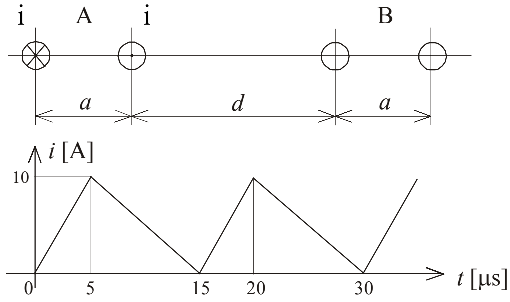
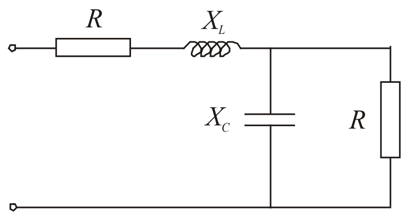
OSNOVE ELEKTROTEHNIKE II (UNI)
2. kolokvij, 13. 06.2002

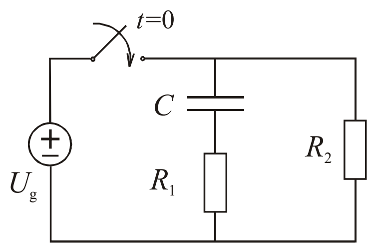
1. V dvovodu A se tok časovno spreminja po podanem diagramu. Določite efektivno vrednost napetosti, ki se inducira v dvovodu (tokovni zanki) B! Vsi vodniki potekajo vzporedno in so dolgi 1 km.
Ostali podatki:
$a = 30\,\text{cm}, d = 100\,\text{cm}, \mu_0 = 4\pi 10^{-7}\,\dfrac{\text{V}\cdot\text{s}}{\text{A}\cdot\text{m}}.$



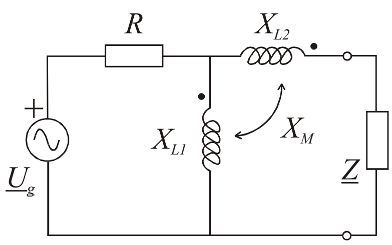
2. Določite jalovo moč v vezju, če ga vzbujamo z napetostjo $u(t) = 100\sin(200t + 70^\circ)$ V!
Ostali podatki: $R = 10\,\Omega,\ X_L = 10\,\Omega,\ X_C = 5\,\Omega.$



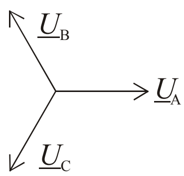
3. Stikalo sklenemo ob času t=0. V kolikšnem razmerju sta napetosti na elementih C in R_2 po preteku dveh časovnih konstant polnjenja kondenzatorja?

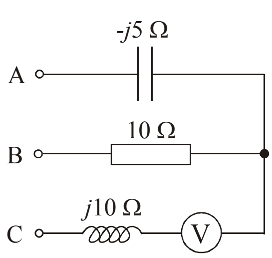


4. Določite največjo delovno moč, ki jo more prejeti impedanca $\underline{Z}$!
Vrednosti elementov vezja so:
$\underline{U}_g = 100\,\text{V},\ X_{L1} = 10\,\Omega,\ X_{L2} = 10\,\Omega,\ X_M = 5\,\Omega,\ R = 10\,\Omega.$



5. Kolikšen je odčitek na idealnem voltmetru, če tripolno vezje priključimo na negativen simetričen sistem napetosti 3x400/230 V_{ef}?





OSNOVE ELEKTROTEHNIKE II (UNI)
2. kolokvij, 13.06.2002, rešitve

1.

$$u_i = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 l}{2\pi} i(t)\left(\int_d^{a+d}\frac{dr}{r} - \int_{a+d}^{2a+d}\frac{dr}{r}\right)\right] = -\frac{\mu_0 l}{2\pi}\ln\left(\frac{a+d}{d}\cdot\frac{a+d}{2a+d}\right)\frac{di}{dt}$$

$$t[\mu s]:\ 0<t<5 \qquad \frac{di}{dt} = 2\cdot 10^6\ \text{A/s} \ \Rightarrow\ u_{i(1)} \cong -21{,}89\ \text{V}$$

$$t[\mu s]:\ 5<t<15 \qquad \frac{di}{dt} = -10^6\ \text{A/s} \ \Rightarrow\ u_{i(2)} \cong 10{,}94\ \text{V}$$

$$U_{ef} = \sqrt{\frac{1}{15\cdot 10^{-6}\ \text{s}}\left(\int_0^{5\cdot 10^{-6}\ \text{s}} u_{i(1)}{}^2(t)dt + \int_{5\cdot 10^{-6}\ \text{s}}^{15\cdot 10^{-6}\ \text{s}} u_{i(2)}{}^2(t)dt\right)} \cong 15{,}5\ \text{V}$$

