OSNOVE ELEKTROTEHNIKE I
zapiski avditornih vaj

Šolsko leto	2007 / 2008
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UREJANJE DOKUMENTA

VERZIJA	01	**REVIZIJA**	00
DATUM	30. 1. 2009		
ZADNJI POPRAVLJAL	/		
PREGLEDAL	/		

OPOMBE

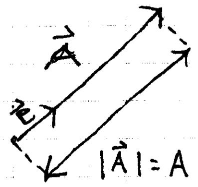
Dokument je načeloma popoln.

POPRAVKI

OSNOVE ELEKTROTEHNIKE
I. DEL

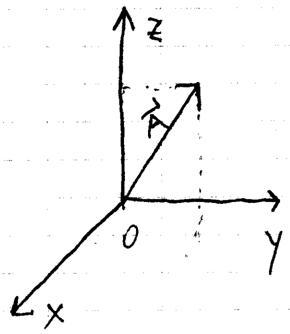
Vektorski račun

- vektor



- enotski vektor $\vec{e}$

$$\vec{e}_A = \frac{\vec{A}}{|\vec{A}|}$$



$$\vec{A} = \vec{e}_x A_x + \vec{e}_y A_y + \vec{e}_z A_z$$

komponente vektorja A

operacije z vektorji

1) $\vec{C} = \vec{A} + \vec{B}$

$$\vec{C} = \vec{e}_x(A_x + B_x) + \vec{e}_y(A_y + B_y) + \vec{e}_z(A_z + B_z)$$

2) $\vec{C} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

$$\vec{C} = \vec{e}_x(A_x - B_x) + \vec{e}_y(A_y + B_y) + \vec{e}_z(A_z - B_z$$

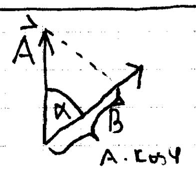
3) množenje s skalarjem

$$k \cdot \vec{A} = k(\vec{e}_A A) = \vec{e}_A(k \cdot A)$$

4) množenje vektorjev

- skalarni produkt $\vec{A} \cdot \vec{B} = C$
- vektorski produkt $\vec{A} \times \vec{B} = C$

$$\boxed{\vec{A} \cdot \vec{B} \stackrel{!}{=} AB\cos(\alpha)}$$



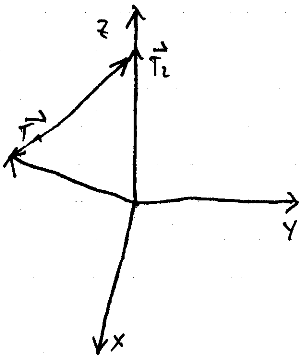
$$\cos\alpha = \frac{A(\text{v smeri } B)}{A}$$

$$\boxed{\begin{aligned}\vec{A}\cdot\vec{B} &= (e_x A_x + e_y A_y + e_z A_z)(e_x \vec{B_x} + e_y \vec{B_y} + e_z \vec{B_z}) \\ &= A_x \cdot B_x + A_y B_y + A_z B_z\end{aligned}}$$

$$\vec{A}\cdot\vec{A} = A_x^2 + A_y^2 + A_z^2 = A^2$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Določite vektor $\vec{A}$, ki kaže iz točke $T_1(2, -4, 1)$ m v $T_2(0, 0, 4)$ m v kartezijskih koordinatah in izračunajte enotski vektor tega vektorja.



$$\vec{T_1} + \vec{A} = \vec{T_2}$$
$$\vec{A} = \vec{T_2} - \vec{T_1}$$

$$\vec{A} = (0-2,\ 0+4,\ 4-1)\,m = (-2, 4, 3)\,m$$

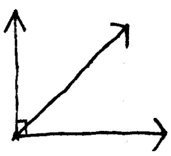
enotski vektor

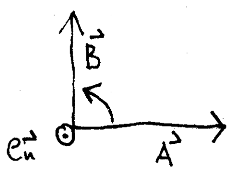
$$|\vec{A}| = \sqrt{(-2)^2 + 4^2 + 3^2}\ m = \sqrt{29}\ m$$

$$\vec{e_A} = \frac{(-2, 4, 3)\cancel{m}}{\sqrt{29}\ \cancel{m}}$$

Vektorski produkt

$$\vec{A}\times\vec{B} \stackrel{!}{=} A\cdot B\cdot\sin(\alpha)\ \vec{e_n}$$





- $\vec{A} \times \vec{B} = (A_x, A_y, A_z) \times (B_x, B_y, B_z)$

$$= \vec{e_x} \vec{A_x} \times \vec{e_x} B_x, \; + \vec{e_y} \vec{A_y} \times \vec{e_y} B_y + \ldots$$

$$= \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ \vec{A_x} & \vec{A_y} & \vec{A_z} \\ \vec{B_x} & \vec{B_y} & \vec{B_z} \end{vmatrix} = \boxed{\vec{e_x}(A_y B_z - A_z B_y) - \vec{e_x}(A_x B_z - A_z B_x) + \vec{e_z}(A_x B_y - A_y B_x)}$$

- $\vec{F} = \vec{F_e} + \vec{F_m} = Q \cdot \vec{E} + Q \vec{v} \times \vec{B}$

Elektron v homogenem polju potuje.

$\vec{B} = \vec{e_x}\, 10\, mT$

S hitrostjo $\vec{v} = 10^5\, m/s$. Določite silo na elektron ob času $t = 0$, ko leti v smeri $\vec{e_y}$.

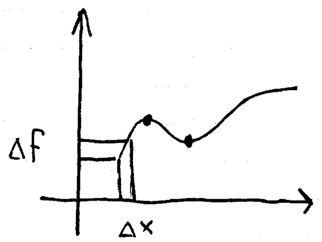
$$\vec{F_m} = Q \vec{v} \times \vec{B} = (-1{,}6 \cdot 10^{-19}\, As) \cdot \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 0 & 10^5 m/s & 0 \\ 10\, mT & 0 & 0 \end{vmatrix} =$$

$$= -1{,}6 \cdot 10^{-19}\, As \cdot (\emptyset, \emptyset, -10^5\, m/s \cdot 10\, mT) =$$

$$= -1{,}6 \cdot 10^{-19}\, As\, (+10^5 \cdot 10\, mT \cdot m/s)\, \vec{e_z}$$

$$= \underline{1{,}6 \cdot 10^{-16}\, N\, \vec{e_z}}$$

Odvod (infinitizemalni račun)



$$i = \frac{dQ}{dt}$$

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{x + \Delta x - x}$$

$$\frac{d}{dx}(C) = 0$$

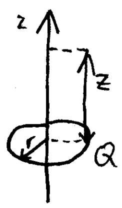
$$\frac{d}{dx}(kx + n) = k\frac{dx}{dx} = k$$

$$\frac{d}{dx}(x^2) = 2x \qquad \frac{d}{dx}(x^n) = n \cdot x^{n-1}$$

$$\frac{d}{dx}\cos x = -\sin x \qquad \frac{d}{dx}\sin x = \cos x$$

Električna poljsku jakost na obroču (na osi obroča)

$$\vec{E} = \vec{e_z} \frac{Q \cdot z}{4\pi\varepsilon_0 (z^2 + r^2)^{3/2}}$$



Pri katerem z je $E = E_{max}$

$$\frac{dE}{dz} = k\frac{d}{dz}\left(z(z^2+r^2)^{-3/2}\right) =$$

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f'g + g'f$$

$$= 1\cdot(z^2+r^2)^{-3/2} + \left(-\frac{3}{2}(z^2+r^2)^{-3/2-1}\right)z\cdot 2z = 0$$

$$\frac{1}{(z^2+r^2)^{3/2}} - \frac{3z^2}{(z^2+r^2)^{-5/2}} = 0$$

$$\frac{(z^2+r^2)-3z^2}{(z^2+r^2)^{3/2}(z^2+r^2)} = 0$$

$$z^2 + r^2 - 3z^2 = 0$$
$$2z^2 = r^2$$
$$z = \boxed{r/\sqrt{2}}$$

tu je polje največje.

Integralni račun

$$I = \int f(x)\,dx = F(x) + C$$

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

$$I = \lim_{\Delta x \to 0} \sum_{x=a}^{b} f(x)\cdot\Delta x$$

$$\int k\,dx = kx + C$$

$$\int x^2\,dx = \frac{x^3}{3} + C \qquad \int x^n\,dx = \frac{x^{n+1}}{n+1}$$

17.10.07

$$\int_{y=0}^{1}\left[\int_{x=0}^{\pi/2}\left(e^{ay}+\sin x\right)dx\right]dy =$$

$$= \int_{y=0}^{1}\left[e^{ay}x\Big|_0^{\pi/2} + (-\cos x)\Big|_0^{\pi/2}\right] dy$$

$$= \int_0^1 \left(e^{ay}\frac{\pi}{2} - 0 + 0 + 1\right) dy$$

$$= \int_0^1 \left(e^{ay}\frac{\pi}{2} + 1\right) dy$$

$$= \frac{\pi \cdot 1}{2 \cdot a}\, e^{ay}\Big|_0^1 + y\Big|_0^1$$

$$= \underline{\frac{\pi}{2a}\cdot\left(e^{a}-1\right) + 1}$$

Coulombov zakon

$$Q_1 \xleftrightarrow{\;r_{12}\;} Q_2$$

300 μC

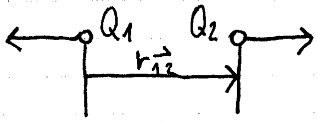
elektronov:

$$n = \frac{300\ \mu C}{1{,}6 \cdot 10^{-19}\ C} = 1{,}8 \cdot 10^{15} \text{ elektronov}$$

$$F_{12} = k\,\frac{Q_1 Q_2}{r_{12}^2}$$

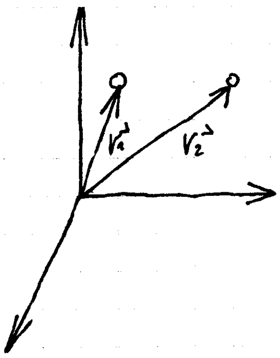
$$k = 1/4\pi\varepsilon_0$$

$\varepsilon_0 = 8{,}854 \cdot 10^{-12}$ F/m $\cancel{(Nm^2/C^2)^{-1}}$ (C^2/Nm^2)



$$\vec{F_{12}} = F_{12} \cdot \vec{e_{r_{12}}} = F_{12} \cdot (\vec{r} / |\vec{r}|)$$

$$\vec{F_{12}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0} \frac{\vec{r_{12}}}{r_{12}^3} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r_{12}^2} \cdot \vec{e_{r_{12}}}$$



$$\vec{r_{12}} = (\vec{r_2} - \vec{r_1})$$

√ V točki $T_1(1,1,1)$ m se nahaja elektrina $Q_1 = 100\,\mu C$. V točki $T_2(-1,0,-2)$ m pa $Q_2 = 50\,\mu C$. Določite silo na naboj Q_2.

$\vec{r_1} = (1,1,1)$ m $\qquad \vec{r_2} = (-1,0,-2)$ m

$\vec{r_{21}} = (-2,-1,-3)$ m $\qquad |\vec{r}| = \sqrt{14}$ m $\qquad \vec{e_r} = \dfrac{(-2,-1,-3)\,\cancel{m}}{\sqrt{14}\,\cancel{m}}$

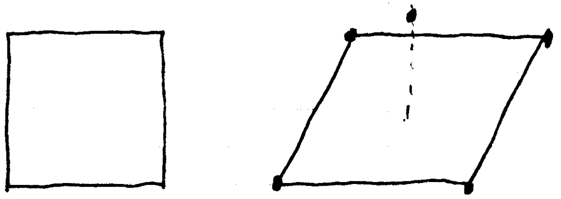
$$\vec{F_{21}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r_{12}^2} \vec{e_r} = \frac{100 \cdot \cancel{10^{-6}}\,\cancel{C} \cdot 50 \cdot \cancel{10^{-6}}\,\cancel{C}}{4\pi\; 8{,}854 \cdot \cancel{10^{-12}}\,\cancel{C^2}} \frac{N\cancel{m^2}}{14\,\cancel{m^2}} \vec{e_r}$$

$$= 3{,}2\,N\,\vec{e_r} = \boxed{\frac{3{,}2\,(-2,-1,-3)}{\sqrt{14}}\,N}$$

√ 4 enake elektrine $Q = 30\,\mu C$ se nahajajo na robovih kvadrata z diagonalo 8 m. Kolikšna je sila na elektrino $150\,\mu C$, ki se nahaja $h = 3\,m$ nad središčem kvadrata.

√

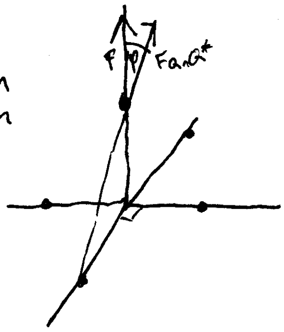
1) način



$Q = 30\,\mu C$
$d = 8\,m$
$Q^* = 150\,\mu C$
$h = 3\,m$

$F_{Q^*} = F_{Q_1Q^*} + F_{Q_2Q^*} + F_{Q_3Q^*} + F_{Q_4Q^*}$

$T(4,0,0)\,m$
$T^*(0,0,3)\,m$

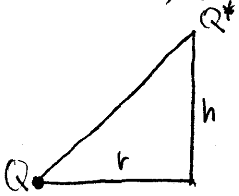


√ $\vec{r_{2\to *}} = (-4,0,3)$

$\vec{F_{Q^*}} = 4 F_{Q_1Q^*} \cdot \cos\varphi \cdot \boxed{\vec{e_z}}$! smer

2) način

$$F = \frac{30 \cdot 10^{-6}\,\cancel{C} \cdot 150 \cdot 10^{-6}\,\cancel{C}}{4\pi \cdot 8{,}854 \cdot 10^{-12}\,\cancel{C^2}/\cancel{Nm^2} \cdot 25\,\cancel{m^2}} \cong 1{,}6\,N$$



$\cos(\alpha) = h/r$
$\cos(\alpha) = 3/5$

$$\vec{F_{Q^*}} \cong 4 \cdot 1{,}62\,N \cdot \frac{3}{5} \cdot \vec{e_z} = 3{,}888\,\vec{e_z}\,N$$

El. poljska jakost na mestu naboja Q^*.

$$E_{Q^*} = \frac{\vec{F_{Q^*}}}{Q_*} = 25{,}9 \cdot 10^3\,N/C = \boxed{25{,}9\ V/m\ \vec{e_z}}$$

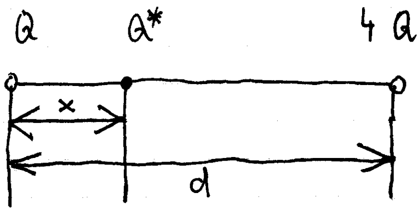
Določite $\vec{E}$ v koor. izhodišču (središču kvadrata)

$$\vec{E_0} = \vec{F_T} / Q_T = \frac{1}{Q_T} \underbrace{\left(F_{Q1} + F_{Q2} + F_{Q3} + F_{Q4} \right)}_{= 0}$$ testni naboj

$$= F_{Q^*} / Q_T = \frac{Q^* \, Q_T}{4\pi \varepsilon_0 r^2} \frac{1}{Q_T} \vec{e_r} = \frac{Q^*}{4\pi\varepsilon_0 r^2} \vec{e_r}$$

$$E_0 = -\vec{e_z} \frac{150 \cancel{\mu C} \cdot 10^{-6}}{4\pi \, 8{,}854 \, 10^{-12} \, C^2/N\cancel{m^2} \, \cancel{3m^2}} = 150\,000 \ V/m$$

V zraku imamo dve pozitivni elektrini: Q in 4Q na razdalji d. Kolikšno elektrino in kam jo moramo postaviti, da bo sistem v ravnovesju.



Q_1 Q_2 Q_3

$$\vec{E_1} = -\vec{E_2}$$

$$\frac{Q}{4\pi\varepsilon_0 r^2} = \frac{4Q}{4\pi\varepsilon_0 r_2^2}$$

$$\cancel{Q} \cancel{4\pi} \cancel{\varepsilon_0} r_2^2 = 4 \cancel{Q} \cancel{4\pi} \cancel{\varepsilon_0} r^2$$

$$r_2^2 = 4 r^2 \quad / \sqrt{\ }$$

$$r_2 = 2r$$

$$x = d/3$$

Porazdelitve nabojev

.- volumska, površinska, linijska

- volumska ρ - gostota volumske porazdelitve
- površinska σ -
- linijska q

$\rho = Q/V$ $\sigma = Q/S$ $q = Q/l$

- kocka

$a = 1\,cm$ $\rho = 10^{-12}\,C/m^3$

$Q = 10^{-12}\,C/m^3 \cdot 10^{-6}\,m^3 = 10^{-18}\,C$

- $Q = 1{,}6 \cdot 10^6\,C$ gostota površinsko porazdeljenega naboja na zemlji.

