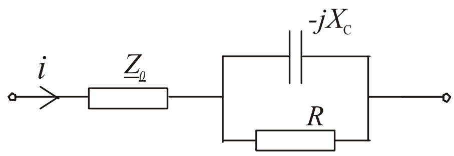
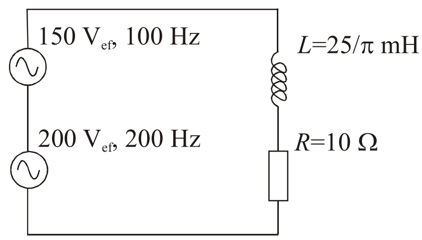
OSNOVE ELEKTROTEHNIKE II (UNI)
2. kolokvij, 02. 06.2003

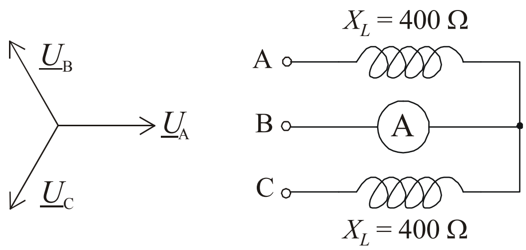
1. Dvopolno vezje priklopimo na vir harmonske napetosti $u(t)=30\cos(\omega t-15^{\circ})$ V. Kolikšna mora biti impedanca $\underline{Z}_0$, da bo vhodni tok $i(t)=5\cos(\omega t+30^{\circ})$ A? Ostali podatki: $X_C=5\ \Omega$, $R=5\ \Omega$.



2. Izračunajte povprečno moč na uporu $R=10\ \Omega$!

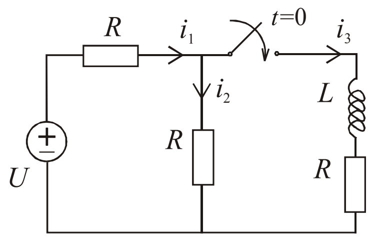


3. Kolikšen je odčitek na idealnem ampermetru, če tripolno vezje priključimo na negativen simetričen sistem napetosti 3x400/230 V_{ef}?



4. Vzporedno k enofaznemu motorju z navidezno močjo S=5000 VA in $\cos\varphi$=0,6 priključimo kondenzator, s čimer povečamo $\cos\varphi$ na 0,85. Za koliko se zmanjša jalova moč?

5. Zapišite izraz za čas t_1, v katerem bo tok i_3 dosegel polovico svoje končne vrednosti!



Rešitve bodo objavljene na http://torina.fe.uni-lj.si/oe

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Rešitve

1. Tok in napetost zapišemo kot kompleksorja: $\underline{U} = 30e^{-j15^\circ}$ in $\underline{I} = 5e^{j30^\circ}$. Impedanco dvopola sprva zapišemo s pomočjo toka in napetosti: $\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{30}{5} e^{j(-15^\circ - 30^\circ)} = 6e^{-j45^\circ}$, nato pa še v obliki: $\underline{Z} = \underline{Z}_0 + \underline{Z}_{RC} = \underline{Z}_0 + \frac{1}{\underline{Y}_{RC}} = \underline{Z}_0 + \frac{1}{\frac{1}{R} + j\frac{1}{X_C}} = \underline{Z}_0 + \frac{5}{2}(1-j)$. Iskano impedanco $\underline{Z}_0$ zapišemo kot: $\underline{Z}_0 = a + jb$ in oba zapisa impedance $\underline{Z}$ izenačimo:

$$6e^{-j45^\circ} = 6\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) = \frac{6\sqrt{2}}{2} - j\frac{6\sqrt{2}}{2} = \left(\frac{5}{2} + a\right) - j\left(\frac{5}{2} - b\right) \Rightarrow a = \frac{6\sqrt{2} - 5}{2} \cong 1,74 \text{ in } b = \frac{-6\sqrt{2} + 5}{2} \cong -1,74.$$

$$\boxed{\underline{Z}_0 \cong (1,74 - j1,74)\ \Omega}$$

2. Vira sta nekoherentna, zato uporabimo teorem o superpoziciji moči:

$$P = P_1 + P_2$$

$$P_1 = RI_{1,ef}^2 = R\left(\frac{U_{1,ef}}{Z_1}\right)^2 = R\left(\frac{U_{1,ef}}{\sqrt{R^2 + (\omega_1 L_1)^2}}\right)^2 = 10\ \Omega\left(\frac{150\ \text{V}}{5\sqrt{5}\ \Omega}\right)^2 = 1800\ \text{W}$$

$$P_2 = RI_{2,ef}^2 = R\left(\frac{U_{2,ef}}{Z_2}\right)^2 = R\left(\frac{U_{2,ef}}{\sqrt{R^2 + (\omega_2 L_2)^2}}\right)^2 = 10\ \Omega\left(\frac{200\ \text{V}}{10\sqrt{2}\ \Omega}\right)^2 = 2000\ \text{W}$$

