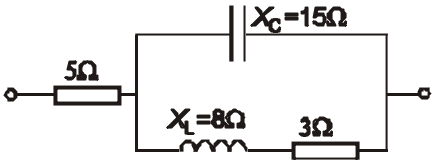
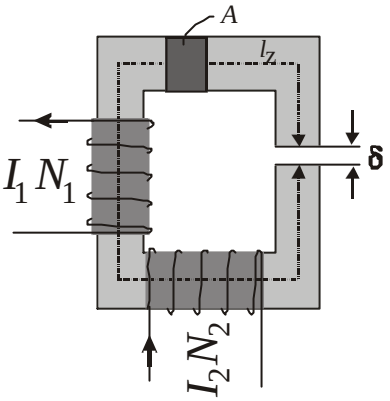
OSNOVE ELEKTROTEHNIKE II (UNI)
Izpit 19.9.2001

1. Dolocite izraza za gostoto magnetnega pretoka $B(r)$ v notranjosti in zunanjosti premega vodnika polnega krožnega preseka polmera a=2cm, ce je vektor gostote toka v vodniku podan z izrazom $\vec{J}_z = \vec{1}_z(2a^2 + 4r^2)\cdot 10^9\,\mathrm{A/m^4}$! Vodnik je vzporeden z osjo z.

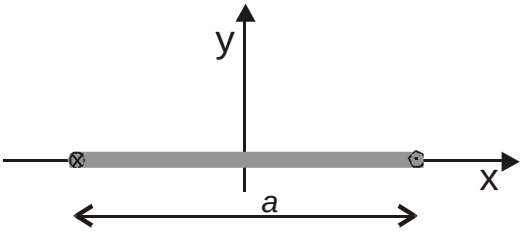
2. Dolocite cos(j) vezja na sliki!



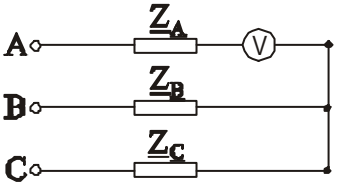
3. Dolocite medsebojno induktivnost tuljav, ce so N_1=100, N_2=400, l_z=20 cm, d=0,5mm, A=10 cm^2, μ_z=10^{-1} Vs/Am!



4. Izracunajte inducirano napetost v pravokotni tuljavici s 100 ovoji in s stranicami a=5cm in b=10cm, ki se nahaja v izmenicnem magnetnem polju gostote $B_y = B_0 \cos(p\frac{x}{a})\cos(w\,t)$! (B_0=0,2T, ω=10^{3}s^{-1})



5. Trifazno breme na sliki prikljucimo na pozitivni simetricni trifazni sistem napetosti 3x400 V. Koliko kaže idealni voltmeter? ($\underline{Z}_A$=50 Ω, $\underline{Z}_B$=100e^{-j60}Ω, $\underline{Z}_C$=200e^{j60}Ω)



OSNOVE ELEKTROTEHNIKE II (UNI)

Izpit 19.9.2001, Rešitve

1. Uporabimo Amperov zakon in za $r<a$ pišemo

$$B(r).2p\,r = m_0.\int_A \vec{J}\cdot d\vec{A} = \left(10^9\,\mathrm{A/m^4}\right) m_0 \int_0^r \vec{1}_z (2a^2 + 4r'^2)\cdot\vec{1}_z 2p\,r'\,dr'$$

$$B(r) = \left(10^9\,\mathrm{A/m^4}\right)\frac{m_0\cdot 4p}{2p\,r}\int_0^r (a^2 r' + 2r'^3)dr' = \left(10^9\,\mathrm{A/m^4}\right)\frac{2m_0}{r}\left(a^2\frac{r^2}{2} + 2\frac{r^4}{4}\right)$$

$$B(r) = \left(10^9\,\mathrm{A/m^4}\right) m_0\left(a^2 r + r^3\right) = \underline{\underline{400p\,(4\cdot 10^{-4}\mathrm{m^2}\cdot r + r^3)\,\mathrm{T/m^3}}}$$