$r = 6400\,km$

$\sigma = Q/A = 1{,}6 \cdot 10^6\,C / (4\pi \cdot (6400 \cdot 10^3)^2\,m^2) \cong 3 \cdot 10^{-9}\,C/m^2$

- V žici je linijsko porazdeljena gostota naboja $q = 10\,nC/m$. Koliko je celotnega naboja na žici dolžine 2km

$Q = 20\,\mu C$

$$\boxed{\text{neenakomerna porazdelitev: } Q = \int q\,dl} \quad \lim_{\Delta l \to 0} \sum q \cdot \Delta l = \int q\,dl$$

$dQ = q \cdot dl \rightarrow q = dQ/dl$

Vzdolž palice dolžine $l = 0{,}5\,m$ se naboj spreminja z enačbo

$q = q_0 \sin\left(\frac{\pi x}{l}\right)$ $q_0 = 10^{-9}\,C/m$

$$Q = \int_{l} q \, dl = \int_{0}^{l} q(x)\, dx$$

$$Q = \int_{0}^{0,5} q_0 \sin\left(\frac{\pi x}{l}\right) dx = q_0 \left(-\cos \pi x / l\right) \cdot \frac{l}{\pi} \Big|_{0}^{0,5\,m}$$

$$= -q_0 \frac{l}{\pi} \left(-1 - 1\right) = q_0 \frac{2l}{\pi} \longrightarrow \boxed{3,18 \cdot 10^{-10}\, C}$$

$$\int \sin(ax) = -\cos(ax) \cdot \frac{1}{a} + C$$

$ax = t$

$a\,dx = dt$

Krogla $R_0 = 0,2$ cm podana z enačbo

$\rho = 200\, r$ [C/m^4] Koliko je celotnega naboja

$$V = \frac{4\pi R^3}{3}$$

$$dQ = \rho\, dV$$

$$\int dQ = \int \rho\, dV$$

$$\frac{dV}{dR} = \frac{4\pi}{3} 3R^2 = 4\pi R^2$$

$$Q = \int_{0}^{R_0} 200\, r \, \frac{C}{m^4} \, 4\pi r^2 dr = 200 \frac{C}{m^4} 4\pi \int_{0}^{R_0} r^3 dr =$$

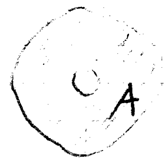
$$= 800\pi \left(\frac{r^4}{4}\right)\Big|_{0}^{R_0} = 800\pi \cdot \frac{R_0^4}{4} = 200 \cdot \pi \cdot (2 \cdot 10^{-3}\, m)^4 \frac{C}{m^4}$$

$= 10,$[redacted] nC

Na zgoščenki $r_n = 2\,cm$ $r_z = 6\,cm$, površinsko porazdeljen naboj:

$$\sigma = k\left(1 - \frac{r}{2r_z}\right), \text{ kjer je } k = 10^{-10}\ C/m^2$$

$$Q = \int_A \sigma\, dA$$



$$A = \pi r^2 \quad /\int$$

$$\frac{dA}{dr} = 2\pi r$$

$$dA = 2\pi r\, dr$$

$$Q = \int_{r_n}^{r_z} k \cdot \left(1 - \frac{r}{2r_z}\right) 2\pi r\, dr = 2\pi k \left[\int_{r_n}^{r_z} r\, dr - \int_{r_n}^{r_z} \frac{r^2}{2r_z}\, dr\right]$$

$$= 2\pi k \left[\frac{r^2}{2}\Big|_{r_n}^{r_z} - \left(\frac{1}{2r_z}\,\frac{r^3}{3}\right)\Big|_{r_n}^{r_z}\right] =$$

$$= 2\pi k \left(\frac{r_z^2}{2} - \frac{r_n^2}{2} - \frac{1}{2 \cdot 3 r_z}\left(r_z^3 - r_n^3\right)\right) =$$

$$= \pi k \left[r_z^2 - r_n^2 - \frac{1}{3} r_z^2 + \frac{1}{3}\,\frac{r_n^3}{r_z}\right] =$$

$$= 10^{-10}\,\frac{C}{m^2}\,\pi \left(\frac{2}{3}(6\,cm)^2 - (2\,cm)^2 + \frac{(2\,cm)^3}{3 \cdot 6\,cm}\right)$$

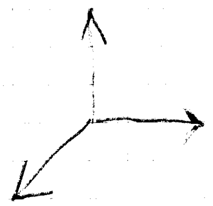
$$= 6{,}4 \cdot 10^{-13}\ C$$

Koordinatni sistemi

- kartezični koordinatni sistem

x, y, z

točka v prostoru je presek treh ravnin



$\vec{r} = (x_1, y_1, z_1)$

$dA = dx\,dy$

$dV = dx\,dy\,dz$

$d\vec{A}_x = \vec{e}_x\,dy\,dz$

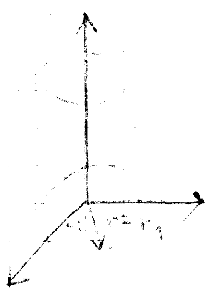
$d\vec{A}_y = \vec{e}_y\,dx\,dz$

$d\vec{A}_z = \vec{e}_z\,dx\,dy$

$d\vec{l} = \vec{e}_x\,dx + \vec{e}_y\,dy + \vec{e}_z\,dz$

- cilindrični (valjni) k.s. (r, φ, z)

tri ravnine



$r = r_1$

$\varphi = \varphi_1$

$z = z_1$

$d\vec{A} = r\,d\varphi \cdot dr$

$d\vec{A}_z = \vec{e}_z\,r\,dr\,d\varphi$

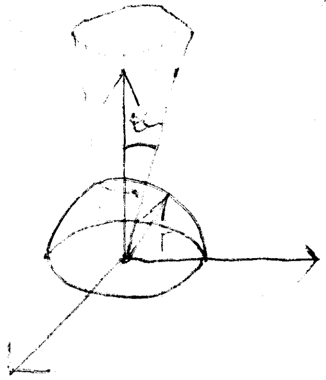
$d\vec{A}_r = \vec{e}_r\,r\,d\varphi\,dz$

$d\vec{A}_\varphi = \vec{e}_\varphi\,dz\,dr$

$dV = dr\;r\,d\varphi\,dz$

$d\vec{l} = \vec{e}_r\,dr + \vec{e}_\varphi\,r\,d\varphi + \vec{e}_z\,dz$

3) sferični (krogelni) (r, ϑ, φ)



$r = r_1$
$\vartheta = \vartheta_1$
$\varphi = \varphi_1$

$$d\vec{l} = \vec{e_r}\, dr + \vec{e_\vartheta}\, r\, d\vartheta + \vec{e_\varphi}\, r \sin\vartheta\, d\varphi$$

$$dA_\varphi = \vec{e_\varphi}\, dr\, r\, d\vartheta$$

$$dA_\vartheta = \vec{e_\vartheta}\, r \sin\vartheta\, dr\, d\varphi$$

$$dA_r = \vec{e_r}\, r^2 \sin\vartheta\, d\vartheta\, d\varphi$$

Površina

$$dV = dr\, r\, d\vartheta\, r \sin\vartheta\, d\varphi$$

volumen

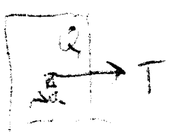
El. poljska jakost porazdeljenih nabojev

polje točkastega naboja

$$E = (Q / 4\pi\varepsilon_0 r^2) \cdot \vec{e_r}$$

$$\Delta E = \vec{e_r} \frac{\Delta Q}{4\pi\varepsilon_0 r^2}$$

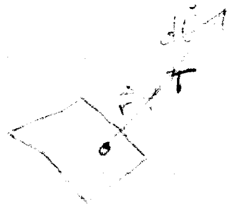
dQ
$\sigma\, dA$
$\rho\, dV$



$$d\vec{E} = \vec{e_r} \frac{dQ}{4\pi\varepsilon_0 r^2}$$

$$\vec{E} = \int d\vec{E} = \int \vec{e_r} \frac{dQ}{4\pi\varepsilon_0 r^2}$$

Postopek:



1) poznamo porazdelitev naboja
se odločimo za koor. sistem
- postavimo izhodišče

2) postavimo dQ v neko splošno lego in ga izrazimo kot $q\,dl$, $\sigma\,dA$ ali $\rho\,dV$.

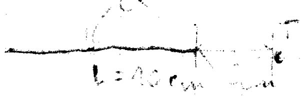
3) zapišemo $\vec{E}$ od dQ

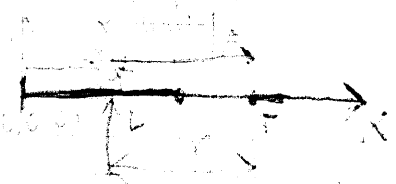
4) zapišemo integral ter določimo meje integracije

5) rešimo integral

Primer:

Tanka palica dolžine 10 cm je enakomerno naelektrena z nabojem 10 pC. Določi $\vec{E}$ v točki 2 cm stran od konca.





$$dQ = q\,dl = q\,dx = \frac{Q}{l}\,dx$$

$$d\vec{E} = \vec{e}_r \frac{dQ}{4\pi\varepsilon_0 r^2}$$

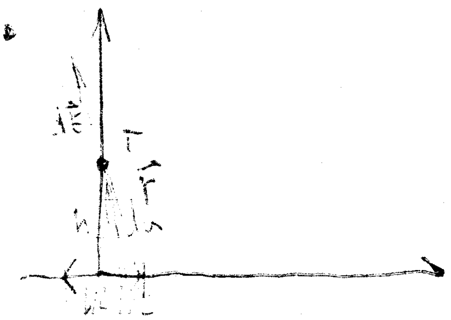
$$d\vec{E} = \vec{e}_x \frac{\frac{Q}{l}\,dx}{4\pi\varepsilon_0 r^2} = \frac{\vec{e}_x\, \frac{Q}{l}\, dx}{4\pi\varepsilon_0 (l - x + a)^2}$$

$$\vec{E} = \frac{\vec{e}_x Q}{l\, 4\pi\varepsilon_0} \int_0^l \frac{dx}{(l+a-x)^2} \qquad (l+a-x) = t, \quad -dx = dt$$

$$= \frac{Q}{l\,4\pi\varepsilon_0} \int t^{-2}\,dt$$

$$\vec{E} = \vec{e_x} \frac{Q}{l\, 4\pi\varepsilon_0} \left(\frac{1}{l+a-x} \right) \Big|_0^l =$$

$$= \vec{e_x} \frac{Q}{l\, 4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{l+a} \right) =$$



2 cm nad središčem palice

$dQ = Q/L \, dx$

$$dE = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{Q/L \; dx}{4\pi\varepsilon_0 (h^2 + x^2)}$$

$$dE_y = dE \cos\alpha = dE \frac{h}{\sqrt{h^2+x^2}}$$

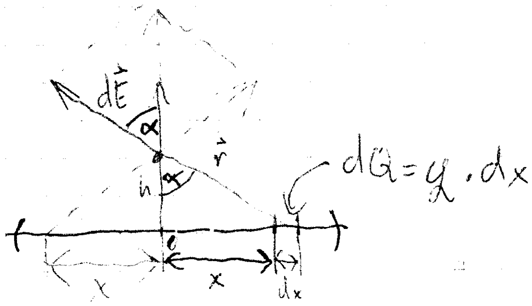
$$dE_y = \frac{Q/L \; dx \; h}{4\pi\varepsilon_0 (h^2 + x^2)^{3/2}}$$

$$\vec{E}_y = \frac{Q/L \cdot h}{4\pi\varepsilon_0} \int_{-l/2}^{l/2} \frac{dx}{(h^2 + x^2)} =$$

$$= \frac{Q/L \cdot h}{4\pi\varepsilon_0} \frac{2x}{h^2 \sqrt{x^2 + h^2}} \Big|_0^{l/2}$$

$$\vec{E} = e_{\vec{r}} \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\vec{E} = \int_{\text{po } Q_{i}} e_{\vec{r}} \frac{dQ}{4\pi\varepsilon_0 r^2}$$



$$r = \sqrt{x^2 + h^2}$$

$$d\vec{E_y} = d\vec{E} \cdot \cos\alpha = d\vec{E} \cdot \frac{h}{r}$$

$$d\vec{E_y} = \frac{q \cdot dx}{4\pi\varepsilon_0 (x^2+h^2)} \cdot \frac{h}{\sqrt{x^2+h^2}}$$

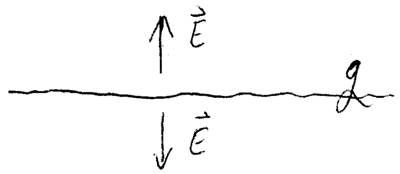
$$\vec{E_y} = \frac{qh}{4\pi\varepsilon_0} \int_{-l/2}^{l/2} \frac{dx}{(x^2+h^2)^{3/2}} = \frac{q \cdot h}{4\pi\varepsilon_0} \cdot \frac{x}{h^2 \sqrt{x^2+h^2}} \Bigg|_{-l/2}^{l/2} =$$

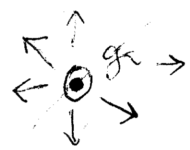
$$= \frac{qh}{4\pi\varepsilon_0} \cdot \left(\frac{l/2}{\sqrt{(l/2)^2+h^2}} - \frac{-l/2}{\sqrt{}} \right) = \frac{q\,l}{4\pi\varepsilon_0 h \sqrt{(l/2)^2+h^2}}$$

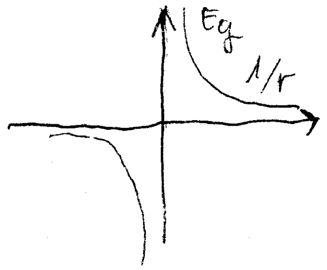
Polje tokovne premice

$$\vec{E} = \vec{e_y} \frac{q\, l}{4\pi\varepsilon_0 h \cdot l/2} = \vec{e_y} \frac{q}{2\pi\varepsilon_0 h}$$

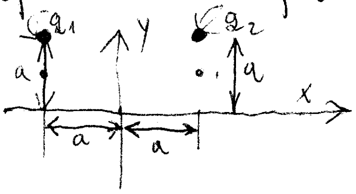
$$\boxed{\vec{E} = \vec{e_r} \frac{q}{2\pi\varepsilon_0 r}}$$







- Vzporedni daljnovodni vrvi q_1 in q_2 sta položeni

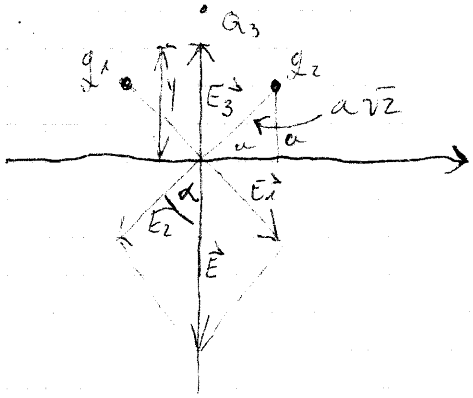


Kam moramo postaviti točkasto elektrino Q_3, da bo vektor EPJ v izhodišču koor. sis. enak 0?