2.

$$Q = \text{Im}\{\underline{S}\} = \text{Im}\left\{\frac{1}{2}U^2\underline{Y}^*\right\}$$

$$\underline{Z} = R + jX_\text{L} + (-jX_\text{C})\|R = \left(10 + j10 + \frac{(-j5)\cdot 10}{-j5+10}\right)\ \Omega = (12+j6)\ \Omega$$

$$\underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{12+j6}\ \text{S} \cong (0{,}067\text{-j}0{,}033)\ \text{S} \ \Rightarrow\ \underline{Y}^* \cong (0{,}067+\text{j}0{,}033)\ \text{S}$$

$$u(t) = 100\sin(200t + 70^\circ)\ \text{V} \ \Rightarrow\ \underline{U} = 100\ \text{V}\ e^{-j20^\circ}, \quad U = 100\ \text{V}$$

$$Q = \text{Im}\left\{\frac{1}{2}U^2\underline{Y}^*\right\} \cong \text{Im}\left\{\frac{1}{2}(100)^2(0{,}067+j0{,}033)\right\}\ \text{VAr} \cong \text{Im}\{330+j165\}\ \text{VAr} = 165\ \text{VAr}$$

3.

$t>0$:

$u_{\text{R}2} = U_\text{g}$

$U_\text{g} - u_\text{C} - u_{\text{R}1} = U_\text{g} - u_\text{C} - R_1 i_\text{C} = U_\text{g} - u_\text{C} - R_1 C\frac{du_\text{C}}{dt} = 0 \Rightarrow$ linearna nehomogena diferencialna enačba 1. reda :

$u_\text{C}'CR_1 + u_\text{C} = U_\text{g} \Rightarrow$ rešitev v obliki $u_\text{C}(t) = Ae^{\lambda t} + B$ vstavimo v diferencialno enačbo : $A\lambda e^{\lambda t}CR_1 + Ae^{\lambda t} + B = U_\text{g}$

Enačba bo izpolnjena za vsak $t>0$, če bo $\lambda = -\frac{1}{CR_1}$ in $B = U_\text{g} \Rightarrow u_\text{C}(t) = Ae^{-\frac{t}{\tau}} + U_\text{g}, \quad \tau = CR_1$

Za določitev konstante A uporabimo začetni pogoj $u_\text{C}(0) = u_\text{C}(t<0) = 0;\ u_\text{C}(0) = Ae^0 + U_\text{g} \Rightarrow A = -U_\text{g}$

$$u_\text{C}(t) = U_\text{g}\left(1 - e^{-\frac{t}{\tau}}\right)$$

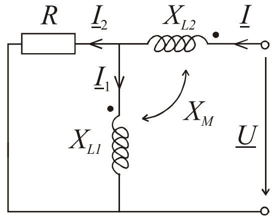
$$\frac{u_\text{C}(t=2\tau)}{u_{\text{R}2}(t=2\tau)} = \frac{U_\text{g}\left(1-e^{-2}\right)}{U_\text{g}} = 1 - e^{-2} \cong 0{,}865$$

4.

Če vezje brez impedance $\underline{Z}$ (med sponkama te impedance) ponazorimo s Theveninovim nadomestnim virom, je na tej impedanci maksimalna moč:

$P_{\max} = \frac{U_T^{\ 2}}{8R_T}$, kjer je $R_T = \operatorname{Re}\{\underline{Z}_T\}$.

