac{dP_b}{dR_b} = \frac{U^2}{(R_b+R_T)^4}\left[(R_b+R_T)^2 - 2(R_b+R_T)\cdot R_b\right] = 0$$

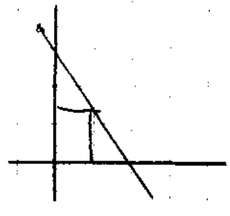
$$(R_{b0}+R_T)^2 = 2R_{b0}(\cancel{R_{b0}}+R_T)$$

$$R_{b0} = R_T$$

govorimo, da je breme prilagojeno našemu vezju

$$P_{b\,max} = \frac{R_{b0}U_T^2}{(R_{b0}+R_T)^2} = \frac{U_T^2}{4R_T}$$

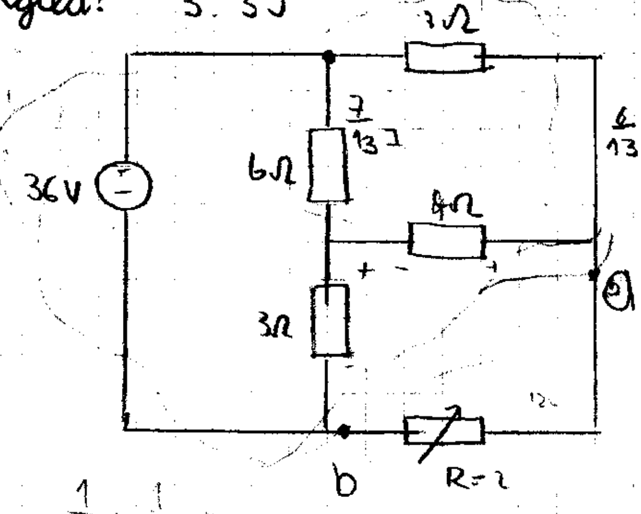




$$P_{b\,max} = \frac{I_K}{2} = I \qquad U = \frac{U_T}{2}$$

$$I = \frac{U_T}{2R_T}$$

Zgled: 3.39



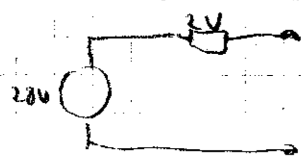
$\frac{1}{7}+\frac{1}{6}=\frac{}{42}$

$R' = ((3+4) \| 6) + 3 = \frac{42}{13} + 3 = \frac{81}{13}\Omega$

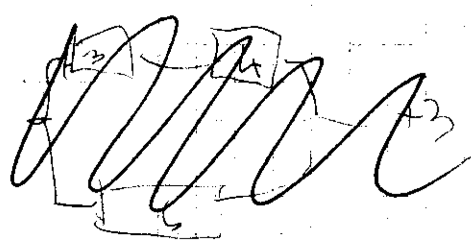
$I = \frac{36V}{R'} = \frac{36 \cdot 4}{81 \cdot 9} \cdot 13 = \frac{52}{9} A$

$U_T = 4\Omega \cdot \frac{6}{13} I + 3\Omega\, I = = \Big(\underbrace{\frac{24}{13}+3}_{\frac{63}{13}}\Big) I \quad V = 28V$

$R_{not} = 3 \| (6 \| 3) + 4)\Omega = 2\Omega$



$P_{max} = 14 \cdot 7 = 98W \qquad P = \frac{V_T^2}{4R_T}$



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