$q_1 = q_2 = 1\,\mu C/m$

$Q_3 = -2\,\mu C$

$a = 4\,m$



$$\vec{E_1} + \vec{E_2} = \frac{q}{2\varepsilon_0 \pi a\sqrt{2}} \cdot \cos\alpha \cdot 2(-\vec{e_y}) =$$

$$\vec{E_3} = \left|\frac{Q}{4\pi\varepsilon_0 y^2}\right| \cdot \vec{e_y}$$

$$\vec{E_1} + \vec{E_2} + \vec{E_3} = 0$$

$$\frac{q}{2\pi\varepsilon_0 a\sqrt{2}} \frac{\sqrt{2}}{2} 2(-\vec{e_y}) + \frac{Q_3}{4\pi\varepsilon_0 y^2}(-\vec{e_y}) = 0$$

$$\frac{q \cdot 2}{a\sqrt{}} + \frac{Q_3}{y^2} = 0$$

$$y^2 = \frac{-Q_3 a}{2q} = \frac{2\cdot 10^{-6}\,C \quad 4\,m}{2\cdot 10^{-6}\,C/m} = 4\,m^2$$

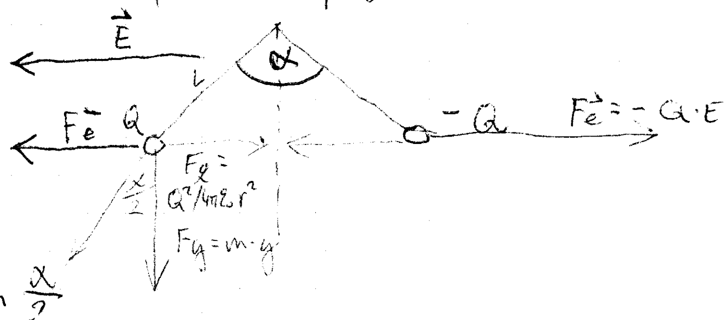
$$\boxed{y = 2\,m}$$

- Ko smo naelektreni kroglici

$Q = 10^{-7}\,C$
$m = 0{,}2\,g$

ki visita na meterskih nitkah vnesli v homogeno el. polje, sta niti oklepali kot $\alpha = 30°$. Kolikšno E je imelo polje

$Q = 10^{-7}\,C$
$m = 0{,}2\,g$
$\alpha = 30°$
$E = ?$



$$\frac{r}{2} = l \cdot \sin\frac{\alpha}{2}$$

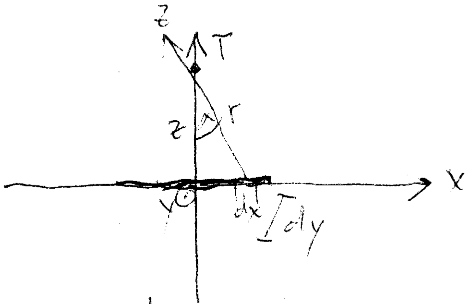
$$r = 2l\,\sin(\alpha/2) = 0{,}52\,m$$

$$tg(\alpha/2) = \frac{Q \cdot E - (Q^2/4\pi\varepsilon_0 r^2)}{mg}$$

$$0{,}27 = \frac{10^{-7}\,C}{0{,}2 \cdot 10^{-3}\,kg \cdot 9{,}8\,m\,kg/s^2}\left(E - \frac{10^{-7}\,C}{4\pi\varepsilon_0 (0{,}52\,m)^2}\right)$$

$$E = 3{,}3 \cdot 10^3\,V/m + 3300\,V/m = \underline{8600\,V/m}$$

Izpeljite izraz za EPJ v točki T, ki jo povzroča naelektren trak širine a z enakomerno površinsko gostoto σ.



$$d\vec{E} = \vec{e_r}\,\frac{dQ}{4\pi\varepsilon_0 r^2}$$

$$dQ = \sigma\,dA = \sigma\,dx\,dy$$

$$r = \sqrt{x^2 + z^2 + y^2}$$

$$dE = \frac{\sigma\,dx\,dy}{4\pi\varepsilon_0 (x^2 + z^2 + y^2)}$$

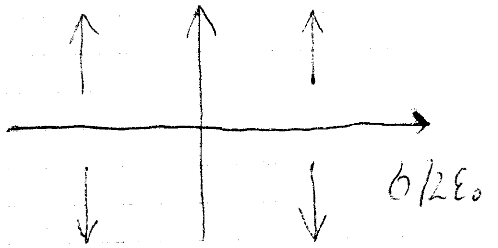
$$dE_z = dE \cdot \cos\alpha = dE \cdot \frac{z}{r}$$

$$dE_z = \frac{\sigma\,dx\,dy \cdot z}{4\pi\varepsilon_0 (x^2 + z^2 + y^2)^{3/2}}$$

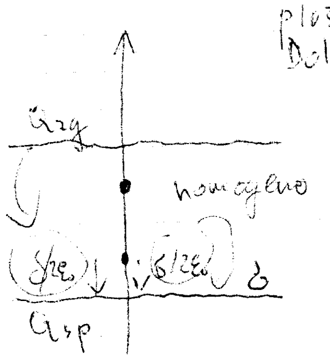
$$E_z = \frac{\sigma z}{4\pi\varepsilon_0} \int_{-\infty}^{\infty}\int_{-a/2}^{a/2} \frac{dx\,dy}{(x^2 + y^2 + z^2)^{3/2}}$$

Naelektrena ravnina

$$\vec{E} = \vec{e_z} \cdot \frac{\sigma}{\pi \varepsilon_0} \cdot \frac{\pi}{2} = \boxed{\vec{e_z} \frac{\sigma}{2\varepsilon_0}}$$



Med ploščama zračnega kondenzatorja se nahajata naboja -Q in +Q. Razdalja je 2 cm, med ploščama pa 4 cm. Površina plošče je 50 cm². Naboj zgornje plošče je enak -Q spodnje je 5nC
Določite silo na naboj



$\pm Q = \pm 2nC$

$h = 2cm$

$d = 4cm$

$A = 50\, cm^2$

$Q_{zg} = -Q_{sp} = 5nC$

$$\vec{F}(Q = +2nC) = ?$$

$$\vec{E}_{+Q} = \vec{E_{zg}} + \vec{E_{sp}} + \vec{E}_{-Q}$$

ker $A \gg d^2$

$$\sigma = Q/A = 5nC / 0{,}005\, m^2 = 5 \cdot 10^{-9}\, C / 5 \cdot 10^{-3} = 10^{-6}\, C/m^2$$

$$E = \sigma/\varepsilon_0 + Q \cdot \cancel{Q} / 4\pi\varepsilon_0 r^2$$

$$E = \sigma/\varepsilon_0 + 2 \cdot 10^{-9} / 4\pi\varepsilon_0 \cdot 4 \cdot 10^{-4}$$

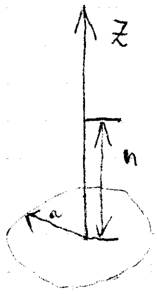
$$E = \sigma/\varepsilon_0 + \cancel{2/4} \cdot 10^{-5} / 4\pi\varepsilon_0$$

$$E = 10^{-6}/\varepsilon_0 + 10^{-5} / 4\pi\varepsilon_0$$

$$E = 8{,}85 \cdot 10^{-6} + 10^{-5} / 4\pi\, 8{,}85 \cdot 10^{-12}$$

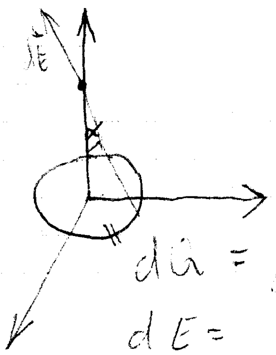
$$E = 8{,}85 \cdot 10^{-6}\, V/m + 9 \cdot 10^{-5}\, V/m$$

Polje v osi naelektrenega prstana



Določite Q tako, da bo epj (z=10cm) znaša 20 kV/m

$a = 5cm$



$dQ = q \cdot dl = q \cdot \boxed{r\,d\varphi}$ del loka

$dE = dQ / 4\pi \varepsilon_0 r^2$

$$dQ = a \cdot d\varphi \cdot q$$

$$r = \sqrt{z^2 + a^2}$$

$$dE_z = dE \cdot \cos\alpha = \frac{q \cdot a\, d\varphi}{4\pi \varepsilon_0 r^2} \cdot \frac{z}{r} =$$

$$E_z = \int \dots$$

$$E_z = \frac{q\, a\, z}{4\pi \varepsilon_0 r^3} \int_0^{2\pi} d\varphi = \frac{q\, a\, z}{4\pi \varepsilon_0 r^3}\, 2\pi$$

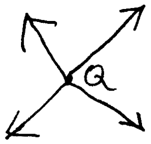
$$\vec{E} = \vec{e_z} \frac{q\, a\, z}{2\varepsilon_0 (z^2 + a^2)^{3/2}}$$

14.11.07

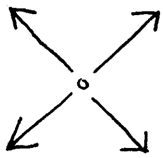
prema elektrina (naelektrena premica)

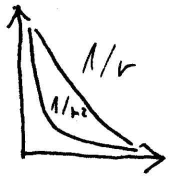
točkasti naboj

$\vec{E} = (Q/4\pi\varepsilon_0 r^2)\, \vec{e_r}$

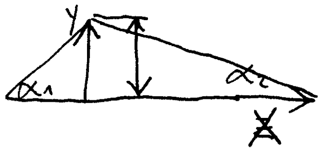


$\vec{E} = q/2\pi\varepsilon_0 r$





naelektrena daljica

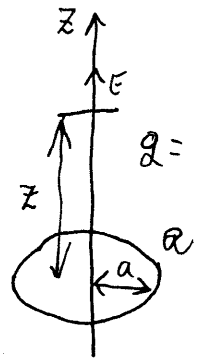


$E = \frac{q}{4\pi\varepsilon_0 r}\left(\left(\cos\alpha_1 + \cos\alpha_2\right)\vec{e_y} + \left(\sin\alpha_1 - \sin\alpha_2\right)\vec{e_x}\right)$

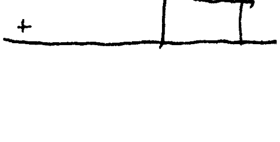
Naelektren obroč

$\vec{E} = \frac{qaz}{2\varepsilon_0(a^2+z^2)^{3/2}} \cdot \vec{e_z}$

$q = Q/l = Q/2\pi a$



Naelektrena ravnina



$\vec{E} = \vec{e_z}\,\frac{\sigma}{2\varepsilon_0}$

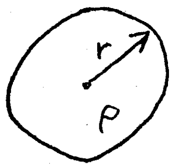
Gaussov zakon

$$\boxed{\oint_A \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}}$$

integral polja po zaključeni površini je enak zaobjetemu naboju deljeno z ε_0.

$$= \frac{\int_V \rho \, dV}{\varepsilon_0}$$

za izračun je primerna, kadar je porazdelitev nabojev simetrična



enakomerno naelektrena krogla

$$\vec{E}(r) = \vec{e_r} \cdot E(r)$$

$$d\vec{A} = \vec{e_r} \cdot dA$$

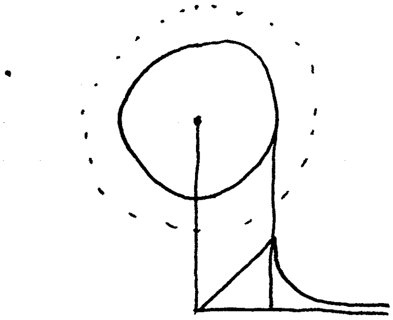
$$\vec{E} \, d\vec{A} = e_r \, E(r) \quad e_r \, dA = E(r) \, dA = E(r) r^2 \sin\vartheta \, d\varphi$$

$$\oint_{A(r)} \vec{E} \, d\vec{A} = E(r) \oint d\vec{A} = E(r) \, 4\pi r^2$$

$$\frac{Q_{\text{znotraj } A}}{\varepsilon_0} = \frac{\rho \cdot V}{\varepsilon_0} = \rho \, \frac{4\pi r^3}{3 \, \varepsilon_0}$$

$$E(r) \cancel{4\pi r^2} = \frac{\rho \, \cancel{4\pi} r^{\cancel{3}}}{3 \varepsilon_0}$$

$$E(r) = \rho r / 3\varepsilon_0 \qquad r \leq R$$



$$r \geq R$$

$$E(r)\cdot 4\pi r^2 = \frac{\rho\cdot\frac{4\pi R^3}{3}}{\varepsilon_0} \Big\} \frac{Q}{\varepsilon_0}$$

$$E(r) = \vec{e}_r \frac{\rho R^3}{3\varepsilon_0 r^2} = \vec{e}_r \frac{Q}{4\pi\varepsilon_0 r^2}$$

Sferični ionski oblak polmera $a = 10\,cm$ ima prostorninsko gostoto elektrine podano z enačbo

$$\rho = \frac{r}{a}\, 10^9\ C/m^3$$

Izračunajte EPJ v oblaku na oddaljenosti $b = a/2$ od središča oblaka.

$$\oint_A E\,dA = \frac{Q}{\varepsilon_0}$$

$$E(r)\oint_A d\vec{A} = E(r)\cdot 4\pi r^2$$

$$\frac{Q}{\varepsilon_0} = \frac{\int \rho\, dV}{\varepsilon_0} = \cancel{\frac{\rho\cdot\frac{4\pi r^3}{3}}{\varepsilon_0}} = \cancel{\frac{\frac{r}{a}\,10^9\,\frac{C}{m^3}\cdot\frac{4\pi a^3}{24}}{\varepsilon_0}} =$$

$$\cancel{E(r)\cdot\frac{4\pi a^2}{4} = r}$$

$$V = \frac{4\pi r^3}{3}$$

$$\frac{dV}{dr} = \boxed{4\pi r^2}$$

$$E = \frac{1}{\varepsilon_0 4\pi r^2}\int \rho\, dV$$

$$\int \rho\, dV = \int \frac{r}{a}\, 10^9 \frac{C}{m^3}\, dV = \int \frac{r}{a}\cdot 10^9\cdot 4\pi r^2\, dr =$$

$$= \int \frac{r^3}{a}\, 4\pi\, 10^9\, dr = \frac{4\pi\, 10^9 \frac{C}{m^3}}{a}\cdot\frac{1}{4} r^4 \Big|_0^b$$

$$= \frac{4\pi 10^9}{a}\left(\frac{a}{8}\right) = \frac{4\pi 10^9}{\cancel{a}} \cancel{\frac{\pi 10^9}{2}} \cdot \frac{a^4}{4\cdot 2^4} = \frac{\pi \cdot 10^9 a^3}{16} = \frac{\pi 10^6}{16} =$$

$= \pi 10^6/16$

$$E = \frac{\int \rho dV}{4\pi\varepsilon_0 r^2} =$$

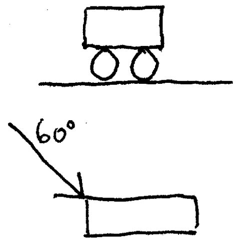
- $F = 100N$
  $s = 5m$

  $A = ?$

  $A = F\cdot s$
  $A = 500\ J$

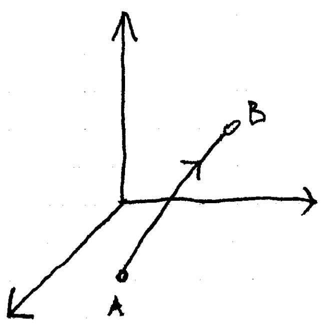
  $A = F\cdot s \cdot \cos 60$

  $A = 250\ J$



Koliko dela opravi električno polje $\vec{E}(-20, 10, 30)\ kV/m$ pri premiku točkaste $Q(1\mu C)$ iz $A(10, 10, 40)cm$ v točko $B(0,30,50)cm$

$$\int_A^B F\cdot dl = A = \int_L Q\cdot\vec{E}\cdot d\vec{l} = Q\cdot\vec{E}\cdot\vec{l} = +20\cdot 0{,}1\ kV + 10\cdot 0{,}2\ kV + 30\cdot 0{,}1 =$$
$$= 2\ kV + 2kV + 3kV$$
$$= 7kV = U_{AB}$$



$A = U\cdot Q$
$A = 7\cdot 10^3 \cdot 10^{-6} = 7\cdot 10^{-3}\ J$

$$A = Q \int_A^B (-20, 10, 30)\, kV/m\, (\vec{e_x} dx, \vec{e_y} dy, \vec{e_z} d\vec{z}) =$$

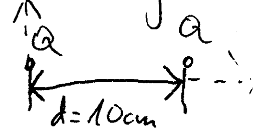
$$= Q \int_A^D (-20 dx + 10 dy + 30 dz)$$

$$= Q \left[\int_{10\,cm}^{0\,cm} -20 dx + \int_{10\,cm}^{30\,cm} 10 dy + \int_{40}^{50} 30 dz \right] kV/m$$

$$= Q \left[-20(0-10)cm + 10(30-10)cm + 30(50-40)cm \right] kV/m$$

$$= 7 \cdot 10^{-3}\, J$$

- 2 naboja



stisk na polovično razdaljo

$Q = 2nC$

$$A = \int_A^B \vec{F}\, d\vec{l} = \int_A^B Q\vec{E}\, d\vec{l} =$$

$$\vec{E} = Q / 4\pi\varepsilon_0 x^2\, \vec{e_x}$$

$$d\vec{l} = \vec{e_x} dx$$

$$\vec{E} \cdot d\vec{l} = Q / 4\pi\varepsilon_0 x^2 \cdot \vec{e_x} \cdot \vec{e_x}\, dx = \frac{Q}{4\pi\varepsilon_0 x^2} dx$$

$$A = Q \int_d^{d/2} \frac{Q}{4\pi\varepsilon_0 x^2} dx = \frac{Q^2}{4\pi\varepsilon_0} \int_d^{d/2} x^{-2} dx = \frac{Q}{4\pi\varepsilon_0} \left(-x^{-1}\right)_d^{d/2} =$$

$$= \frac{Q^2}{4\pi\varepsilon_0 d} (1-2) = -\frac{Q^2}{\pi\varepsilon_0 \pi d}$$

21.11.07

- Homogeno el. polje $\vec{E}=(3,-4,5)$ kV/m. Izračunajte potencial v točki B(2, 3, -1) dm, če je potencial v točki A(3, 2, 0) dm enak $V_A = 1540$ V

$$U = \frac{A(T_1 \to T_2)}{Q} = \int_{T_1}^{T_2} \vec{E}\, d\vec{l} = V(T_1) - V(T_2)$$

$$\vec{E}\, d\vec{l} = (3,4,5)\,\mathrm{kV/m} \cdot (dx, dy, dz) = (3dx - 4dy + 5dz)\,\mathrm{kV/m}$$

$$U_{AB} = -\left(\int_{0{,}2\,m}^{0{,}3\,m} 3\,dx - \int_{0{,}3\,m}^{0{,}2\,m} 4\,dy + \int_{-0{,}1\,m}^{0\,m} 5\,\frac{kV}{m}\,dz\right) =$$

lahko tudi za nehomogeno polje (ker se integrira)

$$= -(0{,}1 \cdot 3\,\mathrm{kV} + 4 \cdot 0{,}1\,\mathrm{kV} + 0{,}5\,\mathrm{kV}) = \underline{-1{,}2\,\mathrm{kV}}$$