Ker sta v vezju sklopljeni tuljavi, računamo impedanco $\underline{Z}_{VH} = \underline{Z}_T$ kot kvocient napetosti in toka pri deaktiviranem napetostnem viru, ob uporabi naslednjih enačb:



$$\underline{Z}_T = \frac{\underline{U}}{\underline{I}}$$

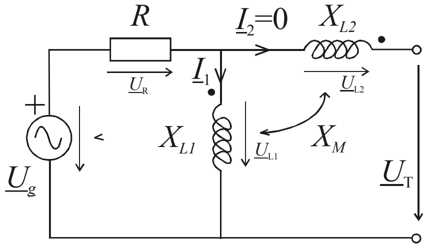
$$\underline{U} = (\underline{I}\ \mathrm{j}X_{L2} + \underline{I}_1 \mathrm{j}X_M) + (\underline{I}_1 \mathrm{j}X_{L1} + \underline{I}\ \mathrm{j}X_M) = \underline{I}(\mathrm{j}X_{L2} + \mathrm{j}X_M) + \underline{I}_1(\mathrm{j}X_{L1} + \mathrm{j}X_M)$$

$$\left.\begin{aligned} &\underline{I}_1 \mathrm{j}X_{L1} + \underline{I}\ \mathrm{j}X_M = \underline{I}_2 R \\ &\underline{I} = \underline{I}_1 + \underline{I}_2 \end{aligned}\right\} \Rightarrow \quad \underline{I}_1 = \underline{I}\frac{R - \mathrm{j}X_M}{R + \mathrm{j}X_{L1}}$$

$$\underline{U} = \underline{I}(\mathrm{j}X_{L2} + \mathrm{j}X_M) + \underline{I}(\mathrm{j}X_{L1} + \mathrm{j}X_M)\frac{R - \mathrm{j}X_M}{R + \mathrm{j}X_{L1}} \quad \Rightarrow \quad \underline{Z}_T = \frac{\underline{U}}{\underline{I}} = \left(\frac{45}{4} + \mathrm{j}\frac{75}{4}\right)\ \Omega \quad \Rightarrow \quad \operatorname{Re}\{\underline{Z}_T\} = R_T = 11{,}25\ \Omega$$

$$\underline{U}_T = \underline{U}_{L1} - \underline{U}_{L2} = (\underline{I}_1 \mathrm{j}X_{L1} - 0) - (0 - \underline{I}_1 \mathrm{j}X_M) = \underline{I}_1(\mathrm{j}X_{L1} + \mathrm{j}X_M) = \frac{\underline{U}_g}{R + \mathrm{j}X_{L1}}(\mathrm{j}X_{L1} + \mathrm{j}X_M) = 75 + \mathrm{j}75 \quad \Rightarrow$$

$$U_T = 106{,}07\ \mathrm{V}$$



$$P_{\max} = \frac{U_T^{\ 2}}{8R_T} = \frac{11250\ \mathrm{V}^2}{8 \cdot 11{,}25\ \Omega} = 125\ \mathrm{W}$$

5.

$$\underline{U}_A = 230\ \mathrm{V}$$

$$\underline{U}_B = \underline{U}_A e^{j120^\circ} = 230\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\ \mathrm{V}$$

$$\underline{U}_C = \underline{U}_A e^{-j120^\circ} = 230\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\ \mathrm{V}$$

$$\underline{U}_V = \underline{U}_R - \underline{U}_{BC}$$

$$\underline{U}_R = \underline{I}_B R = \frac{-\underline{U}_{AB}}{\underline{Z}_{AB}} R = \frac{200(-\sqrt{3} + j)}{10 - j5} 10\ \mathrm{V} = \left(-356{,}8 + j21{,}6\right)\ \mathrm{V}$$

$$\underline{U}_{BC} = \underline{U}_B - \underline{U}_C = 230\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2} + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\ \mathrm{V} = j230\sqrt{3}\ \mathrm{V} \cong 400j\ \mathrm{V}$$

$$\underline{U}_V = \underline{U}_R - \underline{U}_{BC} = \left(-356{,}8 + j21{,}6 - j400\right)\ \mathrm{V} = \left(-356{,}8 - j378{,}4\right)\ \mathrm{V}$$

$$U_{Vef} = U_V = \sqrt{(-356{,}8)^2 + (-378{,}4)^2}\ \mathrm{V} \cong 520\ \mathrm{V}$$