$$\boxed{P = 3800\ \text{W}}$$

3. Impedanca idealnega A-metra je 0 Ω, zato je $\underline{V}_{\text{ZV}} = \underline{U}_{\text{B}}$. A-meter meri tok v fazi B:

$$\underline{I}_B = -(\underline{I}_A + \underline{I}_C) = -\left(\frac{\underline{U}_{AB}}{\underline{Z}_A} + \frac{\underline{U}_{CB}}{\underline{Z}_C}\right)$$

$$\underline{U}_{AB} = U_{mf} e^{-j30^\circ} = 400\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)\ \text{V},\ U_{mf} = U_{mf,ef}$$

$$\underline{U}_{CB} = U_{mf} e^{-j90^\circ} = -j400\ \text{V},\ U_{mf} = U_{mf,ef}$$

$$\underline{Z}_A = \underline{Z}_C = j400\ \Omega$$

$$\underline{I}_B = \left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right)\ \text{A}$$

$$I_{B,ef} = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\ \text{A} = \sqrt{3}\ \text{A}$$

$$\boxed{I_{B,ef} \cong 1,73\ \text{A}}$$

4. Kondenzator za kompenzacijo jalove moči na delovno moč ne vpliva:

$$P_1 = S_1 \cos\varphi_1$$

$$P_2 = P_1 = S_2 \cos\varphi_2 \quad \Rightarrow \quad S_2 = S_1 \frac{\cos\varphi_1}{\cos\varphi_2}$$

$$Q_1 = S_1 \sin\varphi_1$$

$$Q_2 = S_2 \sin\varphi_2$$

$$\varphi_1 = \arccos(0,6) \cong 53{,}13^\circ$$

$$\varphi_2 = \arccos(0,85) \cong 31{,}79^\circ$$

$$S_1 = 5000 \text{ VA}$$

$$S_2 \cong 3529{,}4 \text{ VA}$$

$$Q_1 = S_1 \sin\varphi_1 \cong 4000 \text{ VAr}$$

$$Q_2 = S_2 \sin\varphi_2 \cong 1860 \text{ VAr}$$

$$\boxed{\Delta Q = Q_1 - Q_2 \cong 2140 \text{ VAr}}$$

5. Končna vrednost toka i_3 bo: $I_3 = \frac{1}{2} I_1$, $I_1 = \frac{U}{R + R\|R} = \frac{2}{3}\frac{U}{R} \quad \Rightarrow \quad I_3 = \frac{1}{3}\frac{U}{R}$.

Pred vklopom stikala je tok i_3=0 A, po vklopu pa prične naraščati. To opišemo z rešitvijo diferencialne enačbe, ki jo zapišemo s pomočjo dveh zančnih in ene spojiščne enačbe:

$$U = u_1 + u_2 = i_1 R + i_2 R$$

$$u_2 = u_L + u_3 = L\frac{di_3}{dt} + R i_3 = R i_2$$

$$i_1 = i_2 + i_3$$

$$\Downarrow$$

$$\frac{di_3}{dt} + i_3 \frac{3}{2}\frac{R}{L} = \frac{1}{2}\frac{U}{L}$$

Za reševanje te nehomogene linearne diferencialne enačbe prvega reda uporabimo nastavek $i(t) = Ae^{\lambda t} + B$ (s pomočjo katerega izračunamo λ in B) ter začetni pogoj $i(t=0) = i(t=0^+) = 0 \text{ A}$ (s pomočjo katerega izračunamo še konstanto A):

$$\lambda + \frac{3}{2}\frac{R}{L} = 0 \quad \Rightarrow \quad \lambda \left(= -\frac{1}{\tau} \right) = -\frac{3}{2}\frac{R}{L}$$

$$B\frac{3}{2}\frac{R}{L} = \frac{1}{2}\frac{U}{L} \quad \Rightarrow \quad B = \frac{1}{3}\frac{U}{R}$$

$$i_3(t=0) = 0 = Ae^0 + B = A + \frac{1}{3}\frac{U}{R} \quad \Rightarrow \quad A = -B = -\frac{1}{3}\frac{U}{R}$$

$$\Downarrow$$

$$i_3(t) = \frac{1}{3}\frac{U}{R}\left(1 - e^{-t\frac{3}{2}\frac{R}{L}}\right)$$

$$i_3(t_1) = \frac{1}{3}\frac{U}{R}\left(1 - e^{-t_1\frac{3}{2}\frac{R}{L}}\right) = \frac{1}{2}\left(\frac{1}{3}\frac{U}{R}\right) \quad \Rightarrow \quad e^{t_1\frac{3}{2}\frac{R}{L}} = 2 \quad \Rightarrow \quad \boxed{t_1 = \frac{2}{3}\frac{L}{R}\ln 2}$$