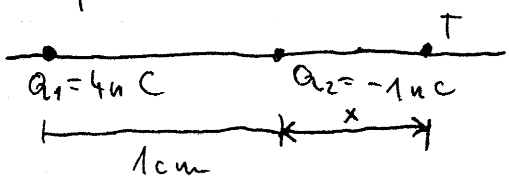
$U_{AB} = V_A - V_B$

~~$V_B = V_A - U_{AB}$~~

$\boxed{V_B = 340\,V}$

lahko tudi $U_{AB} = \vec{E} \cdot \vec{l}_{AB}$, ker je homogeno polje

- Na premici skozi dve naelektreni kroglici je točka T_0, kjer je EPJ = 0. Določite potencial v tej točki



$$E = \frac{Q}{4\pi\varepsilon_0 r^2}\,\vec{e_r}$$

$$E_1 = E_2$$

$$\frac{Q}{\cancel{4\pi\varepsilon_0}(r+1)^2} = -\frac{Q}{\cancel{4\pi\varepsilon_0}\, r^2}$$

$$\frac{4 Q_1}{r^2+2r+1} = -\frac{|Q_2|}{r^2}$$

$$Q_1 \cdot r^2 = |Q_2|(r^2+2r+1)$$

$$4 \cdot r^2 = +r^2 + 2r + 1$$

$$0 = +3r^2 - 2r - 1$$

$x_1 = 1$ cm

$x_2 = -1/3$ cm X polje spremeni smer

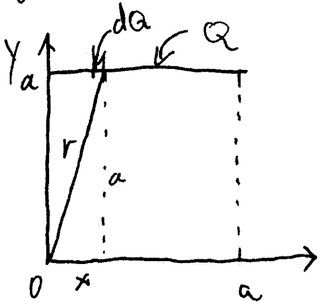
$$\boxed{V(T_0) = \frac{Q}{4\pi\varepsilon_0 r}}$$

$$V(T_0) = \frac{Q_1}{4\pi\varepsilon_0 (2cm)} + \frac{Q}{4\pi\varepsilon_0 \, 1cm}$$

$$= 9\cdot 10^9 \frac{Vm}{As}\left[\frac{4nC}{2\cdot 10^{-2}m} - \frac{1nC}{10^{-2}m}\right] = 9\cdot 10^{11}\cdot 10^{-9}\ V =$$

$$= \boxed{900\ V}$$

- Elektrina množine ~~k~~Q je enakomerno porazdeljena vzdolž tanke niti dolžine a. Določite potencial v izhodišču koor. sistema.



$$dE = \frac{dQ}{4\pi\varepsilon_0 \cdot r^2}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$V = \frac{Q}{4\pi\varepsilon_0 r} \rightarrow dV = \frac{dQ}{4\pi\varepsilon_0 r}$$

$$dQ = q\cdot dl = q\cdot dx = \frac{Q}{a}\,dx$$

$$dV = \frac{\frac{Q}{a}\,dx}{4\pi\varepsilon_0\sqrt{a^2+x^2}}$$

$$V = \int_0^a \frac{\frac{Q}{a}\,dx}{4\pi\varepsilon_0\sqrt{a^2+x^2}} = \frac{Q}{4\pi\varepsilon_0 a}\left(\log\frac{a+\sqrt{a^2+a^2}}{0+\sqrt{0+a^2}}\right) = \qquad \int\frac{dx}{\sqrt{a^2+x^2}} = \ln(x+\sqrt{x^2+a^2})+C$$

$$= \frac{Q}{4\pi\varepsilon_0 a}\log\left(\sqrt{2}+1\right)$$

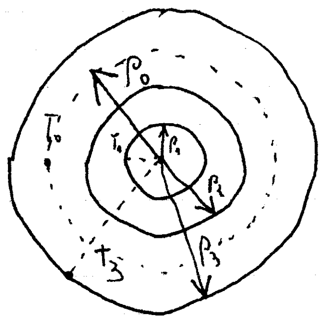
3 koaksialne valjne lupine polmerov

$\rho_1 = 1mm$
$\rho_2 = 2mm$
$\rho_3 = 4mm$

So naelektrene z vzdolžnimi gostotami elektrine

$q_1 = 10 nC/m$
$q_2 = -q_1/2$
$q_3 = -q_1/2$

Določite vektor EPJ. v točki T_0, ki je od z osi oddaljena za $\rho_0 = 3mm$



In določite še napetost med ρ_1 in ρ_3

$U_{13} = ?$

$$\oint \vec{E} d\vec{A} = \frac{Q_{znotraj\,A}}{\varepsilon_0}$$

$$\vec{E} \oint_A dA = EA = E\, 2\pi \rho_0 \cdot l$$

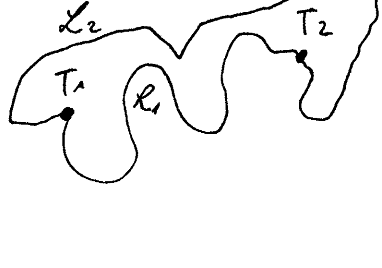
$$Q_{znotraj} = (q_1 + q_2)\, l$$

$$E\, 2\pi \rho_0 \cancel{l} = (q_1 + q_2) \cancel{l} \quad / \varepsilon_0$$
$$E\, 2\pi \rho_0 = q_1 + q_2 \quad / \varepsilon_0$$
$$\boxed{E = \frac{q_1 + q_2}{\varepsilon_0 2\pi \rho_0}} = 3kV/m$$

$$U_{13} = \int_{T_1}^{T_3} \vec{E} d\vec{l} =$$

nevrtinčno, konzervativno polje

$$\int_{\mathcal{L}_1} \vec{E} d\vec{l} = \int_{\mathcal{L}_2} \vec{E} d\vec{l} \rightarrow \oint \vec{E} d\vec{l} = 0$$



$$U_{13} = U_{12} + U_{23} = \int_{T_1}^{T_2} \vec{E} d\vec{l} + \int_{T_2}^{T_3} \vec{E} d\vec{l}$$

$$d\vec{l} = \vec{e_\rho}\, d\rho$$

$$= \int_{T_1}^{T_2} (q_1/2\pi\rho\varepsilon_0)\, d\rho + \int_{T_2}^{T_3} ((q_1+q_2)/2\pi\rho\varepsilon_0)\, d\rho =$$

$$= \frac{q_1}{2\pi\varepsilon_0}\left(\ln \rho \Big|_{\rho_1}^{\rho_2}\right) + \frac{q_1+q_2}{2\pi\varepsilon_0}\left(\ln \rho \Big|_{\rho_1}^{\rho_2}\right) =$$

$$= \frac{q_1}{2\pi\varepsilon_0} \ln \rho_2/\rho_1 + \frac{q_1+q_2}{2\pi\varepsilon_0}\left(\ln \rho_3 - \ln \rho_2\right) =$$

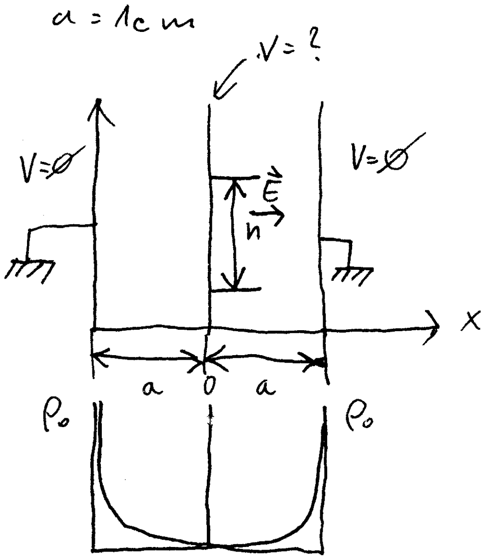
$$= \boxed{\frac{q_1}{2\pi\varepsilon_0}\left(\ln \frac{\rho_2}{\rho_1} + \frac{1}{2} \ln \frac{\rho_3}{\rho_2}\right) =} \quad \leftarrow q_2 = -q_1/2$$

$= 187\,V$

koliko je $U_{1,4}$, če je $\rho_4 = 5\,mm$ $V = 187\,V$.

- Gostota elektrine je razporejena volumsko

$\rho = \rho_0 (x/a)^2$
$\rho_0 = 10^{-6}\ C/m^3$
$a = 1\,cm$



$$\oint \vec{E}\, d\vec{A} = \frac{Q_{znotrajA}}{\varepsilon_0}$$

$$E\cancel{A} = \frac{\int_V \rho\, dV}{\varepsilon_0} = \frac{1}{\varepsilon_0} \cdot \cancel{\underbrace{h \cdot l}_{A}} \cdot \int_{\emptyset}^{x} \rho_0 \left(\frac{x}{a}\right)^2 dx$$

$$\boxed{E = \frac{1}{\varepsilon_0} \rho_0 \frac{1}{a^2} \frac{x^3}{3}}$$

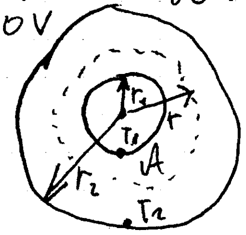
$$V(T) = \int_{T}^{T(V=\emptyset)} \vec{E}\, d\vec{l} = \int_0^a \left(-\vec{e}_x \frac{\rho_0 x^3}{a^2 3 \varepsilon_0}\right) \vec{e}_x\, dx =$$

$$= -\frac{\rho_0}{3\varepsilon_0 a^2} \frac{a^4}{4} = \frac{-\rho_0 a^2}{12\varepsilon_0} = -0{,}94\,V$$

28. 11. 07

GAUSSOV STAVEK

- Med prevodnima koncentričnima krogelnima lupinama s polmerom $r_1 = 2\,cm$ in $r_2 = 5\,cm$ je oblak elektrin konstantne prostorske gostote je $\rho(r) = \rho_0$. Določite ρ_0, če je napetost med lupinama $U = 10\,V$



$$\oint_A \vec{E}\, d\vec{A} = \frac{Q_{znotraj\ A}}{\varepsilon_0}$$

$$\underbrace{E(r)\, A(r)}_{A(r) = 4\pi r^2} = \frac{\rho_0\left(\frac{4\pi r^3}{3} - \frac{4\pi r_1^3}{3}\right)}{\varepsilon_0}$$

$$E(r) = \frac{\rho_0 \cancel{4\pi}(r^3 - r_1^3)}{\cancel{4\pi} r^2\, 3\varepsilon_0} = \frac{\rho_0}{3\varepsilon_0}\left(r - \frac{r_1^3}{r^2}\right)$$

$$U = \int_{T_1}^{T_2} \vec{E}\, d\vec{\ell} = V(T_1) - V(T_2)$$

$$d\vec{\ell} = \vec{e_r}\, dr$$

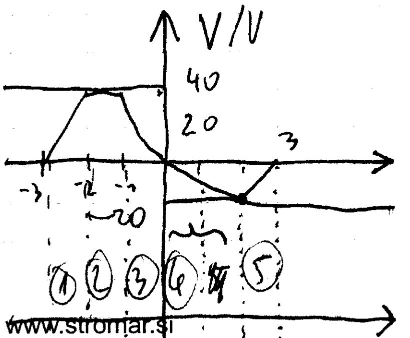
$$U_{12} = \int_{r_1}^{r_2} \frac{\rho_0}{3\varepsilon_0}\left(r - \frac{r_1^3}{r^2}\right)\vec{e_r}\, dr\, \vec{e_r} = \frac{\rho_0}{3\varepsilon_0}\left(\frac{r^2}{2} - \left(-\frac{r_1^3}{r}\right)\right)\Bigg|_{r_1}^{r_2} =$$

$$= \frac{\rho_0}{3\varepsilon_0}\underbrace{\left(\frac{r_2^2 - r_1^2}{2} + r_1^3\left(\frac{1}{r_2} - \frac{1}{r_1}\right)\right)}_{k}$$

$$U_{12} = \frac{\rho_0}{3\varepsilon_0}\, k \rightarrow \rho_0 = \frac{U_{12}\, 3\varepsilon_0}{k} = 3 \cdot 10^{-7}\ As\, m^{-3}$$

- EPJ

Določite epj.



$$n = \vec{e}_n$$

$$dV = -\vec{E} \cdot d\ell$$

$$\vec{E} = -n\, \frac{\partial V}{\partial n}$$

$$E_x = -\frac{\partial V}{\partial x} \approx -\frac{\Delta V}{\Delta x} = -\frac{V(x+\Delta x) - V(x)}{\Delta x}$$

$$\vec{E} = -\text{grad}\, V = -\vec{\nabla} V = -\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right) = -\left(\frac{\partial V}{\partial r}, \frac{1}{r}\frac{\partial V}{\partial \varphi}, \frac{\partial V}{\partial z}\right) =$$

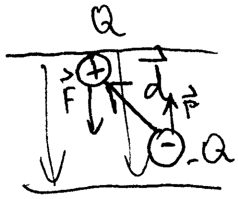
$$-\left(\frac{\partial V}{\partial r}, \frac{1}{r\, \cancel{\sin\vartheta}}\frac{\partial V}{\partial \vartheta} = \frac{1}{r}\frac{\partial V}{\partial \varphi}\frac{1}{\sin\varphi}\right)$$

$$\vec{E} = -\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) V$$

$$-\frac{40V - 0V}{-2mm - (-3mm)} = -40\, kV/m \quad ①$$

② 0V/m ③ 40kV/m ④ 10kV/m ⑤ -20kV/m

Potencial v prostoru je podan z enačbo $V(x,y) = 10^6 \frac{V}{m^2}\, x \cdot y$. Določite navor na dipol z električnim momentom $\vec{p} = (\vec{e_x} + 2\vec{e_y}) \cdot 10^{-9}$ Cm, ki se nahaja v T(1m, 1m, 0m)



$\vec{p} = Q \cdot \vec{d}$

$\vec{M} = \vec{r} \times \vec{F} \rightarrow \vec{M} = \vec{p} \times \vec{E}$

$$\vec{M} = \left(\frac{\vec{d}}{2}\right) \times Q\vec{E} + \left(-\frac{\vec{d}}{2}\right) \times (-Q)\vec{E} = \vec{d} \times Q\vec{E} = \underbrace{Q\vec{d}}_{\vec{p}} \times \vec{E}$$

$$\vec{E} = -\text{grad}\, V = -\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right) = -10^6 \frac{V}{m^2}(y, x, \emptyset)$$

$$\vec{E}(1m, 1m, 0m) = 10^6 \frac{V}{m^2}(1m, 1m, 0m)$$

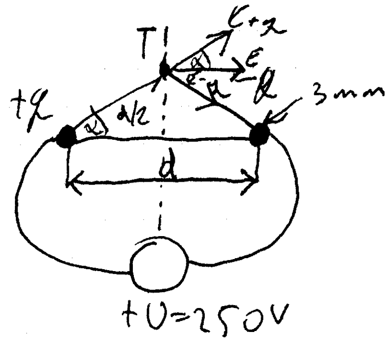
$$\vec{M} = \vec{p} \times \vec{E} = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 1 & 2 & 0 \\ 1 & 1 & 0 \end{vmatrix} \cdot 10^{-9}\, Cm \cdot \left(-10^6 \frac{V}{m}\right) =$$

$$(2 \cdot 0 - 0 \cdot 1), (1 \cdot 0 - 0 \cdot 1), (1 \cdot 1 - 2 \cdot 1)$$

$$= (0, -0, -1)\cdot(-10^{-9}\ AVs) = \vec{e}_z 10^{-3}\ Nm$$

Okovinjanje ekvipotencialnih ravnin

U=250V zračni dvovod, sestavljata dva vodnika polmerov a=3mm in medosne razdalje d=40cm. Izračunajte E(T), ki je za 25 cm oddaljena od oben vodnikov.