Za $r>a$ moramo vzeti celoten tok skozi vodnik in velja

$$B(r).2p\,r = m_0.\int_A \vec{J}\cdot d\vec{A} = \left(10^9\,\mathrm{A/m^4}\right)\frac{m_0\cdot 4p}{2p\,r}\int_0^a (a^2 r' + 2r'^3)dr' =$$

$$= \left(10^9\,\mathrm{A/m^4}\right)\frac{2m_0}{r}\left(a^2\frac{a^2}{2} + 2\frac{a^4}{4}\right)$$

$$B(r) = 2\cdot\left(10^9\,\mathrm{A/m^4}\right) m_0 a^4/r = 2\cdot\left(10^9\,\mathrm{A/m^4}\right) m_0 a^4/r = \underline{\underline{4{,}02\cdot 10^{-4}\,\frac{1}{r}\,\mathrm{Tm}}}$$

2. Dolocimo impedanco vezja in iz nje cos(j):

$$\underline{Z} = \left(5 + \left((3 + j8)\,\|\,(-j15)\right)\right)\,\Omega$$

$$\underline{Z} \simeq (16{,}64 + \mathrm{j}12{,}16)\,\Omega$$

$$\cos(j) \simeq \frac{16{,}64}{\sqrt{16{,}64^2 + 12{,}16^2}} \simeq \underline{\underline{0{,}8}}$$

3. Dolocimo fluks, ki ga v drugi tuljavi povzroca tok prve tuljave in iz razmerja medsebojno induktivnost:

$$L_{12} = L_{21} = \frac{N_2 F_{21}}{I_1}$$

$$I_1 N_1 = F_{21}(R_{l_z} + R_d)$$

$$R_{l_z} + R_d = \frac{l_z}{m_z A} + \frac{d}{m_0 A} = \frac{1}{A}\left(\frac{l_z}{m_z} + \frac{d}{m_0}\right) \simeq 4\cdot 10^5\,\mathrm{A/Vs}$$

$$L_{21} = \frac{N_2 I_1 N_1}{I_1(R_{l_z} + R_d)} = \frac{100\cdot 400}{4\cdot 10^5\,\mathrm{A/Vs}} \simeq \underline{\underline{100\mathrm{mH}}}$$

4. Tuljavica miruje (se ne vrti), spreminja pa se polje

$$u_i = -N\frac{dF}{dt} = -N\frac{d}{dt}\int_A \vec{B}\cdot d\vec{A} = -N\frac{d}{dt}\left(2B_0\int_0^b\int_0^{a/2}\cos(p\,\frac{x}{a})\cos(wt)dx\cdot dz\right)$$

$$u_i = -N\frac{d}{dt}\left(2B_0\left(b\frac{a}{p}\sin(p\,\frac{x}{a})\cos(wt)\right)\Big|_0^{a/2}\right) = -NB_0\frac{2ab}{p}\frac{d}{dt}\left(\cos(wt)\right)$$

$$u_i = 2NB_0\frac{ab}{p}w\sin(wt) = \frac{200}{p}\sin(wt)\ \mathrm{V} \simeq \underline{\underline{63{,}66\sin(wt)\ \mathrm{V}}}$$

5. Idealen voltmeter ima neskoncno notranjo upornost, zato v fazi A ni toka. Voltmeter meri efektivno vrednost napetosti.

$$\underline{U}_V = \underline{U}_{AB} + \underline{U}_{Z_B} = \underline{U}_{AB} + \frac{\underline{U}_{BC}}{\underline{Z}_B + \underline{Z}_C}.\underline{Z}_B =$$

$$= 400.e^{j30} + \frac{400e^{-j90}}{100.(e^{-j60} + 2e^{j60})}.100e^{-j60}$$

$$U_V = \left|\underline{U}_V\right| \simeq \underline{\underline{231\ \mathrm{V}}}$$