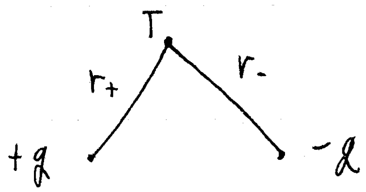
$$E(T) = \vec{E}_{+q}(T) + \vec{E}_{-q}(T)$$

$$|\vec{E}_{+q}(T)| = \frac{q}{2\pi\varepsilon_0 b}$$

$$|\vec{E}(T)| = |E_{+q}(T)|\cos\alpha \cdot 2$$

$$= \frac{q}{2\pi\varepsilon_0 b}\cdot\frac{d/2}{b}\cdot 2$$

Iščemo še q.



$$V = \int_T^{T(V=0)} \vec{E}\,d\vec{l}$$

$$V_{+q} = \int_r^{T\infty} \frac{q}{2\pi\varepsilon_0 r}\,\vec{e_r}\,\vec{e_r}\,dr = \frac{q}{2\pi\varepsilon_0}\ln r\Big|_r^{T\infty} = \frac{q}{2\pi\varepsilon_0}\left(\ln(T\infty) - \ln(r)\right)$$

$$V_{+q} = -\frac{q}{2\pi\varepsilon_0}\ln r + \text{konst}$$

$$V(T) = V_{+q} + V_{-q} = \frac{q}{2\pi\varepsilon_0}\ln r\Big|_{r_+}^{T\infty} + \frac{-q}{2\pi\varepsilon_0}\ln\Big|_{r_-}^{T\infty} =$$

$$\boxed{V(T) = \frac{q}{2\pi\varepsilon_0}\ln\frac{r_-}{r_+}}$$

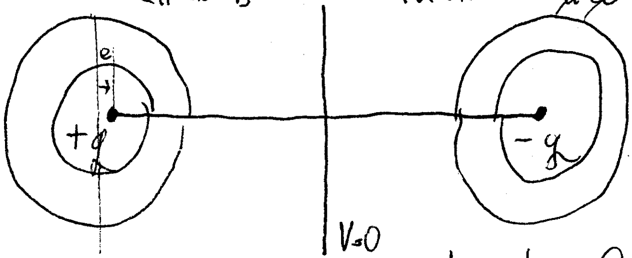
$U = V(+q) - V(-q)$ na sredini $(r_+ = r_-)$ je $V = \emptyset$

$U = 2 \cdot V(+q) =$

$= \frac{2q}{2\pi\varepsilon_0} \ln \frac{d-a}{a} = \frac{q}{\pi\varepsilon_0} \ln \frac{d-a}{a} \approx \frac{q}{\pi\varepsilon_0} \ln \frac{d}{a}$

$q = \frac{U\pi\varepsilon_0}{\ln d/a} = \frac{250\,V \cdot \pi \cdot 8{,}854 \cdot 10^{-12}\,\frac{As}{Vm}}{\ln \frac{400\,mm}{3\,mm}} = 1{,}42\,nC$

$E(T) = \frac{q\,d}{2\pi\varepsilon_0 b^2} = \frac{U\cancel{\pi\varepsilon_0}}{\ln d/a}\,\frac{d}{2\cancel{\pi\varepsilon_0} b^2} = \frac{Ud}{2b^2 \ln d/a} = 163\,V/m$



ekvipotencialne ravnine

$\frac{V(T) \cdot 2\pi\varepsilon_0}{q} = \frac{r^-}{r^+}$

konst $= e$

$r_- = \text{konst} \cdot r_+$

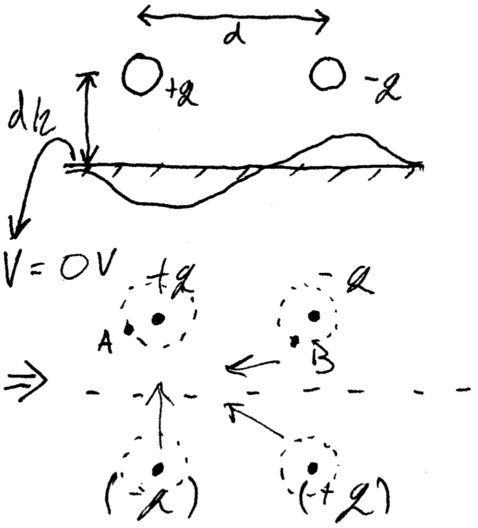
$$\boxed{e = \frac{d - \sqrt{d^2 - 4a^2}}{2}}$$

$U = \frac{q}{\pi\varepsilon_0} \ln \frac{d-a-e}{a-e}$

$Q = C \cdot U$ $\quad \cancel{Q=}$ $\quad C = \frac{l\pi\varepsilon_0}{\ln \frac{d-a-e}{a-e}}$

5. 12. 2007

Dvovod (žici polmera $\rho_0 = 1\,mm$ v medsebojni oddaljenosti $d = 40\,mm$ vzporedno z ravno ozemljeno površino na višini $d/2$. Določite napetost med žicama dvovoda, če je $q_2 = -q_1 = -10^{-3}\,As/m$!



$$V_{ai} = \frac{q_i}{2\pi\varepsilon_0}\ln\frac{1}{\rho_{Ai}}$$

$$V_A = \frac{q}{2\pi\varepsilon_0}\ln\frac{1}{\rho_0} + \frac{-q}{2\pi\varepsilon_0}\ln\frac{1}{d} + \frac{-q}{2\pi\varepsilon_0}\ln\frac{1}{d} + \frac{q}{2\pi\varepsilon_0}\ln\frac{1}{d\sqrt{2}}$$

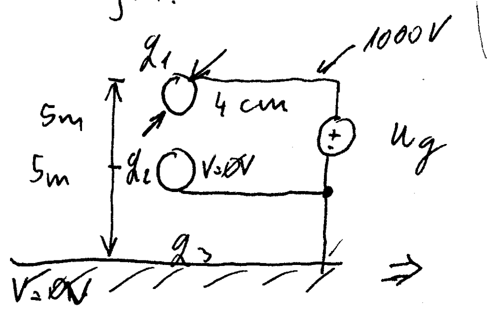
$$= \frac{q}{2\pi\varepsilon_0}\ln\frac{d\,d}{\rho_0\cdot d\cdot\sqrt{2}} = \frac{q}{2\pi\varepsilon_0}\ln\frac{d}{\rho_0\sqrt{2}}$$

$$V_B = \frac{-q}{2\pi\varepsilon_0}\ln\frac{1}{\rho_0} + \frac{q}{2\pi\varepsilon_0}\ln\frac{1}{d} + \frac{-q}{2\pi\varepsilon_0}\ln\frac{1}{d\sqrt{2}} + \frac{q}{2\pi\varepsilon_0}\ln\frac{1}{d} =$$

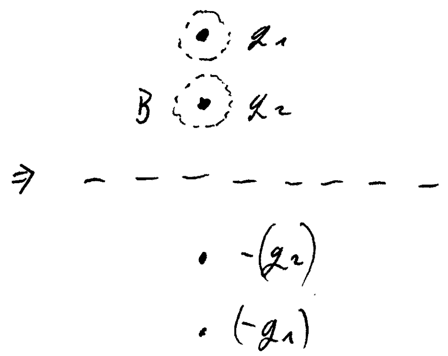
$$= \frac{q}{2\pi\varepsilon_0}\ln\frac{\rho_0\,d\sqrt{2}}{d\,d} = \frac{q}{2\pi\varepsilon_0}\ln\frac{\rho_0\sqrt{2}}{d} = \frac{q}{2\pi\varepsilon_0}\ln\left(\frac{d}{\rho_0\sqrt{2}}\right)^{-1} = -V_A$$

$$U_{AB} = V_A - V_B = 2V_A = \boxed{\frac{q}{\pi\varepsilon_0}\ln\frac{d}{\rho_0\sqrt{2}}} = 120.3\,V$$

Nad zemljo visita dve daljnovodni vrvi - ena vrh druge. Med njiju je priključen vir $U_g = 1kV$, spodnja pa je ozemljena. Določite razmerje q_2/q_3 (razmerje med vzdolžnim nabojem na spodnji vrvi in zemlji).



⇒



$$q_1 + q_2 + q_3 = 0$$

$$q_3 = -q_1 - q_2 \quad /:q_2$$

$$\frac{q_3}{q_2} = -\frac{q_1}{q_2} - 1$$

$$\frac{q_2}{q_3} = -\frac{1}{1 + q_1/q_2}$$

$$V_B = \frac{q_1}{2\pi\varepsilon_0}\ln\frac{1}{5} + \frac{q_2}{2\pi\varepsilon_0}\ln\frac{1}{0.02} + \frac{-q_2}{2\pi\varepsilon_0}\ln\frac{1}{10} + \frac{-q_1}{2\pi\varepsilon_0}\ln\frac{1}{15}$$

$$= \frac{q_1}{2\pi\varepsilon_0}\ln\frac{15}{5} + \frac{q_2}{2\pi\varepsilon_0}\ln\frac{10}{0.02} =$$

$$= \frac{1}{2\pi\varepsilon_0}\underbrace{\left[q_1\ln 3 + q_2\ln 500\right]}_{0}$$

$$= \emptyset \Rightarrow$$

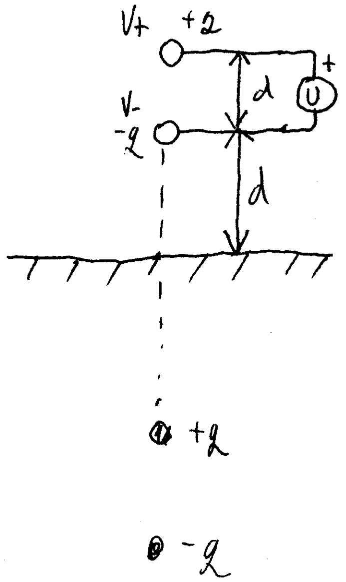
$$\boxed{\frac{q_1}{q_2} = -\frac{\ln 500}{\ln 3}}$$

$$\frac{q_2}{q_3} = 0.215$$

13.12.07

- Gaussov stavek EPL simetrično v prostoru (krogelni, krogelni z luknjo, oblak. Pri 2h nalogah.
- izolant normalna tang. meja, izolant / izolant homogeno polje, listič v (homogenem polju)
- Vodnik nad zemljo, Ploščni kondenzator

(1)



$r_0 = 1\,cm$
$d = 1\,m$
$U = 1\,kV$

določite potenciala žic

$r_0 \ll d$

$V = \frac{q}{2\pi\varepsilon_0} \ln \frac{1}{r}$

$\sum_i V_i$

$U = V_1 - V_2$

$$V_+ = \frac{q}{2\pi\varepsilon_0} \ln \frac{1}{r_0} + \frac{-q}{2\pi\varepsilon_0} \ln \frac{1}{4d} + \frac{q}{2\pi\varepsilon_0} \ln \frac{1}{3d} + \frac{-q}{2\pi\varepsilon_0} \ln \frac{1}{d} =$$

$$= \frac{q}{2\pi\varepsilon_0} \left(\ln \frac{1}{r_0} - \ln \frac{1}{4d} + \ln \frac{1}{3d} - \ln \frac{1}{d} \right) = \frac{q}{2\pi\varepsilon_0} \ln \left(\frac{4d^2}{3r_0 d} \right)$$

$$V_- = \frac{q}{2\pi\varepsilon_0} \left(-\ln \frac{1}{r_0} + \ln \frac{1}{2d} + \ln \frac{1}{d} - \ln \frac{1}{3d} \right) =$$

$$= \frac{q}{2\pi\varepsilon_0} \ln \frac{r_0}{2d} \frac{3d}{d} = \frac{3r_0}{2d}$$

$$U = \frac{q}{2\pi\varepsilon_0} \left(\ln \frac{4d}{3r_0} \frac{2d}{3r_0} \right) = \frac{q}{2\pi\varepsilon_0} \log \frac{8d^2}{9r_0^2} \rightarrow q = \frac{U \cdot 2\pi\varepsilon_0}{\log (8d^2/9r_0^2)}$$

V_+, V_-

$$V_+ = \frac{U}{\ln (8d^2/9r_0^2)} \cdot \ln \frac{4d}{3r_0} = 538\,V$$

$$V_- = -U + V_+ = -462\,V$$

Variante:

- 1) ena žica ozemljena

  ⏚ q_1

  o q_2

2) nevtralna žica $\Rightarrow q_{\text{žice}} = 0$

3) med žicama ni napetosti, znan je potencial ali naboj ene od žic

  o q_1

  o q_2

4) žica v zunanjem homogenem polju

  $V = E \cdot h$ prištejemo žici

5) računanje polja in gostote naboja na površini

$$E = \sum \frac{q}{2\pi\varepsilon_0 r}\, \vec{e}_r$$

$$\sigma = E \cdot \varepsilon_0$$

Polje v snovi

$$\boxed{\vec{D} = \varepsilon_0 \vec{E} + \vec{P}}$$

polarizacija (gostota dipolov)

gostota el. pretoka

$$P = \sum_i \frac{\vec{p}_i}{\Delta V} \qquad \lim \Delta \to 0$$

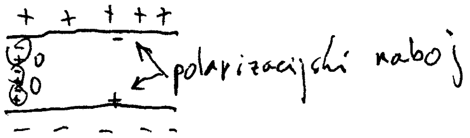
$$\vec{P} = \chi\, \varepsilon_0 \vec{E}$$

električna susceptibilnost

$$\vec{D} = \varepsilon_0 \vec{E} + \chi \varepsilon_0 \vec{E} = \varepsilon_0 \vec{E} \underbrace{(1+\chi)}_{\varepsilon_r} = \varepsilon_0 \varepsilon_r \cdot \vec{E} = \varepsilon \cdot \vec{E}$$

ε_r relativna dielektričnost

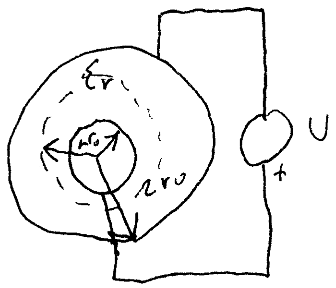
$$\boxed{\oint_A \vec{D}\, d\hat{A} = Q_{znotraj} \text{ prostih nabojev}}$$



Določite prebojno napetost med žilo in ovojem koaksialnega kabla, če se dielektričnost spreminja

$\varepsilon = \varepsilon_0\, r/r_0$ in je $E_p = 100\,kV/m$ ← prebojna trdnost

$r_0 = 0{,}5\,cm$



$$E = D/\varepsilon$$

$$\oint_A \vec{D}\, d\vec{A} = D \oint_A dA = D \cdot 2\pi r\, l$$

$$Q_{znotraj} = q \cdot l$$

$$\vec{D} = \frac{q}{2\pi R}$$

$$E = D/\varepsilon = \vec{e_r} \cdot \frac{q}{2\pi\varepsilon_0 \frac{r}{r_0} \cdot r} = \vec{e_r} \frac{q\, r_0}{2\pi\varepsilon_0 r^2}$$

$$U = \int_{r_0}^{2r_0} \vec{E}\, d\hat{l} = \int \vec{e_r} \frac{q\, r_0}{2\pi\varepsilon_0 r^2} \vec{e_r}\, dr =$$

$$= \frac{q\, r_0}{2\pi\varepsilon_0} \left(-\frac{1}{r}\right)\Bigg|_{r_0}^{2r_0} = \frac{q}{4\pi\varepsilon_0}$$

$$E_{preb} = E(r = r_0) = \frac{q\, r_0}{2\pi\varepsilon_0 r_0^2} = \frac{q}{2\pi\varepsilon_0 r_0}$$

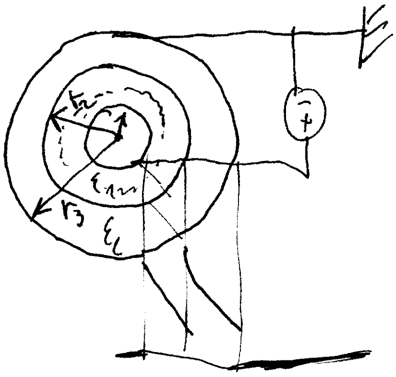
$$\frac{q}{2\pi\varepsilon_0} = E_{preb} \cdot r_0$$

$$U = 1/2\, E_{preb} r_0 = 250\ V$$

Določite potencial na meji dielektrikov dvoplastnega

• $\varepsilon_{r1} = 4$ $\varepsilon_{r2} = 2$, če je $U = 10\,kV$

$r_3 = 2r_2 = 4r_1 = 4\,cm$



D isti v obeh prostorih

$$D = \vec{e}_r \frac{q}{2\pi r}$$

$$E_1 = \frac{D}{\varepsilon_1} = e_r \frac{q}{2\pi \varepsilon_1 r}$$

$$E_2 = \frac{D}{\varepsilon_2} = e_r \frac{q}{2\pi \varepsilon_2 r}$$

$$E_1 \varepsilon_1 = E_2 \varepsilon_2$$

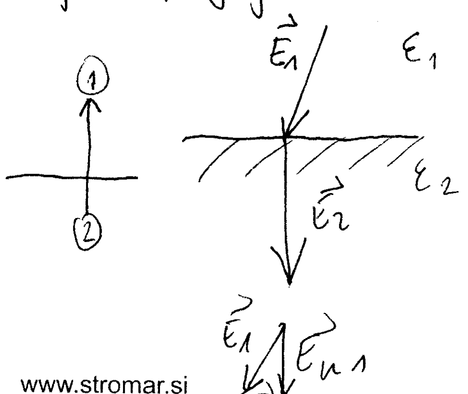
$$U = U_1 + U_2$$

$$= \int_{r_1}^{r_2} \frac{q}{2\pi\varepsilon_1 r}\,dr + \int_{r_2}^{r_3} \frac{q}{2\pi\varepsilon_2 r}\,dr$$

$$U = \frac{q}{2\pi\varepsilon_0}\left[\frac{1}{\varepsilon_{r1}} \ln\frac{r_2}{r_1} + \underbrace{\frac{1}{\varepsilon_{r2}} \ln\frac{r_3}{r_2}}_{U_2}\right)$$

$$V(r=r_2) = U_2 = \frac{q}{2\pi\varepsilon_0}\frac{1}{\varepsilon_{r2}} \ln\frac{r_3}{r_2} =$$

Mejni pogoji



$$\oint_A \vec{D}\cdot d\vec{A} = Q_{not}$$

$\boxed{D_{n1} = D_{n2}}$ če $\sigma = 0$

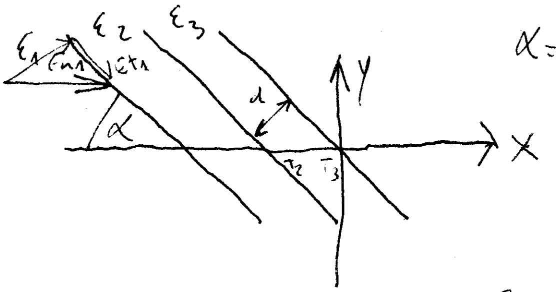
$\sigma \neq 0$ $\vec{e}_n \cdot (\vec{D}_1 - \vec{D}_2) = \sigma_{prosti}$

$$E_{n1}\varepsilon_1 = E_{n2}\varepsilon_2$$

dvoplastni dielektrik vstavimo v homogeno polje

$\varepsilon_2 = 5\varepsilon_0$ Določite EPJ E_3 v snovi z

$E_1 = 150 V/m$ dielektričnostjo $\varepsilon_3 = 2\varepsilon_0$



$\alpha = 60°$ $\varepsilon_1 = \varepsilon_0$

$D_{n1} = D_{n2} = D_{n3}$

$D_{n2} = D_{n3}$

$E_{n1}\varepsilon_1 = E_{n3}\varepsilon_3$ $E_{n1} = E_1 \cdot \sin 60°$

$E_{n3} = \frac{E_{n1}\varepsilon_1}{\varepsilon_3} = E_1 \sin 60 \frac{\varepsilon_1}{\varepsilon_3} = 150\frac{V}{m}\frac{\sqrt{3}}{2} \cdot \frac{\varepsilon_0 \, 1}{2\varepsilon_0} = \frac{150\sqrt{3}}{4} = \frac{75\sqrt{3}}{2} V/m$

$E_{t1} = E_{t2} = E_{t3}$ $E_{t1} = E_1 \cdot \cos 60° = \boxed{75 \; V/m}$

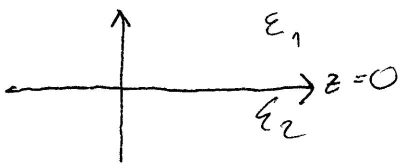
$E_3 = \sqrt{E_{n3}^2 + E_{t3}^2} = 100 V/m$

$d = 4cm$ $U_{23} = E_{3x} \cdot (d/\sin\alpha)$ (1)

$U_{23} = \cancel{D_n} E_{3n} \cdot d + E_{t3} \cdot d/tg\alpha$ (2)

Ravnina $z = 0$ je meja snovi z dielektričnostima $\varepsilon_1 = 3\varepsilon_0$ in $\varepsilon_2 = 5\varepsilon_0$. V točki T_+ je vektor EPJ $\vec{E_1}(10^3 e_\rho + 0{,}5 \cdot 10^3 e_\varphi - 0{,}5 \cdot 10^3 \vec{e_z})$ V/m

Določite E_2 v točki T_-.

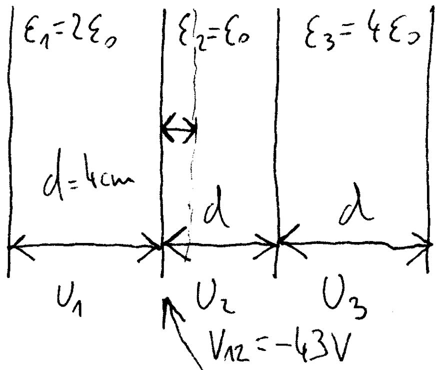


$\vec{e_n} = \vec{e_z}$

$\varepsilon_1 E_1 = \varepsilon_2 E_2$

$E_2 = E_1 \, \varepsilon_1/\varepsilon_2 \rightarrow \vec{E_2}(10^3 e_\rho + 0{,}5 \cdot 10^3 e_\varphi - 0{,}3 \cdot 10^3 \vec{e_z})$

V ploščatem triplastnem kondenzatorju določite ravnino z $V=0$. $V_{23} = 71V$



$V_d = 100\,V$

$V_e = -100\,V$

$U = V_D - V_e = 200V = U_1 + U_2 + U_3$

$U = E_1 \cdot d + E_2 \cdot d + E_3 \cdot d =$

$D_1 = D_2 = D_3 \rightarrow E_1 \varepsilon_1 = E_2 \varepsilon_2 = E_3 \varepsilon_3$

$= E_1 d + E_1 \frac{\varepsilon_1}{\varepsilon_2} d + \frac{\varepsilon_1}{\varepsilon_3} E_1 d$

$U = E_1 d \left(1 + \frac{\varepsilon_1}{\varepsilon_2} + \frac{\varepsilon_1}{\varepsilon_3}\right)$

$200V = E_1 \cdot 4cm \left(1 + 2 + \frac{1}{2}\right)$

$E_1 = \frac{200 \cdot 2}{7 \cdot 4} V/cm = 100/7\ V/cm$

$U_1 = 100/7 \cdot 4cm = 57V$

$V_{12} = -43\ V$

$U_2 = E_2 \cdot d = \frac{\varepsilon_1}{\varepsilon_2} E_1 d = 2 E_1 d = \boxed{114\,V}$

$V_0 = 0V = V_{12} + \boxed{E_2 x}$

$x = \frac{-V_{12}}{E_2} = \frac{43V \cdot 7}{200\ V/cm} = \underline{\underline{1{,}5\ cm}}$

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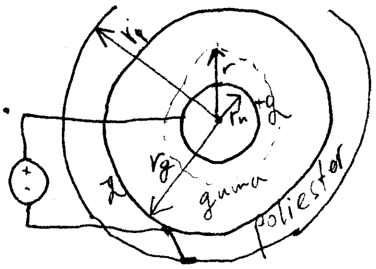


Dimenzioniranje coax kabla

$U = 20\,kV$

$r_n = 4\,mm$

$r_g = ?$

$r_p = ?$

$\varepsilon_{r\,gume} = 3,2$

$\varepsilon_{r\,poli} = 2,6$

Prebojne trdnosti

$E_{pg} = 25\,kV/mm$

$E_{pp} = 20\,kV/mm$

$E_{max} = 25\,\%\,E_{preb}$
predimenzioniramo

E_{max} v gumi $= 0{,}25 \cdot 25\,kV/mm = 6{,}25\,kV/mm$

E_{max} v poli $= 5\,kV/mm$

$$\boxed{\oint \underbrace{\vec{D}}_{\text{gostota pretoka}} d\vec{A} = Q \text{ prosti znotraj } A}$$

pretok el. polja

$D(r) \cdot A(r) = q \cdot l$

$D(r) \cdot 2\pi r \cdot l = q \cdot l$

$D(r) = q / 2\pi r$

$$\boxed{\vec{D} = \varepsilon \cdot \vec{E}}$$

$$E_{gume} = \frac{D(r)}{\varepsilon_{rg}\,\varepsilon_0} = \frac{q}{2\pi\,\varepsilon_{rg}\,\varepsilon_0\, r}$$

$$E_{poli} = \frac{q}{2\pi\,\varepsilon_{rp}\,\varepsilon_0\, r}$$

$E_{max\,gume}$ = pri notranjem ~~gum~~ radiju

$$= \frac{q}{2\pi\,\varepsilon_{rg}\,\varepsilon_0\, r_n} = 6{,}25\,kV/mm \rightarrow \text{lahko } q \text{ izračunamo}$$

$$E_{max\,poli} = \frac{q}{2\pi\,\varepsilon_{rp}\,\varepsilon_0\, r_g} = 5\,kV/mm \rightarrow r_g = \frac{6{,}25}{5}\,\frac{\varepsilon_{rg}}{\varepsilon_{rp}}\, r_n$$

$= 6{,}2\,mm$

$$U = \int E\,dl = \int_{r_n}^{r_g} \frac{q\,dr}{2\pi \varepsilon_{rg} \varepsilon_0 r} + \int_{r_g}^{r_p} \frac{q\,dr}{2\pi \varepsilon_0 \varepsilon_{rp} r}$$

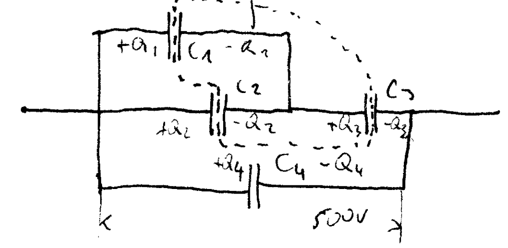
$$U = \frac{q}{2\pi\varepsilon_0\varepsilon_{rg}} \log\frac{r_g}{r_n} + \frac{q}{2\pi\varepsilon_{rp}\varepsilon_0} \log\frac{r_p}{r_g} \rightarrow \text{iščemo}$$

$$\frac{q}{2\pi\varepsilon_{rg}\varepsilon_0 r_n} = 6{,}25\ \mathrm{kV/mm}$$

$$U = \underbrace{6{,}25\ \mathrm{kV/mm} \cdot r_n \cdot \log\frac{r_g}{r_n}}_{a} + 6{,}25\ \mathrm{kV/mm} \cdot r_n \frac{\varepsilon_{rg}}{\varepsilon_{rp}} \log\frac{r_p}{r_g}$$

$$r_p = r_g\, e^{\frac{U-a}{5\,\mathrm{kV/mm}\, r_g}} = \boxed{7{,}7\ \mathrm{mm}}$$

V kondenzatorskem vezju določite elektrino na kondenzatorju C_1

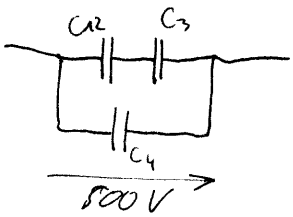


$U = 500\ V$

$C_1 = C_2 = 2\ \mu F$
$C_3 = 4\ \mu F$
$C_4 = 6\ \mu F$

$\boxed{Q = C \cdot U}$ naboj se ohranja

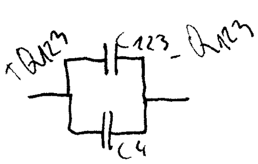
$-Q_1 - Q_2 + Q_3 = 0$
$Q_1 + Q_2 = Q_3$



$Q = Q_1 + Q_2$
$CU = C_1 U + C_2 U \rightarrow C = C_1 + C_2$ vzporedno

$C = \varepsilon A/d$ poveča se površina plošč

$C_{12} = C_1 + C_2 = 4\ \mu F$



$U = U_{12} + U_3$

$$\frac{Q}{C_{123}} = \frac{Q}{C_{12}} + \frac{Q}{C_3}$$

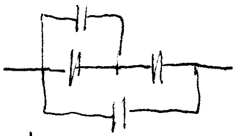
$\dfrac{1}{C_{123}} = \dfrac{1}{C_{12}} + \dfrac{1}{C_3}$ zap. vezava

$$C = \frac{4\mu F \cdot 4\mu F}{4\mu F + 4\mu F} = 2\mu F$$

$$Q_{123} = C_{123} \cdot U = 2\mu F \cdot 500\,V = 1\,mC$$

$$Q_{12} = Q_3 = Q_{123}$$

$$\rightarrow Q_{12} = 1\,mC$$



$$Q_1 = \frac{1}{2} Q_{12} = 0{,}5\,mC$$

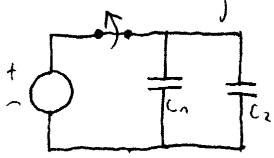
$$\boxed{U_1 = \frac{Q_{12}}{C_{12}} \rightarrow Q_1 = C_1 \cdot U_1}$$ če ne bi bila ista.

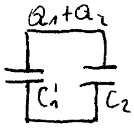
Dva zračna kondenzatorja, kapacitativnosti

$C_1 = 100\,nF$
$C_2 = 200\,nF$

sta priključena $U = 90\,V$. Po izklopu stikala

Razmaknemo prvemu plošči na 2x oddaljenost, koliko naboja se izmenja med njima





$$Q = C \cdot U$$

$$Q_1 = 9\mu C \qquad Q_2 = 18\mu C$$

$$C_1' = C_1/2 = 50\,nF$$

$$C_{12}' = C_1' + C_2 = 250\,nF$$

$$Q_{12}' = Q_{12} = Q_1 + Q_2 = 27\mu C$$

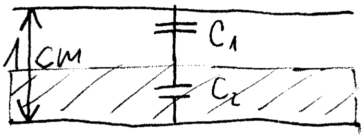
$$U_{12}' = \frac{Q_{12}'}{C_{12}'} = \frac{27\,\mu C}{250\,nF} = \underline{108\,V}$$

$$Q_1' = C_1' \cdot U = \cancel{\ldots}\ 5{,}4\,\mu C$$

$$\Delta Q = Q_1 - Q_1' = \boxed{3{,}6\,\mu C}$$

Na ploščni zračni kondenzator $A = 10\,cm^2$, $d = 1\,cm$ priključimo $U = 12V$.

Kolikšna je U' na kondenzatorju, če vir odklopimo in v kondenzator vložimo dielektrični listič debeline $a = 0{,}5\,cm$ z $\varepsilon_r = 3$



$$C_0 = \varepsilon_0 A/d$$

$$Q_0 = C_0 U$$

$$C_1 + C_2 = C$$

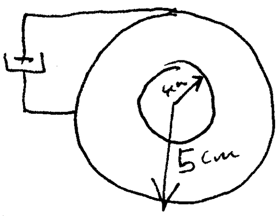
$$C_1 = \varepsilon_0 \frac{A\,2}{d} \qquad C_2 = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C'} \rightarrow$$

$$U' = \frac{Q'}{C'} = Q\left(\frac{d}{2\varepsilon_0 A} + \frac{d}{2\varepsilon_0 \varepsilon_r A}\right) = C_0 U \left(\frac{d}{2\varepsilon_0 A} + \frac{d}{2\varepsilon_0 \varepsilon_r A}\right) =$$

$$= U\left(\frac{1}{2} + \frac{1}{2\varepsilon_r}\right) = 12V\left(\frac{2}{3}\right) = \boxed{8V} \quad \checkmark$$

Sferični kondenzator z dielektrikom $\varepsilon_r = 5$ naelektrino z $U = 300V$



Kolikšna je akumulirana energija v polju?

gostota energije

$$\boxed{w = \frac{1}{2} \varepsilon E^2} \quad J/m^3$$

$$W = \int_V w\, dV$$

$$4\pi r^2 D = Q \rightarrow D = \frac{Q}{4\pi r^2} \rightarrow E = \frac{Q}{4\pi \varepsilon_r \varepsilon_0 r^2}$$

$$U = \int E\,dl = \int \frac{Q\,dr}{4\pi r^2 \varepsilon_r \varepsilon_0} = \frac{Q}{4\pi\varepsilon_r\varepsilon_0}\left(\frac{1}{r_n} - \frac{1}{r_z}\right)$$

$$U = Q \cdot \frac{1}{C} \rightarrow C = \frac{4\pi\varepsilon_r\varepsilon_0}{\frac{1}{r_n} - \frac{1}{r_z}}$$

$$E = \frac{U}{\frac{1}{r_n} - \frac{1}{r_z}} \cdot \frac{1}{r^2} = \frac{k}{r^2}$$

$$w = \frac{1}{2}\varepsilon_r\varepsilon_0\left(\frac{k}{r^2}\right)^2 \qquad \boxed{dV = 4\pi r^2\,dr}$$

$$W = \int w\,dV = \int_{r_n}^{r_z} \frac{1}{2}\varepsilon_r\varepsilon_0\left(\frac{k}{r^2}\right)^2 4\pi r^2\,dr = \frac{1}{2}\varepsilon_r\varepsilon_0 k^2 4\pi \int_{r_n}^{r_z}\frac{dr}{r^2} =$$

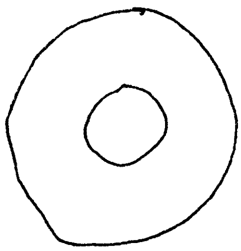
$$= \frac{1}{2}\varepsilon_r\varepsilon_0 k^2 4\pi\left(\frac{1}{r_n} - \frac{1}{r_z}\right) = \frac{\frac{1}{2}\varepsilon_r\varepsilon_0 4\pi U^2}{\left(\frac{1}{r_n} - \frac{1}{r_z}\right)^{\cancel{2}}}\cancel{\left(\frac{1}{r_n} - \frac{1}{r_z}\right)} =$$

$$= \frac{\varepsilon_r\varepsilon_0 U^2 2\pi}{\cancel{2}\left(\frac{1}{r_n} - \frac{1}{r_z}\right)} = CU^2 \;.\!= 5 \text{ nJ}$$

$$\boxed{W = \frac{QU}{2} = \frac{CU^2}{2} = \frac{Q}{2C}}$$

če je ε_r konstanten, če ni
konst | integriramo

? je kapacitivnost, če je $\varepsilon_r = k/r$



$$D = \frac{Q}{4\pi r^2}$$

$$E = \frac{D}{\varepsilon} = \frac{Q\,r}{4\pi k \cdot \varepsilon_0 r^2} = \frac{Q}{4\pi k \varepsilon_0 r}$$

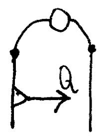
$$U = \int_{r_n}^{r_z} E\,dl = \int \frac{Q}{4\pi k\varepsilon_0 r}\,dr = \frac{Q}{4\pi k\varepsilon_0}\log\frac{r_z}{r_n}$$

$$C = \frac{Q}{U} = \frac{4\pi k\varepsilon_0}{\log(r_z/r_n)} \qquad W = \longrightarrow$$

$$\underline{\underline{W = \frac{CU^2}{2}}}$$

Potencialna energija

prelet naboja v polju



$$\frac{mv^2}{2} = QU \qquad A = \int_{T_1}^{T_2} \vec{F_e}\, dl = Q \int_{T_1}^{T_2} \vec{E}\, dl = Q\,U_{12}$$

$W_k = W_p$ ni $\frac{Q \cdot U}{2}$!

elektrenje kondenzatorja

$$dW = U\,dQ$$

$$dW = \frac{Q}{C}\,dQ$$

$$W = \frac{1}{C}\int Q\,dQ = \frac{Q^2}{C \cdot 2}$$

potencialna energija sistema nabojev

$$\boxed{W = \frac{1}{2}\sum_{i=1}^{n} Q_i V_i}$$

V_i potencial na mestu naboja Q_i, ki ga povzročajo vsi ostali naboji.

$$V_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{Q_j}{4\pi\varepsilon_0 r_{ij}}$$

Določeno potencialno energijo sistema treh nabojev, ki se nahajajo v ogliščih $\triangle$ s stranico $a = 10\,cm$.

$Q = Q_1 = Q_2 = Q_3 = 20\,nC$.

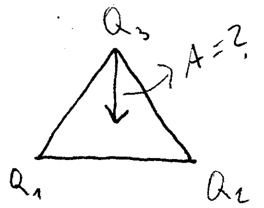
$n = 3$

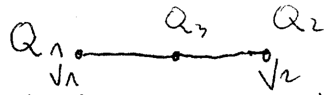
$$W = \frac{1}{2}\left(Q_1V_1 + Q_2V_2 + Q_3V_3\right) = \frac{3}{2}\,Q \cdot V_1 = \frac{3}{2}\,Q \cdot 2\frac{Q}{4\pi\varepsilon_0 a} = \boxed{\frac{3Q^2}{4\pi\varepsilon_0 a}}$$

$$V_1 = \frac{Q_2}{4\pi\varepsilon_0 a} + \frac{Q_3}{4\pi\varepsilon_0 a} = \frac{2Q}{4\pi\varepsilon_0 a}$$

$$V_1 = V_2 = V_3$$

9. 1. 08





$$A\,(T_1 \to T_2) = W_1 - W_2$$

$$W_{končna} = \frac{1}{2}\left(Q_1 V_1 + Q_2 V_2 + Q_3 V_3\right)$$

$$V_1 = V_2 = \frac{Q_3}{4\pi\varepsilon_0 \, a/2} + \frac{Q_3}{4\pi\varepsilon_0 a} = \frac{Q}{4\pi\varepsilon_0 a}\cdot 3$$

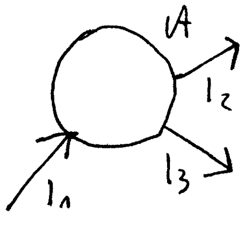
$$V_3 = \frac{Q}{4\pi\varepsilon_0 \, a/2}\cdot 2 = \frac{Q}{4\pi\varepsilon_0 a}\cdot 4$$

$$W_{končna} = \frac{1}{2} Q\left(2\cdot 3\,\frac{Q}{4\pi\varepsilon_0 a} + 4\,\frac{Q}{4\pi\varepsilon_0 a}\right) = \frac{Q^2}{2\cdot 4\cdot \pi\varepsilon_0 a} = \frac{5Q^2}{4\pi\varepsilon_0 a}$$

$$A = W_1 - W_2 = -2\,\frac{Q^2}{4\pi\varepsilon_0 a} = -72\ \mu J$$

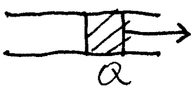
tokovno polje (časovno konstantno)

$$\boxed{-\frac{dQ}{dt} = i(t) = \oint_A \vec{J}\, d\vec{A}} = 0$$ kontinuitetna enačba



$$I_1 - I_2 - I_3 = 0$$

$$\frac{dQ}{dt} = \frac{d\int \rho\, dV}{dt} = 0 \Rightarrow \rho = konst$$



$$\vec{J} = \rho \vec{v}$$

gostota toka

$$\boxed{I = \int \vec{J}\, d\vec{A}}$$

$$\boxed{J = \frac{dI}{dA}}$$

če je tokovno polje homogeno: (in pravokotno)

$$I = J\cdot A \Leftrightarrow J = I/A$$

kondukitivni tok

$$\boxed{\vec{v} = \mu \cdot \vec{E}}$$ V mnogih snoveh ~ mobilnost

gostota naboja

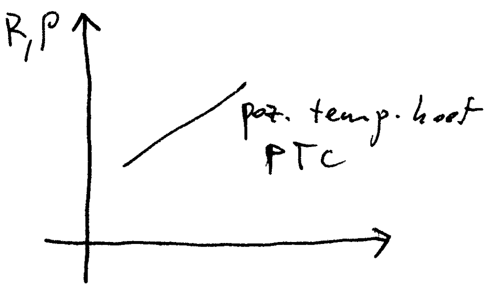
$$\boxed{J = (\rho \cdot \mu) \cdot \vec{E} = \gamma \cdot \vec{E}}$$

γ → specifična prevodnost ← ohmov zakon

$$U = \int \vec{E}\, d\vec{l} = \int_0^l \frac{J}{\gamma}\, dx = \int_0^l \frac{I/A}{\gamma}\, dx = \frac{1}{\gamma A}\, l = \boxed{I \cdot R}$$

$$R = \frac{l}{\gamma \cdot A} = \rho \frac{l}{A} \qquad G = \frac{\gamma A}{l}$$

$\boxed{\frac{1}{\gamma} \text{ specifična upornost}}$



$$\rho = \rho(T = 20°C)\,(1 + \alpha\,(T - T_0))$$

α ← temp. koeficient

MOČ

$$P = \int p \cdot dV = (p \cdot V)^{\text{za homogeno polje}}$$

p → gostota moči $[W/m^3]$

$\boxed{p = \vec{J} \cdot \vec{E}}$ Joulov zakon

$$p = \gamma \cdot \vec{E} \cdot \vec{E} = \gamma \vec{E}^2$$

$$P_{\text{za hom. polje}} = p \cdot V = \gamma E^2 \cdot A \cdot l = \gamma \left(\frac{U}{l}\right)^2 A \cdot l = \gamma \frac{U^2}{l} \cdot A =$$

$$= U^2 \cdot G = \frac{U^2}{R} = U \cdot I = I^2 \cdot R$$

- Bakrena žica AWG 12 ima premer 0,0808 in. Vodnik je dolg 50ft in v njem teče tok 20A. Določimo $\vec{E}$, U, R in $\vec{v}$.

$1\,in = 2{,}54\,cm$
$1\,ft = 12\,in$

$\gamma_{cu} = 5{,}8 \cdot 10^7\ S/m$
$\mu_{cu} = 0{,}0032\ m^2/Vs$

$J = \frac{I}{A} = \frac{20A}{\pi r^2} = 6{,}04 \cdot 10^6\ A/m^2$

E dobimo iz ohmovega zakona

$E = \frac{J}{\gamma} = 0{,}104\ V/m$

$U = E \cdot l$ ker je v žici homogeno $= 0{,}104\,V/m \cdot 50\,ft = 1{,}59\,V$

$R = \frac{U}{I} = \frac{1{,}59\,V}{20\,A} = \boxed{7{,}95 \cdot 10^{-2}\ \Omega}$ $\boxed{R = \frac{l}{\gamma A}}$

$\vec{v} = \mu \cdot \vec{E}$
$\vec{v} = 0{,}0032\ m^2/Vs \cdot 0.104\ V/m = 3{,}34 \cdot 10^{-4}\ m/s$

- Bakrena cev dolga 15 m ima premer 1 mm. Kolikšna je $R(T = 50°)$?

$l = 15\,m \quad 2r = 1\,mm$
$\gamma_{cu} = 5 \cdot 10^7\ S/m$
$\alpha(T = 20°) = 3{,}9 \cdot 10^{-3}\ K^{-1}$

$R = R(T_0)(1 + \alpha(T - T_0))$

$\underset{\substack{\shortparallel \\ 20°}}{R(T_0)} = \frac{l}{\gamma A} = \frac{15\,m}{5{,}8 \cdot 10^7\,S/m \cdot \pi(0{,}5 \cdot 10^{-3}\,m)^2} = 0{,}33\ \Omega$

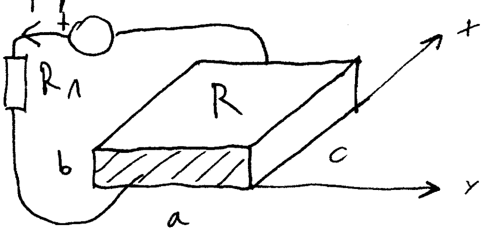
$R(T) = 0{,}33\,\Omega\,(1 + 3{,}9 \cdot 10^{-3} \cdot 30) = 0{,}37\ \Omega$

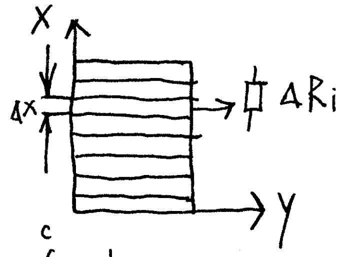
Določite moč, ki se troši v uporovnem traku, če ga preko upora 2000 Ω priklopimo na 12 V.

Specifična upornost se spreminja

$$\gamma = \gamma_0 \frac{x+c}{c}$$

$\gamma_0 = 10^{-3}$ S/mm $a = 10$ mm $b = 2$ mm $c = 100$ mm





$$R = \sum \Delta R_i$$

$$R = \int_0^c dR$$

$$R = \int_0^c \frac{dx}{\gamma_0 \frac{x+c}{c} \cdot a \cdot b} = \frac{c}{\gamma_0 ab} \cdot \int_0^c \frac{dx}{x+c} =$$

$$R = \frac{l}{\gamma A} \qquad dR = \frac{dx}{\gamma A}$$

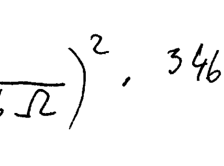
$$= \frac{c}{\gamma_0 ab} \ln(x+c)\Big|_0^c = \frac{c}{\gamma_0 ab} \ln \frac{2\cancel{c}}{\cancel{c}} =$$

$$= \frac{c}{\gamma_0 ab} \ln 2 = \frac{100\, \cancel{mm} \cdot \cancel{mm}}{10^{-3}\, S \cdot 10\, \cancel{mm} \cdot 2\, \cancel{mm}} \ln 2 = 10^4 \frac{1}{2} \ln 2\ \Omega = 3466\ \Omega$$

$$P_R = I^2 R \longleftarrow I = \frac{U}{R_1 + R}$$

$$P_R = \left(\frac{12\ V}{5466\ \Omega}\right)^2 \cdot 3466\ \Omega = 16{,}7\ mW$$

• primer:



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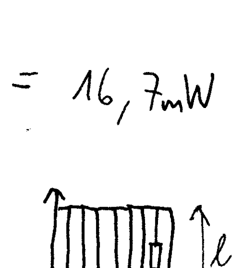


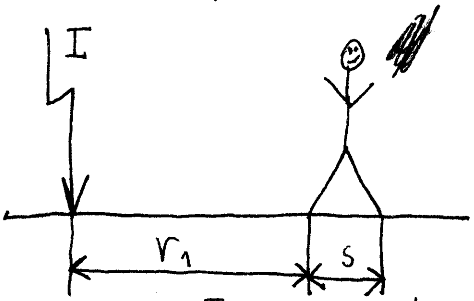
$$\Delta G = \frac{\gamma \Delta A}{l} = \frac{\gamma \Delta x b}{l}$$

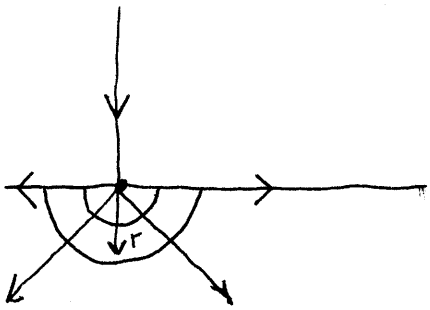
$$dG = \frac{\gamma b dx}{l}$$

$$G = \int_0^a \frac{\gamma_0 \frac{x+c}{c} dx\, b}{c}$$

Določimo napetost koraka pri udaru strele s tokom 20kA, če je človek oddaljen za $r_1 = 15\,m$ od udara.

$\gamma_{zemlje} = 10^{-4}\ S/m$
$s_{koraka} = 0{,}8\ m$





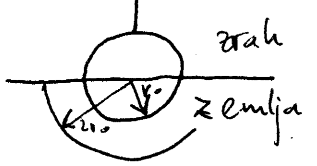
$$\vec{J}(r) = \frac{I}{A(r)} = \frac{I}{\frac{4\pi r^2}{2}} = \frac{I}{2\pi r^2}$$

$$\vec{E}(r) = \frac{J(r)}{\gamma} = \frac{I}{2\pi\gamma r^2}$$

$$U = \int_{r_1}^{r_1+s} \vec{E}\,d\vec{l} = \int_{r_1}^{r_1+s} \frac{I}{2\pi\gamma r^2}\,dr = \frac{I}{2\pi\gamma}\int_{r_1}^{r_1+s}\frac{1}{r^2}\,dr = \frac{I}{2\pi\gamma}\left(-\frac{1}{r}\right)\Big|_{r_1}^{r_1+s} =$$

$$= \frac{I}{2\pi\gamma}\left(\frac{1}{r_1} - \frac{1}{r_1+s}\right) \qquad U = \frac{20\,kA}{2\pi\,10^{-4}\,S/m}\left(\frac{1}{15\,m} - \frac{1}{15{,}8\,m}\right) \cong 107\,kV$$

V Do polovice vkopano kovinsko krogelno ozemljilo polmera 0,5 cm vstopa tok 2kA.



Koliko toplote se sprosti v zemlji s spec. upornostjo $\rho = 200\,\Omega m$ v pasu med r_0 in $2r_0$ v času 5 s?

- 1. varianta
  - gostota moči → moč → energija
- 2. varianta
  - napetost → moč → energija
- 3. varianta
  - upornost → moč → energija

$$p = \vec{J} \cdot \vec{E} = \vec{J} \cdot \frac{\vec{J}}{\gamma} = \frac{\vec{J}^2}{\gamma}$$

$$J(r) = \frac{1}{A(r)} = \frac{1}{4\pi r^2/2}$$

$$p = \frac{1}{\gamma} \cdot \left(\frac{1}{2\pi r^2}\right)^2$$

$$P = \int p \, dV = \int_{r_0}^{2r_0} p \cdot 2\pi r^2 \, dr$$

$$P = \int \frac{1}{\gamma}\left(\frac{1}{2\pi r^2}\right)^2 2\pi r^2 \, dr = \frac{I^2}{\gamma 2\pi} \int_{r_0}^{2r_0} \frac{dr}{r^2} = \frac{I^2}{2\pi\gamma}\left(\frac{2 \cdot 1}{2r_0} - \frac{1}{r_0}\right) = \frac{I^2}{4\pi\gamma r_0}$$

• 2 varianta

$$U = \int_{r_0}^{2r_0} \vec{E}(r)\,dr = \int_{r_0}^{2r_0} \frac{J}{\gamma} \, dr = \int_{r_0}^{2r_0} \frac{I}{\cancel{4}\,2\pi r^2 \gamma} \, dr = \frac{I}{2\pi\gamma}\left(\frac{1}{r_0} - \frac{1}{2r_0}\right) =$$

$$= \frac{I}{4\pi\gamma r_0} \qquad P = U \cdot I$$

• 3. varianta

$$R = \frac{U}{I} = \frac{1}{4\pi\gamma r_0} \qquad P = I^2 R$$

$$W = \int_0^t P\,dt = P \cdot t = \frac{I^2}{4\pi\gamma r_0} \cdot t = \frac{(2\,kA)^2\, 200\,\Omega m \cdot 5\,s}{4\pi\ 5 \cdot 10^{-3}\,m} = \boxed{\frac{1}{\gamma} = \rho}\ !$$

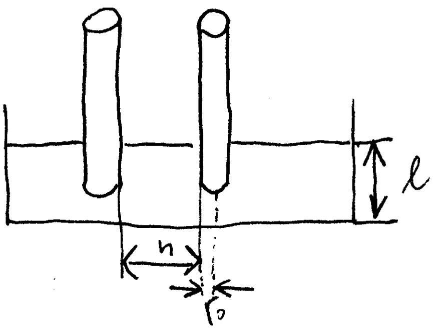
$$= 637\ MJ$$

Ozemljitvena upornost ?

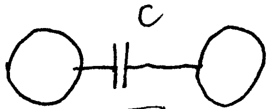
$$U = \int_{r_0}^{\infty} E\,dr = \frac{I}{2\pi\gamma r_0}$$

Med dve prevodni palici okroglega preseka polmera

• $r_0 = 2\ mm$ razmaknjeni za $h = 2cm$, priključimo $10V$. Potopimo v prevoden medij in izmerimo tok $0{,}2\ mA$. Določimo upornost med elektrodama in spec. upornost medija.



R, γ ?



$$\frac{C}{G} = \boxed{R \cdot C} = \frac{\cancel{U}}{I} \cdot \frac{Q}{\cancel{U}} = \frac{\int \vec{D}\, d\vec{A}}{\int \vec{J}\, d\vec{A}} =$$

$$= \frac{\varepsilon \int \vec{E}\, d\vec{A}}{\gamma \int \vec{E}\, d\vec{A}} = \boxed{\frac{\varepsilon}{\gamma}} = \boxed{\rho \cdot \varepsilon}$$

$$\boxed{G = C \frac{\gamma}{\varepsilon}}$$

$$R = \frac{1}{C} \rho \varepsilon$$

$$G = \frac{\varepsilon A}{d} \frac{\gamma}{\varepsilon} = \frac{\gamma A}{d}$$

$$\boxed{C = \pi \varepsilon l \left(\ln \frac{d + \sqrt{d^2 - 4r^2}}{2r} \right)^{-1}}$$

$$\boxed{R = \frac{1}{C} \rho \varepsilon}$$

ali, če je $d \gg r$

$$C = \pi \varepsilon l \left(\ln \frac{d + \cancel{\sqrt{d^2}}}{\cancel{2} r} \right)$$

$$d = h + 2r_0 = 2{,}4\ cm$$

$$\frac{I}{U} = G = \pi \gamma l \left(\ln \frac{d + \sqrt{d^2 - 4r^2}}{2r} \right)$$

iz tega se izračuna upornost ali prevodnost

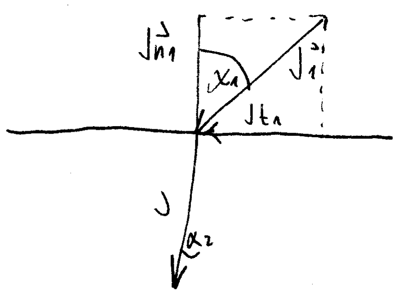
$$G = 20\ \mu S \Rightarrow \gamma = 509\ S/m$$

Mejni pogoji v tokovnem polju

$$\oint \vec{D}\, d\vec{A} = 0 \Rightarrow D_{n1} = D_{n2}$$

$$\boxed{\oint \vec{J}\, d\vec{A} = 0 \Rightarrow J_{n1} = J_{n2}}$$

$$\oint \vec{E}\, d\vec{l} = 0 \Rightarrow E_{t1} = E_{t2} \rightarrow \text{ohm} \rightarrow \frac{J_{t1}}{\gamma_1} = \frac{J_{t2}}{\gamma_2}$$



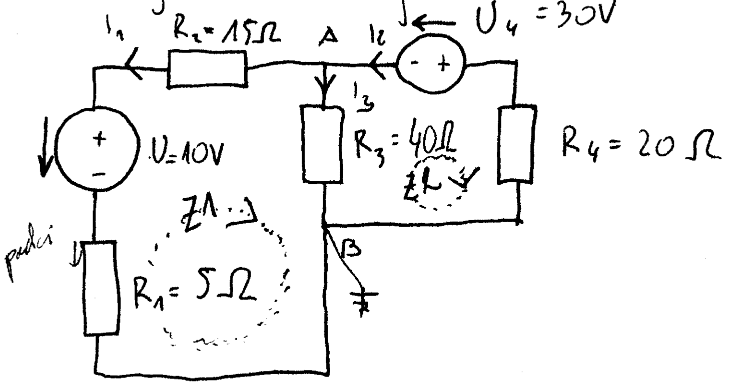
$$\frac{J_{t1}}{J_{n1}} = \frac{\gamma_1}{\gamma_2} \quad \frac{J_{t2}}{J_{n2}}$$

$$\boxed{\frac{tg\,\alpha_1}{tg\,\alpha_2} = \frac{\gamma_1}{\gamma_2}}$$

I ♥ FE

11.1.08 – VAJE
DODATEK

Z uporabo Kirchoffovih zakonov izračunajte toke v vejah vezja



1. Kirchoffov zakon: $\sum_{k=1}^{n} I_k = 0$ v spojišču

2. Kirchoffov zakon: $\sum_{\alpha=1}^{m} U_\alpha = 0$ v zaključeni zanki

- smeri tokov v vejah
- 3 neznanke

1) $-I_1 + I_2 - I_3 = 0$ vozlišče A

$I_1 - I_2 + I_3 = 0$ /(-1) vozlišče B. brez veze [odvisna enačba!]

eno vozlišče ozemljimo

smer obhoda

Z1: $I_3 R_3 - I_1 R_1 - U_1 - I_1 R_2 = 0$

Z2: $-I_3 R_3 - U_4 - I_2 R_4 = 0$

$-I_1 + I_2 - I_3 = 0$
$-I_1 R_2 - I_1 R_1 + I_3 R_3 = U_1$
$-R_4 I_2 - R_3 I_3 = U_4$

$\rightarrow [R][I] = [U]$

$$
\begin{bmatrix} -1 & 1 & -1 \\ -R_2-R_1 & 0 & R_3 \\ 0 & -R_4 & -R_3 \end{bmatrix}
\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}
=
\begin{bmatrix} 0 \\ U_1 \\ U_4 \end{bmatrix}
$$

$$
\begin{bmatrix} -1 & 1 & -1 \\ -20 & 0 & 40 \\ 0 & -20 & -40 \end{bmatrix}
\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 10 \\ 30 \end{bmatrix}
$$

$I_1 = \dfrac{D_1}{D}$ ← determinanta matrike

$I_2 = \dfrac{D_2}{D}$ $\qquad I_3 = \dfrac{D_3}{D}$

$$
D = \begin{vmatrix} -1 & 1 & -1 \\ -20 & 0 & 40 \\ 0 & -20 & -40 \end{vmatrix} \begin{matrix} -1 & 1 \\ -20 & 0 \\ 0 & -20 \end{matrix} =
$$

$= -1(0\cdot(-40) - (-20\cdot 40)) + 1(40\cdot 0 - (-20)(-40) + -1(-20\cdot(-20) - 00$

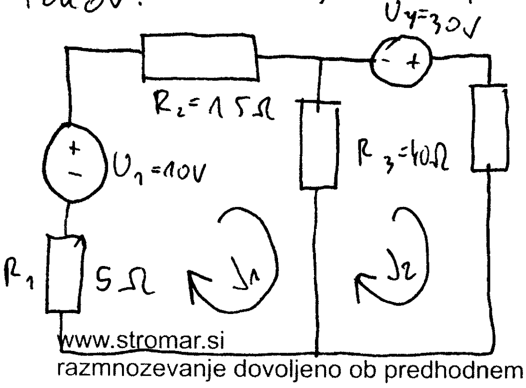
$$
D_1 = \begin{vmatrix} 0 & 1 & -1 \\ 10 & 0 & 40 \\ 30 & -20 & -40 \end{vmatrix}
$$

Metodologija reševanja vezij. Metoda zančnih tokov

Za isto vezje zapišite enačbe po metodi zančnih tokov.

temelji na II. kirchovem zakonu

$\sum U = 0$



Zamislimo si zančne navidezne tokove. npr. Izračunamo J_1 in J_2. sledi

$J_1 = -I_1$

$J_2 = -I_2$

$I_3 = J_1 - J_2$

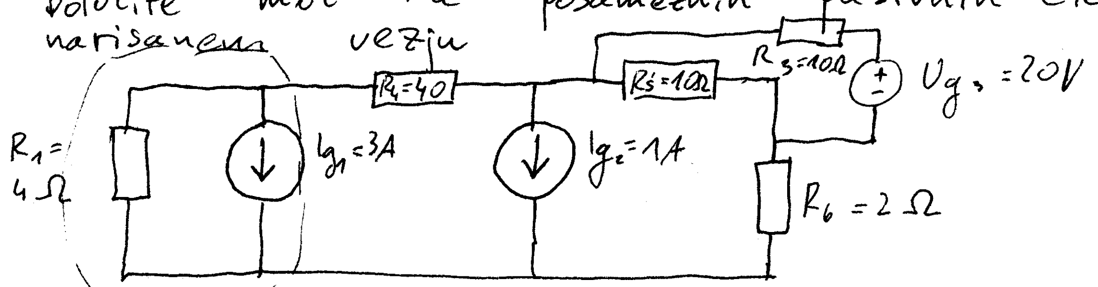
Zanka 1: ob zapisu tega zančnega toka se naslonimo samo na smer tega zančnega toka

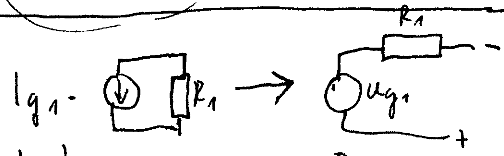
$$J_1 \cdot R_2 + J_1 \cdot R_3 + J_1 R_1 - U_1 \boxed{- J_2 R_3} = 0$$

Zanka 2: $J_2 R_4 + J_2 R_3 - U_4 \boxed{- J_1 R_3} = 0$

$$\begin{bmatrix} R_2+R_3+R_1 & -R_3 \\ -R_3 & R_4+R_3 \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} = \begin{bmatrix} U_1 \\ U_4 \end{bmatrix} \Rightarrow \text{REŠIMO}$$

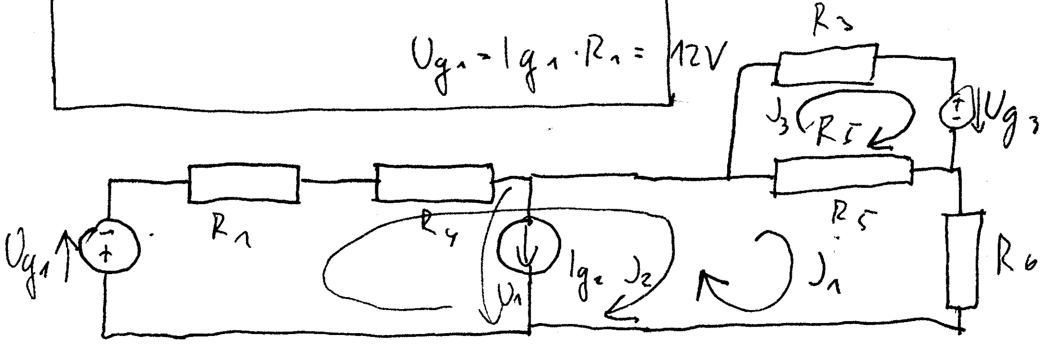
Določite moč na posameznih pasivnih elementih v narisanem vezju





Norton Thevenin

$U_{g1} = I_{g1} \cdot R_1 = 12V$



J_1 je takoj znan: $\boxed{J_1 = -I_{g2} = -1A}$

J_2: $U_{g1} + J_2 R_1 + J_2 R_4 + J_2 R_5 + J_2 R_6 - J_3 R_5 + J_1 R_5 + J_1 R_6 = 0$

J_3: $J_3 R_3 + U_{g3} + J_3 R_5 - J_2 R_5 - J_1 R_5 = 0$

izračunamo enačbi ...

$J_3 = -2A$

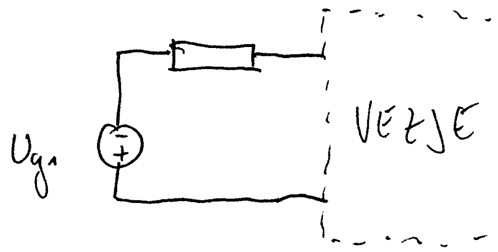
$J_2 = -1A$

Moči: $P = U \cdot I = I^2 R = \frac{U^2}{R}$

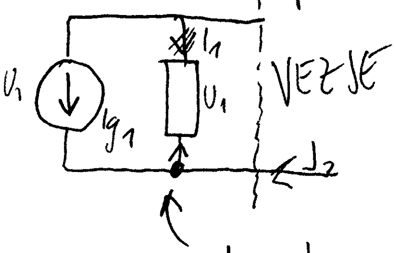
$P_{R4} = J_2^2 R_4$ $\qquad$ $P_{R5} = (J_1 + J_2 - \cancel{J_3})^2 R_5$

$P_{R6} = (J_2 + J_1)^2 R_6$

Pri transformaciji se moči ne ohranjajo



Realno vezje



$I_1 = -I_{g1} - J_2 = -3 - (-1) = \boxed{-2A}$

pozor: smer I_1 je obratna

$I_1 + I_{g1} + J_2 = 0$

$P_{R1} = I_1^2 \cdot R_1 =$

Moč, ki jo oddajajo v vezje viri

$P = U \cdot I$ $\qquad$ U_1 je tudi napetost na uporu

$P_{g1} = U_1 \cdot I_1 = (R_1 \cdot I_1) \cdot I_{g1}$

$P_{g3} = U_{g3} \cdot I_3 = U_{g3} \cdot J_3 = U_{g3} \cdot I_3 = 20 \cdot 2 = 40W$

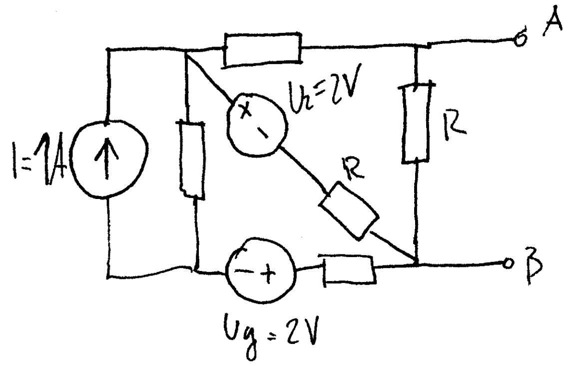
Moč na I_{g2}

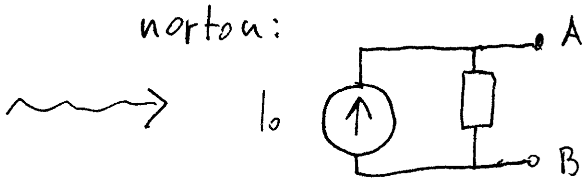
$U_{R4} + U_1 - U_{g2} = 0$

$I_4 R_4 - I_1 R_1 = U_{g2}$

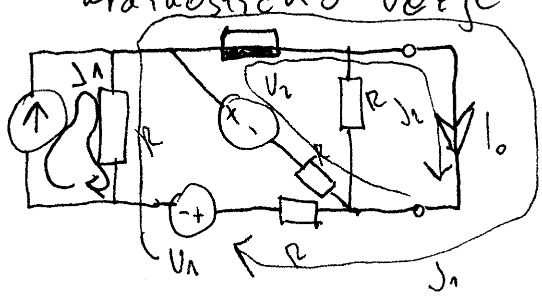
$2 \cdot 40 - 2 \cdot 4 = U_{g2} \rightarrow U_{g2} = 72V$

Določite Nortonovo nadomestno vezje na sponkah AB





Določimo tok I_0 : Med točki A in B vstavimo kratkostično vezje



$I_0 = J_1 + J_2$

zanka 1: $J_1 3R + U_1 + J_2 R - 1R$

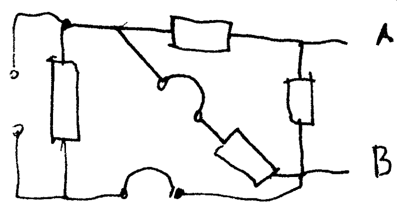
zanka 2: $J_2 2R - U_2 + J_1 R = 0$

Dobimo zančne tokove

$J_1 = 1/4 A$, $J_2 = 0A$, $I = 1A$

$I_0 = \frac{1}{4} A$

Določimo R_0, da napetostne vire nadomestimo s kratkim stikom, tokovne vire z odprtimi sponkami



$R_0 = [(2R || R) + R] || R = 5/8 R$

VELIKO SREČE!